Name of Paper	**:**	**Thermal Physics**
Name of the Course	**:**	**B.Sc. (Hons.) Physics**
Semester	**:**	**III**
Duration	**:**	**3 Hours**
Maximum Marks	**:**	**75**

Dec. 2019

Attempt five questions in all.
Question No. 1 is compulsory.
Answer any four of the remaining six,
attempting any two parts from each question.

Q. 1. Attempt all parts.

(*a*) Which of the two, an isothermal or an adiabatic, has greater slope? Prove mathematically. **(2)**

Ans. For isothermal process

$$PV = \text{constant}$$

Differentiating this we get

$$PdV + VdP = Q$$

$$\frac{dP}{dV} = \frac{-P}{V}$$

So, slope isothermal $= \frac{-P}{V}$

For adiabatic process

$$PV^r = \text{constant}$$

Differentiating this we get :

$$(dP)V^r + P\,rV^{r-1}\,dV = 0$$

$$\frac{dP}{dV} = -r\,\frac{PV^{r-1}}{V^r} = -r\left(\frac{P}{V}\right)$$

So, slope of adisbatic $= -r\,\frac{P}{V}$

Since, $r > 1$ the slope of adisbatic curve is greater.

(*b*) A Carnot's engine whose sink is at 27°C has an efficiency of 50%. By how much the temperature of the source be changed to decrease its efficiency to 40%? **(2)**

Ans. Efficiency $= n = 1 - \frac{T_2}{T_1}$

T_1 = Temperature of source

T_2 = Temperature of sink

$T_2 = 27°C \;= 300$ K (given)

$$n = 50\% = \frac{50}{100} = \frac{1}{2}$$

So, $$\frac{1}{2} = 1 - \frac{300}{T_1}$$

$$\frac{300}{T_1} = 1 - \frac{1}{2} = \frac{1}{2}$$

$T_1 = 600$ K → old temperature of source

$$\text{new effiancy} = 40\% = \frac{40}{100} = \frac{2}{5}$$

So, $$\frac{2}{5} = 1 - \frac{300}{T_1}$$

$$\frac{300}{T_1} = 1 - \frac{2}{5} = \frac{3}{5}$$

$$T_1 = \frac{300 \times 5}{3} = 500$$

$T_1 = 500$ K → New temperature of source

So, Difference = 600 – 500 = 100 K

(*c*) One kilogram of water is heated from 0°C to 100°C and converted into steam at the same temperature. Calculate the increase in entropy. Given that specific heat of water is 4.18 × 103Jkg^{-1}K^{-1} and latent heat of vaporisation is 2.24 × 106Jkg^{1}. (3)

Ans. Heating is carried out then entrops increase is given by

$T_1 = 0°C = 273K$

$T_2 = 100°C = 373$ K

$$\Delta S_1 = MC \ln\left(\frac{T_2}{T_1}\right)$$

$$= 1 \times 4.18 \times 10^3 \times 2.303 \times \log_{10}\left(\frac{373}{273}\right)$$

$\Delta S_1 = 1305$ JK^{-1}

Entrapy when water is converted into

$$\text{Steam} = \Delta S_2$$

$$\Delta S_2 = \frac{\delta Q}{T} = \frac{mL}{T}$$

$$\Delta S_2 = \frac{1 \times 2.24 \times 10^6}{373} = 6 \times 10^3$$

$$\Delta S_2 = 6 \times 10^3 \text{ JK}^{-1}$$

$$\text{Total at entropy} = \Delta S_1 + \Delta S_2$$

$$S = 1305 + 6000$$

$$S = 73.05 \text{ JK}^{-1}.$$

(*d*) Using Carnot's cycle derive Clausius-Clapeyron latent heat equation. (4)

Ans. The solution to this questions is in this textbook.

(*e*) A substance has volume expansivity = 2*b*T/V and isothermal compressibility = *a*/V, where '*a*' and '*b*' are constant. Find the equation of state. (3)

Ans. $$\text{Volume expansivity} = \frac{2bT}{V} = \alpha$$

$$\text{Isothermal compressibility} = B_T = \frac{a}{V}$$

$$V = V(P, T)$$

$$dV = \left(\frac{\partial V}{\partial P}\right)_T dP + \left(\frac{\partial V}{\partial T}\right)_P dT$$

In terms of α and B_T

$$\frac{dV}{V} = B_T dP + \alpha\, d\, T$$

$$\frac{dV}{V} = \frac{-a}{V} dP + \frac{2bT}{V} dT$$

$$dV + a\, dP - 2\, b\, T\, d\, T = 0$$

Integrate both sides

$$V + a\,P - b\,T^2 = \text{constant} \rightarrow \text{equation of state}$$

(*f*) Define Boyle Temperature. Give relation between Boyle temperature, Temperature of inversion and Critical temperature. (2)

Ans. The solution to this questions is in this textbook.

(*g*) What is Brownian motion? Give its characteristics. (3)

Ans. The solution to this questions is in this textbook.

Q. 2. (*a*) (*i*) State first law of thermodynamics. What are its physical significance and limitations? Write first law of thermodynamics for an adiabatic, isobaric and isochoric processes. (4)

(*ii*) Derive the work done by an ideal gas in expanding adiabatically from initial state (P_i, V_i, T_i) to the final state (P_f, V_f, T_f}). (3)

Ans. (*i*) The solution to this questions is in this textbook.

(*ii*) The solution to this questions is in this textbook.

(*b*) Using first law of thermodynamics, prove that

(*i*) $$\left(\frac{\partial U}{\partial P}\right)_V = \frac{C_V K_T}{\beta}$$

(*ii*) $$\left(\frac{\partial U}{\partial V}\right)_P = \frac{C_P}{\beta V} - P$$

where β and K_T are volume expansion coefficient and isothermal compressibility respectively. (3.5, 3.5)

Ans. Write U in terms of V and P

$$U = U(V, P)$$

$$dU = \left(\frac{\partial U}{\partial V}\right)_P dV + \left(\frac{\partial U}{\partial P}\right)_V dP$$

$$\left(\frac{\partial U}{\partial P}\right)_V = \left(\frac{\partial U}{\partial V}\right)_P \left(\frac{\partial V}{\partial P}\right)_V + \left(\frac{\partial V}{\partial P}\right)_V \frac{\partial P}{\partial P}$$

So, $$\left(\frac{\partial U}{\partial P}\right)_V = \left(\frac{\partial V}{\partial P}\right)_V \times \frac{\partial T}{\partial T}$$

$$\left(\frac{\partial U}{\partial P}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V \times \left(\frac{\partial T}{\partial P}\right)_V \quad (a)$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V \quad \text{and} \quad \beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right) \quad \dots (1)$$

$$K_T = +\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)$$

$$\frac{K_T}{\beta} = \left(\frac{\partial T}{\partial P}\right)_V \quad (3)$$

Put (1) and (2) in (*a*)

$$\left(\frac{\partial U}{\partial P}\right)_V = \frac{C_V K_T}{\beta}$$

(*ii*) $$\left(\frac{\partial U}{\partial V}\right)_P = \left(\frac{\partial U}{\partial T}\right)_P \times \left(\frac{\partial T}{\partial V}\right)$$

Now, from first law

$$\partial U = \partial\theta - \partial w$$

So, $$\left(\frac{\partial U}{\partial V}\right)_P = \left(\frac{\partial\theta - \partial w}{\partial T}\right)_P \; V \, \frac{\partial T}{\partial V}$$

$$= \left(\frac{\partial\theta}{\partial T}\right)_P \left(\frac{\partial T}{\partial V}\right)_P - \left(\frac{\partial w}{\partial T}\right) \times \left(\frac{\partial T}{\partial V}\right)$$

$$C_P = \left(\frac{\partial\theta}{\partial T}\right)_P \text{ and } \beta = \frac{1}{V}\frac{\partial V}{\partial T}$$

$$8\; \partial w = P\, \partial V$$

Using this

$$\left(\frac{\partial U}{\partial V}\right)_P = \frac{C_P}{\beta V} - P$$

(*c*) Find ΔW and ΔU for an iron cube of side 6 cm as it is heated from 20°C to 300°C. For iron C = 0.11 cal/g°C and volume coefficient of expansion is $\beta = 3.6 \times 10^{-5}$°C^{-1}. Given, Mass of the cube is 1700 gm. **(7)**

Ans.

$$T_1 = 20°C = 293 \text{ K}$$

$$T_2 = 300°C = 573 \text{ K}$$

$$C = 0.11 \text{ cal/g°C}. \;\; \beta = 3.6 \times 10^{-5}\text{°C}^{-1}$$

$$M = 1700 \text{ gm}$$

$$\Delta Q = \text{Heat} = m\, C\, \Delta T$$

$$= 1700 \times 0.11 \times [573 - 293]$$

$$= 52360 \text{ Cal}$$

$$P = 1 \text{ atm} = 10^{-5} \text{ N/m}^2$$

$$w = P\, dV$$

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)$$

So, $$dV = \beta\, V\, dT$$

$$= 3.6 \times 10^{-5} \times (6)^3 \times 280$$

$$= 2.177 \text{ cm}^3 = 2.177 \times 10^{-6} \text{ m}^3$$

$$w = \text{P } d \text{ V} = 105 \times 2.177 \times 10\text{–}6$$

$$w = 0.21775$$

$$= \frac{0.2177}{4184} \text{ col} = 5 \times 10^{-5} \text{ cal}$$

$$\Delta\text{U} = \Delta\text{Q} - \Delta w$$

$$\Delta\text{V} = 52360 \text{ cal}$$

Q. 3. (*a*) What are reversible and irreversible processes? Give one example of each. Prove that if Kelvin-Planck statement of second law is violated then Clausius statement is also violated. **(7)**

Ans. The solution to this questions is in this textbook.

(*b*) If, two Carnot engines R and S are operated in series such as engine T absorbs heat at temperature T_1 and rejects heat to the sink at temperature T_2, while Engine S absorbs half of the heat rejected by engine R and rejects heat to the sink at temperature T_3. If the work done in both the cases is equal, show that $T_2 = (T_3 + 2T_1)/3$. **(7)**

Ans. The solution to this questions is in this textbook.

(*c*) (*i*) A refrigerator freezes 6 kg of water at 0°C into ice in a time interval of 20 min. Assume that room temp, is 25°C, calculate the power needed to accomplish it.

(*ii*) If coefficient of performance of a refrigerator is 5 and operates at the room temperature 27°C, find the temperature inside the refrigerator. **(3.5, 3.5)**

Ans. $m = 6$ kg   time = 20 min

Heat required to convert 0°C water to 0°C ice

$$\text{Q} = 6 \times 80 = 480 \text{ K cal}$$

Room temperature T = 25°C = 298 K

convert it to 0°C (T^1) = 273 K

work done = w

$$\frac{\theta}{w} = \frac{T^1}{T - T^1}$$

$$w = \theta \frac{(T - T^1)}{T^1}$$

$$w = 480 \left(\frac{298 - 273}{273} \right)$$

$$= 43.95 \text{ K cal}$$

$$w = 43.95 \times 10^3 \times 4.25$$

$$P = \frac{w}{t} = \frac{43.95 \times 10^3 \times 4.2}{20 \times 60}$$

$$P = 153.825 \text{ watt.}$$

(*ii*) $\beta = 5$, Room temperature = 27°C

$$T = 300 \text{ K}$$

$$T^1 = \text{temperature inside}$$

$$S = \frac{T^1}{200 - T^1}$$

$$1500 - 5T^1 = T^1 \Rightarrow 6T^1 = 1500$$

$$T^1 = 250 \text{ K}$$

Q. 4. (*a*) Define entropy. What is principle of increase of entropy? Find increase in entropy for reversible and irreversible processes. (7)

Ans. The solution to this questions is in this textbook.

(*b*) If two bodies have equal mass *m* and heat capacity *c*, are kept at different temperature T_1 and T_2 respectively, taking $T_1 > T_2$ and the first body as source of heat for reversible engine and the second as sink, find out the maximum work done. (7)

Ans. The solution to this questions is in this textbook.

(*c*) (*i*) The temperature variation of C_P is given by the relation $C_P = 0.4T - 0.05T^2 - 0.25$, in the temperature range 50K to 100 K in cal/K. If 4 moles of the substance is heated from 50K to 100 K, calculate the change in entropy.

(*ii*) An ideal gas is confined to a cylinder by a piston. The piston is slowlyn pushed such that the gas temperature remains at 20°C. During compression, 730J of work is done on the gas. Find the entropy change of the gas. (3.5, 3.5)

Ans. (*i*) Change in intrapy $= \Delta S = \int \frac{\partial\theta}{T}$

$$C_P = \frac{d\theta}{dT} \Rightarrow \partial\theta = C_P dT$$

$$\Delta S = n\int \frac{C_P\, dT}{T} \qquad n = 4 \text{ mol}$$

$$\Delta S = 4\int_{T_1}^{T_2} \frac{0.4T - 0.05T^2 - 0.25dT}{T}$$

$$= 4\int_{T_1}^{T_2} (0.4 - 0.05T - \frac{0.25}{T})\, dT$$

$$= 4\,[0.4\,(T_2 - T_1) - \frac{0.05}{2}(T_2^2 - T_1^2) - 0.25 \ln T\,T]_{T_1}^{T_2}$$

$$\Delta S = 4\,[0.4\,(100 - 50) - \frac{0.05}{2}[100]^2 - (50)^2] - 0.25 \ln \frac{100}{50}]$$

$$= 4\left[20 - \frac{0.05}{2} \times 7500 - 0.25 \ \ln 2\right]$$

$$\Delta S = 4\left[20 - 187.5 - 0.25 \ln 2\right]$$

(ii) $T = 20°C = 293\ K$

$w = 730\ S$

Temperature is fixed. So, $\Delta U = 0$

$$\Delta\theta = \Delta w$$

$$\lambda S = \frac{d\theta}{T} = \frac{dw}{T}$$

$$\text{Change in entrapy} = \frac{dw}{T} = \frac{730}{293}$$

$$\Delta S = 2.49 \text{ cal K}^{-1}.$$

Q. 5. (*a*) What are thermodynamic potentials? Why are they so called? Give relations for them. Write physical significance of Gibb's free energy. (7)

Ans. The solution to this questions is in this textbook.

(*b*) Apply Maxwell's relation to prove that the difference of isothermal compressibility and adiabatic compressibility is equal to $TV\beta^2/C_P$.

Ans. The solution to this questions is in this textbook.

(*c*) Minute droplets of water are slowly pushed out of an atomizer into air. The average radius of the droplets is 10^{-4} cm. If 1 kg of water is atomized isothermally at 25°C, calculate the amount of heat transferred. The specific volume of water at 25°C is $1.00187 \times 10^{-3}\ m^3kg^{-1}$ and the rate of change of surface tension of water with temperature is $-0.152 \times 10^{-3}\ Nm^{-1}K^{-1}$. (7)

Ans. $r = 10^{-4}$ cm, $m = 1$ kg, temperature = 25°C

Specific volume = $1.00187 \times 10^{-3} m^3kg^{-1}$

$$\frac{\Delta S}{T} = -0.152 \times 10^{-3}\ Nm^{-1}k^{-1}$$

$$\text{Excess pressure} = P = \frac{2 \times \Delta S}{r}$$

$$P = \frac{2 \times (-0.152 \times 10^{-3}) \times 298}{10^{-4}}$$

$$= 905.9$$

$$\text{Volume} = \text{Specific Volume} \times \text{mass}$$

$$= 1.00187 \times 10^{-3} \times 1 = 1.00187 \times 10^{-3} m^3$$

$$\text{Work done} = P\Delta V = -\, 905.9 \times 1.00187 \times 10^{-3}$$

$$w = -\, 0.90761$$

$$\text{Energy} = \text{Surface tension} \times \text{Area}$$

$$= 0.152 \times 10^{-3} \times 298 \times 4 \times 3.14 \times 10^{-8} \times 10^{-8}$$

$$= 568.91 \times 10\text{–}19S$$

$$\Delta S = \Delta U - \Delta w$$

$$\Delta\theta = 0.90761.$$

Q. 6. (*a*) Define mean free path (λ) of molecules of a gas. Derive the expression $\lambda = \dfrac{3}{4\pi\sigma^2 n}$. Where σ is the diameter of the gas molecules and n is the no. of molecules per unit volume. (Assuming that all molecules move with the same velocity *i.e.,* that the average velocity of the gas. (7)

Ans. The solution to this questions is in this textbook.

(*b*) (*i*) Plot Maxwell distribution function for molecular speeds temperatures T_1, T_2 and T_3 such as $T_1 < T_2 < T_3$. Write the necessary inference from these curves. Write the necessary inference from these curves.

(*ii*) Calculate the value of v_x for which the probability of a molecule having *x*-velocity falls to half of its maximum value. (3, 4)

Ans. (*i*) The solution to this questions is in this textbook.

(*ii*) The solution to this questions is in this textbook.

(*c*) (*i*) Calculate the probability that the speed of oxygen molecule lies between 109.5 and 110.5 metre/sec. at 300 K.

(*ii*) Hydrogen and Nitrogen are maintained under identical conditions of temperature and pressure. Calculate the ratio of their coefficients of viscosity if the diameters of these molecules are 2.5×10^{-10}m and 3.5×10^{-10}m respectively. (4, 3)

Ans. (*i*) The probability that a molecule will have speed between VS V + *d*V is

$$\frac{dN_V}{N} = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} V^2 \, e - \left(\frac{mV^2}{2k_B T}\right)_{dV}$$

$$m = 32a = \frac{32}{6 \times 10^{26}} kg$$

$$V = 109.5$$

$$dV = 110.5 - 109.5 = 1 m/s$$

$$k_B = 1.38 \times 10^{-23}$$

$$T = 300\ k$$

$$\frac{d^N V}{N} = 4\pi \left(\frac{\frac{32}{6} \times 10^{26}}{2 \times 3.14 \times 1.38 \times 10^{-23} \times 300} \right) \times (109.5)^2$$

$$\times \exp \left[\frac{-\frac{32}{6} \times 10^{-26} \times 109 \times 109}{2 \times 1.38 \times 10^{-23} \times 300} \right] \times 1$$

$$\frac{d^N V}{N} = 8.11 \times 10^{-4}.$$

(ii)

$$n = \text{viscosity} = \frac{1}{3\sqrt{2}} \frac{1}{\pi d^2} m\overline{V}$$

Since both have at same temperature

So, $\overline{V}$ same.

So,

$$\frac{n_1}{n_2} = \frac{d_2^2}{d_1^2}$$

$$\frac{n_H}{n_N} = \frac{d_N^2}{d_H^2} = \frac{(3.5)^2 \times (10^{-10})^2}{(2.5)^2 \times (10^{-10})^2}$$

$$\frac{n_H}{n_N} = 1.96$$

Q. 7. (*a*) Discuss Joule-Thomson porous plug experiment. Obtain equation for Joule-Thomson co-efficient. (7)

Ans. The solution to this questions is in this textbook.

(*b*) What are the limitations of Van der waal's equation of state. Draw and discuss similarities and dis-similarities of theoretical experimental curves for CO_2 gas.

Ans. The solution to this questions is in this textbook.

(*c*) The Van der Waal's constant for Hydrogen are $a = 0.247$ atm. litre^{2}mol^{-2} and $b = 2.65 \times 10^{-2}$ litre/mol. Calculate :

(*i*) The temperature of inversion

(*ii*) Joule Thomson coefficient for 2 atm fall of pressure, initial temp, being 100 K. Given R = $^{224}/_{273}$ atoms litre/mol/K. **(7)**

Ans. (*i*) $a = 0.247$ atm, $b = 2.65 \times 10^{-2}$ litre^{2}mol^2

$$\text{Inversion temperature} = T_i = \frac{2a}{12b}$$

$$T_i = \frac{2 \times 0.247 \times 273}{224 \times 2.65 \times 10^{-2}} = 22.7\ k$$

$$T_i = 22.7\ k$$

(*ii*)

$$\Delta P = 2\ \text{atm}$$

$$\text{Initial temperature} = 100k$$

$$\text{Change in temperature} = 100 - 22.7$$

$$\Delta T = 77.3k$$

$$\Delta T = \mu \Delta P$$

$$\mu = \frac{\Delta T}{\Delta P} = \frac{77.3}{2}$$

$$\mu = 38.65\ \text{katm}^{-1}.$$

■■