B.Sc. (h) physics
Statistical mechanics-2018
Semester-VI

S. No. of Paper : 6686 HC

Unique Paper Code : 32221602

Name of the Paper : Statistical Mechanics

Name of the Course : B.Sc. (Hons.) Physics

Semester : VI

Duration : 3 hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **five** questions in all.*
*Question No. 1 is compulsory. Answer **one** question from each Section.*
Symbols have their usual meanings.

1. Answer any *five* of the following:

(a) Write two properties of photons which make them different from other bosons.

(b) Derive Stefan's law using thermodynamics.

(c) What is the significance of partition function in statistical mechanics?

(d) List three characteristics of liquid Helium at low temperature.

(e) What do you mean by ultraviolet catastrophe? Explain with the help of a diagram.

(f) Let Ω_{MB}, Ω_{BE} and Ω_{FD} be the number of ways in which 3 particles can be distributed in 3 energy states

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according to M-B, B-E and F-D statistics respectively. Find $\Omega_{MB}:\Omega_{BE}:\Omega_{FD}$

(g) Calculate the Fermi energy of electrons in silver at absolute zero. Given electron density $= 5.86 \times 10^{28}$ m^{-3}.

(h) Give two examples where we can use equipartition of energy theorem. $5 \times 3 = 15$

Section A

2. (a) Establish the relation between entropy and thermodynamic probability. Show that the constant occurring in the relation is the Boltzmann constant. 7

(b) A Maxwell-Boltzmann system consisting of 4 particles has a total energy 5ϵ. The permitted energy levels are equally spaced with energies $0, \epsilon, 2\epsilon, 3\epsilon$. (i) Write all the possible macrostates. (ii) Determine the thermodynamic probability of each macrostate. 8

3. (a) Discuss the concept of negative temperature from the statistical point of view. Show that it is possible to attain a state of negative temperature for a system of paramagnetic dipoles subjected to an external magnetic field induction ***B***. 10

(b) Differentiate between 100K and −100K on the basis of entropy-energy diagram. 2

(c) Consider a system of 13 classical particles distributed initially in three energy states of energies $\varepsilon_1=0$, $\varepsilon_2=2\varepsilon$, $\varepsilon_3=4\varepsilon$, such that $n_1=6$, $n_2=4$, $n_3=3$. If $\delta n_3=-2$, find δn_1 and δn_2, and the new distribution of the particles. 3

Section B

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4. (a) Show that the number of modes of vibrations per unit volume of an enclosure in the frequency range ν to $\nu+d\nu$ is given by $N_\nu d\nu = \frac{8\pi\nu^2 d\nu}{c^3}$ and hence derive Rayleigh-Jeans formula for blackbody radiation. 12

(b) A black body is placed inside an evacuated chamber maintained at temperature 27°C. If the surface area of the black body is 500 cm^2, find energy radiated per time per unit area from the black body when its temperature is 127°C. 3

5. (a) Derive Saha's ionization formula stating its basic assumptions, and highlight its two applications. 10

(b) Show that the pressure exerted by a diffuse radiation is given by $p = \frac{u}{3}$, where u is the radiation density. 5

SECTION C

6. (a) Give the Bose derivation of Planck's radiation law. 7

(b) Derive the expression for entropy (S) and specific heat capacity (C_v) for photon gas and hence show that $C_v = 3S$ 8

7. (a) What is Bose-Einstein condensation and how is it different from ordinary condensation? Derive the expression for the temperature (T_c) at which the Bose-Einstein condensation sets in. 10

(b) Obtain the expressions for fractions of degenerate Bosons in the ground state (N_0/N) and in the excited state (N_{exc}/N) as functions of temperature T. Plot them. 5

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SECTION D

8. (a) Given that the chemical potential for a strongly degenerate Fermi gas ($0<T<<T_F$) is :

$$\mu(T) \simeq \epsilon_F \left[1 - \frac{\pi^2}{12}\left(\frac{T}{T_F}\right)^2\right]$$

Obtain the expressions for the internal energy, pressure, entropy and specific heat capacity. 12

(b) The Fermi energy of metal A is 8.4 eV. Find its value for metal B if the free electron density in metal B is 27 times that in metal A. 3

9. (a) Give two typical characteristics of a white dwarf star. Show that electron gas inside a white dwarf star is strongly degenerate and relativistic. 5

(b) Obtain an expression for the mass-radius relationship for a white dwarf star and hence discuss the importance of Chandrasekhar mass limit. 10

Q1 a)

Some of the important properties of the photons are :

(i) Photons are particles of zero rest mass.

(ii) Photons are indistinguishable from one another the their number in a system is not necessarily constant; because in every emission process in an atom a new light quantum is formed and if a photon of frequency ν is absorbed by the wall of the enclosure, it can be replaced by the emission of several photons of frequencies $\nu_1, \nu_2 \ldots$, provided the total energy of the system is constant *i.e.*

$$h\nu = h\nu_1 + h\nu_2 + \ldots$$

Thus is this case, we have

$$\Sigma\, \delta n_i \neq 0 \qquad \ldots(1)$$

Hence we must drop the auxiliary condition $\sum_i \delta n_i = 0$, and therefore, the undetermined multiplier α is equal to zero.

(iii) The photons are Bose particles with spin 1, having two modes of propagation.

Q1 b) refer chapter 2

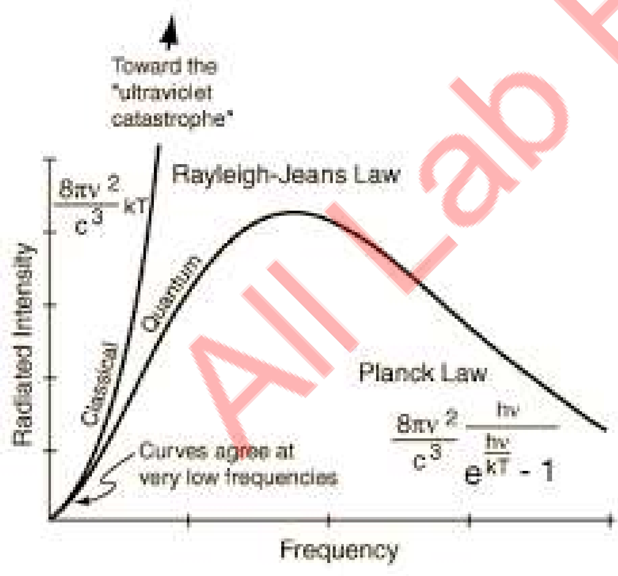
All Lab Experiments

Q1 c) a partition function describes the statistical properties of a system in thermodynamic equilibrium. Partition functions are functions of the thermodynamic state variables, such as the temperature and volume. Most of the aggregate thermodynamic variables of the system, such as the total energy, free energy, entropy, and pressure, can be expressed in terms of the partition function or its derivatives. The partition function is dimensionless, it is a pure number.

Q1 d) the temperature required to produce liquid helium is low because of the weakness of the attractions between the helium atoms. These interatomic forces in helium are weak to begin with because helium is a noble gas, but the interatomic attractions are reduced even more by the effects of quantum mechanics. These are significant in helium because of its low atomic mass of about four atomic mass units. The zero point energy of liquid helium is less if its atoms are less confined by their neighbors. Hence in liquid helium, its ground state energy can decrease by a naturally occurring increase in its average interatomic distance. However at greater distances, the effects of the interatomic forces in helium are even weaker.

Because of the very weak interatomic forces in helium, the element remains a liquid at atmospheric pressure all the way from its liquefaction point down to absolute zero. Liquid helium solidifies only under very low temperatures and great pressures. At temperatures below their liquefaction points, both helium-4 and helium-3 undergo transitions to super fluids.

Q1 e) refer chapter 2



Q1 f)

All Lab Experiments

Maxwell - Boltzmann: $N = 3$ $g = 3$ $n = 3$

$$W = N! \frac{g^n}{n!}$$

$$= 3! \frac{3^3}{3!} = 27 \text{ ways}$$

Bose - einstein:

$$W = \frac{(n_i + g_i - 1)!}{n_i! \, (g_i - 1)!}$$

$$= \frac{(3 + 3 - 1)!}{3! \, (3 - 1)!}$$

$$= \frac{5!}{3! \, 2!} = \frac{5 \times 4}{2} = 10 \text{ ways}$$

All Lab Experiments

fermi-dirac:

$$w = \frac{g_i!}{n_i!\,(g_i - n_i)!}$$

$$= \frac{3!}{3!\;0!} = 1$$

Q1 g)

$$\epsilon_f = \frac{h^2}{8m}\left[\frac{3N}{\pi V}\right]^{2/3}$$

$$\epsilon_f = \frac{(6.63\times10^{-34}\ Js)^2}{8\times(9.11\times10^{-31}\ kg)}\left[\frac{3\times5.86\times10^{28}\ m^{-3}}{\pi}\right]^{2/3}$$

$$= 0.60\times10^{-37}\times14.63\times10^{18}$$

$$= 8.78\times10^{-19}\ J$$

$$= 5.49\ eV$$

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Q1 h) Diatomic gases

A diatomic gas can be modeled as two masses, m1 and m2, joined by a spring of stiffness a, which is called the rigid rotor-harmonic oscillator approximation.[19] The classical energy of this system is

$$H = \frac{|\mathbf{p}_1|^2}{2m_1} + \frac{|\mathbf{p}_2|^2}{2m_2} + \frac{1}{2}aq^2,$$

Where p1 and p2 are the momenta of the two atoms, and q is the deviation of the inter-atomic separation from its equilibrium value. Every degree of freedom in the energy is quadratic and, thus, should contribute 1⁄2kBT to the total average energy, and 1⁄2kB to the heat capacity. Therefore, the heat capacity of a gas of N diatomic molecules is predicted to be 7N·1⁄2kB: the momenta p1 and p2 contribute three degrees of freedom each, and the extension q contributes the seventh. It follows that the heat capacity of a mole of diatomic molecules with no other degrees of freedom should be (7/2)$N_A k_B$ = (7/2)R and, thus, the predicted molar heat capacity should be roughly 7 cal/(mol·K).

Extreme relativistic ideal gases

Equipartition was used above to derive the classical ideal gas law from Newtonian mechanics. However, relativistic effects become dominant in some systems, such as white dwarfs and neutron stars and the ideal gas equations must be modified. The equipartition theorem provides a convenient way to derive the corresponding laws for an extreme relativistic ideal gas. In such cases, the kinetic energy of a single particle is given by the formula

$$H_{\rm kin} \approx cp = c\sqrt{p_x^2 + p_y^2 + p_z^2}.$$

Taking the derivative of H with respect to the p_x momentum component gives the formula

$$p_x \frac{\partial H_{\rm kin}}{\partial p_x} = c\frac{p_x^2}{\sqrt{p_x^2 + p_y^2 + p_z^2}}$$

and similarly for the p_y and p_z components. Adding the three components together gives

$$\begin{aligned}\langle H_{\rm kin}\rangle &= \left\langle c\frac{p_x^2 + p_y^2 + p_z^2}{\sqrt{p_x^2 + p_y^2 + p_z^2}}\right\rangle \\ &= \left\langle p_x \frac{\partial H^{\rm kin}}{\partial p_x}\right\rangle + \left\langle p_y \frac{\partial H^{\rm kin}}{\partial p_y}\right\rangle + \left\langle p_z \frac{\partial H^{\rm kin}}{\partial p_z}\right\rangle \\ &= 3k_B T\end{aligned}$$

Where the last equality follows from the equipartition formula. Thus, the average total energy of an extreme relativistic gas is twice that of the non-relativistic case: for N particles, it is 3 Nk_BT

Q2 a) refer book.
Q2 b)

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2) (b) Maxwell – Boltzmann system –

4 particles.

total Energy = 5ϵ.

macrostates. $E = \sum_i n_i \epsilon_i$.

3ϵ

2ϵ

ϵ

0

$E = 0 \longrightarrow 4 \times 0$

$E = \epsilon \longrightarrow 3 \times 0 + 1\epsilon$.

$E = 2\epsilon \longrightarrow \begin{cases} 3 \times 0 + 1 \times 2\epsilon \\ 2 \times 0 + 2 \times \epsilon \end{cases}$ (also) → microstates/arrangements

$E = 3\epsilon \longrightarrow \begin{cases} 3 \times 0 + 1 \times 3\epsilon + 0 \times \\ 3 \times \epsilon + 0 \times 2\epsilon + 0 \times \\ 1 \times \epsilon + 1 \times 2\epsilon + 2 \times 0 \end{cases}$.

Similarly $\left.\begin{array}{l} E = 4\epsilon \\ E = 5\epsilon \end{array}\right\}$ try by yourself.

so, there are 5 macrostates.

microstates.

All Lab Experiments

Total Energy $= 5\epsilon$

macrostate	Thermodynamic prob.
$E = 0$	$P_1 = 0/5\epsilon = 0$
$E = \epsilon$	$P_2 = \epsilon/5\epsilon = 1/5$
$E = 2\epsilon$	$P_3 = 2\epsilon/5\epsilon = 2/5$
$E = 3\epsilon$	$P_4 = 3\epsilon/5\epsilon = 3/5$
$E = 4\epsilon$	$P_5 = 4\epsilon/5\epsilon = 4/5$
$E = 5\epsilon$	$P_6 = 5\epsilon/5\epsilon = 1$

Q3 a)

only substitute, $E = \vec{\mu}\cdot\vec{B}$ Same Concept.

$$E = \begin{cases} -\mu H \text{ or } -\mu B \rightarrow -\epsilon \\ +\mu H \text{ or } +\mu B \rightarrow \epsilon \end{cases}$$

Q3 b)

Draw the graphs of entropy vs temperature and energy vs temperature; from graphs we can infer the negative and positive temperature

Q4 a) refer chapter 2

Q4 b)

Q5 a)refer chap 2

Q5 b)refer your book

Q6 a)

All Lab Experiments

Q6b)

Q7 a)refer book

Q7b)refer your text book

The number of particles in the ground state N_g, is given by

$$N_g = N - N_{ex} = N\left[1 - \frac{N_{ex}}{N}\right] = N\left[1 - \left(\frac{T}{T_c}\right)^{3/2}\right] \tag{15.13}$$

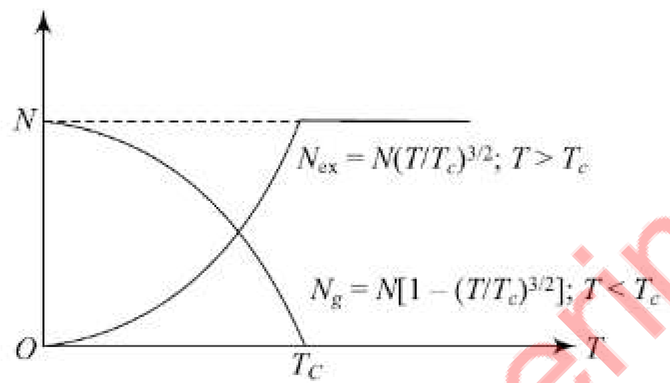

Fig. 15.1 Distribution of bosons as a function of temperature below and above the Bose–Einstein condensation temperature.

Q8 a)refer chapter-5

Q8 b)

All Lab Experiments

Q8 b)

fermi energy :

$$E_f = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$$

$$E_{fA} = 8.4\,eV$$

$$\left(\frac{N}{V}\right)_B = 27\left(\frac{N}{V}\right)_A$$

$$\frac{E_{fB}}{E_{fA}} = \left(\frac{N}{V}\right)_B^{2/3} \left(\frac{V}{N}\right)_A^{2/3}$$

$$= \cancel{\left(\frac{N}{V}\right)}^{A} (27)^{2/3} \cancel{\left(\frac{V}{N}\right)}_{A}$$

$$= 3^2 = 9$$

$$E_{fB} = 9\,E_{fA}$$

$$= 9 \times 8.4\,eV = \underline{\underline{75.6\,eV}}$$

Ans.

Q9:refer chapter 5

All Lab Experiments