

**Name of Paper** : Solid State Physics  
**Name of the Course** : B.Sc. (Hons.) Physics  
**Semester** : V  
**Duration** : 3 Hours  
**Maximum Marks** : 75

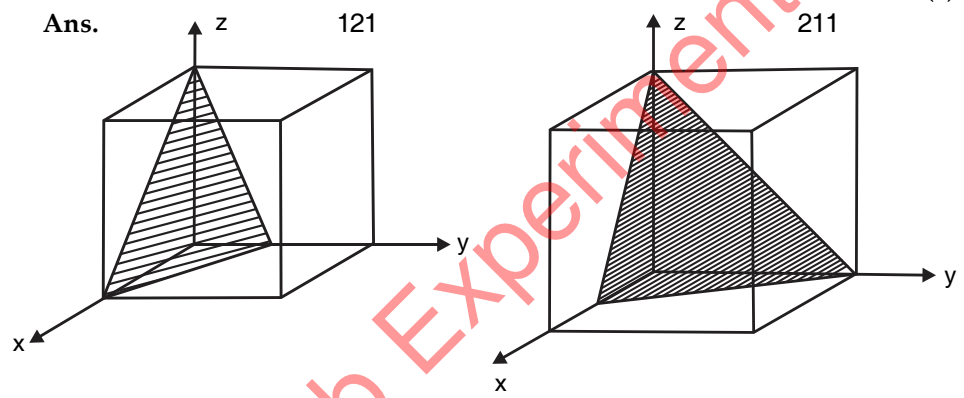
**Nov. / Dec. 2019**

*Question No. 1 with all its parts is compulsory.  
 Out of three parts of question Nos. 2 - 7 attempt any two parts.  
 Non-programmable scientific calculator is allowed.*

**Q. 1. Attempt all parts of this question.**

**(a) Draw the lattice planes (121) and (211) in a crystal with a cubic unit cell.**

**(2)**



**(b) What is an essential condition for the Bragg Diffraction to occur? Determine the maximum wavelength for which Bragg Diffraction can be observed from a crystal with an atomic separation of 0.2 nm.**

**(3)**

Ans. Essential condition for bragg diffraction to occur is

$$2d \sin\theta = n\lambda$$

$d \rightarrow$  Interplanar spacing  
 $\lambda \rightarrow$  Wavelength  
 $\theta \rightarrow$  Angle of incident ray with plan

$$(\sin\theta)_{\max} = 1$$

$$2d \geq n\lambda \quad d = 0.2 \text{ nm}$$

$$\frac{n\lambda}{2d} \leq 1$$

For  $n=1$ ,  $\lambda = \lambda_{\max}$

$$\lambda_{\max} = 2d$$

$$\lambda_{\max} = 2 \times 0.2 = 0.4 \text{ nm}$$

$$\lambda_{\max} = 4 \text{ \AA}$$

(c) What is the most important feature of dispersion curve that distinguishes diatomic lattice from a monoatomic lattice? State the reason. (2)

Ans. In monoatomic lattice the dispersion curve is sinusoidal with a period of  $2\pi/a$  in  $k$ -space

$$w = w_m \sin \left| \frac{ka}{2} \right|$$

In diatomic lattice dispersion relation is

$$w^2 = k \left( \frac{1}{M} + \frac{1}{m} \right) \pm k \left[ \left( \frac{1}{M} + \frac{1}{m} \right)^2 - \frac{4 \sin^2 ka}{Mm} \right]^{1/2}$$

Since, there is positive and negative term on right side, each  $k$  corresponds to two values of  $w$ . So, it has two branches, one is lower which comes from negative sign and one is upper which comes from positive sign lower one is acoustic and upper one is optical.

(d) How are metals, semiconductors and insulators differentiated on the basis of band theory? (3)

Ans. Find the expression in the text book.

(e) As number of domains in a ferromagnetic material increases, the total energy stored in the system decreases. Why do domains with single atom do not occur preferably? (3)

Ans. As the number of domains in a ferromagnetic material increases, total energy stored in the system decreases. A part of this energy is the energy taken by domain to maintain the boundary of domain walls. The energy required for the domain wall is proportional to the area of the wall. The number of domains keeps on increasing as long as the energy required for the formation of domain wall is available from the energy stored in the system. However after a certain point the energy required by domain wall is equal to the energy left in the system after this formation and more atoms do not occur preferably as the energy required for such a system is greater than energy available.

(f) A superconducting lead has a critical temperature of 7.26 K at zero magnetic field and a critical field of  $8 \times 10^5$  A/m at 0 K. Find the critical field at 5 K. (3)

Ans.  $T_c = 7.26$  K,  $H_c(0) = 8 \times 10^5$  A/m  
 $H_c(T) = ?$

$$H_c(T) = H_0(T) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]$$

$$= 8 \times 10^5 \left[ 1 - \left( \frac{5}{7.26} \right)^2 \right]$$

$$= 8 \times 10^5 [0.5376]$$

$$H_c(T) = 4.3 \times 10^5 \text{ A/m}$$

(g) The susceptibility of a paramagnetic material is  $1.2 \times 10^{-5}$  at 300K. Find the susceptibility at 400 K. (3)

Ans.  $\chi_{\text{para}} = \frac{C}{T}$  [C = Curie constant]

$$\chi \propto \frac{1}{T}$$

$$\frac{\chi_1}{\chi_2} = \frac{T_2}{T_1} \Rightarrow \chi_2 = \chi_1 \times \frac{T_1}{T_2}$$

$$\chi_2 = 1.2 \times 10^{-5} \times \frac{300}{400}$$

$$\chi_2 = 0.9 \times 10^{-5}$$

Q. 2. (a) What is the significance of reciprocal lattice as compared to normal lattice? Write the relation of the primitive translational vectors of the reciprocal lattice in terms of primitive translational vectors of the normal lattice. Prove that the reciprocal lattice vector  $G = ha^* + kb^* + lc^*$  is perpendicular to the  $(hkl)$  plane. (2, 2, 3)

Ans. Find the expression in the text book.

(b) What is the importance of Geometrical Structure Factor in the analysis of the crystal structure? Calculate it for BBC and illustrate diagrammatically the absence of (100) reflection for a BBC lattice. (2, 3, 2)

Ans. The scattering of electrons in the same atom is called atomic scattering factor and summation of this scattering from all atoms in the unit cell is called geometrical structure factor (F).

The atomic scattering factor is a measure of the efficiency of an atom in scattering x-rays. it is defined as the ratio of amplitude scattered by actual electron distribution in an atom to that scattered by one electron localized at a point.

For calculating the intensity of x-ray beam scattered by one unit cell in a particular direction where there is diffraction maximum it is necessary to sum to waves that arise from all atoms in unit cell.

If position of atom in unit cell is given by  $x$ ,  $y$  and  $z$ , the amplitude for given  $(h, k, l)$

$$F(h, k, l) = \sum_{i=1}^N f_i \exp\{2\pi j(hx_i + ky_i + lz_i)\}$$

$$I \propto |F(h, k, l)|$$

$$|F(h, k, l)|^2 = \left( \sum_{i=1}^N f_i \cos 2\pi j(hxi + kyi + lzi) \right)^2 +$$

$$\left( \sum_{i=1}^N f_i \sin 2\pi j(hxi + kyi + lzi) \right)^2$$

For BCC : Position of atom are  $(0, 0, 0)$  and  $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

$$x_1 = y_1 = z_1 = 0 \quad \text{and} \quad x_2 = y_2 = z_2 = \frac{1}{2}$$

$$|F|^2 = f_i^2 \left[ \cos 2\pi \cdot 0 + \cos 2\pi \left( \frac{h}{2} + \frac{k}{2} + \frac{l}{2} \right)^2 \right]$$

$$+ f_i^2 \left[ \sin 2\pi \cdot 0 + \sin 2\pi \left( \frac{h}{2} + \frac{k}{2} + \frac{l}{2} \right)^2 \right]$$

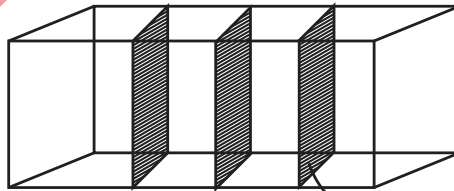
$$|F|^2 = f_i^2 \left[ \{1 + \cos \pi (h + k + l)\}^2 + \sin^2 \pi (h, k, l) \right]$$

When,  $h + k + l = \text{odd}$

$$F = 0 \quad \text{and} \quad I = 0$$

So, it does not show (100) and other lines because there exists parallel plane at between two 100 planes.

The diffraction from these two planes are out of phase by  $\pi$  with respect to the first plane.



→ Additional 100 planes

(c) Show that the concept of Brillouin zone (B.Z.) in crystals emerges from the interpretation of the Bragg's law  $2\bar{K} \cdot \bar{G} + G^2 = 0$ . What significance one can attach to the Brillouin Zone with respect to the observations made from  $x$ -ray diffraction. (5, 2)

**Ans.** From Bragg's law :

$$n \lambda = 2a \sin \theta$$

$\theta$  = angle of incidence of beam motion of  $e^-$  is in terms of wave no.

$$\lambda = \frac{2\pi}{k} \Rightarrow k = \frac{2\pi}{\lambda}$$

$$|k| = \frac{n\pi}{a \sin \theta} \equiv k$$

For two dimensional lattice

$$k_x = k \sin \theta = \frac{n\pi}{a}$$

$$k_y = \frac{n\pi}{a}$$

These wave number defined the Brillouin zone.

For  $n = \pm$  [ first zone]

$$k_x = \pm \frac{\pi}{a} \quad \text{and} \quad k_y = \pm \frac{\pi}{a}$$

Now, consider square lattice

$$a = a \hat{i} \quad \text{and} \quad b = b \hat{j}$$

**Reciprocal lattice :**

$$a^* = \frac{2\pi}{a} \hat{i} \quad \text{and} \quad b^* = \frac{2\pi}{a} \hat{j}$$

$$G = n_1 a^* + n_2 b^* = \frac{2\pi}{a} [n_1 \hat{i} + n_2 \hat{j}]$$

$$k = k_x \hat{i} + k_y \hat{j}$$

Using Bragg's condition  $2k \cdot G + G^2 = 0$

$$2(k_x \hat{i} + k_y \hat{j}) \cdot \frac{2\pi}{a} (n_1 \hat{i} + n_2 \hat{j}) + \frac{4\pi^2}{a^2} (n_1 \hat{i} + n_2 \hat{j})^2 = 0$$

$$\frac{4\pi}{a} (n_1 k_x + n_2 k_y) + \frac{4\pi^2}{a^2} (n_1^2 + n_2^2) = 0$$

$$n_1 k_x + n_2 k_y = \frac{\pi}{a} (n_1^2 + n_2^2)$$

$n_1$  and  $n_2$  are integers for diffraction from horizontal vertical row.

**For first zone :**  $n_1 = \pm 1, n_2 = 0$  and  $k_x = \pm \frac{\pi}{a}$  and  $n_1 = 0, n_2 = \pm 1$  and  $k_y = \pm \frac{\pi}{a}$

The region of  $k$  space that electron occupy without diffracted is called first Brillouin zone.

Hence, the concept of BZ emerges from Bragg's law.

Brillouin zones are parallel to the reflecting planes which are responsible for x-ray diffraction.

**Q. 3. (a) Explain quantization of lattice vibrations. Obtain dispersion relation for elastic waves in a linear monoatomic chain and show that in the short wavelength limit lattice behave as continuum and no dispersion takes place.**

(2, 5)

**Ans.** Find the solution to this question in the text book.

**(b) How does Einstein assumption lead to an improvement in the specific heat of a solid over the classical theory? Explain its demerits.**

(5, 2)

**Ans.** Find the solution to this question in the text book.

**(c) Copper has an atomic weight of 63.5, a density of  $8.9 \times 10^3 \text{ kg/m}^3$  and frequency of longitudinal and transverse modes are  $4.76 \times 10^3 \text{ m/s}$  and  $2.32 \times 10^3 \text{ m/s}$ . Estimate the specific heat of copper at 30K.**

(7)

**Ans.** Atomic weight of Cu = 63.5

$$\rho = 8.9 \times 10^3 \text{ kg/m}^3$$

$$V_t = 2.32 \times 10^3 \text{ m/s}$$

$$V_l = 4.76 \times 10^3 \text{ m/s}, T = 30\text{K}$$

$$C_v = ?$$

$$Q_D = \frac{h}{k} \left[ \frac{9N}{4\pi V \left[ \frac{1}{V_t^3} + \frac{1}{V_l^3} \right]} \right]^{1/3}$$

$$h = 6.6 \times 10^{-34} \text{ J s}, k = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Put all values :

$$Q_D = 340 \text{ K}$$

$$C_v = \frac{12}{5} \pi^4 R \left( \frac{T}{Q_D} \right)^3$$

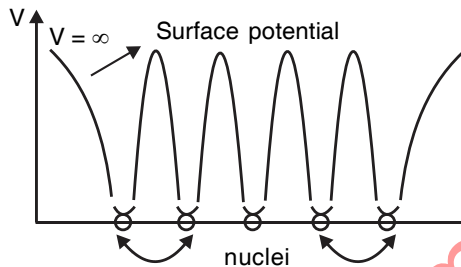
$$= \frac{12 \times \pi^4 \times 8.31 \times 10^3}{\left(\frac{30}{340}\right)^3}$$

$$C_V = 1.33 \text{ KJmol}^{-1}\text{K}^{-1}$$

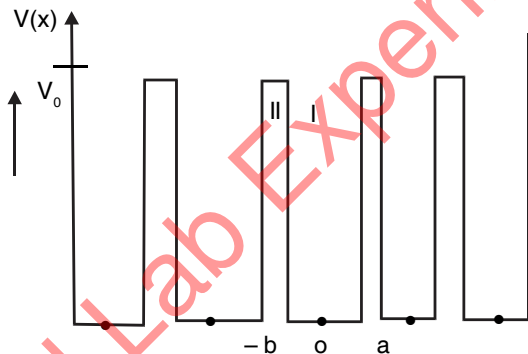
Q. 4. (a) Draw the potential experienced by an electron in a 1D monoatomic crystal lattice. How is this modified in Kronig Penney model? Set up the Schrodinger equation for an electron in this model and obtain the E-k relationship.

(1, 1, 5)

Ans.



1D periodic potential in monoatomic crystal lattice



Schrodinger equation for two regions I and II :

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}E\psi = 0 \quad 0 < x < a$$

$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \quad \alpha = \frac{8\pi^2mE}{h^2} \quad \dots(1)$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}(E - V_0)\psi = 0 \text{ for, } -b < x < 0$$

$$\frac{d^2\psi}{dx^2} + \beta^2\psi = 0 \quad \beta = \frac{8\pi^2m}{h^2}(V_0 - E)$$

$$\psi_1(x) = A \exp(i\alpha x) + B \exp(-i\alpha x) \quad \dots (3)$$

$$\psi_2(x) = C \exp(\beta x) + D \exp(-\beta x), E < V_0 \quad \dots (4)$$

A, B, C and D are const in I and II

I+ can be obtained by boundary condition

$$\left| \psi_1(x) \right|_{x=0} = \left| \psi_2(x) \right|_{x=0}$$

$$\left[ \frac{d\psi_1}{dx} \right]_{x=0} = \left[ \frac{d\psi_2}{dx} \right]_{x=0}$$

$$\left| \psi_1(x) \right|_{x=0} = \left| \psi_2(x) \right|_{x=-b}$$

$$\left[ \frac{d\psi_1}{dx} \right]_{x=0} = \left[ \frac{d\psi_2}{dx} \right]_{x=-b}$$

$$V(x+a) = V(x)$$

$$\psi_k(x+a+b) = \psi_k(x) \exp[ik(a+b)]$$

$$\psi_k(x) = \psi_k(x+a+b) \exp[-ik(a+b)]$$

$$x = -b \text{ and } x = a$$

$$\psi_2(-b) = \psi_1(a) \exp[-ik(a+b)] \quad \dots (5)$$

$$\left[ \frac{d\psi_2}{dx} \right]_{x=-b} = \left[ \frac{d\psi_1}{dx} \right]_{x=a} \exp[-ik(a+b)] \quad \dots (6)$$

After applying boundary condition

$$A + B = C + D \quad \dots (7)$$

$$i\alpha(A - B) = \beta(C - D) \quad \dots (8)$$

$$C e^{-\beta b} + D e^{\beta b} = e^{-ik(a+b)} [A e^{i\alpha a} + B e^{-i\alpha a}] \quad \dots (9)$$

$$\beta C e^{-\beta b} - \beta D e^{\beta b} = i\alpha e^{-ik(a+b)} [A e^{i\alpha a} - B e^{-i\alpha a}] \quad \dots (10)$$

Now making determinants :

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ i\alpha & -i\alpha & -\beta & \beta \\ -e^{-ik(a+b)+i\alpha a} & -e^{-ik(a+b)-i\alpha a} & e^{-\beta b} & e^{\beta b} \\ -i\alpha e^{ik(a+b)+i\alpha a} & i\alpha e^{-ik(a+b)-i\alpha a} & \beta e^{-\beta b} & -\beta e^{\beta b} \end{vmatrix}$$

On simplifying this determinant :



$$\cos k(a+b) = \left( \frac{\beta^2 - \alpha^2}{2\alpha\beta} \right) \sin \alpha a \sin \beta b + \cos \alpha a \cos \beta b$$

$$\sin \beta b \rightarrow \beta b \text{ and } \cos \beta b \rightarrow 1 \text{ as } b \rightarrow 0$$

$$\cos ka = \left( \frac{\beta^2 - \alpha^2}{2\alpha\beta} \right) \beta b \sin \alpha a + \cos \alpha a \quad \dots (9)$$

where,

$$\beta^2 - \alpha^2 = \frac{8\pi^2 m}{h^2} (V_0 - E) - \frac{8\pi^2 m E}{h^2}$$

$$V_0 \gg E \text{ So, } \beta^2 - \alpha^2 = \frac{8\pi^2 m}{h^2} V_0$$

Put this in (xi)

$$\cos ka = \frac{8\pi^2 m}{2\alpha\beta h^2} V_0 \beta b \sin \alpha a + \cos \alpha a$$

$$= \frac{4\pi^2 m a}{h^2} V_0 b \frac{\sin \alpha a}{\alpha a} + \cos \alpha a$$

$$\cos ka = \frac{P \sin \alpha a}{\alpha a} + \cos \alpha a$$

$$P = \frac{4\pi^2 m a}{h^2} V_0 b$$

Now,

$$\frac{\sin \alpha a}{\alpha a} \rightarrow 0 \Rightarrow \sin \alpha a = 0$$

$$\alpha a = \pm n\pi \quad \text{and} \quad \alpha^2 = \frac{n^2 \pi^2}{a^2}$$

$$\alpha^2 = \frac{n^2 \pi^2}{a^2} = \frac{8\pi^2 m E}{h^2}$$

$$E = \frac{n^2 h^2}{8ma^2}$$

If

$$P = 0$$

$$\cos \alpha a = \cos k \alpha \quad \Rightarrow \quad \alpha = k = \frac{8\pi^2 m E}{h^2}$$

Find the solution in this text book.

(b) Explain with the help of diagrams how the concept of effective mass is inherent to band theory. If energy of an electron in a crystal is given by  $E = 7h^2k^2/m$ . Calculate its effective mass. (4, 3)

Ans. The solution to this question is comprised in this text book.

(c) For an intrinsic semiconductor with gap width,  $E_g = 0.7eV$ , determine the position of the Fermi level at 300 K if  $m_h^* = 6m_e^*$  also calculate the density of electrons and holes at 300 K. (3, 4)

Ans.

$$E_g = 0.7eV,$$

$$T = 300 \text{ K}$$

$$m_h^* = 6m_e^*$$

$$E_F = \frac{E_C + E_V}{2} + \frac{3kT}{4} \ln \frac{m_h^*}{m_e^*}$$

$$= \frac{E_g}{\alpha} + \frac{3kT}{4} \ln \left( \frac{m_h^*}{m_e^*} \right)$$

$$= \frac{0.7}{2} + \frac{3 \times 1.38 \times 10^{-23}}{4 \times 1.6 \times 10^{-19}} \ln \frac{6m_e^*}{m_e^*}$$

$$= 0.35 + 0.64 \times 10^{-4} \ln 6$$

$$= 0.35 + 1.146 \times 10^{-4}$$

$$E_F = 0.3501 \text{ eV}$$

$$n = P = ni = 2 \left( \frac{2\pi kT}{h^2} \right)^{3/2} (m_e^* \cdot m_h^*)^{3/4} \exp \left( \frac{-E_g}{2kT} \right)$$

$$= 2 \times \left[ \frac{2 \times 3.14 \times 1.38 \times 10^{-23} \times 300}{(6.6 \times 10^{-34})^2} \right]$$

$$[6 \times (9.1 \times 10^{-31})^2]^{3/4} \exp \left( \frac{-E_g}{2kT} \right)$$

$$m = P = 8.4 \times 10^{18} / \text{m}^3.$$

Q. 5. (a) Obtain an expression for diamagnetic susceptibility using Langevin's theory. What is the significance of negative susceptibility? (6, 1)

Ans. Find the solution to this question in magnetism chapter of this book.

(b) Ferromagnetic materials lose their magnetism on heating. Give a qualitative idea how various energies, involved in the total energy of a ferromagnet as per domain applied heat energy resulting in the density of magnetism. (7)

Ans. According to domain theory main contribution of energy are :

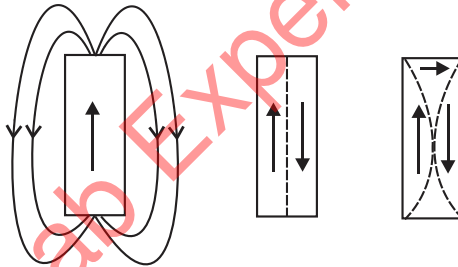
- (i) The exchange energy.
- (ii) Magnetic field energy.
- (iii) Anisotropy energy.

(i) Exchange Energy :  $E_{xc} = -2J S_i \cdot S_j$

$S_i$  and  $S_j$  are spin,  $J \Rightarrow$  strength of interaction.

$E_x$  is minimum when  $S_i$  and  $S_j$  are parallel on heating, spin are random and value of this energy increases. Hence, Ferramagnetism will be reduced.

(ii) Manetic Field Engery : In this by dividing a crystal into two domains the magnetic energy can be reduced minimum or even zero.



The lost two have zero energy.

As we heat the domains are randomly aligned. Hence, ferramagnetism decreases.

(iii) The Anisotropy Energy : This energy depends on the direction. A very small magnetic field along  $\langle 100 \rangle$  is sufficient to produce a large magnetization while to produce the same magnetization along  $\langle 111 \rangle$  a large field is required.

The difference in magnetic energy to produce saturation in an easy and hard direction is called anisotropy energy.

For cubic crystal :

$$E_{an} = k_1(\alpha^2\beta^2 + \beta^2r^2 + r^2\alpha^2) + k_2\alpha^2\beta^2r^2$$

On heating this difference also decreases.

(c) A magnetic substance has  $10^{28}$  atom/ $m^3$ . The magnetic moment of each atom is  $1.8 \times 10^{-23}$  Am<sup>2</sup>. Calculate the paramagnetic susceptibility at 300K. What would be the dipole moment of a bar of this material 0.1 m long and 1sq. cm cross section in a field of  $4 \times 10^4$  Am<sup>-1</sup>. (4, 2)

Ans.

$$N = 10^{28} \text{ atom/m}^3$$
$$\mu m = 1.8 \times 10^{-23} \text{ A m}^2$$
$$T = 300 \text{ K}$$

$$\chi = \frac{\mu_0 N \mu m^2}{3 k T} = \frac{4\pi \times 10^{-7} \times 10^{28} \times (1.8 \times 10^{-23})^2}{3 \times 1.38 \times 10^{-23} \times 300}$$

$$\chi = 3.28 \times 10^{-4}$$

$$M = \chi H = 3.28 \times 10^{-4} \times 8 \times 10^4$$
$$= 26.24 \text{ A/m}$$

Dipole moment =  $M \times \text{Area} \times \text{length}$

$$= 26.24 \times 10^{-4} \times 0.1$$

$$= 2.62 \times 10^{-4} \text{ Am}^2$$

**Q. 6. (a) Explain Polarizability of atoms and molecules. Derive Clausius Mosotti relation between polarizability and dielectric constant of a solid. (2, 5)**

Ans. Find the solution to this question in ferroelectric chapter of this book.

**(b) The value of dielectric constant for water is 81 at very low frequency (near to zero) and 1.8 at optical frequencies. Explain with diagram the possible reasons for this variation in the dielectric constant. (7)**

Ans. From Clausius-Mosetti relation :

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{1}{3\epsilon_0} \sum_j N_j \alpha_j$$

$\epsilon_0$  = Dielectric constant

$\alpha_j$  = atomic polarizability

This polarizability comes from three contribution :

(i) Electronic polarizability

(ii) Ionic polarizability

(iii) Dipolar or orientational

The extent to which a particular polarizability contributes depends on the nature of dielectric and frequency of applied field.

**Electronic :** It arises due to displacement of electron cloud of an atom relative to its nucleus in presence of applied field.

At optical frequency both dielectric constant and polarizability comes from this.

$$\frac{n^2 - 1}{n^2 + 2} = \frac{1}{3\epsilon_0} \sum_j N_j \alpha_j \text{ (Electronic)}$$

At optical frequency.

**Ionic Polarizability :** It arises due to displacement of charged ions relative to other ions in solid.

$$\alpha_i = \frac{P}{E} = \frac{e\Delta x}{E} = \frac{e^2}{\beta}$$

$\beta = F$  are constant

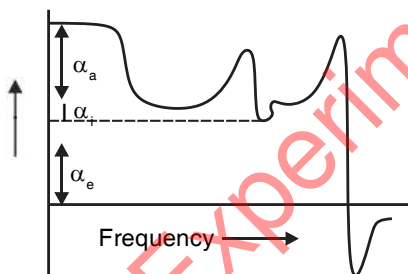
$\Delta x =$  Displacement

This is important at low temperature.

**Dipolar:** The molecule which have permanent dipole moment is called dipolar polarizability is property of dipolar substances ( $\alpha_d$ ).

$$\text{Total polarizability} = \alpha = \alpha_e + \alpha_i + \alpha_d$$

At low frequency all three parts contribute in case of  $H_2O$ . So, its dielectric constant is 81. A optical frequency only  $\alpha_e$  contributes. So it is 1.8.



(c) Expand the Langevin's function for small values of the  $pE/kT$  and show that the zeroth order is absent. Explain why this makes physical sense. (7)

Ans. Langevin's function is given by

$$L(a) = \text{Coth}(a) - \frac{1}{a}$$

$$\text{Coth}(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$= \frac{\left(1 + x + \frac{x^2}{2!} + \dots\right) + \left(1 - x + \frac{x^2}{2!} + \dots\right)}{\left(1 + x + \frac{x^2}{2!} + \dots\right) - \left(1 - x + \frac{x^2}{2!} + \dots\right)}$$

$$\begin{aligned}
&= \frac{2 \left( 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \right)}{2 \left( x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \right)} \\
&= \frac{1}{x} \left( 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \right) \left( 1 - \left( \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \right) \right) \\
&= \frac{1}{x} \left( 1 + \frac{x^3}{3} + \frac{x^4}{45} + \dots \right) \\
&= \frac{1}{x} + \frac{x}{3} + \frac{x^3}{45} + \dots
\end{aligned}$$

$$\text{Cath}(a) = \frac{1}{a} + \frac{a}{3}$$

$$L(a) = \frac{1}{a} + \frac{a}{3} - \frac{1}{a} \Rightarrow L(a) = \frac{a}{3}$$

For small value of  $a = \frac{PE}{kT}$

ie.,  $PE \ll kT$

$$L(a) = \frac{a}{3}$$

There is no zero order term.

Physically for  $a$  to be 1,  $PE = kT$ . We need the field strength of about  $10^4$  V/m at room temperature. For a moderate field of  $10^5 \sim 10^6$  at room temperature  $a$  turns out to be order  $[10^{-3} - 10^{-4}]$  which is much less than 1 and thus about approximation  $L(a) = a/3$  could be used freely without any loss of accuracy.

**Q. 7. (a) Explain Meissner Effect in superconductors with suitable diagram. Does it contradict Maxwell's equations? Support your answer with suitable reason.**

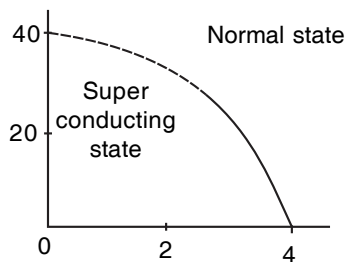
(2, 1, 4)

**Ans.** Find the solution to this question in the textbook.

**(b) What do you understand by critical magnetic field of a superconductor? How is it dependent on temperature? A superconducting material has a critical temperature of 3.8 K in zero magnetic field and critical field of 0.0306 T at 0 K. Find the critical field at 2 K.**

(2, 1, 4)

**Ans.** The critical magnetic field is the value at which superconductivity is destroyed. It is a function of temperature. It decreases with increase in temperature.



$$H_c(T) = H_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]$$

$$T_c = 3.8\text{K},$$

$$T = 2\text{K},$$

$$H_c(0) = 0.0306\text{ T}$$

$$H_c(T) = ?$$

$$H_c(T) = H_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]$$

$$= 0.0306 \left[ 1 - \left( \frac{2}{3.8} \right)^2 \right]$$

$$H_c(T) = 0.0216\text{T}$$

(c) (i) Calculate the Hall coefficient for sodium (Na) whose lattice constant is 0.428 nm. Sodium is bcc and will have two atoms per unit cell. (3)

(ii) How one can show and validate the idea of the presence of electrons and holes in semiconductors using Hall effect, Explain with the help of diagram. (4)

$$k_B = 1.3807 \times 10^{-23} \text{ JK}^{-1}$$

$$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$$

$$h = 6.63 \times 10^{-34} \text{ JS}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$$

$$\mu_B = 9.2732 \times 10^{-24} \text{ Am}^2$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

**Ans. (i)**

$$a = 0.428 \text{ nm} = 4.28 \text{ \AA} = 4.28 \times 10^{-10} \text{ m}$$

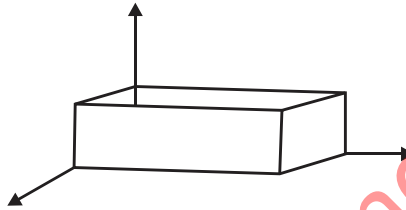
$$n = \frac{2}{a^3} = \frac{2}{(4.28)^3 \times 10^{-30}}$$

$$n = 2.55 \times 10^{28} / \text{m}^3$$

$$R_H = -\frac{1}{ne} = \frac{1}{2.55 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$R_H = 2.45 \times 10^{-9} \text{ m}^3 \text{C}^{-1}$$

(ii)



In this semiconductor both electron and holes are present. These charges are different and they move in opposite direction in electric field, the Lorentz force  $F_L = qV \times B$ , deflects them in small direction. However, the Hall field developed by the deflection of electrons is in the opposite direction to Hall field developed by the deflection of holes. The two charge carriers flow in  $x$ -direction under the electric field  $E_x$  and current is

$$\sigma_r = \frac{J_e}{E} = \frac{neVe}{E} = ne\mu_e$$

$$\sigma_n = ne\mu_n$$

two carrier Hall mobility :

$$\mu_H = \sigma R_H = \frac{p\mu_n^2 - n\mu_e^2}{n\mu_e + p\mu_n}$$

$$\mu_H = \mu_n - \mu_e$$

○○



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