Electromagnetic Theory

BSc. (H) Physics 2018

This question paper contains 4 printed pages]

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S. No. of Question Paper : 6685

Unique Paper Code : 32221601 HC

Name of the Paper : Electromagnetic Theory

Name of the Course : B.Sc. (H) Physics-CBCS

Semester : VI

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt total 5 questions.

All questions carry equal marks.

Question No. 1 is compulsory.

Scientific calculator is allowed.

1. Answer any *five* of the following questions : 5×3=15

(*a*) Write the boundary conditions satisfied by the electric and magnetic field vectors at the boundary of the two dielectrics.

(*b*) Explain the terms loss tangent and intrinsic impedance.

P.T.O.

(c) Explain why in high frequency circuits current flows only on the surface of conductors.

(d) What do you understand by homogenous and isotropic medium ?

(e) A parallel polarized wave propagates from air into dielectric at Brewster angle of 75°. Find relative permittivity.

(f) For an optical fibre with refractive index of the core 1.47 and of its cladding 1.46. Calculate the pulse dispersion per kilometre.

(g) Given $E = E_0 \sin(\omega t - \beta z) a_y$ V/m in free space. Find H.

(h) Write constitutive relations in electrodynamics.

2\. (a) State and prove Poynting's theorem. 2.5

(b) What are Lorentz and Coulomb gauges ? Show that Lorentz transformation remains invariant if gauge function ϕ satisfies :

$$\nabla^2\phi - \mu_0 \epsilon_0 \frac{\partial^2 \phi}{\partial t^2} = 0.$$ 2.6

3\. (a) Derive em wave equation satisfied by E field using Maxwell's equation in an isotropic, linear and homogenous dielectric material (no free charges or free currents).

(b) Show that the em waves are transverve in nature.

(c) Calculate the characteristic impedance of the medium.

(d) Show that the energy is equally shared between the electric and magnetic fields in free space. 4,4,3,4

4\. (a) Derive Fresnel's formula for the case of propagation of em waves in an anisotropic medium. 9

(b) Show that critical frequency for the propagation of em waves in plasma is $f_c = 9\sqrt{n_0}$, where n_0 is number of electrons/m^3. 6

5\. (a) Derive Fresnel's relations for reflection and refraction of plane em waves at an interface between two dielectric media when electric vector of the incident wave is parallel to the plane of incidence. Also find the expressions for R and T. 8,4

(b) Find the expression for the Brewster's angle at which the reflected wave is completely extinguished. 3

P.T.O.

6\. (*a*) State Biot's laws for rotatory polarization.

(*b*) Using the Fresnel's theory of optical rotation, obtain the formula for the angle of rotation of plane of vibration.

(*c*) Discuss the construction and working of Laurent's half shade device. 4,6,5

7\. (*a*) Determine the change of phase in the reflected ray when it suffers a total internal reflection for the case when E is parallel to the plane of incidence. 5

(*b*) What is step index optical fibre ? Derive relation between the numerical aperture and the angle of acceptance. 6

(*c*) An optical fibre has a core of refractive index 1.48 and cladding of refractive index 1.46, calculate the acceptance angle and the numerical aperture of the fibre. 4

8\. (*a*) For the case of propagation of em waves in conducting medium, derive an expression for complex intrinsic impedance and the ratio u_e/u_m, where u_e is electric energy density and u_m is magnetic energy density. 8,4

(*b*) What is the significance of the equation : 3

$$\vec{\nabla}.\vec{B} = 0$$

Ans: 1(a) Refer to question no. 5 in chapter no. 1

Ans: 1(b)

Intrinsic impedance

Intrinsic impedance of a medium can be defined as the impedance which an electromagnetic wave faces while travelling in a medium. An electromagnetic wave comprises of both electric and magnetic field, and thus mathematically the intrinsic impedance is defined as, n= (E/H) where E is the electric field and H is the magnetic field component. The unit is ohms.

It can also be defined as, n=√(u/e), where u= permeability of that medium while e= permitivitty of that medium. Remember that u=(relative permeability of that medium)*(permeability of free space) and same goes for e.

For air, the intrinsic impedance by using the standard values of e and u comes out to be 377 ohms.

Ans: 1(c)

This is termed skin effect. It is due to induced emf created by the alternating magnetic field as a result of the flowing current. This emf is strongest at the center of the conductor hence it forces the electrons away from the center towards the surface. Since current is the flow of electrons which are now at the surface, it can be said that current flows at the surface.

Ans: 1(d)

Homogenous materials

One of the characteristics of a homogenous material is that it's made of a single compound or element.

Or

A material is said to be homogenous if the materials elastic properties (E,μ) are same at all the points or throughout the body.

Eg: A uniform electric field

Isotropic materials

A material whose properties are direction independent.i.e., a material whose properties don't depend on orientation.

Eg: rubber is a very isotropic material. Take a rubber ball, and it will feel the same and bounce the same however you rotate it.

Ans: 1(e)

Ans: 1(e) The Brewster angle θ_B is given as

$$\theta_B = \tan^{-1}\frac{n_2}{n_1}$$

$$\tan\theta_B = \frac{n_2}{n_1}$$

$$\tan 75 = \frac{n_2}{n_1}$$

$$\frac{n_2}{n_1} = 3.73$$

and $\frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$ [For non-magnetic mat

Relative permittivity, $\varepsilon = \left(\frac{n_2}{n_1}\right)^2$

$$= (3.73)^2$$

$$= 13.91$$

Ans: 1(f)

Que: 1(f) The fractional refractive-index change Δ is given b

$$\Delta = \frac{n_1 - n_2}{n_1}$$

Pulse dispersion in a step index optical fibre is given by,

$$\text{pulse dispersion} = \frac{\Delta n_1 \ell}{c}$$

$$= \frac{n_1 - n_2}{n_1} \frac{n_1 \ell}{c}$$

$$\frac{\text{pulse dispersion}}{\ell} = \frac{(1.47 - 1.46)}{3 \times 10^8}$$

$$= \frac{0.01}{3} \times 10^{-8} \ \text{s}\,\text{m}^{-1}$$

$$= 3.33 \times 10^{-10} \ \text{m}^{-1}$$

Ans: 1(g)

Ans:- 1 (g) Given $E = E_0 \sin(\omega t - \beta z)\,\hat{a}_y$ V/m

$$H_x = -\sqrt{\frac{\varepsilon}{\mu_0}}\,\sin(\omega t - \beta z)\,\hat{a}_x$$

as Electric field is in y-direction and wave is travelling in Z direction, magnetic field vector will point in x direction to obey transverse nature of light.

Ans: 1(h)

There are four of Maxwell's equations that describe classical electromagnetics. Maxwell's equations define the fields that are generated by currents and charges. However, they do not describe how these currents and charges are generated. Thus, to find a self-consistent solution for the electromagnetic field, Maxwell's equations must be supplemented by relations that describe the behavior of matter under the influence of fields. These material equations are known as constitutive relations. They are

$$\nabla \times \vec{E} = -j\omega \vec{B}$$

$$\nabla \times \vec{H} = \vec{J} + j\omega \vec{D}$$

$$\nabla \bullet \vec{D} = 0$$

$$\nabla \bullet \vec{B} = 0$$

The constitutive relations describe how the electromagnetic fields interact with matter, although the specifics of the interaction is not obvious just by looking at the equations. These are

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

In these equations, ϵ is the permittivity of the medium and characterizes how well the medium stores electric energy. It has units of Farads/meter. μ is the permeability of the medium and characterizes how well the medium stores magnetic energy. It has units of Henries per meter

Ans: 2(a) Refer to questions no. 5 and 6 in chapter no. 1

Ans: 2(b) Refer to question no. 6 in chapter no. 1

Ans: 3(a) Refer to question no. 1 in chapter no. 2

Ans: 3(b) Refer to question no. 1 in chapter no. 2

Ans: 3(c) Intrinsic impedance of a medium can be defined as the impedance which an electromagnetic wave faces while travelling in a medium. An electromagnetic wave comprises of both electric and magnetic field, and thus mathematically the intrinsic impedance is defined as,

n= (E/H) where E is the electric field and H is the magnetic field component. The unit is ohms.

For any medium, it can be written as,

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_R}{\varepsilon_0 \varepsilon_R}} = 376.73 \times \sqrt{\frac{\mu_R}{\varepsilon_R}}\ Ohms$$

$\mu_0 = permeability\ of\ freespace = 4\pi \times 10^{-7}\ Henries/meter$

$\varepsilon_0 = permittivity\ of\ freespace = 8.854 \times 10^{-12} =\sim (1/36\pi) \times 1E-9\ Farads/meter$

Ans: 3(d) Refer to question no. 1 in chapter no. 2

Ans: 4(a) Refer to question no. 2 in chapter no. 4

Ans: 4(b) Refer to question no. 3 in chapter no. 2

Ans: 5(a) Refer to question no. 2 in chapter no. 3

Ans: 5(b) Refer to question no. 3 in chapter no. 3

Ans: 6(a) Refer to question no. 2 in chapter no. 5

Ans: 6(b) Refer to question no. 3 in chapter no. 5

Ans: 6(c) Refer to question no. 6 in chapter no. 5

Ans: 7(a)

Ans: 7(a) The reflection coefficient for TE mode wave, denoted by $r_{\perp}$ is given by the Fresnel equation as,

$$r_{\perp} = \frac{-\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \quad \text{(i)}$$

$$= \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \quad \text{(ii)}$$

$$= \frac{\cos\theta_i - \sqrt{(n_{ti}^2 - \sin^2\theta_i)}}{\cos\theta_i + \sqrt{(n_{ti}^2 - \sin^2\theta_i)}} \quad \text{(iii)}$$

where $n_{ti} = n_t / n_i$

To compute the phase changes $\delta\phi_r$ on reflection from an interface as a function of the angle of incidence, we rewrite Fresnel's equation in the form,

$$r_{-\perp} = r_\perp e^{j\delta\phi_\perp} \qquad \text{(iv)}$$

In case of total internal reflection, because $[(n_t/n_i) - \sin^2\theta_i]^{1/2}$ becomes imaginary, eqn (iii) can be rewritten as

$$r_{-\perp} = \frac{E_{r\perp}}{E_{i\perp}} = \frac{A+jB}{A-jB} = \frac{Ce^{j\phi}}{Ce^{-j\phi}} = e^{j\delta\phi'_{r\perp}} \quad \text{(v)}$$

where $A = \cos\theta_i$, $B = [\sin^2\theta_i - (n_t/n_i)^2]^{1/2}$

and $\delta\phi'_{r\perp}$ is the TE phase shift for internal reflection, it follows that

$$\tan\frac{\delta\phi'_\perp}{2} = \frac{[\sin^2\theta_i - (n_t/n_i)^2]^{1/2}}{\cos\theta_i}$$

Ans: 7(b) Refer to question no. 2 and 4 in chapter no. 7

Ans: 7(c) Refer to question no. 2 chapter no. 7

Ans: 8(a) Refer to question no. 2 in chapter no. 2

Ans: 8(b) Refer to question no. 2 in chapter no. 1