Electromagnetic Theory

BSc. (H) Physics 2019

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **2267** **IC**

Unique Paper Code : 32221601

Name of the Course : **B.Sc. (Hons.) Physics**

Name of the Paper : Electromagnetic Theory

Semester : VI

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **five** questions.

(c) Question **No. 1** is compulsory.

(d) **All** questions carry equal marks.

(e) Scientific calculator is allowed.

1. Answer any **five** of the following questions : 3×5=15

(a) In a lossy dielectric of relative permittivity 12 the displacement current is 25 times greater than the conduction current at 100MHz. Calculate the conductivity of dielectric.

(b) Mention any two differences between half and quarter wave plates.

P.T.O.

(c) Calculate the minimum thickness of calcite plate which would convert plane polarized light into circularly polarized light. Given n_0=1.568, n_e=1.468 and λ=5890 Å^0.

(d) In an optical fibre the core refractive index is 1.5 and cladding refractive index is 1.47. Determine critical angle at core clad interface and numerical aperture.

(e) Using Faraday's law, find the intrinsic impedance of free space.

(f) In what respect does an electrically anisotropic medium differ from an isotropic medium. Mention at least **two** points.

(g) Show that in plasma electron current lags the electric field by $\pi/2$.

(h) Can perfectly static fields possess momentum and angular momentum ?

2. (a) Show how Maxwell modified Ampere's law to make it consistent with the equation of continuity. 3

(b) Show that the Maxwell's equations can be expressed as two coupled second order differential equations in term of scalar and vector potentials. How does these two equations get modified after Lorentz gauge ? 8,4

3. (a) Derive wave equation for E of an em wave in a conducting medium. 4

(b) Show that the amplitude of electric field of em wave attenuates as it propagates in a conducting medium. 8

(c) Find the expression for skin depth. 3

4. (a) Show that in an electrically anisotropic dielectric medium the permittivity tensor is symmetric. 6

(b) Show that in anisotropic dielectric medium the electric field, magnetic field and the Poynting's vector on one hand and the electric displacement, magnetic field and the wave normal on the other hand form orthogonal triplets. 9

5. (a) Derive Fresnel's relations for reflection and refraction of plane em wave at an interface between dielectric media when the electric field vector of the incident wave is normal to the plane of incidence. Also find the expressions for R and T. 8

(b) If a parallel polarized em wave is incident from air onto distilled water with $\mu_r = 1$ and $\epsilon_r = 81$, find the Brewster angle θ_B. 3

6. (a) How would you optically distinguish between circularly polarized light and plane polarized light ? 4

(b) Explain the construction and working of a Nicol prism. 8

(c) What is graded index optical fibre ? Give its one advantage over step index fibre in optical communication. 3

7. (a) Derive wave equation for E of em wave in a symmetric planar dielectric wave guide whose refractive index $[n^2 = n^2(x)]$ profile is :

$n = n_1, \; -d/2 < x < d/2$

$= n_2, \; x < -d/2, x > d/2.$

Using the boundary conditions, obtain the eigenvalue equation for symmetric TE modes. 8,4

(b) Show that there exists only one symmetric TE mode for $0 < V < \pi$, where V denotes the dimensionless wave guide parameter. 3

8. (a) A long straight conducting wire of radius b and conductivity σ is kept along z-axis and it carries a direct current I in +z-direction. Calculate the Poynting's vector on the surface of this wire. 5

(b) Calculate the reflection coefficient at normal incidence for a plane em wave incident on silver from vacuum ($f = 10^{15}$ Hz, $\sigma = 6 \times 10^7$ mho/m). 5

(c) Find the maximum usable frequency for em waves to be transmitted through a distance of 1.5×10^6m by reflection from the ionosphere at a height of 300 km. (number of electrons per unit volume in ionosphere is $6 \times 10^{11} m^{-3}$) 5

Que: 1(a)

Que 1(a) Maxwell displacement current for a plane wave is

$$\vec{J}_D = \varepsilon \frac{\partial \vec{E}}{\partial t} = \varepsilon \frac{\partial}{\partial t} (\vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t)) = \varepsilon \omega \vec{E}$$

and $\vec{J}_c = \sigma \vec{E}$

The ratio between these two currents

$$Q = \frac{\vec{J}_D}{\vec{J}_c} = \frac{\varepsilon \omega}{\sigma}$$

$$25 = \frac{12 \times 100 \times 10^6}{\sigma} \times 8.85 \times 10^{-12} \times 2 \times 3.14$$

$$\sigma = \frac{12}{25} \times 10^8 \times 8.85 \times 10^{-12} \times 2 \times 3.4$$

$$\sigma = 26.67 \times 10^{-4} \text{ mho m}^{-1}$$

$$\sigma = 2.67 \times 10^{-3} \text{ mho m}^{-1}$$

Que: 1(b)

1) A half wave plate is a birefringent material which changes the phase between two perpendicular polarisation by π or half a wave. A quarter wave plate is a birefringent material which changes the phase between the same two polarisation by π/2

(2) A half wave plate can be used to convert one linear polarisation to anothe, or one circular polarisation to another.A quarter wave plate is can be used to convert linear polarised light to circularly polarised light

Que: 1(c)

Ans: 1(C) The phase difference generated by calcite plate is given by,

$$\phi = \frac{2\pi}{\lambda}(n_o - n_e)t$$

$$\Rightarrow \frac{\pi}{2} = \frac{2\pi}{\lambda}(n_o - n_e)t$$

$$t = \frac{\lambda}{4(n_o - n_e)}$$

$$t = \frac{5890}{4(1.568 - 1.468)}$$

$$\boxed{t = 1.47\ \mu m}$$

Que: 1(d)

Que: 1(d) Critical angle for optical fibre is given as,

$$\sin\theta_c' = \frac{n_2}{n_1}$$

$$\sin\theta_c' = \frac{1.47}{1.5}$$

$$\theta_c' = \sin^{-1}\left(\frac{1.47}{1.5}\right)$$

$$\boxed{\theta_c' = 78.52^\circ}$$

Numerical Aperture $NA = n_1 \sin(90 - \theta_c)$

$$= 1.5 \sin(11.48)$$

$$= 0.298$$

Que: 1(e)

The characteristic impedance of free space , also called the Z_o of free space, is an expression of the relationship between the electric-field and magnetic-field intensities in an electromagnetic field (EM field) propagating through a vacuum . The Z_o of free space, like characteristic impedance in general, is expressed in ohm s, and is theoretically independent of wavelength . It is considered a physical constant.

Mathematically, the Z_o of free space is equal to the square root of the ratio of the permeability of free space (μ_o) in henrys per meter (H/m) to the permittivity of free space (ε_o) in farads per meter (F/m):

$$Z_o = (\mu_o / \varepsilon_o)^{1/2}$$

$$= [(1.257 \times 10^{-6} \text{ H/m})/(8.85 \times 10^{-12} \text{ F/m})]^{1/2}$$

$$= 377 \text{ ohms (approximately)}$$

Que: 1(f)

Isotropic refers to the properties of a material which is independent of the direction whereas anisotropic is direction-dependent. These two terms are used to explain the properties of the material in basic crystallography. The mechanical and physical properties can be easily affected based on the atom orientation in crystals. Some examples of isotropic materials are cubic symmetry crystals, glass, etc. Some examples of anisotropic materials are composite materials, wood, etc. Below are a few differences between isotropic and anisotropic materials.

Difference Between Isotropic And Anisotropic

Characteristics	Isotropic	Anisotropic
Properties	Direction independent	Direction-dependent
Refractive index	Only one	More than one
Chemical bonding	Consistent	Inconsistent
Appearance	Dark	Light
Light passes through it	No	Yes
Velocity of light	Same in all directions	Different
Uses	Lenses	Polarizers

Double refraction	No	Yes
Example	Glass	Wood

Que: 1(g)

Que: 1(g) $E = E_0 e^{-i\omega t}$

$$\sigma = \frac{n_e q^2}{m_e(b - i\omega)}$$

$$J = Re\left\{\frac{n_e q^2 E_0 e^{-i\omega t}}{m_e(b - i\omega)}\right\}$$

$$= Re\left\{\frac{n_e q^2 E_0 e^{-i\omega t}}{m_e(b^2+\omega^2) e^{-i\phi}}\right\}$$

$$= Re\left\{\frac{n_e q^2 E_0 e^{-i(\omega t - \phi)}}{m_e(b^2+\omega^2)}\right\}$$

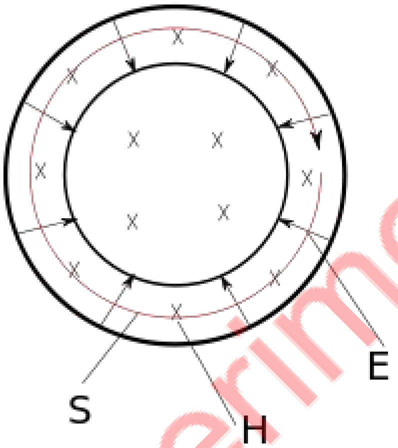
$$\phi = \tan^{-1}\frac{\omega}{b}$$

for plasma, $b \to 0$

$$\phi \to \frac{\pi}{2}$$

Que: 1(h)

The consideration of the Poynting vector in static fields shows the relativistic nature of the Maxwell equations and allows a better understanding of the magnetic component of the Lorentz force, *q*(**v** × **B**). To illustrate, the accompanying picture is considered, which describes the Poynting vector in a cylindrical capacitor, which is located in an **H** field (pointing into the page) generated by a permanent magnet. Although there are only static electric and magnetic fields, the calculation of the Poynting vector produces a clockwise circular flow of electromagnetic energy, with no beginning or end.



While the circulating energy flow may seem nonsensical or paradoxical, it is necessary to maintain conservation of momentum. Momentum density is proportional to energy flow density, so the circulating flow of energy contains an *angular* momentum.[14] This is the cause of the magnetic component of the Lorentz force which occurs when the capacitor is discharged. During discharge, the angular momentum contained in the energy flow is depleted as it is transferred to the charges of the discharge current crossing the magnetic field.

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Que 2(a) :

The Ampere's circuital law states that the path integration of magnetic field $\mathbf{B}$ around any closed path equal to the total current through the surface enclosed by that path times μ_0. The integral form is

$$\oint \mathbf{B}.d\mathbf{l} = \mu_0 \iint \mathbf{J} \cdot d\mathbf{l} = \mu_0 I_{enc} \tag{1}$$

Using stokes theorem we can get the differential form which is

$$\Delta \times \mathbf{B} = \mu_0 \mathbf{J} \tag{2}$$

or for free current density we will get

$$\Delta \times \mathbf{H} = \mathbf{J}_f \tag{3}$$

Where $\mathbf{J}_f$ is the free current. But there are two problem with this

1. taking divergence of the equation 2 we get

$$\delta \cdot (\Delta \times \mathbf{B}) = 0$$

So we get $\Delta.J = 0$. Which is not true and the correct relationship is

$$\Delta \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

2. One more problem from above equation we get is when $\mathbf{J} = 0$ then we get $\Delta \times \mathbf{B} = 0$ which is also not correct. Because we know that in space

$$\Delta \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

To solve this problem Maxwell introduced displacement current which is given by

$$\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E} = \epsilon \mathbf{E} \tag{4}$$

and displacement current density as

$$\mathbf{J}_D = \epsilon \frac{\partial \mathbf{E}}{\partial t} \to \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \text{ (in space)} \tag{5}$$

Then he included that term in the Amperes circuital law which then become as

$$\boxed{\oint \mathbf{B}.d\mathbf{l} = \mu_0 \iint \left(\mathbf{J} \cdot d\mathbf{l} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \right) \cdot d\mathbf{S}} \tag{6}$$

And the differential equation becomes

$$\boxed{\Delta \times \mathbf{B} = \mu_0 \mathbf{J} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}} \tag{7}$$

Which solved the problem.

Que: 2(b)

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho \quad \Rightarrow \quad \nabla \cdot \left(-\nabla V - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} \right) = 4\pi\rho$$

$$\Rightarrow \nabla^2 V + \frac{1}{c}\frac{\partial}{\partial t}\nabla \cdot \boldsymbol{A} = -4\pi\rho \quad ,$$

which with $\nabla \cdot \boldsymbol{A} = 0$ still leaves us with a Poisson equation, but Ampère's law gives

$$\nabla \times (\nabla \times \boldsymbol{A}) = \frac{4\pi}{c}\boldsymbol{J} - \frac{1}{c}\frac{\partial}{\partial t}\nabla V - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2}$$

$$\nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A} =$$

or $$\left(\nabla^2 \boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} \right) - \nabla\left(\nabla \cdot \boldsymbol{A} + \frac{1}{c}\frac{\partial V}{\partial t} \right) = -\frac{4\pi}{c}\boldsymbol{J}$$

Lorentz gauge

It's hard to compute $\boldsymbol{A}$ in Coulomb gauge. On the other hand, we could choose λ such that

$$\boxed{\nabla \cdot \boldsymbol{A} + \frac{1}{c}\frac{\partial V}{\partial t} = 0 \quad ,}$$ Lorentz gauge

for which the second-order PDEs become,

$$\nabla^2 V + \frac{1}{c}\frac{\partial}{\partial t}\nabla \cdot \boldsymbol{A} = -4\pi\rho$$

$$\Rightarrow \boxed{\nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = -4\pi\rho}$$

and

$$\left(\nabla^2 \boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} \right) - \nabla\left(\nabla \cdot \boldsymbol{A} + \frac{1}{c}\frac{\partial V}{\partial t} \right) = -\frac{4\pi}{c}\boldsymbol{J}$$

$$\Rightarrow \boxed{\nabla^2 \boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\frac{4\pi}{c}\boldsymbol{J}}$$

- Not utterly simple, but at least V and $\mathbf{A}$ are separately determined, and the four equations are very similar to one another.

Que: 3(a) Refer to question no. 2 in chapter no. 2

Que: 3(b) Refer to question no. 2 in chapter no. 2

Que: 3(c) Refer to question no. 2 in chapter no. 2

Que: 4(a) Refer to question no. 2 in chapter no. 4

Que: 4(b)

The electromagnetic field consists of coupled electric and magnetic fields that are described by the vectors **E** and **B**, known as *electric* and *magnetic induction* vectors, respectively. In order to describe the effects of these two fundamental fields on matter, it is necessary to introduce the *electric displacement* and *magnetic* vectors, denoted by **D** and **H**, respectively[1]. If the *electric current density* **J** is also introduced, the four fields **E**, **B**, **D** and **H** are linked by Maxwell's equations[2]

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}}{\partial t}(\mathbf{r},t), \quad (1)$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \mathbf{J}(\mathbf{r},t) + \frac{\partial \mathbf{D}}{\partial t}(\mathbf{r},t), \quad (2)$$

$$\nabla \cdot \mathbf{D}(\mathbf{r},t) = \rho(\mathbf{r},t), \quad (3)$$

$$\nabla \cdot \mathbf{H}(\mathbf{r},t) = 0, \quad (4)$$

For such a monochromatic plane wave, the substitution $\frac{\partial}{\partial t} \to j\omega$ and $\nabla \to -j\omega \frac{n}{c}\mathbf{u}$ can be performed into Maxwell's equations (1)-(4), leading to

$$\frac{n}{c}\mathbf{u} \times \mathbf{E} = \mathbf{B}$$

$$\frac{1}{\mu_0}\frac{n}{c}\mathbf{u} \times \mathbf{B} = -\mathbf{D}$$

$$\mathbf{u} \cdot \mathbf{D} = 0$$

$$\mathbf{u} \cdot \mathbf{B} = 0$$

It has been further assumed that the medium is non magnetic, M = 0, hence B = μ_0H. Under this assumption, the Poynting vector becomes

$$\mathbf{S} = \mathbf{E} \times \frac{\mathbf{B}}{\mu_0}.$$

Considering equations given above, it can easily be established that, at any given time

- $(\mathbf{u}, \mathbf{D}, \mathbf{B})$ is a right handed orthogonal vector triplet.

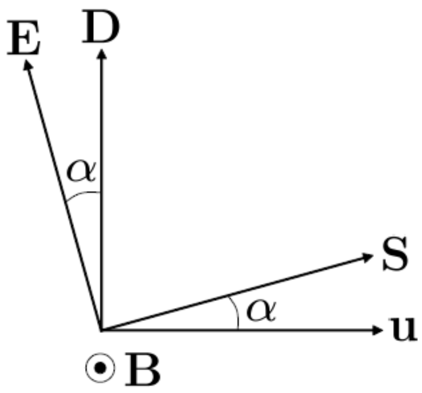


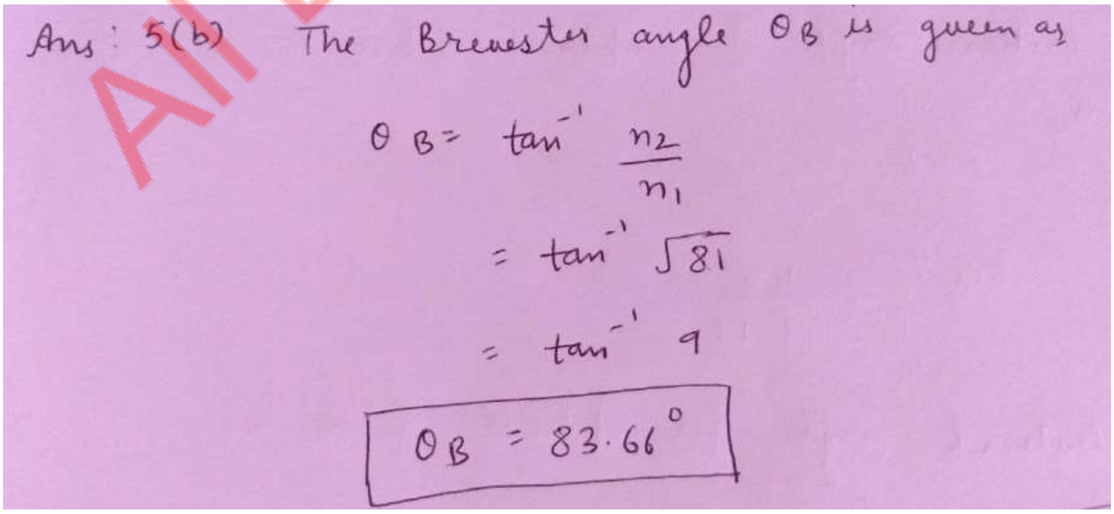
Figure 2 Illustration of the relative directions of the vectors and fields associated with the propagation of an electromagnetic wave in an anisotropic medium. Note that the relative magnitude of the vectors is arbitrary in this representation.

- $(\mathbf{S}, \mathbf{E}, \mathbf{B})$ is also a right handed orthogonal vector triplet.

- However, the tensor relation imposes that, contrarily to the isotropic case, D is not parallel to E, and consequently S is not parallel to u. Therefore the two right handed orthogonal vector triplets (u, D, B) and (S, E, B) are distinct.
- Consequently, the direction of the energy flow, as defined by the Poynting's vector, is no longer parallel to u, direction of propagation of the phase.
- D and E are no longer parallel, as in the isotropic case. Hence $u \cdot E$ is not equal to zero and the E field is no longer transverse. However D and B are transverse.

Que: 5(a) Refer to question no. 1 in chapter no. 3

Que: 5(b)



Ans: 5(b) The Brewster angle θ_B is given as

$$\theta_B = \tan^{-1} \frac{n_2}{n_1}$$

$$= \tan^{-1} \sqrt{81}$$

$$= \tan^{-1} 9$$

$$\boxed{\theta_B = 83.66^\circ}$$

Que: 6(a) Refer to question no. 7 in chapter no. 4

Que: 6(b) Refer to question no. 5 in chapter no. 4

Que: 6(c) Refer to question no. 4 in chapter no. 7

Que: 7(a) and (b) Refer to question no. 2,3 in chapter no. 6

Que: 8(a)

Que: 8(a) According to Ohm's Law

$$\vec{J} = \sigma \vec{E} \Rightarrow \vec{E} = \frac{I}{\sigma}\hat{z} = \frac{I}{\sigma \pi b^2}\hat{z} \quad \text{— (i)}$$

By Ampere's circuital law,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$\vec{B} = \frac{\mu_0 I}{2\pi b}\hat{\phi} \quad \text{— (ii)}$$

Poynting vector, $S = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$ — (iii)

Substituting (i) and (ii) in (iii),

$$\vec{S} = \frac{1}{\mu_0}\left\{\frac{I}{\sigma \pi b^2}\hat{z} \times \frac{\mu_0 I}{2\pi b}\hat{\phi}\right\}$$

$$\vec{S} = \frac{-I^2 \hat{r}}{2\pi^2 \sigma b^3}$$

Que: 8(b)

Que: 8(b) For Metallic Reflection,

$$R = 1 - 2\sqrt{\frac{2\omega \varepsilon_0}{\sigma}}$$

$$= 1 - 2\sqrt{\frac{2 \times 2\pi \times 10^{15} \times 10^{-9}}{6 \times 10^{7} \times 36 \times \pi}}$$

$$= 1 - 2\sqrt{\frac{1}{540}}$$

$$= 1 - 2 \times 0.043$$

$$= 0.914$$

~ 91.4% reflected from metal surface

Que: 8(c)

Que:-8(c) Critical frequency for EM waves propagating in plasma is given as,

$$f_c = 9\sqrt{N_e} \qquad N_e = 9 \times 10^{11}\ m^{-3}$$

$$f_c = 9 \times \sqrt{10^{11} \times 6}$$

Maximum usable frequency

$$f_m = f_c\sqrt{1 + \left(\frac{D}{2h}\right)^2}$$

where $D = 1.5 \times 10^{6}\ m$

and $h = 300\ km$

$$f_m = 9\sqrt{6 \times 10^{11}}\sqrt{1 + \left(\frac{1.5 \times 10^{6}}{2 \times 3 \times 10^{5}}\right)^2}$$

$$= 9\sqrt{6 \times 10^{11}}\sqrt{7.25}$$

$$\simeq 18\ MHZ$$

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