This question paper contains 6 printed pages.]

Your Roll No.....

Sr. No. of Question Paper: 1116

Unique Paper Code : 32221601

Name of the Paper : Electromagnetic Theory

Name of the Course : B.Sc. (Hons) Physics-CBCs

senester : VI

puration: 3 Hours

Maximum Marks: 75

(2022)

Instructions for Candidates

- 1. Write your Roll No. on the top immediately on receipt of this question paper.
- 2. Question No. 1 is compulsory.
- 3. Answer any four of the remaining six.
- 4. Use of non-programmable calculator is allowed.
- 1. Attempt any 5 parts of this question. (5x3)

(a) A conductor of square cross section and having conductivity 3.8×10^7 S/m is 50 m long. It measures 0.5 cm on either side. Calculate the skin depth if conductor carries current at 150 kHz.

3.

- (b) The conduction current density in a material is given as $\overline{J_C} = 0.02 \sin{(10^9 \text{t})} \text{ A/m}^2$. Find the displacement current density if $\sigma = 10^5 \text{ mho/m}$, $\epsilon_r = 6.5$.
- (c) What is plasma frequency and minimum penetration depth for collision free plasma having 10¹² electrons/m³.
 - (d) A 300 mm long tube containing 56 cm³ of sugar solution produces an optical rotation of 12° when placed in a polarimeter. If the specific rotation of sugar solution is 66°, calculate the quantity of sugar contained in the tube.
 - (e) For glass air interface $(n_1 = 1.5, n_2 = 1.0)$, find the reflection and transmission coefficients for normal incidence.
- A left circularly polarized wave ($\lambda = 5893$ A) is incident normally on calcite crystal of thickness 0.00514 mm. Find the state of polarization of the emergent beam. Take $n_0 = 1.65836$ and $n_0 = 1.48641$.
- (g) A circularly polarized electromagnetic wave is propagating in the z-direction in free space and is described by the following equation

 $\vec{E} = 5\cos(\omega t - kz)\hat{x} + 5\sin(\omega t - kz)\hat{y} Vm^{-1}$

The wavelength is 6×10^{-7} m. Find the corresponding magnetic field and the average of the Poynting vector.

- (a) State and establish the Poynting theorem for conservation of energy for electromagnetic fields.
 Explain the physical significance of each term in the equation.
 - (b) Using electromagnetic scalar and vector potentials, show that the four Maxwell equations can be written as two coupled second order differential equations.

 (6)
 - (e) What is the ratio of amplitudes of conduction current density and displacement current densities density if applied field is $\vec{E} = \vec{E}_0 e^{-t/\tau}$, where τ is real.
- Discuss the propagation of high frequency electromagnetic wave in plasma. Show that the critical frequency for the propagation of electromagnetic wave in plasma is given by $f_c = 9\sqrt{n_0}$, where n_0 is the electron density. (8)

- (b) A material has $\sigma = 6.0 \times 10^{-2} \Omega^{-1}/m$, $\mu = \mu_0$ and $\varepsilon_r = 7.0$. A plane wave of frequency 10^9 Hz with amplitude 200 V/m is propagating along positive z direction. Find (a) E_x at (x = 0 cm, y = 0 cm, z = 3 cm. t = 0.16 ns) (b) H_y at (x = 0 cm, y = 0 cm, z = 3 cm. t = 0.16 ns).
- 4. (a) Derive the Fresnel's equation for reflection and transmission of a plane electromagnetic wave at the boundary separating between two dielectric media when electric field vector is perpendicular to the plane of incidence. (9)
 - An electromagnetic wave propagating in a dielectric medium with $\varepsilon = 16\varepsilon_0$ along the z direction. It strikes another dielectric medium with $\varepsilon = 4\varepsilon_0$ at z = 0. If the incoming wave has a maximum value of 0.2 V/m at the interface, and its angular frequency is 300 M rad/s, determine the power densities of the incident, reflected, and transmitted waves. (6)
- 5. (a) Derive Fresnel's formula for the propagation of light in Anisotropic crystals. Also explain how this leads to the phenomenon of double refraction.

c Stati

d

- (b) Discuss the propagation of light in uniaxial crystals. Explain the difference between a positive uniaxial crystal and a negative uniaxial crystal. Also give one example of each. (6)
- 6. (a) Explain the construction and working of a Babinet Compensator. How is it used to analyze the elliptically polarized light? (9)
 - (b) A right-handed circularly polarized plane wave with electric field magnitude of 3 mV/m is travelling in the +Y direction in a dielectric medium with $\varepsilon = 4\varepsilon_0$ and $\mu = \mu_0 = 0$. If the frequency is $100 \, \mathrm{MHz}$, obtain expressions for $\overline{E}(y,t)$ and $\overline{H}(y,t)$.

7. (a) Starting with Maxwell's equations, derive the wave equations for a symmetric planar dielectric waveguide with refractive index profile as:

$$n = n_1$$
 $-d/2 < x < d/2$
 $n = n_2$ $-d/2 > x > d/2$

Using the boundary conditions, obtain the eigenvalue equation for symmetric TE modes.

(b) Consider a symmetric planar waveguide with the follow ing parameters.
n₁ = 1.5; n₂ = 1.48; d = 3.912 μm
How many TE modes exist for λ₀ = 1 μm? Determine the corresponding propagation constants.

Q1a

Date		Page No.
(Da) Ahres	Conductivity of conductor (o	-)=3.8 x 10 5/m
	length = som	y
	breadth= 0.5 cm width = 0.5cm	
	frequency = 150 kHz	H= Usho
	Skin depth (8) = P	Mi=1
	N II fu	
	3.8mo7x3.14x150x	103 x 4x3.14x10-7
	z 1 m	
	\$150x103	

Q1b

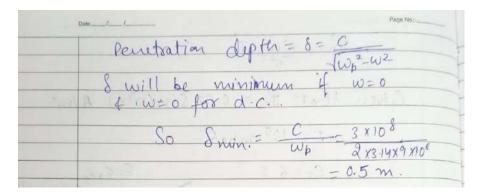
Q1c

Ohns- Plasma frequency (fp) =
$$\omega p$$

$$2\pi$$

$$\omega p = \left(\frac{n_e e^2}{\epsilon_o m}\right)^{1/2}$$

$$f_p = \frac{\omega p}{2\pi} \sim 9\sqrt{n_e}$$
Given
$$n_e = 10^{12} e^{-/m^3}$$



Q1d

d.) Arize	- l= path length = 300 mm = 300 x/ dm
	= 3dm
3 ml P	Specific rotation = [x], = 60°
	and x=12°
	Now we know [x] = x
	Here C is the concentration
	> 60° - 18 3x C
	$\Rightarrow C = \frac{12}{60 \times 3} = \frac{1}{15} \frac{9/mL}{1}$
	V= 48 56 cm3 = 56 mL

50 mass = 56 g of = 3.73g

Q1e

e) and we know
$$R = \begin{bmatrix} n_1 - n_2 \\ n_1 + n_2 \end{bmatrix}^2$$
 and $T = \underbrace{\frac{U}{n_1 + n_2}}^2 = \underbrace{\frac{N_1 - N_2}{(n_1 + n_2)^2}}^2$
Here $M_1 = 1.5$ and $M_2 = 1.0$ (qive $M_2 = 1.0$) $M_2 = 1.0$ (qive $M_2 = 1.0$) $M_2 = 1.0$ (qive $M_2 = 1.0$) $M_2 = 1.0$ $M_$

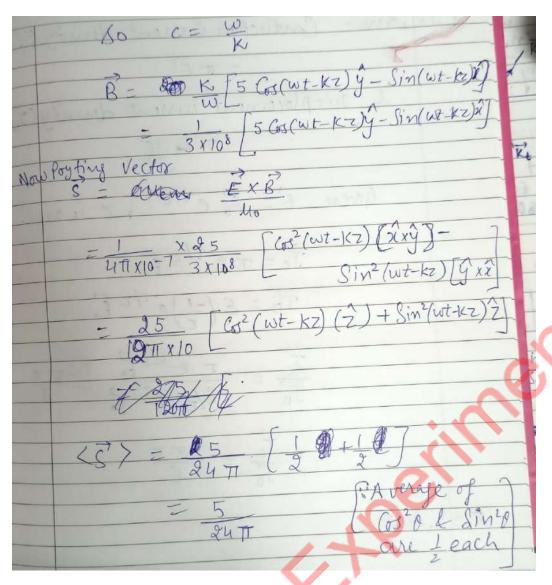
f)Ansr Given A = 5893 A t = 0.00514mm no= 1.65836 ne=1.48641 boths of Path difference between ordinary and extradicionary bay = = (1.65836-1.48641) t (1.65836-1.48641) x (0.00514 8838.23 XID-10 m Phase diff 2TT x 8838,83×10-10 - 2TT X (1.499) Atth smile, for the unito is the beginning of love."—Mother Teresa 377 Chitra

Now the thickness of the plate is such that the phase diff (d)=3TT. So the enreigent light will be plane-polarised light.

Q1g

AT ST X 107 [Cos (wt-kz) y - Sin(wt-kz) x]

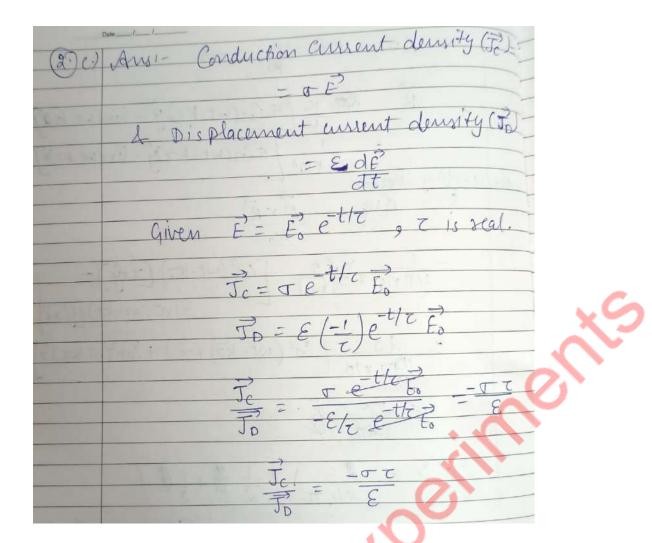
The medium is free space



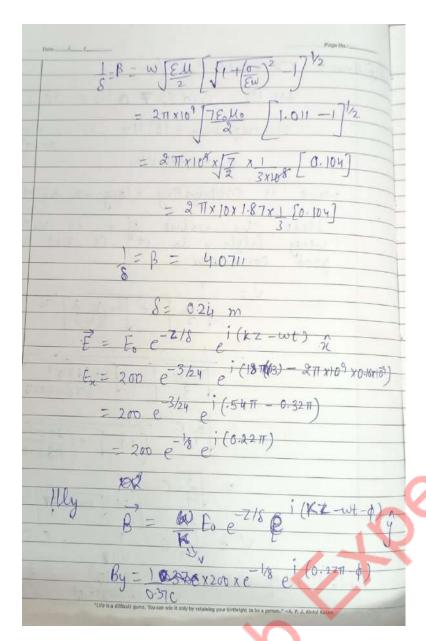
Q2a

Q2b

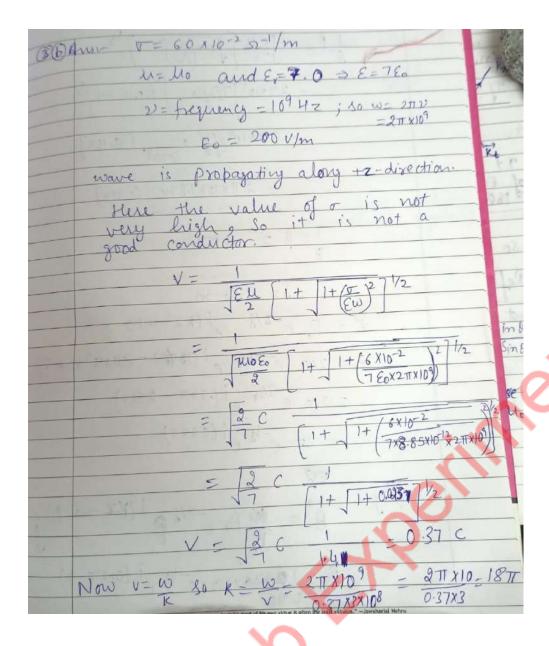
Q2c



Q3a



Q3b



Q4a

Refer to chapter 2, page 12 for the solution to this question

Q4b

Refer to chapter 2, page 12 for the solution to this question

Q5a

2.2 Optics of (simple) anisotropic media

In order to understand the principal behavior of anisotropic materials, it is best to first ignore any dispersive property (frequency dependence) of the material. Owing to the Kramers-Kronig relation the material will then be lossless. This is what we mean by a simple anisotropic material.

For a given material, the values $\epsilon_1, \epsilon_2, \epsilon_3$ and the corresponding principal axes are determined by the microscopic arrangements and symmetries of the material's constituents (atoms, molecules, etc). It is one of the tasks of solid-state physics to determine these quantities. In optics we take these quantities as given input.

We consider time-harmonic plane waves, i.e., E, D, B, H depend on r and t through the common factor $e^{i(k\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}-\omega t)}$ as in $\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0e^{i(k\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}-\omega t)}$ with a constant vector \mathbf{E}_0 (same goes for D, B, and H). We also ignore (free) charges and (free) currents, hence we set $\rho = 0$ and j = 0. Then we can deduce from the Maxwell equations (1.45) and (1.47):

$$k\hat{\mathbf{k}} \times \mathbf{H} + \omega \mathbf{D} = 0 \Rightarrow \mathbf{D} \perp \hat{\mathbf{k}}, \mathbf{H}$$
 (2.13)

$$k\hat{\mathbf{k}} \times \mathbf{H} + \omega \mathbf{D} = 0 \implies \mathbf{D} \perp \mathbf{k}, \mathbf{H}$$

$$k\hat{\mathbf{k}} \times \mathbf{E} - \mu_0 \omega \mathbf{H} = 0 \implies \mathbf{H} \perp \hat{\mathbf{k}}, \mathbf{E}$$

$$\Rightarrow \mathbf{H} \perp \hat{\mathbf{k}}, \mathbf{E}, \mathbf{D}$$

$$(2.13)$$

$$(2.14)$$

$$\Rightarrow \mathbf{H} \perp \hat{\mathbf{k}}, \mathbf{E}, \mathbf{D} \tag{2.15}$$

$$\hat{\mathbf{k}} \times \mathbf{H} = \frac{k}{\mu_0 \omega} \underbrace{\hat{\mathbf{k}} \times (\hat{\mathbf{k}} \times \mathbf{E})}_{=-\mathbf{E} + (\mathbf{E} \cdot \hat{\mathbf{k}}) \hat{\mathbf{k}}} = -\frac{\omega}{k} \mathbf{D}$$

$$\Rightarrow \mathbf{D} = \frac{k^2}{\mu_0 \omega^2} \left[\mathbf{E} - (\mathbf{E} \cdot \hat{\mathbf{k}}) \hat{\mathbf{k}} \right]$$
(2.16)

$$\Rightarrow \mathbf{D} = \frac{k^2}{\mu_0 \omega^2} \left[\mathbf{E} - \left(\mathbf{E} \cdot \hat{\mathbf{k}} \right) \hat{\mathbf{k}} \right] \tag{2.17}$$

Eq. (2.17) implies $\hat{\mathbf{k}} \cdot \mathbf{D} = 0$, i.e., the divergence condition (2.13) for the displacement field is consistent with (2.17).

Furthermore, $\mathbf{E} \cdot \hat{\mathbf{k}}$ can be non-zero now. If the dispersion relation is known, this allows to determine the polarization (i.e., the direction) of D.

Note:

$$\mathbf{H} = \frac{k}{\mu_0 \omega} \hat{\mathbf{k}} \times \mathbf{E} \tag{2.18}$$

$$\Rightarrow \mathbf{S} = \frac{1}{2} \operatorname{Re}(\mathbf{E} \times \mathbf{H}^*) \tag{2.19}$$

$$= \frac{1}{2} \frac{k}{\mu_0 \omega} \operatorname{Re} \left[(\mathbf{E} \cdot \mathbf{E}^*) \,\hat{\mathbf{k}} - \left(\mathbf{E} \cdot \hat{\mathbf{k}} \right) \mathbf{E}^* \right]$$
 (2.20)

 \Rightarrow The Poynting-vector S is not necessarily parallel to the wave vector $\hat{\mathbf{k}}$.

We now insert the constitutive relation $\mathbf{D} = \epsilon_0 \epsilon \mathbf{E}$ into Eq. (2.17). This gives

$$\underbrace{\left(\frac{\epsilon}{=} - \frac{c^2 k^2}{\omega^2} \mathbb{1} + \frac{c^2 k^2}{\omega^2} \hat{\mathbf{k}} : \hat{\mathbf{k}}\right)}_{\text{det}=0 \text{ for non trivial solutions}} \mathbf{E} = 0 \text{ independent of Coordinate system}$$
(2.21)

Without loss of generality, we can assume that we are in a coordinate system that coincides with the material's principal axes:

$$\stackrel{\epsilon}{=} = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix}, \qquad \hat{k} = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix}.$$
(2.22)

Then, the requirement of the determinant being zero gives the celebrated Fresnel Equation:

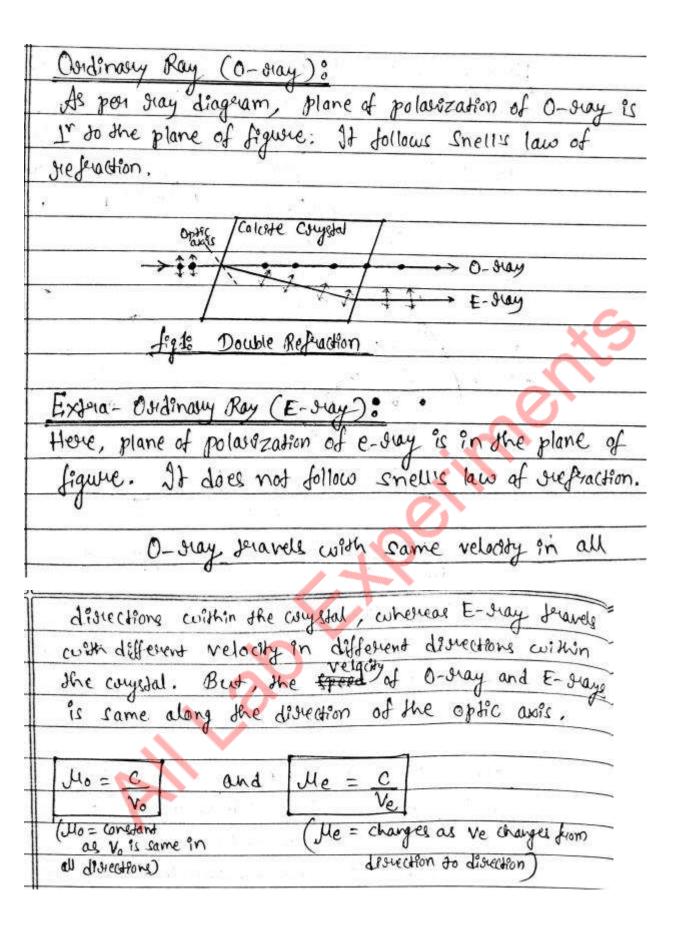
$$\left(\frac{c^2k^2}{\omega^2}\right)\left(\frac{\epsilon_1c^2k_1^2}{\omega^2} + \frac{\epsilon_2c^2k_2^2}{\omega^2} + \frac{\epsilon_3c^2k_3^2}{\omega^2}\right)$$
(2.23)

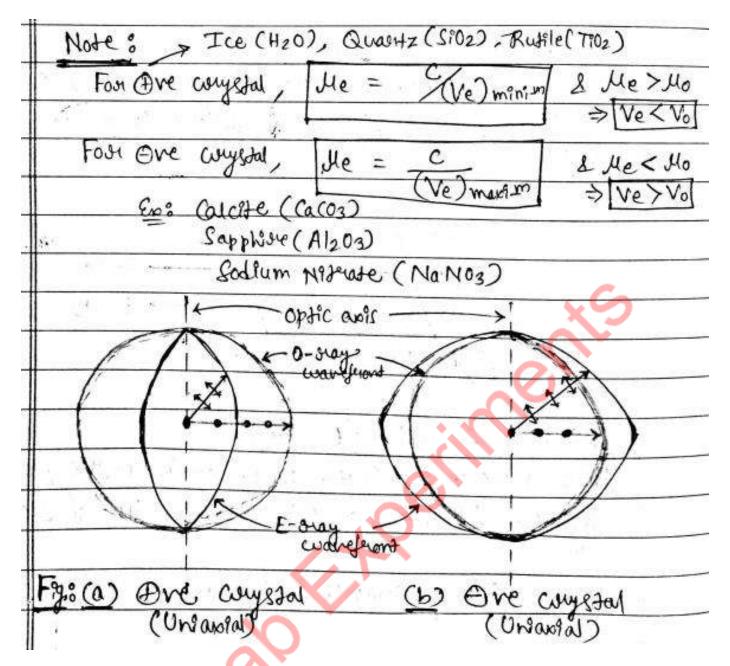
$$-\left(\frac{c^2k_1^2}{\omega^2}\epsilon_1\left(\epsilon_2+\epsilon_3\right)+\frac{c^2k_2^2}{\omega^2}\epsilon_2\left(\epsilon_1+\epsilon_3\right)+\frac{c^2k_3^2}{\omega^2}\epsilon_3\left(\epsilon_1+\epsilon_2\right)\right) \tag{2.24}$$

$$+ \epsilon_1 \epsilon_2 \epsilon_3 = 0$$
 (2.25)

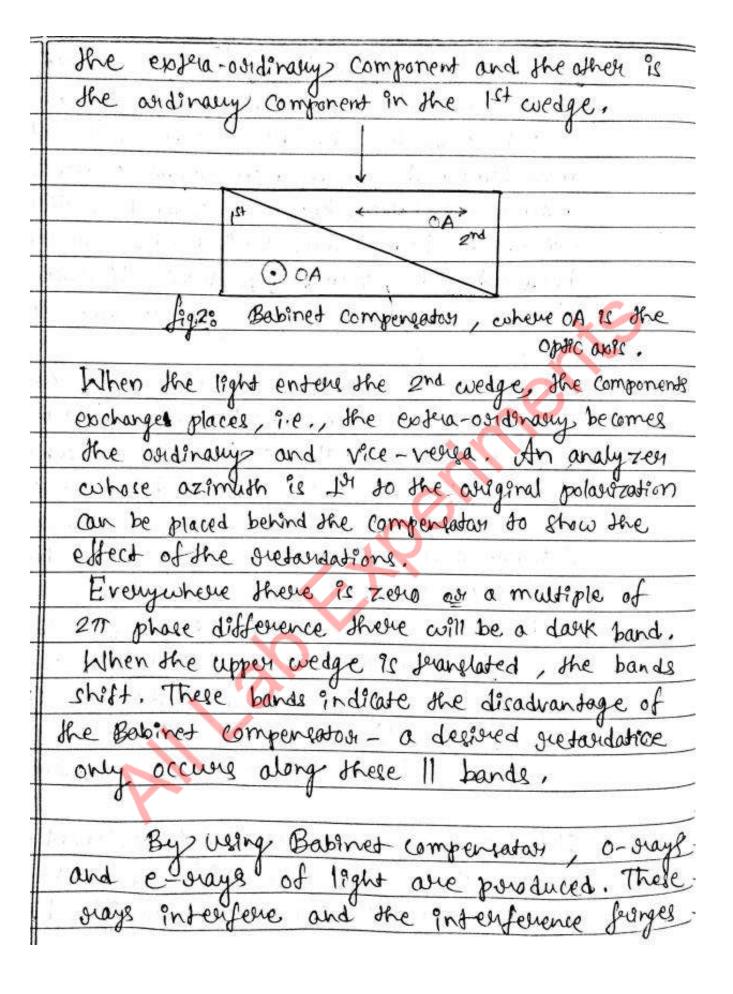
where, $k^2 = k_1^2 + k_2^2 + k_3^2$.

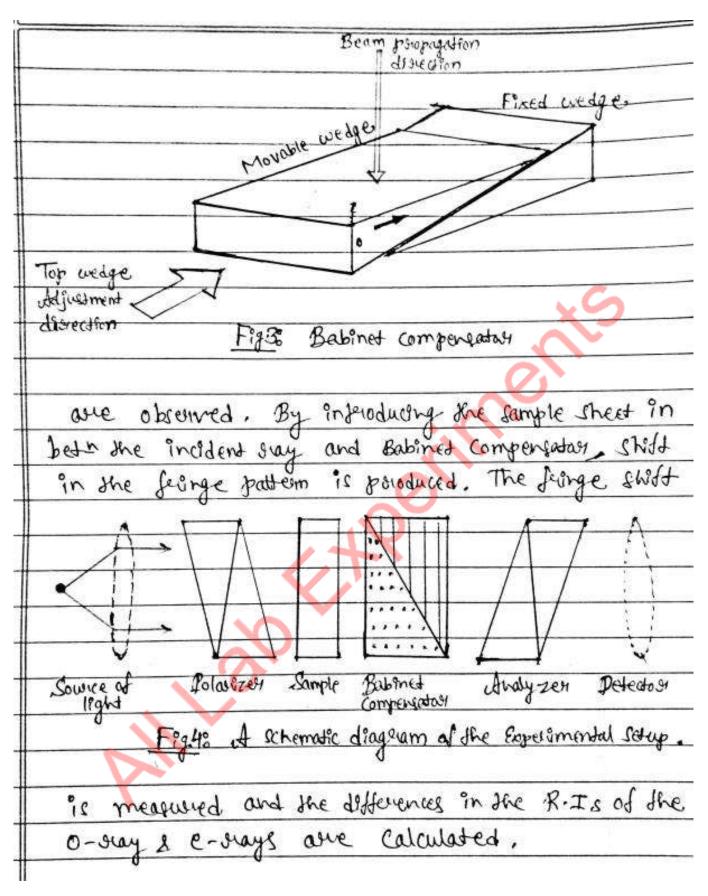
Q5b





"Babinet Compensators & [Jocques Babinet] Retaindens have been constructed of movable elements In osides to posoduce variable ordandance. Two of the most Common designs based on movable wedges are the Babinet and Soleil, known as Babinet - Soleil Compensa-- took. The term compensation is used for these elements because they are often used to allow adjustable compen.
- Sation of retardance originating in a sample under test. The Babinet Compensator, Shown in fig. Consists of two wedges of a (uniavial) bise-fringent material (quartz). The bottom evedge is fixed while the top evedge slides over the bottom by means of a miconometer. The optic axes of both the wedges are Il to the outer for faces of the wedge pair but are 131 to one another. At any particular location across the face of the Babinet compensators, the net retardation is φ = 20 (d1-d2) (Me-Mo) - 0 where did do one the Hickness at that location. If monocheromodic polarized light oriented at 45° to one of the optic axes is incident on the Babinet compensation, one component of the light becomes





Q6b

Please refer to chapter 5 for the solution to this question

Q7a

Refer chapter 6 for its solution

Q7b Refer chapter 6 for its solution