11

Polarization Of Light Waves And Double Refraction

11.1. NATURE OF LIGHT WAVES

The experiments illustrating interference and diffraction of light studied in previous chapters have shown beyond doubt that *light like sound is some form of wave motion.* These experiments do not reveal whether the light waves are longitudinal or transverse because phenomenon of interference and diffraction can occur with both longitudinal and transverse waves. We shall, therefore, investigate the nature of light waves and to begin with, we shall describe an important feature which distinguishes the two types. From the study of sound, we know that sound travels in the form of *longitudinal waves and properties of such a wave motion are the same with respect to any plane through its line of propagation while a transverse wave behaves differently in different planes.* The statement can be illustrated by a simple mechanical analogy given below.

Take a stretched rubber cord *CD* threading through two narrow slits S_1 and S_2 cut in card board pieces and placed parallel to each other in the vertical planes as shown in Fig. 11.1 (*a*). End *D* of the cord is fixed. *Now set up a longitudinal wave in CD by moving the end C forward and backward along the cord.* Rotate any of the slits about *CD* as axis. It will be found that this rotation does not affect the passage of the wave, *i.e.*, the wave passes through the first and second slits without being affected at all in whatever position the slits may be arranged. Thus, a longitudinal wavemotion has the same properties with respect to all planes througout its line of advance.

On the other hand, if we set up a transverse wave in the cord by moving the end *C* up and down in the vertical plane, it will be found that the wave passes through the slits undisturbed as shown in Fig. 11.1 (*a*) and reach the point *D* with undisturbed amplitude. If however, the slit S_2 is rotated to become perpendicular to slit S_1, it will be found that the wave passes through slit S_1 undisturbed as before but is not able to pass through slit S_2 so that beyond S_2 the cord remains practically undisturbed as shown in Fig. 11.1 (*c*). In the intermediate positions of the slit S_2, the vibrations will be partly transmitted and partly stopped, *i.e.*, they will reach the point *D* with diminished amplitude, Fig. 11.1 (*b*). Thus, we conclude that the transverse waves can pass through the slits only when they are parallel to each other and lie in the same plane in which the wave lies.

Now, continuously change the direction of vibrations of the end *C* at random always keeping the vibrations perpendicular to the cord. A number of transverse waves will be produced which will travel along the string in rapid succession and the vibration in each wave, though perpendicular to the length of the string will be in a different direction. In the portion CS_1, Fig. 11.2(*a*) it will appear asif the string

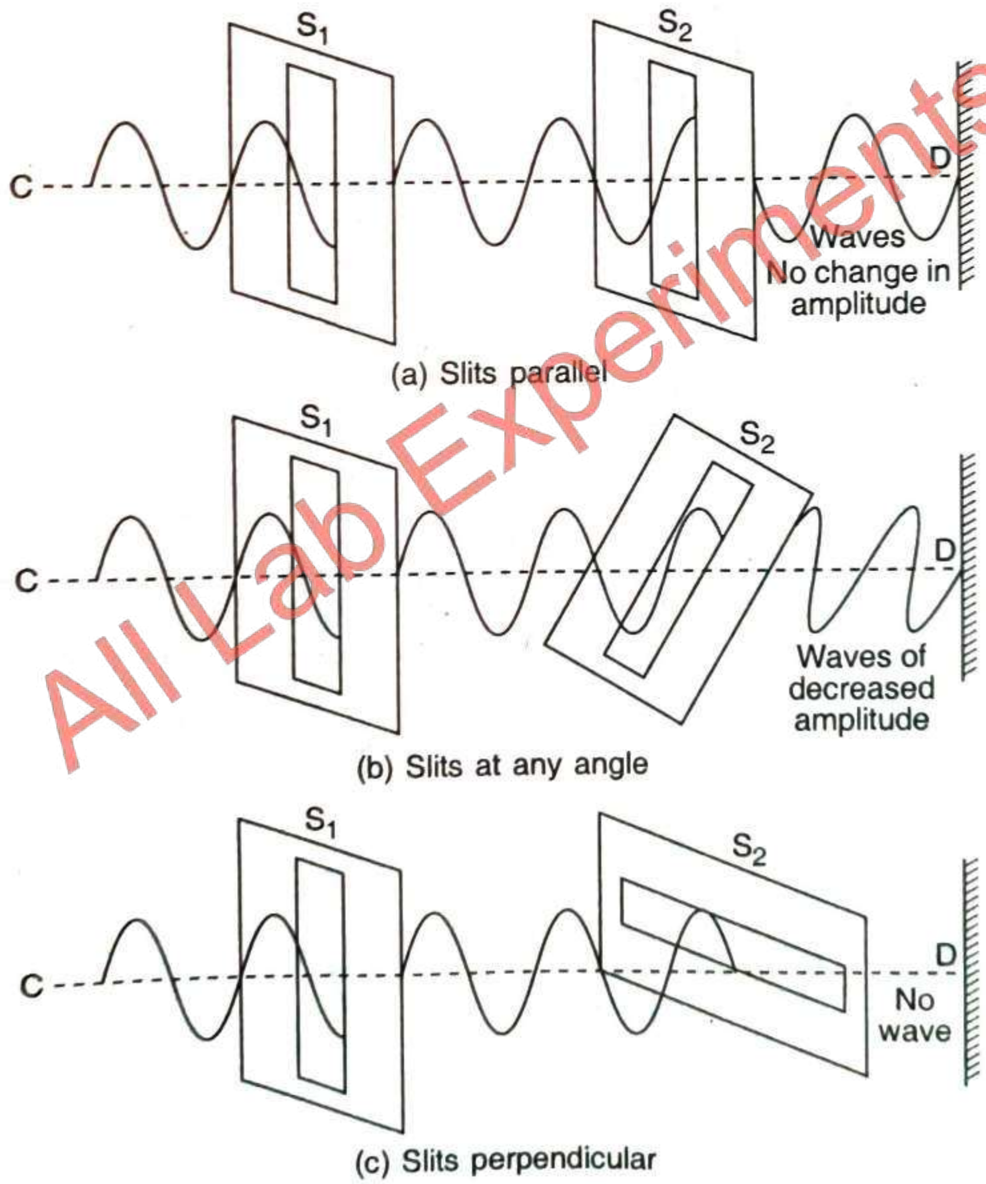


Fig. 11.1.

11.2. PLANE POLARIZED LIGHT

In case of light emerging out from crystal *P*, the vibrations are confined to only one direction in a plane perpendicular to the direction of propagation. Such light which has acquired the property of one sidedness is called *linearly polarized.* Since the vibrations constituting the beam of light are confined to only one plane containing the direction of propagation of the wave, the light is generally said to be *plane polarized.*

The properties of plane polarised beam differ w.r.t. two planes, one containing the vibrations and the other at right angles to it. The plane *EFGH* (Fig. 11.4) containing the crystallographic axis in which

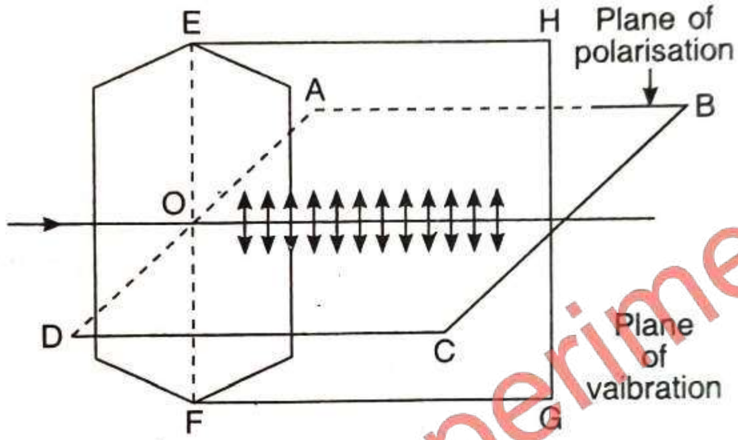


Fig. 11.4. Vibration and polarization planes.

vibrations are occurring is called *plane of vibrations* and the plane *ABCD* (Fig. 11.4) perpendicular to plane of vibrations of light wave is called *plane of polarization.*

11.3. ORDINARY OR NATURAL LIGHT IS UNPOLARIZED

The transverse character of light waves was known in early years of the nineteenth century, however, the nature of the displacement associated with a light wave was known only after Maxwell had put forward his famous electromagnetic theory, according to which the light waves are electromagnetic. An electromagnetic wave consists of a vibrating electric and a vibrating magnetic field, each field being perpendicular to the other and both being perpendicular to the direction of propagation of the waves as shown in Fig. 11.5. The electric vector $\vec{E}$ represents the electric field and the magnetic vector $\vec{B}$, the magnetic field. Now, according to all available theoretical and experimental evidence, it is the electric vector $\vec{E}$ of light wave which produces all the observed effects of light, *i.e.*, optical effects of light waves are due to the electric field vibrations only. So whenever we talk of vibrations in a light wave, we mean the vibrations of the celectric vector $\vec{E}$.

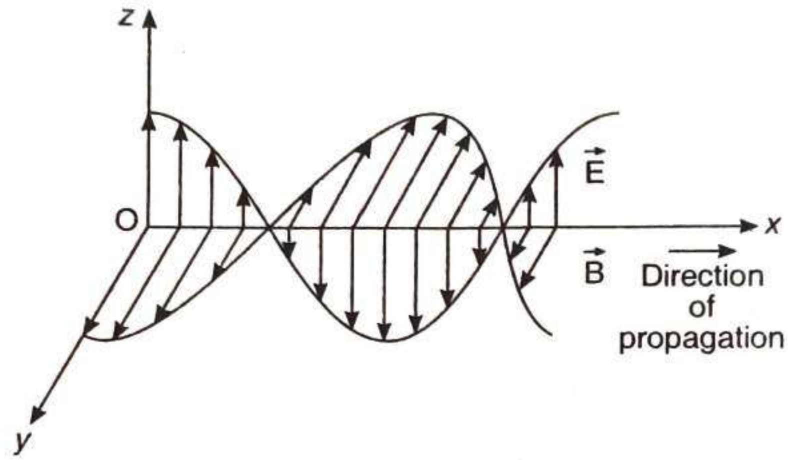


Fig. 11.5. Electromagnetic wave.

Light is emitted by excited atoms and molecules. Any actual source of light contains millions and millions of excited atoms oriented at random. Each excited atom emits light for 10^{-8} sec and the emitted wave is linearly polarized. So in a ray of light millions of waves follow each other in rapid succession, the waves being linearly polarized in all possible directions perpendicular to the ray. So in general an ordinary light beam (like the one coming from a sodium lamp or from sun) is *unpolarized, i.e.,* the electric vector (in a plane perpendicular to the direction of propagation) keeps on changing its direction in a random manner, Fig. 11.6 (*a*). When such a beam is

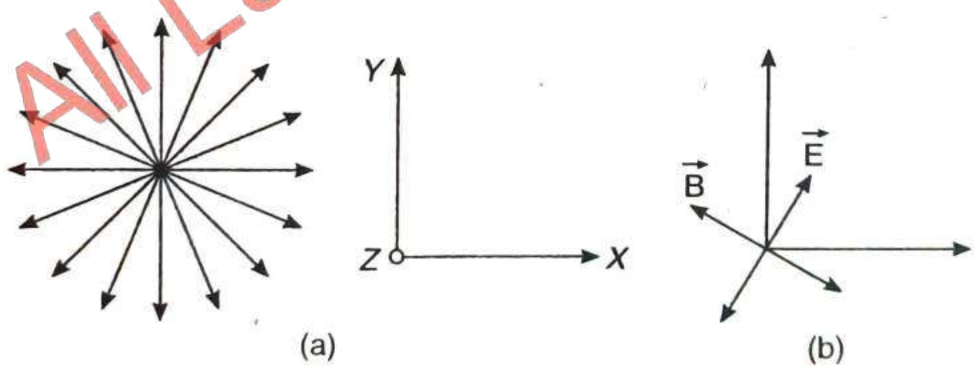


Illustration an unpolarized wave propagating in +Z-direction, the electric vector $\vec{E}$ (lying in x-y plane) keeps on changing its direction randomly.

Illustration a linearly polarised wave vector $\vec{E}$ or $\vec{B}$ which oscilates in a particular direction.

Fig. 11.6.

incident on a polaroid (tourmaline crystal) the emergent light is linearly polarised with its electric vector oscillating in a particular direction as shown in Fig. 11.6(*b*). The direction of the electric vector of the emergent beam will depend on the orientation of polaroid. As will be seen in article (11.5.1) that the component of $\vec{E}$ along a particular direction gets absorbed by the polaroid and the component perpendicular to it passes through. The direction of the electric vector of the emergent wave is generally known as *pass axis* of the polaroid.

Brewster's Law

In 1811, Sir Brewster by his researches showed that the tangent of the angle of polarization i_p is numerically equal to the refractive index μ of refractive medium, *i.e.*,

$$\mu = \tan i_p \qquad \ldots(11.1)$$

This relation is called *Brewster's Law* and the polarising angle i_p is also known as Brewster's angle. As the refractive index of the medium depends on the wavelength of incident light, so the polarising angle varies with the wavelength of light. So the polarization can only be completed for one particular wavelength at a time. Hence, complete polarization is possible with monochromatic light and not with white light.

Relation between i_p and angle of reflection 'r' in denser medium. A direct consequence of Brewster law is that *when light is incident at the polarising angle, the reflected and refracted rays are mutually perpendicular to each other.*

As shown in Fig. 11.10, *xy* is the plane surface separating glass and air *IO* is the incident ray of unpolarized light incident at the polarising angle i_p; *OR* is the refracted ray, *r* being the corresponding angle of refraction; *OS* is the reflected ray. The refracted ray is only partially polarized and the reflected ray is completely polarized.

From Snell's Law,

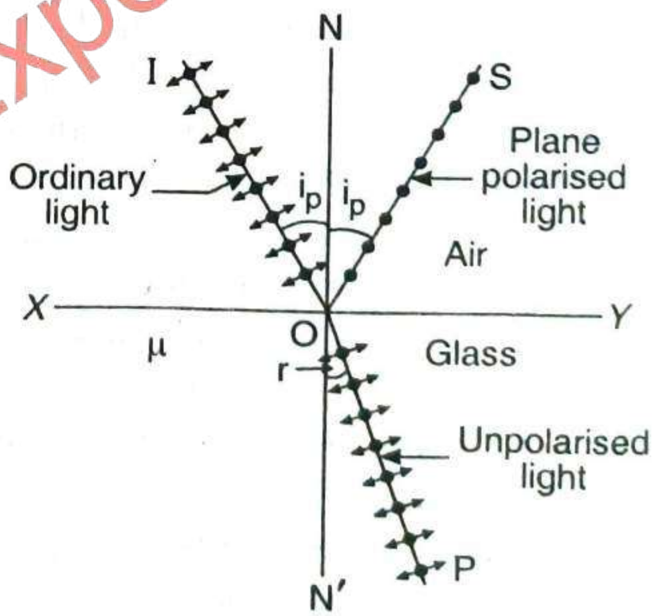


Fig. 11.10. Polarization by reflection.

$$\mu = \frac{\sin i_p}{\sin r} \qquad \ldots(11.2)$$

and from Brewster's Law,

$$\mu = \tan i_p = \frac{\sin i_p}{\cos i_p} \qquad \ldots(11.3)$$

Comparing eqns. (11.2) and (11.3),

$$\sin r = \cos i_p$$
$$= \cos (90 - i_p)$$
$$\therefore \quad r = 90 - i_p$$

or

$$\boxed{r + i_p = 90^\circ} \qquad \ldots(11.4)$$

that is, at *polarizing incidence, the refracted and reflected rays are mutually perpendicular to each other.*

Explanation. The production of polarized light by reflection from glass or any other transparent medium can be explained as follows :

When ray of light is incident on a transparent medium, the displacements are in all directions perpendicular to the incident ray. These vibrations can be divided into two sets/groups.

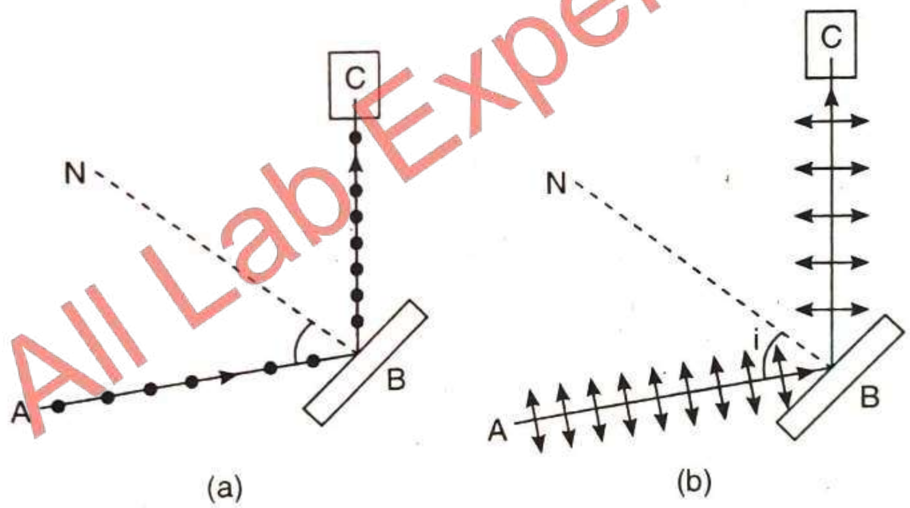


Fig. 11.11.

(*i*) One perpendicular to the plane of incidence *ABC*. Fig. 11.11(*a*)

and

(*ii*) The other in the plane of incidence Fig. 11.11(*b*).

The displacements which are perpendicular to plane of incidence ($\perp$ to the plane of paper) would always remain parallel to the reflecting surface whatever the angle of incidence may be. These conditions of reflection are, therefore, not affected by any change in the angle of incidence. Thus, a glass plate will reflect a ray of light with vibration to the plane of incidence whatever be the angle of incidence. For a particular angle of incidence (polarising angle) it may refuse to reflect a ray in which vibrations are parallel to the plane of incidence.

The ray reflected from glass at polarising angle is plane polarized or to have acquired one sidedness as its vibrations are confined to the plane perpendicular to the incident plane. The ray is said to be polarised in the plane of incidence.

The ray in which the vibrations are parallel to the plane of incidence and which have been refused reflection (when angle of incidence is 57.5° in case of glass) is refracted and transmitted through glass. Thus, transmitted ray is said to be polarised in plane perpendicular to the plane of incidence since as we have seen the vibrations are confined to the plane of incidence.

The reflected ray is wholly or completely polarised if the angle of incidence is equal to polarizing angle.

The transmitted ray is, however, only partially polarised since some rays have vibrations perpendicular to the plane of incidence are also transmitted.

The reflected light can be detected or analysed by a tourmaline crystal, or by another glass surface as explained below :

it.

Example 1. *A ray of light strikes a glass plate at an angle of incidence* 60°. *If the reflected and refracted rays are perpendicular to each other, find the refractive index of glass.* *(K.U. 1994 A)*

Solution. When the reflected and the refracted rays are at rt. angle to each other, the angle of incidence is equal to the polarizing angle $= i_p$.

So, $\mu = \tan i_p = \tan 60°$

or $\mu = \mathbf{1.732}$

Example 2. *A ray of light is incident on the surface of a glass plate of refractive index* 1.5 *at the polarizing angle. What is the angle of refraction?* *(K.U. 1993 S)*

Solution. Let i_p be the polarising angle of incidence and r the corresponding angle of refraction.

From Brewster's Law,

$$\mu = \tan i_p$$

$$\therefore \quad i_p = \tan^{-1}(\mu) = \tan^{-1}(1.5) = 56.3°$$

Since $\quad i_p + r = 90°$

$$\therefore \quad r = 90° - i = 90° - 56.3° = 33.7°.$$

11.6. LAW OF MALUS

When a beam of light, polarized by reflection at one plane surface is allowed to fall on the second plane surface at the polarizing angle, the intensity of the twice reflected beam varies with the angle between the planes of the two surfaces. It has been found that when these two planes are parallel, the intensity of twice reflected beam is maximum, and when these two planes are perpendicular to each other, the intensity of the twice reflected beam is zero. The same is true for twice transmitted beam from the polarizer and analyser. It was experimentally shown by Etiennie Louis Malus in 1809, that *when a completely plane polarized light beam is incident on an analyser, the intensity of the polarized light transmitted through the analyser varies as the square of the cosine of the angle between the planes of transmission of the analyser and the polarizer.* This is called **Law of Malus** and holds good for a combination of the reflecting planes, Nicol prisms, polaroids but not for the pile of plates (because in this case the light emerging from the polarizer is not completely plane polarized). It can be proved as follows :

Let OP = 'a' be the amplitude of the vibrations transmitted or reflected by the polarizer (*i.e.* plane polarized light) and θ be the angle between the plane of transmission of the analyser and that of polarizer as shown in Fig. 11.18. This light is incident on the analyser. Its amplitude can be resolved into two rectangular components :

(*i*) $a \cos \theta$ is parallel to the plane of transmission of the analyser.

(*ii*) $a \sin \theta$ is perpendicular to the plane of transmission of the analyser, *i.e.*, along OA and OB respectively as shown in Fig. 11.18. Only $a \cos \theta$ component is transmitted through the analyser. Therefore, the intensity of light emerging from the analyser is given by

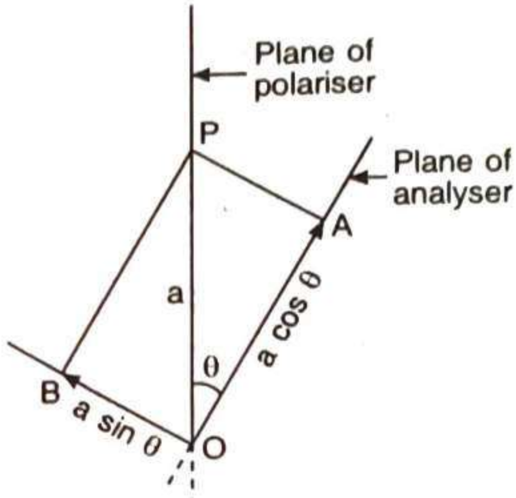


Fig. 11.18.

$$I_0 = k\,(a \cos \theta)^2 = k\,a^2 \cos^2 \theta$$

$$= I_0 \cos^2 \theta \qquad \ldots(11.5)$$

where $I_0 = k\,a^2$ is the intensity of the incident plane polarized light and k is a constant of proportionality. This is, of course, one half of the intensity of unpolarized light striking the polarizer, provided we ignore the losses of light by absorption in passing through it.

Thus, from eqn. (11.5), we have

$$I \propto \cos^2 \theta$$

which is Law of Molus.

It is due to the fact that vibrations are randomly distributed in all directions perpendicular to the direction of propagation and therefore for determining the intensity of transmitted light, the average value of $\cos^2 \theta$ (which is equal to $\frac{1}{2}$) is to be taken in eqn. (11.5).

Special cases

(*i*) If the polarizer and the analyser are parallel,

i.e. $\theta = 0$ or $180°$, then

$$I_0 = I_0 \qquad (\because \cos \theta = 1)$$

Thus, when polarizer and analyser are parallel to each other, the intensity of light transmitted by analyser is equal to that of incident on it.

(*ii*) If the polarizer and the analyser are perpendicular to each other,

i.e., $\theta = 90°$, then

$$I_\theta = 0.$$

Thus, when palarizer and analyser are perpendicular to each other, the intensity of light transmitted by analyser it zero.

Example 3. *Unpolarized light is incident on two plarising sheets placed one on top of the other. What must be the angle between the characteristic directions of the sheets if the intensity of the transmitted light (i) one-third the maximum intensity of the transmitted beam, (ii) one-third the intensity of the incident beam? Assume that the sheet is ideal, i.e., it reduces the intensity of upolarized light by exactly 50%.*

Solution. (*i*) Suppose I_0 be the intensity of unpolarised light. Then the intensity of the (polarized) ray transmitted by the first sheet would be

$$I = \frac{1}{2} I_0 \qquad \ldots(i)$$

Thus, I is the "maximum intensity" transmitted by the second sheet which will be the case when the sheets are parallel.

If θ be the angle between the characteristic directions of two sheets, then by Malus Law, the intensity transmitted by the second sheet is given by

$$I_\theta = I \cos^2 \theta \qquad \ldots(ii)$$

Now, according to question $I_\theta = \frac{1}{3} I$, we have

$$\frac{1}{3} I = I \cos^2 \theta$$

or $$\frac{1}{3} = \cos^2 \theta \text{ or } \cos \theta = \pm \frac{1}{\sqrt{3}}$$

or $$\theta = \cos^{-1} \pm \left(\frac{1}{\sqrt{3}}\right) = \pm 55°$$

From eqns. (*i*) and (*ii*), we have

$$I_\theta = \frac{1}{2} I_0 \cos^2 \theta$$

Now, according to second part of the question

$$I_\theta = \frac{1}{3} I_0.$$

we have

$$\frac{1}{3} I_0 = \frac{1}{2} I_0 \cos^2 \theta \qquad \text{or} \qquad \cos^2 \theta = \frac{2}{3}$$

or $$\cos \theta = \pm \sqrt{\frac{2}{3}} \qquad \text{or} \qquad \theta = \cos^{-1}\left(\pm \sqrt{\frac{2}{3}}\right) = \pm 35°$$

11.7. DOUBLE REFRACTION

In the study of refraction we have been considering isotropic medium which has similar properties in all directions. When light is incident on such a medium, refraction takes place obeying **Snell's Law** and refraction takes place in one direction. However, Erasmus, a Danish scientist, discovered in 1669, that a crystal of calcite (Ice-land spar, *i.e.*, a transparent crystalline form of calcium carbonate) is anisotropic, *i.e.*, possesses different properties in different directions.

A ray of an ordinary unpolarized light when incident on a calcite or quartz crystal, splits up into two refracted rays in place of the usual one as in glass. The crystals having this property are said to be *doubly refracting* and the phenomenon is called **double refraction.** Mica, sugar, topaz are some other examples of doubling refracting crystels.

To illustrate this phenomenon, allow a narrow beam of light from a point source to pass through a crystal of calcite, two images will be obtained on the screen as shown in Fig. 11.19.

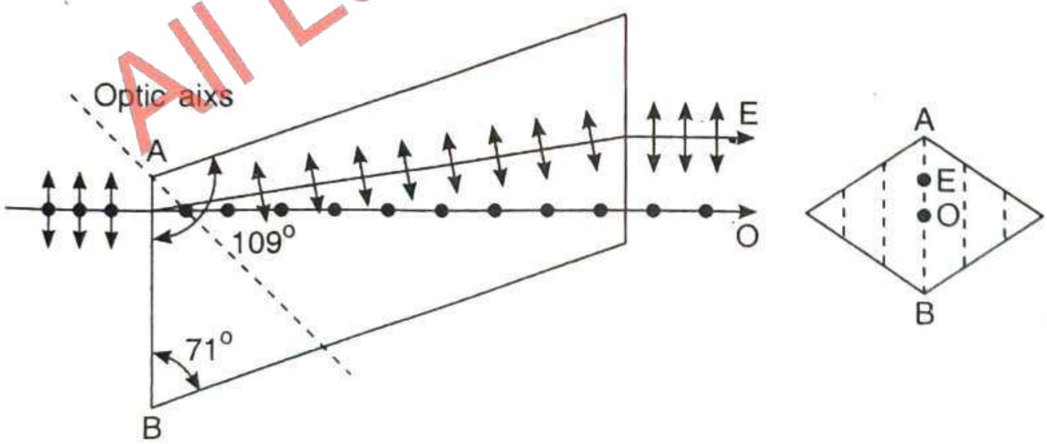


Fig. 11.19.

The phenomenon of double refraction can also be illustrated in a simple manner by putting a black dot on a sheet of paper and viewing it through a calcite crystal. Two images of the dot are seen (Fig. 11.20). On rotating the crystal about incident ray as axis, one of the two images remains stationary and is known as *ordinary image*. The second image rotates round the first and is called *extra-ordinary image*. The refracted rays which produce the ordinary image are called ordinary rays because they obey the ordinary laws of refraction, while the refracted rays which produce extra-ordinary image are called extra-ordinary rays as they do not obey the ordinary laws of refraction. Since the two opposite faces of a calcite crystal are always parallel so that the two refracted rays, ordinary and extra-ordinary, emerge parallel to the incident beam but relatively displaced by a distance proportional to thickness of the crystal. For normal incidence the ordinary ray will pass straight through the crystal without deviation whereas the extra-ordinary ray will be refracted at some angle and will come out parallel to and displaced from the incident ray. As stated above, if we rotate the crystal about the incident beam as axis, the *E*-ray rotates about the fixed *O*-ray, the line joining *O* and *E* images always lies along a line parallel to the shorter diagonal of the emergent crystal face.

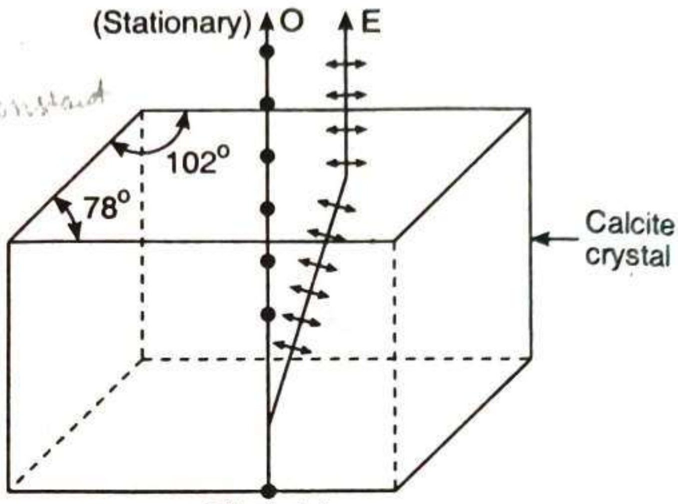


Fig. 11.20

Polarization of Rays. Both the ordinary and extra-ordinary rays obtained by double refraction are plane polarized in perpendicular planes, the vibrations of ordinary ray being normal to the plane of paper while those of the extra-ordinary take place in the plane of paper.

This can be verified by viewing *O* and *E* images through a tourmaline plate. As the tourmaline plate is rotated the intensities of both the images undergo a change. If the intensity of the *O* image increases, then that of extra-ordinary image decreases and *vice versa*. So to say that if in one position of tourmaline, the *O*-image has maximum intensity while the E-image disappears, and on rotating the tourmaline plate through 90°, from this position, the intensity of E-image becomes maximum while the O-image disappears. *Hence the rays by which the O and E images are seen must be polarized.*

Ordinary and extra-ordinary refractive indices. The refractive index for ordinary light is known as ordinary refractive index as denoted by μ_0 and is constant. Its value for a calcite crystal for sodium light is 1.658.

The refractive index for the extra-ordinary ray is known as the extra-ordinary refractive index and is denoted by μ_e. Its value is not constant but varies with the direction of the incident ray. For calcite its value varies between 1.486 and 1.658 for sodium light. Since

$$\mu = \frac{c}{\upsilon} = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}} \quad \ldots(11.6)$$

so it is clear that velocity of ordinary light in a crystal is same in all directions but the velocity of extra-ordinary ray is different in different directions.

11.8. CALCITE CRYSTAL-OPTIC AXIS-PRINCIPAL SECTIONS AND PRINCIPAL PLANE (*i.e.,* Geometry of calcite crystal)

A *calcite crystal,* also known as Ice-land spar ($CaCO_3$) is a colourless crystal transparent to visible as well as to ultraviolet light. It occurs in nature in different forms, all of which readily break up into simple rhombohedrons as shown in Fig. 11.21. Each of the six faces of the crystal is a parallelogram whose angles are 78° and 102° nearly. At the two diametrically opposite corners such as A and B as shown in Fig. 11.21, the three angles of the faces meeting there are obtuse. These corners are called *blunt* corners. At each of the remaining six corners one of the angles is obtuse whereas the other two are acute.

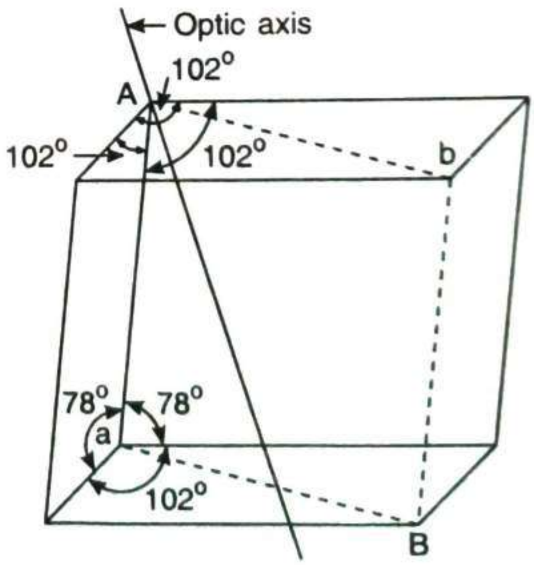


Fig. 11.21. Calcite crystal.

A line passing through any one of the blunt corners (A or B)

and equally inclined to the three edges meeting there, represents the direction of the *optic axis* (Fig. 11.21) and any other line parallel to it is an identically equivalent axis in all respects.

If the rhombohedron is so cut that all its edges are equal, then the line joining *A* and *B*, the opposite blunt corners or any line parallel to it gives the direction of optic axis. It may be clearly noted that

(*i*) Optic axis is a direction and not a particular line.

(*ii*) No separation of a ray into *O* and *E* occurs, if the ray passes along this axis, *i.e.*, velocity of ordinary and the extra-ordinary rays along this axis is the same. *So, optic axsis is a direction through the crystal along which there is no double refraction.*

(*iii*) Crystal is symmetrical about this axis.

Principal plane or Principal section. A plane containing the optic axis and perpendicular to a pair of opposite faces of the crystal is called *principal section* of the crystal for that pair of faces. In the above Fig. 11.21, A*a* B*b* is the principal plane for the side faces of the crystal. Thus, there are three principal sections passing through any point inside the crystal, one corresponding to each pair of opposite faces. A principal section always cuts the surfaces of the calcite crystal in a parallelogram having angles 71° and 109° as shown in Fig. 11.22(*a*). An end view of principal section perpendicular to a pair of opposite faces of the crystal cuts these faces in a line parallel to the shorter diagonal of these faces. Fig. 11.22(*b*) represents a face of the crystal, in which the dotted line A*a* represents the end view of the principal section shown in Fig. 11.22 (*a*). Other parallel lines represent the end views of the other principal sections parallel to that in Fig. 11.22(*a*).

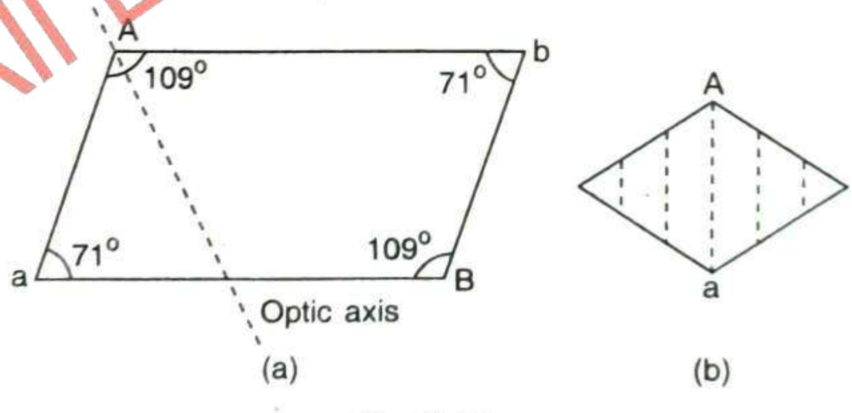


Fig. 11.22.

It is found that the ordinany ray is polarized in the principal plane and hence its displacements (shown by dots) are perpendicular to the principal plane. The extra-ordinary ray is polarized perpendicular to the principal plane with its vibrations (shown by arrows) in the principal plane. Thus, plane polarized light can be obtained by double refraction.

11.9. TYPES OF CRYSTALS

From the standpoint of physical optics, crystals are classified as either *uniaxial* or *biaxial*.

In uniaxial crystals such as calcite, quartz, tourmaline and nitrate of soda, there is a single direction known as optic axis in which all waves are transmitted with one uniform velocity while in any other direction, there are two velocities of transmission. Thus there is no separation of rays when light travels along this direction.

In biaxial crystals such as borax, mica, topaz, aragonite and selenite, there are two such directions for one particular velocity, *i.e.*, these crystals have two optic axes.

We shall confine ourselves to the former class of crystals which is a special case of the latter class. It may be noted that in optical instruments mainly calcite and quartz are used which are uni-axial crystals.

11.10. POLARIZATION BY DOUBLE REFRACTION

When a ray of unpolarized light is passed through a double refracting crystal like calcite it splits into O and E rays. As said above, both these beams are plane polarized having vibrations perpendicular to each other. Thus, polarization of light by double refraction can be demonstrated by sending a beam of light normally through a pair of calcite crystals and rotating the second crystal about incident ray as axis. The following phenomenon is observed :

(*i*) When the principal planes of the two crystals are parallel to each other, two images E_1 and O_1 are seen [Fig. 11.23(*a*)]. The O-ray from the first crystal passes undeviated through the second crystal and emerges as O_1-ray. E-ray from the first passes through the second crystal along a path which is parallel to that inside the first and emerges out as E_1-ray. Hence, two resultant beams or images O_1 and E_1 are separated by a distance equal to the sum of the displacements produced by each crystal, when used separately.

(*ii*) When the second crystal is rotated about the incident ray as axis, each of the two rays O and E from first crystal is split into two parts, giving rise to four images. Thus, besides O_1 and E_1, two new

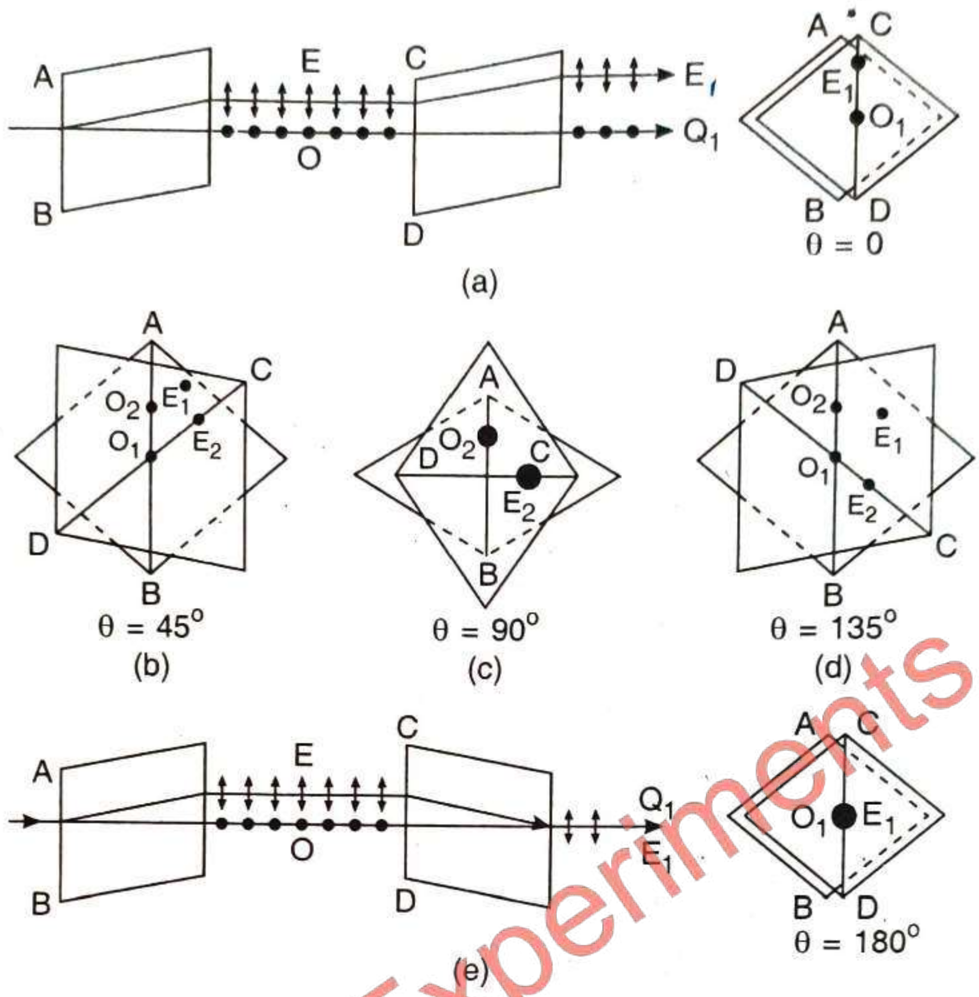


Fig. 11.23.

images O_2 and E_2, appear. As the rotation of second crystal is continued, O_1 and O_2 remain stationary while E_1 and E_2 rotate around O_1 and O_2 respectively and the intensity of O_1 and E_1 goes on decreasing while that of O_2 and E_2 increasing. When the principal section of the second crystal makes an angle of 45° with that of the first, the four images of equal intensity are seen. [Fig. 11.23(*b*)]

(*iii*) At 90° rotation, we again have two images. The original images O_1 and E_1 vanish and new images O_2 and E_2 acquires more intensity. [Fig. 11.23(*c*)]

(*iv*) On further rotation, original images O_1 and E_1 reappear and start gaining intensity while O_2 and E_2 start fading. At 135°, the four images are once again equally intense. [Fig. 11.23 (*d*)]

(*v*) At 180° rotation, the principal planes of the two crystals are once again parallel to each other, but having optic axis oriented in opposite directions. The images O_2 and E_2 vanish while O_1 and E_1 come together into one single image in the centre provided the crystals are of equal thickness.

Explanation. Suppose AB and CD represent the principal sections of the first and second crystals respectively, inclined at an angle θ with each other as shown in Fig. 11.24.

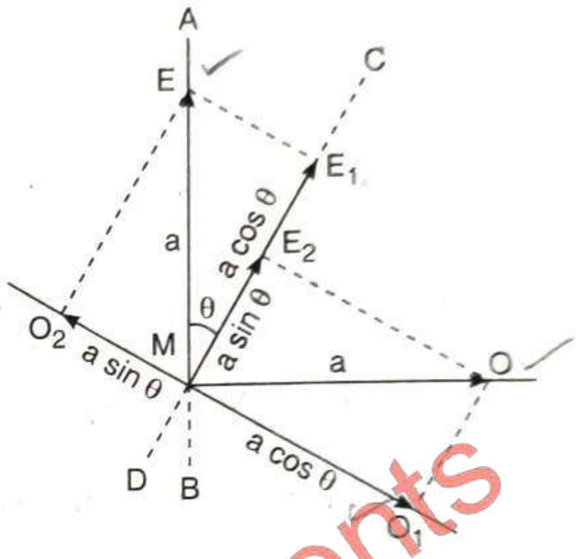


Fig. 11.24

Ordinary light on entering the first crystal is broken into two plane polarized beams, one O-ray vibrating perpendicular to the principal section AB and the other E-ray vibrating in the principal section AB. Let 'a' be amplitude of each ray, represented by MO and ME. The O-ray on entering the second crystal is further split up into an ordinary ray of amplitude $a \cos\theta$ perpendicular to the principal section CD of the second crystal and an extra-ordinary ray of amplitude $a \sin\theta$ in the principal section. These amplitudes are represented by MO_1 and ME_2 respectively. Similarly, the E-ray on entering the second crystal is split up into an extra-ordinary ray of amplitude $ME_1 = a\cos\theta$ in the principal section CD and an ordinary ray of amplitude $MO_2 = a\sin\theta$ perpendicular to CD. The splitting of O-ray and E-ray on entering the second crystal is because of the fact that the second crystal like the first transmits light vibrating in its principal section along one path and that vibrating at right angles along the other path. Now, the intensity of O_1 and E_1 is $a^2\cos^2\theta$ while that of O_2 and E_2 is $a^2\sin^2\theta$.

At $\theta = 0$ and $\theta = 180°$,

We have

$$\cos^2\theta = 1 \text{ and } \sin^2\theta = 0$$

Hence, O_1 and E_1 have a maximum intensity of a^2, while O_2 and E_2 disappear.

At $\theta = 45°$ or $\theta = 135°$, we have

$$\cos^2\theta = \frac{1}{2} \text{ and } \sin^2\theta = \frac{1}{2}$$

Hence, θ_1 and E_1 as well as Q_2 and E_2 have the same insensity $a^2/2$

At $\theta = 90°$ we have

$$\cos^2\theta = 0 \text{ and } \sin^2\theta = 1.$$

Hence, O_1 and E_1 disappear while O_2 and E_2 have maximum intensity equal to a^2.

Thus, all the above observations are theoretically explained. In short, at all positions the sum of the two components $a^2\sin^2\theta + a^2\cos^2\theta$ is just equal to a^2, the intensity of the incident beam.

Thus, plane polarized light may be gainfully obtained by double refraction.

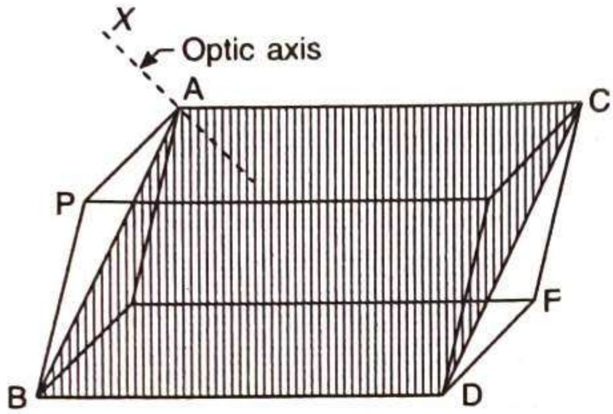
11.11. NICOL PRISM

It is an optical device made from calcite crystal and is used in many optical intruments for producing and analysing plane polarized light. It derives its name from its invertor William Nicol who first designed it in 1828.

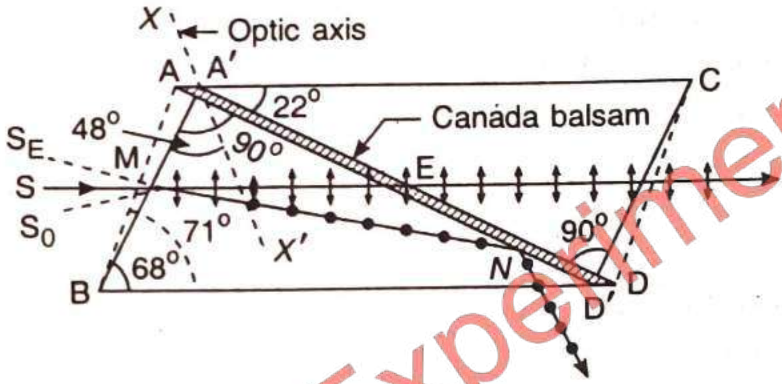
It is based upon the fact that ordinary and extra-ordinary rays in calcite possess unequal refractive index. It gives an intense beam of plane polarized light which cannot be obtained by refraction through tourmaline or by reflection methods.

Construction. A Nicol prism is made by taking a rhombohedron of calcite whose length is three times its thickness with the optic axis in plane *ABCD* [Fig. 11.25(*a*)]. Therefore *ABCD* represents a principal section of the crystal.

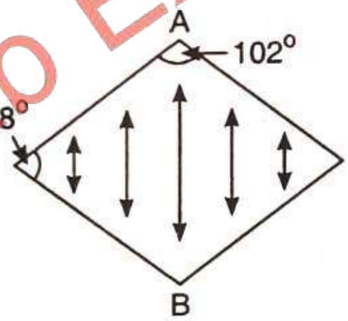
The end faces *BA* and *CD* of this crystal are ground away until they make an angle of 68° instead of 71° with the longitudinal edges, so that these two faces change into *BA′* and *CD′* as shown in Fig [11.25(*b*)]. The resulting crystal is then cut apart into two pieces, along a plane *A′D′* [Fig. 11.25 (*b*)], passing through the blunt corners and perpendicular to both the principal plane and the end faces so that *A′D′* makes an angle of 90° with ends *A′B* and *CD′*. The two cut surfaces are ground and polished optically flat. They are then cemented together with canada balsam which is a clear transparent substance having refractive index midway between the refractive indices of the O and E-rays. For example, sodium yellow light



(a) Calcite rhomb, ABCD is a principal section of calcite.



(b) Nicol prism



End view of nicol showing direction of displacement in emergent transmitted light

Fig. 11.25.

$$\mu_0 = 1.658, \mu_e = 1.486 \text{ and } \mu_{CB} = 1.55.$$

The two end faces of the prism are kept open while its sides are coated with lamp black and kept covered by a brass tube.

Action. If a ray SM of unpolarized light is incident nearly parallel to BD' in the plane of paper on the face $A'B$, it suffers double refraction and gives rise to E-ray ME and O-ray MN. Both the rays are plane polarized. The O-ray has vibrations perpendicular to the principal section of the crystal while the E-ray has vibrations in the principal section, *i.e.*, in the plane of paper. The ordinary ray is travelling from a denser medium (calcite) to an optically rarer medium (canada balsam) and will suffer total internal reflection provided the angle of incidence at the canada balsam layer is greater than the critical angle for the two media, calcite and balsam, *i.e.*, greater than $\sin^{-1}\left(\frac{1.55}{1.658}\right) = 69°$. This totally reflected ray is then absorbed by the blackened side of the crystal. the extra-ordinary ray under the same conditions would be transmitted. The angle of incidence on canada balsam layer depends on the angle which the edge BA' makes with the blunt edge BD and the ratio of the breadth to the length of the crystal. It is for this reason that the natural angle 71° was reduced to 68° and that a crystal of length about three times its breadth was taken. This enables the ordinary ray to be incident on balsam layer at an angle greater than the critical angle and suffers total internal reflection. Thus, we are able to obtain a beam of plane polarized light with vibrations in principal plane, the direction of vibrations in the emergent plane polarised light is parallel to the shorter diagonal AC of the end face of the crystal [Fig. 11.25(c)].

Limitation. The Nicol prism works only when the incident beam is slightly convergent or slightly divergent. If the incident ray makes an angle much smaller than $\angle BMS$ with the face $A'B$, the ordinary ray will strike the balsam layer at an angle less than the critical angle 69° and hence will be transmitted. Therefore, the light emerging from the nicol will not be plane-polarized.

If the incident ray makes an angle greater than $\angle BMS$ the E-ray will become more and more parallel to the optic axis XX' so that its refractive index will increase and will become nearly equal to that of calcite for O-ray*. Then the E-ray will also suffer total internal reflection at the calcite balsam surface and no light will emerge out of the Nicol prism. *Thus a Nicol prism, therefore, cannot be used for highly convergent or divergent beams.* With the dimensions chosen, the angle between the extreme rays of the incoming beam is limited

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*It is because for E-ray that the refractive index of calcite is different in different directions of the crystal. Its value is minimum = 1.486 when the ray is travelling normal to optic axis and along optic axis it increases to 1.658 same as for O-ray. For intermediate angles, its values lies between the two limits 1.486 and 1.658.

to about 28°, *i.e.*, the semi-vertical angle of the cone of incident light $S_E M S_O$ should not exceed 14°.

Uses. The nicol prism can be used both as polarizer and as an analyser.

When an unpolarised ray of light is incident on Nicol prism *P* [Fig. 11.26(*a*)], the ray emerging from *P* is plane polarized with vibrations in the principal section of *P*. If this ray is made to fall on second Nicol prism *A*, whose principal section is parallel to that of *P*, its vibrations will be in the principal section of *A*. Hence the ray will behave as *E*-ray in the prism *A* and will be completely transmitted and the intensity of the emergent light will be maximum.

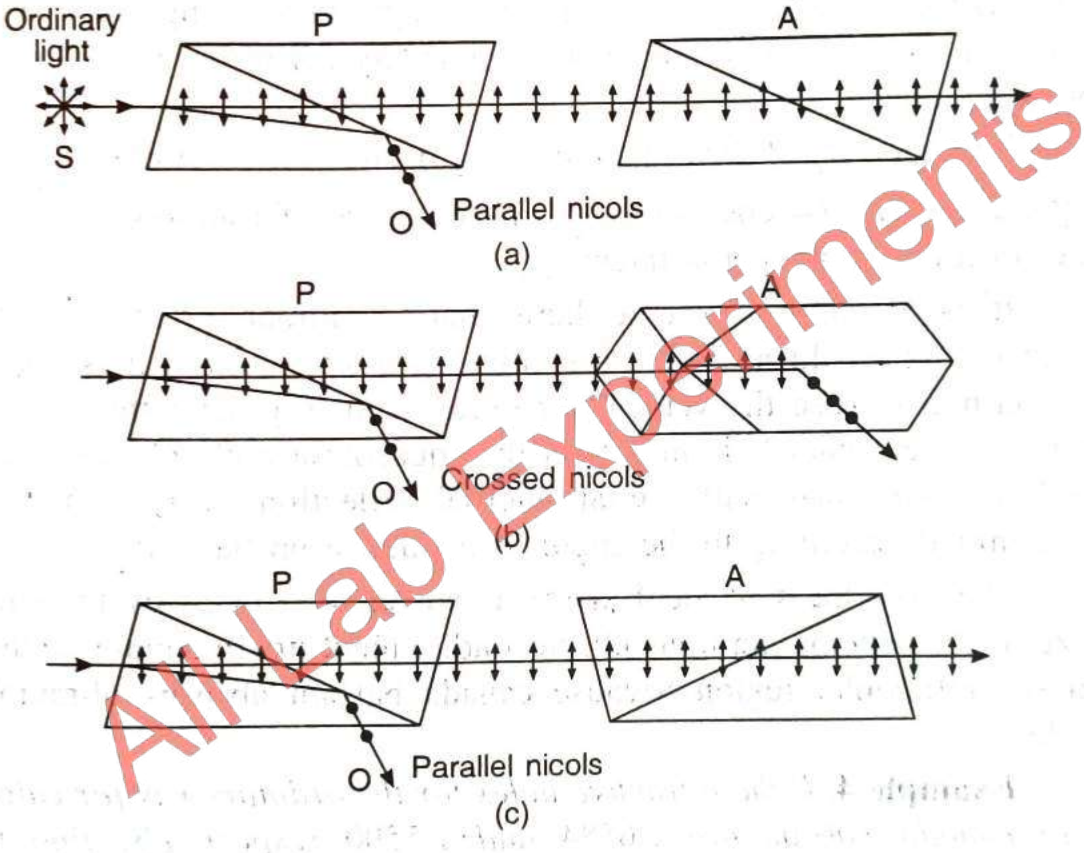

Fig. 11.26.

If one of the nicols is rotated, the intensity of the transmitted beam becomes less and finally no light is transmitted. This will happen when the principal plane of Nicol *A* becomes perpendicular to that of *P* [Fig. 11.26(*b*)], the vibrations in the plane polarized ray incident on A will be normal to the principal section of *A*. It will thus have no component in the principal section of *A*. So the ray will behave as *O*-ray inside *A* and will be lost by total reflection at the balsam layer.

Hence, no light will emerge from A. In this postion, the two nicols are said to be **crossed.**

If nicol A is further rotated, to have its principal section again parallel to that of P [Fig. 11.26 (c)], the intensity of emergent light will again be maximum.

In positions (a) and (c), the two nicols where the angle between the principal sections of the two prisms are 0° and 180° respectively, are referred as 'parallel nicols'.

The first nicol P polarizes the light and is called the **polarizer** and the second nicol A analyses the polarized light and is called **analyser.**

The above facts can be used for analysing the plane polarized light. If the given light on examining through the rotating nicol shows variation of intensity with minimum intensity zero, the given light is plane polarized.

The intensity of the beam emerging from the nicol A obeys cosine square law, *i.e.*, $I \propto \cos^2 \theta$ where θ is the angle of rotation of A from the position of maximum intensity.

It is important to note here that a similar prism prepared from quartz would not serve a similar purpose although it is doubly refracting, because the velocity of O-ray in it is greater than that of E-ray. Since canada balsam is rarer than quartz for both, O and E-rays, both the rays may suffer total internal reflection or both may be transmitted according to the angle of incidence on balsam.

The usefulness of nicol prism, however, is limited by the small size of the beam it can transmit. Secondly, nicol prism cannot be used in the ultraviolet region because canada balsam absorbs ultraviolet light.

Example 4. *If the refractive index for the ordinary ray for calcite and canada balsam are* 1.6584 *and* 1.5500 *respectively, find the maximum allowable angle of incidence with the axis of symmetry of the nicol so that ordinary ray is still quenched.*

Solution. The critical angle for the two media, calcite and balsam is

$$= \sin^{-1}\left(\frac{1.5500}{1.6584}\right) = 69°$$

If the angle of incidence on the balsam surface is > 69°, the

O-ray will be quenched. Hence, the maximum allowable angle of incidence with the symmetry axis of the nicol is

$$= 90° - 69° = 21°.$$

Example 5. *Two nicols are crossed to each other. Now one of them is rotated through* 60°. *What percentage of incident unpolarized light will pass through the system?*

Solution. Initially the nicols are at 90°. When one is rotated through 60°, the angle between their principal sections becomes 30°.

Let I_0 be the intensity of the incident beam. The incident beam breaks up into an E-ray and an O-ray by the first nicol. The O-ray is quenched by reflection, while the E-ray is transmitted. Thus, the transmission of a nicol for incident unpolarized light is 50% (neglecting absorption losses), *i.e.*, the intensity of the transmitted E-ray is $I_0/2$.

The plane polarized E-ray on entering the second nicol again breaks us into an E' component with intensity $\frac{I_0}{2}\cos^2\theta$ and an O'-component with intensity $\frac{I_0}{2}\sin^2\theta$ where θ is the angle between the principal plane of the two nicols. Only the E'-component is transmitted. Thus, the intensity I_0 of the light transmitted by the second nicol is

$$I_\theta = I_0 \frac{1}{2}\cos^2\theta \qquad \text{(by Malus Law)}$$

Here $\theta = 30°$

$$\therefore \quad \frac{I_\theta}{I_0} = \frac{\cos^2 30°}{2} = \frac{(\sqrt{3}/2)^2}{2} = \frac{3}{8}$$

$$\therefore \quad \text{Percentage of transmission} = \frac{I_\theta}{I_0} \times 100$$

$$= \frac{3}{8} \times 1.00 = 37.5\%$$

11.12. NEGATIVE AND POSITIVE CRYSTALS

Uniaxial crystals are of two types : *negative and positive*.

Negative crystals : Calcite and tourmaline are the examples of this type of crystals. The important properties of such crystals are:

(*i*) The velocity of ordinary ray is constant in all directions.

(*ii*) The velocity of extra-ordinary ray varies with the direction. It is minimum and equal to the velocity of the ordinary ray along the optic axis. *It is maximum in a direction perpendicular to the optic axis.*

(*iii*) The refractive index for *E*-ray is less than that for *O*-ray (*i.e.*, $\mu_E < \mu_0$) in all directions except along optic axis where $\mu_E = \mu_0$.

Positive crystals : Quartz and ice are the examples of such crystals. The important properties of these crystals are :

(*i*) The velocity of ordinary ray is constant in all directions.

(*ii*) The velocity of extra-ordinary ray varies with the direction. It is maximum and is equal to velocity of ordinary light along optic axis. It is minimum in a direction perpendicular to the optic axis.

(*iii*) The refractive index for the *E*-ray is more than the refractive index of the ordinary ray, *i.e.*, ($\mu_E > \mu_0$) in all directons except along optic axis where $\mu_E = \mu_0$.

Example 6. *Plane polarized light is incident normally on a plate of doubly refracting uniaxial crystal with faces cut parallel to the optic axis. Calculate the ratio of intensities of E and O-rays if the light is incident with vibrations making an angle of 30° with the face of the crystal. Given* $\lambda = 5993$ Å, $\mu_e = 1.5532$ *and* $\mu_0 = 1.5442$.

Solution. As $\mu_e > \mu_0$, so the given crystal is positive. Now, let '*a*' be the amplitude of incident wave and θ the angle which the incident vibrations make with the optic axis. On entering the crystal the light splits up into two components, an extra-ordinary component $a \cos\theta$ having vibrations parallel to the optic axis and an ordinary component $a \sin\theta$ having vibrations perpendicular to the optic axis. Hence, the ratio of intensities of the extra-ordinary and ordinary rays is

$$\frac{I_E}{I_0} = \frac{a \cos^2\theta}{a \sin^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$$

Here θ = 30°

$$\therefore \qquad \frac{I_E}{I_0} = \frac{\cos^2 30°}{\cos^2 30°} = \frac{(\sqrt{3/2})^2}{\left(\frac{1}{2}\right)^2} \frac{3/4}{1/4} = \frac{3}{1}$$

11.15. ELLIPTICALLY AND CIRCULARLY POLARIZED LIGHTS

In article 11.13, from Figs. 11.29 (*b*) and 11.30 (*b*), we found that when unpolarized light was incident normally on a thin plate of a uniaxial crystal cut with faces parallel to the optic axis, both ordinary and extra-ordinary rays travel along the same path but with different velocities.* Similarly when a beam of plane polarized light (with vibrations making an angle with optic axis) is made to fall normally on such a crystal cut parallel to the optic axis, it is split into ordinary and extra-ordinary components, which travel through the crystal in the same direction but with different velocities in accordance with Huygen's construction of double refraction (Fig. 11.33).

If a plane polarized and monochromatic light of wavelength λ and amplitude A be incident normally on a uniaxial double refracting crystal of calcite with face cut parallel to the optic axis and the plane polarized incident ray be such that the linear vibrations in it be along

*In case of negative crystal like calcite the value of v_e (velocity of E-ray) was greater than as compared to the value v_0 (velocity of ordinary ray). In

PQ making an angle θ with the optic axis, then on entering the crystal, the incident light will be split into two components, ordinary and extra-ordinary. The ordinary component will have vibrations along *PO* and extra-ordinary along *PE*. the amplitudes of vibration of *E* and *O* components are $A \cos\theta$ and $A \sin\theta$ respectively along the optic axis of the crystal and perpendicular to it (Fig. 11.34). According to Huygen's construction of double refraction, both the components travel in the same direction that is perpendicular to the plane of paper (Fig. 11.33), but with different speeds depending on the nature of the crystal. In calcite *E*-ray travels faster than *O*-ray. Hence, it should be clearly born in mind that it is because of unequal velocities of the two components that although on entering the crystal they have the same phase, yet on emergence their phases are different as the two rays will have a path difference depending on the thickness of the crystals*. The emergent light will, therefore, consist of two simple harmonic vibrations in two mutually perpendicular planes having same period but different amplitudes and phases. They can be compounded into a single motion **linear**, **circular** or **elliptical** depending on the phase difference and magnitude of angle θ.

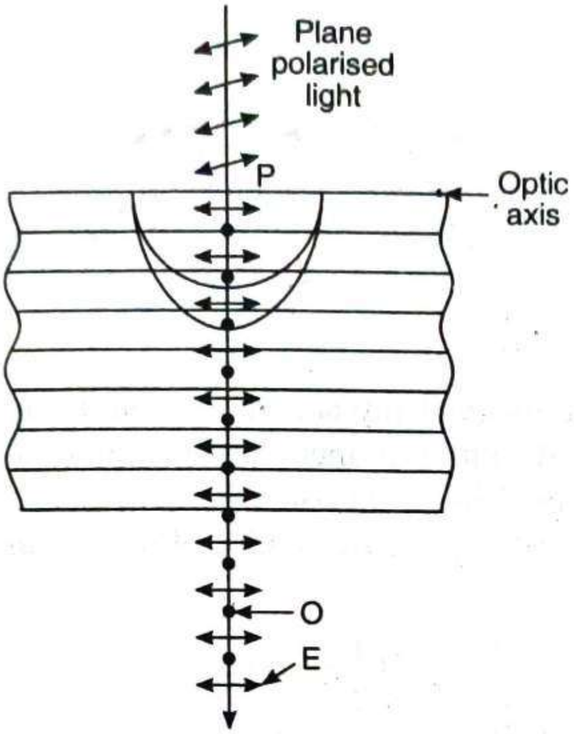

Fig. 11.33.

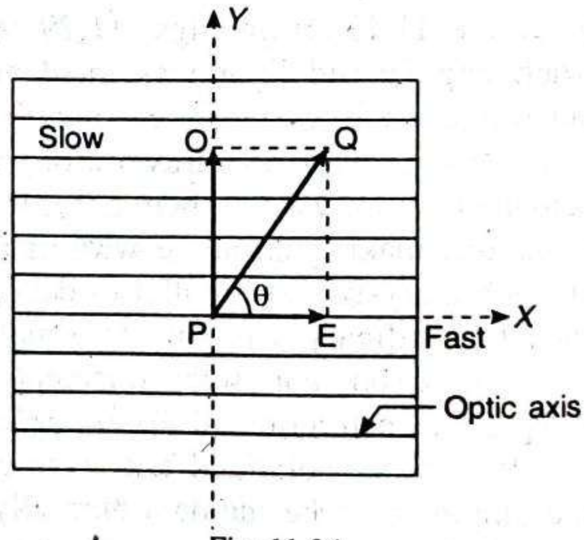

Fig. 11.34.

*Since $v_e \geq v_0$ for calcite, so length of path traversed in crystal by *E*-component is greater than that of *O*-component. Thus the ordinary wave falls behind the extra-ordinary.

Theory

If the phase difference introduced between the two components after travelling a certain thickness of the crystal be δ, then by a convenient choice of the origin of time, their displacements can be written as

$$E\text{-ray} \quad x = A \cos\theta \sin(\omega t + \delta)$$

$$O\text{-ray} \quad y = A \sin\theta \sin \omega t$$

where ω is the angular frequency of vibration of either particles.

Putting $A \cos\theta = a$, $A \sin\theta = b$, we have

$$x = a \sin(\omega t + \delta) \qquad \text{...(11.7)}$$

$$y = b \sin(\omega t) \qquad \text{...(11.8)}$$

To obtain the path of resultant vibration we eliminate ωt between the above two equations.

From eqn. (11.7), we have

$$\frac{x}{a} = \sin(\omega t + \delta) = \sin \omega t \cos\delta + \cos \omega t \sin\delta \qquad \text{...(11.9)}$$

and from eqn. (11.8), we have

$$\frac{y}{b} = \sin \omega t$$

$$\therefore \qquad \cos \omega t = \sqrt{1 - \sin^2 \omega t} = \sqrt{1 - \frac{y^2}{d^2}}$$

Substituting for sin ωt and cos ωt in eqn. (11.9), we get

$$\frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{1 - \frac{y^2}{b^2}}\,\sin\delta$$

$$\frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{1 - \frac{y^2}{b^2}}\,\sin\delta$$

Squaring both sides, we have

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\delta - \frac{2xy}{ab}\cos\delta = \left(1 - \frac{y^2}{b^2}\right)\sin^2\delta$$

or

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}(\cos^2\delta + \sin^2\delta) - \frac{2xy}{ab}\cos\delta - \sin^2\delta = 0$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta \qquad \text{...(11.10)}$$

which is the general equation of an ellipse whose major and minor axes are inclined at certain angle, depending on their relative phases. Hence, the light emerging from the crystal plate, in general, is elliptically polarized.

Special cases. (*i*) If the thickness of the crystal plate is such that $\delta = 0$ or $2n\pi$ or path diffrence is λ or $n\lambda$, where n is an integer then the equation of resultant vibration is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

or

$$\pm\left(\frac{x}{a} - \frac{y}{b}\right) = 0$$

or

$$y = +\left(\frac{b}{a}\right)x$$

which is the equation of a straight line passing through the origin and having a slope $= b/a$ and which is the same as that of incident plane polarized ray. This means that the emergent light from the crystal is plane polarized in the same plane as the incident plane polarized beam. The crystal in such cases has thickness such that it introduces a phase difference 0 or $2n\pi$ between *O*- and *E*-component and so the crystal plate is then **whole or full wave plate.**

(*ii*) If the thickness of the crystal plate is such that $\delta = \pi$ or $(2n+1)\,\pi$ or the path difference is $\frac{\lambda}{2}$ or $(2n+1)\frac{\lambda}{2}$, then the eqn. (11.10) of resultant motion reduces to

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$$

or

$$\frac{x}{a} + \frac{y}{b} = 0$$

or

$$y = -\left(\frac{b}{a}\right)x$$

which again represents a straight line passing through the origin but with a slop $= (-b/a)$. In this case the emergent beam will again be **plane polarized**, with the vibration direction making an angle $2\tan^{-1}(b/a) = 2\theta$ with that of incident light. The plate is then a **half wave plate.**

(*iii*) If the thickness of the plate is such that $\delta = \pi/2$ or $(2n+1)\frac{\pi}{2}$ or path difference is $\lambda/4$ or $(2n+1)\frac{\lambda}{4}$ and $a \neq b$ (*i.e.*, $\theta \neq 45°$) then the equation of resultant motion reduces to

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

which is the equation of an ellipse with its axes along x and y-directions. Hence, the emergent light is **elliptically** polarized, the axes of the ellipse being along and perpendicular to the optic axis. The plate is then a **quarter wave plate**.

If in this case, $\theta = 45°$, *i.e.*, incident vibrations make an angle of 45° with optic axis, then $a = b$, *i.e.*, the amplitudes of the ordinary and extra-ordinary components are equal, then eqn. (11.10) reduces to

$$x^2 + y^2 = a^2$$

which is the equation of a circle with radius a. Hence, in this case emergent beam is **circularly polarized**.

Thus, we see that the plan polarized and circularly polarized lights are the special cases of elliptically polarized light.

(*iv*) If the thickness of the plate is such that $\delta = \pi/4$, we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2}{ab}\frac{xy}{\sqrt{2}} = \frac{1}{2}$$

which represents an ellipse with major axis inclined in the positive direction of x-axis.

Thus, light is said to be **elliptically polarized** when the vibrations of the ether particles are elliptical having a constant period and takes place in a plane perpendicular to the direction of propagation. In this case the amplitude of the vibrations changs in magnitude as well as in direction.

And light is said to be circularly polarized when the vibrations of the ether particles are circular having a constant period and take place in a plane perpendicular to the direction of propagation. In this case the amplitude of vibrations remains constant but the direction changes only.

Fig. 11.35 shows the shape of resultant transmited vibrations for different values of the phase difference between two rectangular components : (*a*) when the angle between the original plane of vibration and optic axis is $\pi/4$ and (*b*) when this angle is less than $\pi/4$. If the angle is more than 45°, the effect will be to increase the vertical axis instead of diminishing it.

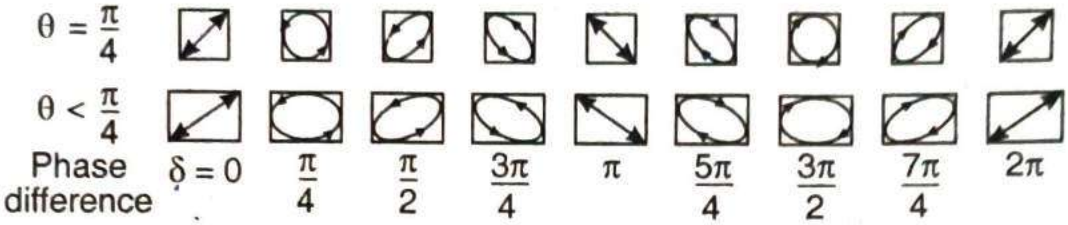

Fig. 11.35.

(*i*) For $0 < \delta < \pi$, the elliptical vibration is described counter-clockwise w.r.t. an observer towards whom the light travels and the light is said to be *right-handed elliptically polarized.*

(*ii*) For $\pi < \delta < 2\pi$, the ellipse is described in the clockwise direction and the light is said to be left-handed elliptically polarized.

(*iii*) For $\delta = \dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$, the ellipse will change into circle if $a = b$ or $\theta = 45^\circ$. The light in that case is said to be circularly polarized.

Distinction between elliptically and circularly polarized light

elliptically polarized light

When the vibrations of the ether particles are elliptical having a constant period and take place in a plane perpendicular to the direction of propagation (transverse plane) light is said to be elliptically polarized. In this case the amplitude of the vibrations changes in magnitude as well as in direction.

Circularly polarized light

When the vibrations of the ether particles are circular having a constant period and take place in the transverse plane, light is said to be circularly polarized. In this case the amplitude of vibration remains constant but the direction changes.

Example 7. *Calculate the thickness of a quarter-wave plate for light of wavelength 6000 Å, the refractive indices for ordinary and extra-ordinary rays being 1.544 and 1.553 respectively.*

(K.U. 1989 A, 95 A, 2003 A; M.D.U. 2000 A)

Solution. Let t be the required thickness of the plate.

Here, $\mu_0 = 1.544$, $\mu_e = 1.553$ and $\lambda = 6000 \times 10^{-8}$ cm.

Since $\mu_e > \mu_0$, the crystal is positive and the required thickness is given by

$$t = \frac{\lambda}{4(\mu_e - \mu_0)} = \frac{6000 \times 10^{-8}}{(1.553 - 1.544) \times 4}$$

$$= \frac{6 \times 10^{-5}}{4 \times 0.009} = \mathbf{16.6 \times 10^{-4} \text{ cm.}}$$

Example 8. *Calculate the thickness of double refracting crystal to introduce a path difference of* $\lambda/2$ *between O and E-rays when*

$\lambda = 5893$ Å, $\mu_0 = 1.5442$ *and* $\mu_e = 1.5533$. *(K.U. 1996 A)*

Solution. Let t be the required thickness of the plate. Here $\mu_0 = 1.5442$, $\mu_e = 1.5553$, $\lambda = 5893 \times 10^{-8}$ cm. Since $\mu_e > \mu_0$, the crystal is positive and the required thickness is given by

$$t = \frac{\lambda}{2(\mu_e - \mu_0)} = \frac{5893 \times 10^{-8}}{2(1.5533 - 1.5442)}$$

$$= \frac{5893 \times 10^{-8}}{2 \times .0091} = \mathbf{0.3238 \times 10^{-4} \text{ cm.}}$$

Example 9. *Plane polarized light is incident on a quartz plate cut parallel to the optic axis. Find the least thickness of the plate for which the ordinary and extra-ordinary ray combine to form plane polarized light on emergence.*

($\mu_0 = 1.5442$, $\mu_e = 1.5533$, $\lambda = 5893$ Å)

Solution. In this case the quartz plate must act as a half-wave ($\lambda/2$) plate. Thus, if t is the required thickness, then

$$t = \frac{\lambda}{2(\mu_e - \mu_0)} = \frac{5893 \times 10^{-8}}{2(1.5533 - 1.5442)} = \mathbf{0.0032 \text{ cm.}}$$

Example 10. *Calculate the thickness of a half-wave plate for light of wavelength* 5893 Å. *(Give* $\mu_0 = 1.658$ *and* $\mu_e = 1.486$*)*

(K.U. 1992 A)

Solution. The required thickness is given by

$$t = \frac{\lambda}{2(\mu_0 - \mu_e)}$$

$$= \frac{5893 \times 10^{-8}}{2(1.658 - 1.486)}$$

$$= \frac{5893 \times 10^{-8}}{2 \times 0.172} = \mathbf{1.713 \times 10^{-4} \text{ cm}}$$