B.Sc. (Physical science)
Mechanics 2016 (Solved Paper)

Q1:

Q.1. (a) What is a position vector? Find modulus of position vector $2\hat{i}+2\hat{j}-3\hat{k}$.

(b) What are polar and axial vectors? Give one example of each.

(c) Solve following equations:

(i) $(x^2+y^2)\,dx = 2xy\,dy$

(ii) $(x^2-y^2)\,dx + 2xy\,dy = 0$

Ans 1 a) In geometry, a **position** or **position vector**, also known as **location vector** or **radius vector**, is a Euclidean vector that represents the position of a point *P* in space in relation to an arbitrary reference origin *O*. Usually denoted **x**, **r**, or **s**, it corresponds to the straight-line from *O* to *P*.

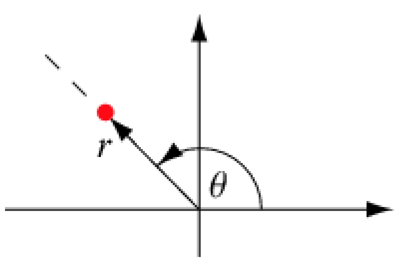
$$\mathbf{r} = \overrightarrow{OP}.$$

$$\vec{r} = 2\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\text{modulus},\ |\vec{r}| = \sqrt{2^2 + 2^2 + (-3)^2}$$

$$= \sqrt{4+4+9} = \sqrt{17}\ \text{units}$$

Ans 1 b) a polar vector is a vector such as the radius vector $\mathbf{r}$ that reverses sign when the coordinate axes are reversed. Polar vectors are the type of vector usually simply known as "vectors." In contrast, pseudovectors (also called axial vectors) do not reverse sign when the coordinate axes are reversed. Examples of polar vectors include $\mathbf{r}$, the velocity vector $\mathbf{v}$, momentum $\mathbf{p}$, and force $\mathbf{F}$. The cross product of two polar vectors is a pseudovector.



Ans 1(c)

1(c) (i) Given equation is

$$(x^2+y^2)\,dx = 2xy\,dy$$

$$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}.$$

This is homogeneous differential equation

Put $y = vx$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2x(xv)} = \frac{x^2(1+v^2)}{2x^2v}$$

$$= \frac{1+v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v} - v = \frac{1-v^2}{2v}$$

$$\Rightarrow \frac{2v}{1-v^2}\,dv = \frac{dx}{x}$$

Integrating both sides:

$$\int \frac{2v}{1-v^2}\,dv = \int \frac{1}{x}\,dx$$

$$\log(1-v^2) + \log x = \log c$$

$$(1-v^2)\,x = c$$

Replacing v by $\frac{y}{x}$

$$\left(1-\frac{y^2}{x^2}\right)x = c$$

Q2:

Q.2. (a) What are the conservative forces? Give two examples of conservative forces.

(b) State and prove work energy theorem.

(c) Two particles of masses 20 g and 40 g are moving opposite to each other with speed 2 m/s and 5 m/s respectively, on a straight line. They collide elastically. Find the change in momentum of each particle.

Ans 2(a) A **conservative force** is a force with the property that the total work done in moving a particle between two points is independent of the taken path. Equivalently, if a particle travels in a closed loop, the net work done (the sum of the force acting along the path multiplied by the displacement) by a conservative force is zero.

Gravitational force is an example of a conservative force. Other examples of conservative forces are: force in elastic spring, electrostatic force between two electric charges, magnetic force between two magnetic poles. The last two forces are called central forces as they act along the line joining the centres of two charged/magnetized bodies. Thus, all central forces are conservative forces.

Ans 2(b)

The work-energy theorem states that the work done by all forces acting on a particle equals the change in the particle's kinetic energy.

Now for infinitesimal displacement calculations and forces whether variable or constant, we can write from the definition of work,

$$dW = Fdx$$

Now let the total work done is **W**, and the displacement is from x_0 to x

Integrating the both sides we get,

$$\int_0^W dW = \int_{x_0}^{x} Fdx$$

$$W = \int_{x_0}^{x} Fdx$$

$$W = m\int_{x_0}^{x} \frac{dv}{dt}dx$$

$$W = m\int_{v_0}^{v} v\,dv$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$W = \Delta E_k$$

(Proved)

2(c) Ans:

2(c) : $m_1 = 20g$ $m_2 = 40g$

$u_1 = 2m/s$ $u_2 = 5m/s$

using momentum conservation

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$20(2) + 40(5) = 20v_1 + 40v_2$$

$$40 + 80 = 20v_1 + 40v_2$$

$$120 = 20v_1 + 40v_2$$

$$6 = v_1 + 2v_2 \quad —①$$

using conservation of K.E.

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$20(2)^2 + 40(5)^2 = \cancel{\frac{1}{2}}\ 20v_1^2 + 40v_2^2$$

$$4 + 50 = v_1^2 + 2v_2^2$$

$$54 = v_1^2 + 2v_2^2 \quad —②$$

solve these two equation to get the value of v_1 and v_2.

Q3:

Q.3. (a) Give the statement of law of conservation of angular momentum. Give two examples of its application where the law of conservation of angular momentum helps us to understand rotational motion.

(b) A solid sphere of mass 100 g and radius 5 cm is rotating about its own axis. It completes 5 revolutions in 2 minutes, calculate its

(i) Moment of inertia

(ii) Angular momentum

(iii) Rotational kinetic energy

(c) State the importance of Newton's first law of motion.

3(a) Ans: **When the net external torque acting on a system about a given axis is zero, the total angular momentum of the system about that axis remains constant.**

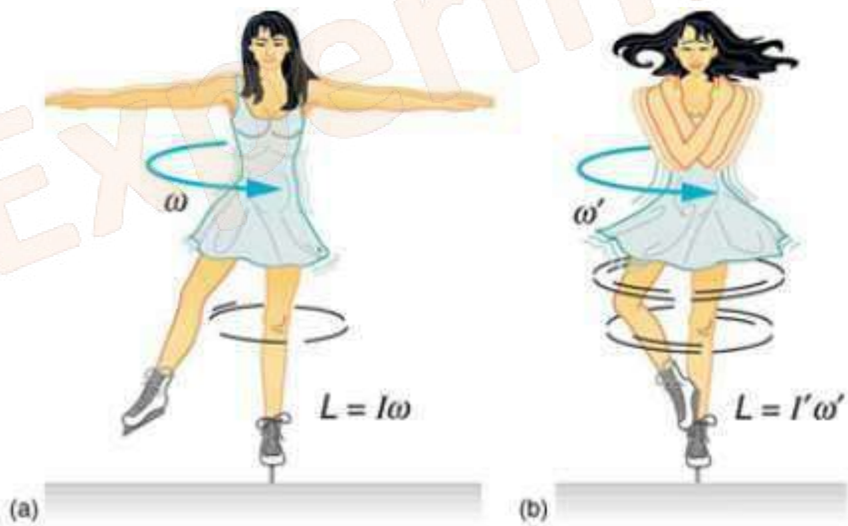
Mathematically,

$$\text{If } \sum \vec{\tau} = 0 \text{ then } \vec{L} = \textbf{constant}$$

An example of conservation of angular momentum is seen in an ice skater executing a spin, as shown in. The net torque on her is very close to zero, because

1) there is relatively little friction between her skates and the ice,

2) the friction is exerted very close to the pivot point.



Conservation of Angular Momentum: An ice skater is spinning on the tip of her skate with her arms extended. Her angular momentum is conserved because the net torque on her is negligibly small. In the next image, her rate of spin increases greatly when she pulls in her arms, decreasing her moment of inertia. The work she does to pull in her arms results in an increase in rotational kinetic energy.

(Both F and r are small, and so $\vec{\tau} = \vec{r} \times \vec{F}$ is negligibly small.) Consequently, she can spin for quite some time. She can also increase her rate of spin by pulling in her arms and legs. When she does this, the rotational inertia decreases and the rotation rate increases in order to keep the angular momentum $L = I\omega$ constant. (I: rotational inertia, ω: angular velocity)

3(b) Ans:
Moment of inertial , I= $(2/5)MR^2$
Angular momentum, L= IѠ
Rotational Kinetic energy , K= (½) I $Ѡ^2$

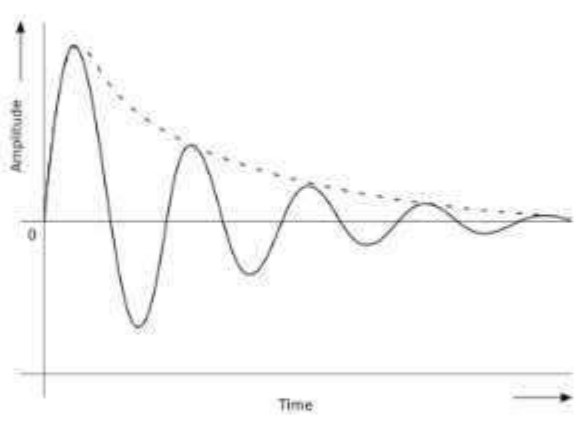
3(c) Ans: First Law of Motion: Every object will remain at rest or constant speed unless an external force is applied.
Significance:

It is thanks to this law if seatbelts in cars are compulsory for car manufacturers and for us to wear, otherwise every time you quickly come to a stop your face would constantly hit the windscreen, the dashboard or the front seat of your car. In more serious accidents the invention of seatbelt heavily decreased the amount of deaths on roads, so I would say it is a significant benefit. It is thanks to your awareness of this law if when you are skateboarding, you jump off the skateboard if there is a rock on your way, because the rock is going to stop the skateboard but not you. You would keep travelling at the same speed until the friction of your body and the ground stops you. It is also thanks to this law if when you are skiing, after coming down the mountain you still have speed even on flat land, of course, until the friction of the snow and the ski stops you.

Q4:

Q.4. (a) Define damped oscillations. Give one example.
(b) Write the differential equation for a damped harmonic oscillator. Solve the differential equation for its displacement for un-damped condition only.

4 (a) Ans: When the motion of an oscillator reduces due to an external force, the oscillator and its motion are **damped**. These periodic motions of gradually decreasing amplitude are damped simple harmonic motion. An example of damped simple harmonic motion is a simple pendulum.
In the damped simple harmonic motion, the energy of the oscillator dissipates continuously. But for small damping, the oscillations remain approximately periodic. The forces which dissipate the energy are generally **frictional forces**.



4(b) Ans

Let's drive our damped spring-object system by a sinusoidal force. Suppose that the x-component of the driving force is given by

$$F_x(t) = F_0 \cos(\omega t) \ , \tag{23.6.1}$$

where F_0 is called the *amplitude* (maximum value) and ω is the *driving angular frequency*. The force varies between F_0 and $-F_0$ because the cosine function varies between $+1$ and -1. Define $x(t)$ to be the position of the object with respect to the equilibrium position. The x-component of the force acting on the object is now the sum

$$F_x = F_0 \cos(\omega t) - kx - b\frac{dx}{dt} \ . \tag{23.6.2}$$

Newton's Second law in the x-direction becomes

$$F_0 \cos(\omega t) - kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2} \ . \tag{23.6.3}$$

We can rewrite Eq. (23.6.3) as

$$F_0 \cos(\omega t) = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx \ . \tag{23.6.4}$$

We shall now use complex numbers to solve the differential equation

$$F_0 \cos(\omega t) = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx \ . \tag{23.D.1}$$

We begin by assuming a solution of the form

$$x(t) = x_0 \cos(\omega t + \phi) \ . \tag{23.D.2}$$

where the amplitude x_0 and the phase constant ϕ need to be determined. We begin by defining the complex function

$$z(t) = x_0 e^{i(\omega t + \phi)} \ . \tag{23.D.3}$$

Our desired solution can be found by taking the real projection

$$x(t)=\mathrm{Re}(z(t))=x_0\cos(\omega t+\phi)\ .$$

Our differential equation can now be written as

$$F_0 e^{i\omega t}=m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz\ .$$

We take the first and second derivatives of Eq. (23.D.3),

$$\frac{dz}{dt}(t)=i\omega x_0 e^{i(\omega t+\phi)}=i\omega z\ . \tag{23.D.6}$$

$$\frac{d^2z}{dt^2}(t)=-\omega^2 x_0 e^{i(\omega t+\phi)}=-\omega^2 z\ . \tag{23.D.7}$$

We substitute Eqs. (23.D.3), (23.D.6), and (23.D.7) into Eq. (23.D.5) yielding

$$F_0 e^{i\omega t}=(-\omega^2 m+bi\omega+k)z=(-\omega^2 m+bi\omega+k)x_0 e^{i(\omega t+\phi)}\ . \tag{23.D.8}$$

We divide Eq. (23.D.8) through by $e^{i\omega t}$ and collect terms using yielding

$$x_0 e^{i\phi}=\frac{F_0/m}{((\omega_0{}^2-\omega^2)+i(b/m)\omega)}\ . \tag{23.D.9}$$

where we have used $\omega_0{}^2=k/m$. Introduce the complex number

$$z_1=(\omega_0{}^2-\omega^2)+i(b/m)\omega\ . \tag{23.D.10}$$

Then Eq. (23.D.9) can be written as

$$x_0 e^{i\phi}=\frac{F_0}{my}\ . \tag{23.D.11}$$

Multiply the numerator and denominator of Eq. (23.D.11) by the complex conjugate $\overline{z}_1=(\omega_0{}^2-\omega^2)-i(b/m)\omega$ yielding

$$x_0 e^{i\phi}=\frac{F_0\overline{z}_1}{mz_1\overline{z}_1}=\frac{F_0}{m}\frac{((\omega_0{}^2-\omega^2)-i(b/m)\omega)}{((\omega_0{}^2-\omega^2)^2+(b/m)^2\omega^2)}\equiv u+iv\ . \tag{23.D.12}$$

where

$$u=\frac{F_0}{m}\frac{(\omega_0{}^2-\omega^2)}{((\omega_0{}^2-\omega^2)^2+(b/m)^2\omega^2)}\ , \tag{23.D.13}$$

$$v = -\frac{F_0}{m}\frac{(b/m)\omega}{((\omega_0{}^2 - \omega^2)^2 + (b/m)^2\omega^2)} \ . \tag{23.D.14}$$

Therefore the modulus x_0 is given by

$$x_0 = (u^2 + v^2)^{1/2} = \frac{F_0/m}{((\omega_0{}^2 - \omega^2)^2 + (b/m)^2\omega^2)} \ , \tag{23.D.15}$$

and the phase is given by

$$\phi = \tan^{-1}(v/u) = \frac{-(b/m)\omega}{(\omega_0{}^2 - \omega^2)} \ . \tag{23.D.16}$$

Oscillator Equation. The solution to is given by the function

$$x(t) = x_0 \cos(\omega t + \phi) \ , \tag{23.6.5}$$

where the amplitude x_0 is a function of the driving angular frequency ω and is given by

$$x_0(\omega) = \frac{F_0/m}{\left((b/m)^2\omega^2 + (\omega_0{}^2 - \omega^2)^2\right)^{1/2}} \ . \tag{23.6.6}$$

The phase constant ϕ is also a function of the driving angular frequency ω and is given by

$$\phi(\omega) = \tan^{-1}\left(\frac{(b/m)\omega}{\omega^2 - \omega_0{}^2}\right) \ . \tag{23.6.7}$$

In Eqs. (23.6.6) and (23.6.7)

$$\omega_0 = \sqrt{\frac{k}{m}} \tag{23.6.8}$$

is the natural angular frequency associated with the undriven undamped oscillator. The x-component of the velocity can be found by differentiating Eq. (23.6.5),

$$v_x(t) = \frac{dx}{dt}(t) = -\omega x_0 \sin(\omega t + \phi) \ , \tag{23.6.9}$$

where the amplitude $x_0(\omega)$ is given by Eq. (23.6.6) and the phase constant $\phi(\omega)$ is given by Eq. (23.6.7).

Q5

Q.5. (a) What is Poisson's ratio? Can it be more than 0.5? Give reasons for your answer using appropriate equation.

(b) Obtain the relation between Poisson's ratio (a) bulk modulus (K) and Young's modulus(Y) for an isotropic material.

5(a) Ans : **Poisson's ratio** is the ratio of transverse contraction strain to longitudinal extension strain in the direction of stretching force. Tensile deformation is considered positive and compressive deformation is considered negative. The definition of Poisson's ratio contains a minus sign so that normal materials have a positive ratio. Poisson's ratio, also called Poisson ratio or the Poisson coefficient, or coefficient de Poisson, is usually represented as a lower case Greek nu, n.

$$\nu = - \varepsilon_{trans} / \varepsilon_{longitudinal}$$

We know,

$$E=3K(1-2n),$$

where E is young's modulus of elasticity, K is bulk modulus of elasticity and n is poission's ratio. If value of poission's ratio is more than half, implies that value of young's modulus is negative which is not possible. So value of poission's ratio is always less than 0.5

5 (b)Ans

The generalized Hooke's law for the x-, y- and z-directions is

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right]$$

and

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

where ε is strain and σ is stress.

Adding up these three equations gives

$$(1)\ \epsilon_x + \epsilon_y + \epsilon_z = \frac{1-2\nu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

Now note in the above equation that $\epsilon_x + \epsilon_y + \epsilon_z$ is volume strain Δ and that $\sigma_x + \sigma_y + \sigma_z$ is just $3\sigma_m$ where σ_m is mean stress. Accordingly, Eq. (1) becomes

$$(2)\ \Delta = \frac{1-2\nu}{E} 3\sigma_m$$

But since bulk modulus is defined as $\frac{\sigma_m}{\Delta}$, we have

$$(3)\ K = \frac{\sigma_m}{\Delta} = \frac{E}{3(1-2\nu)}$$

which is the desired relation between K, E, and ν.

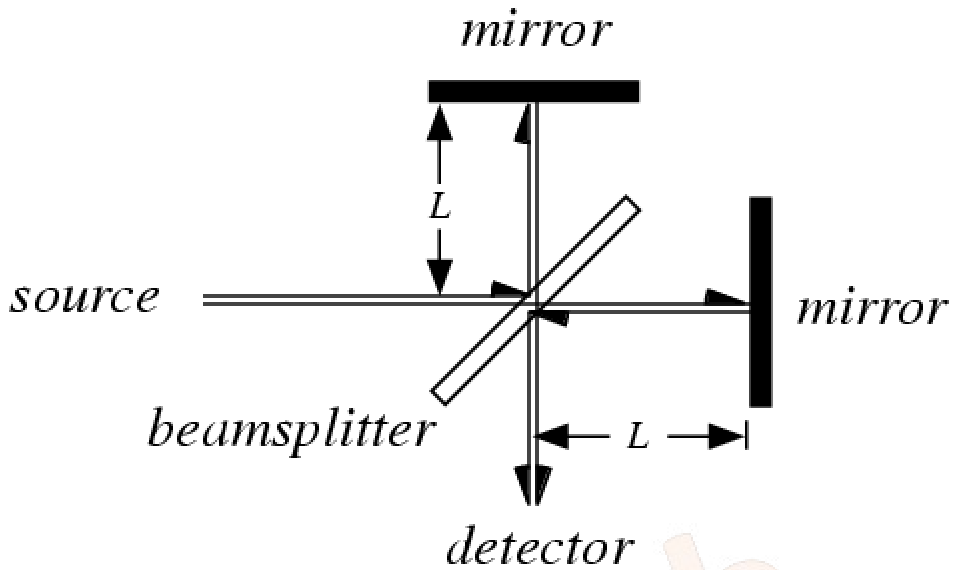
Q6:

Q. 6. (a) Describe the Michelson-Morley experiment and explain the physical significance of its negative result.

(b) Find the velocity of a rod moving in the direction of its length so that appears to be 60% of its original length.

6(a) Ans: Michelson and Morley built a Michelson interferometer, which essentially consists of a light source, a half-silvered glass plate, two mirrors, and a telescope. The mirrors are placed at right angles to each other and at equal distance from the glass plate, which is obliquely oriented at an angle of 45° relative to the two mirrors. In the original device, the mirrors were mounted on a rigid base that rotates freely on a basin filled with liquid mercury in order to reduce friction.

Prevailing theories held that ether formed an absolute reference frame with respect to which the rest of the universe was stationary. It would therefore follow that it should appear to be moving from the perspective of an observer on the sun-orbiting Earth. As a result, light would sometimes travel in the same direction of the ether, and others times in the opposite direction. Thus, the idea was to measure the speed of light in different directions in order to measure speed of the ether relative to Earth, thus establishing its existence.



Michelson and Morley were able to measure the speed of light by looking for interference fringes between the light which had passed through the two perpendicular arms of their apparatus. These would occur since the light would travel faster along an arm if oriented in the "same" direction as the ether was moving, and slower if oriented in the opposite direction. Since the two arms were perpendicular, the only way that light would travel at the same speed in both arms and therefore arrive simultaneous at the telescope would be if the instrument were motionless with respect to the ether. If not, the crests and troughs of the light waves in the two arms would arrive and interfere slightly out of synchronization, producing a diminution of intensity. (Of course, the same effect would be achieved if the arms of the interferometer were not of the same length, but these could be adjusted accurately by looking for the intensity peak as one arm was moved. Changing the orientation of the instrument should then show fringes.)
Although Michelson and Morley were expecting measuring different speeds of light in each direction, they found no discernible fringes indicating a different speed in any orientation or at any position of the Earth in its annual orbit around the Sun.

Following important conclusions can be drawn from the negative results of Michelson-Morley experiment.

1. The velocity of light is constant in all directions.
2. The effects of ether in entire space of the universe are undetectable.
3. A new theory with different concepts of space, time and mass is needed. Thus, we must think of different set of transformation in contract to Galilean transformation which failed to give correct results.

6(b) Ans:

6(b) $\Rightarrow$ $$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$L_0 \rightarrow$ rest length

$L \rightarrow$ contracted length.

$v \rightarrow$ velocity of frame of reference

$$L = 0.6 L_0$$

$$0.6 L_0 = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$\left(\frac{6}{10}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$\frac{9}{25} = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\boxed{v = \frac{4}{5}c}$$

Q7:

Q.7. Write short notes on any two of the following:

(a) Geo-stationary satellite.

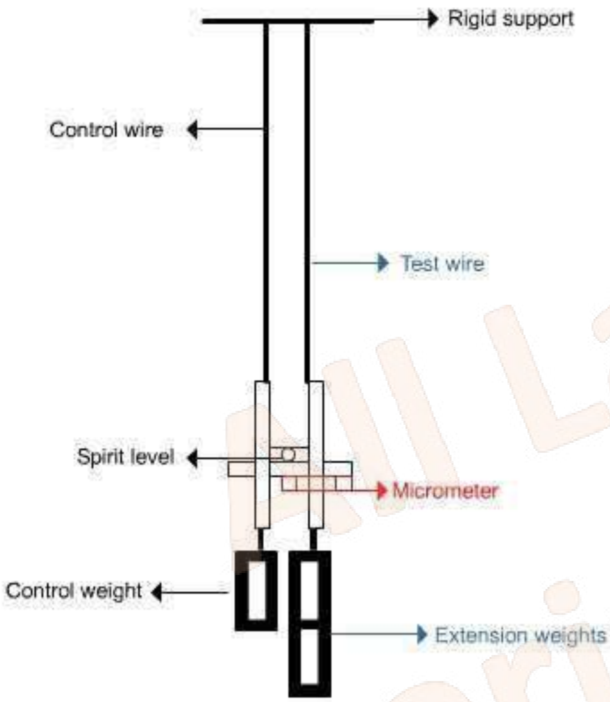
(b) Searle's method of determining Young's modulus.

(c) Time dilation in special theory of relativity.

7(a) Ans: A geostationary satellite is an earth-orbiting satellite, placed at an altitude of approximately 35,800 kilometers (22,300 miles) directly over the equator, that revolves in the same direction the earth rotates (west to east). At this altitude, one orbit takes 24 hours, the same length of time as the earth requires to rotate once on its axis. The term geostationary comes from the fact that such a satellite appears nearly stationary in the sky as seen by a ground-based observer.

A single geostationary satellite is on a line of sight with about 40 percent of the earth's surface. Three such satellites, each separated by 120 degrees of longitude, can provide coverage of the entire planet, with the exception of small circular regions centered at the north and south geographic poles. A geostationary satellite can be accessed using a directional antenna, usually a small dish, aimed at the spot in the sky where the satellite appears to hover. The principal advantage of this type of satellite is the fact that an earthbound directional antenna can be aimed and then left in position without further adjustment. Another advantage is the fact that because highly directional antennas can be used, interference from surface-based sources, and from other satellites, is minimized.

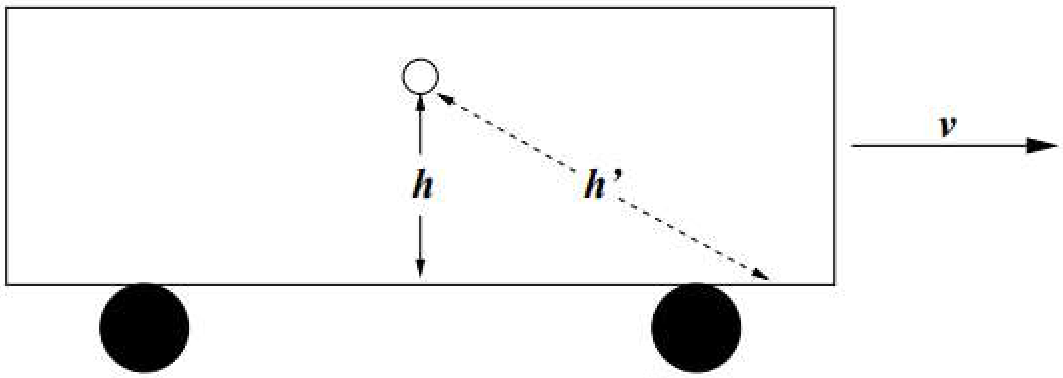
7(b) Ans: The Searle's apparatus consists of two wires (control wire and test wire) of equal length attached to a rigid support. Both control and test wires are attached to the other ends by a horizontal bar supporting a spirit level. The bar is hinged to the control wire so that when the test wire is extended due to the addition of weights on the side of the test wire, the spirit level is tilted by a small amount. We can remove any tilt of the spirit level and restore it to the horizontal position by turning the screw of a micrometer, which is positioned on the test wire side and making the bar mounted spirit level travel in the desired direction.



The test procedure is,

1. Measure the initial length L of the wire by a scale and diameter d of the wire by a screw gauge.
2. Adjust the spirit level so that it is in the horizontal position by turning the micrometer. Record the micrometer reading to use it as the reference reading.
3. Load the test wire with a further weight. The spirit level is tilted due to elongation of test wire.
4. Adjust the micrometer screw to restore the spirit level into the horizontal position. Subtract the first micrometer reading from the second micrometer reading to obtain the extension l of the test wire.
5. Calculate stress and strain from the formulae.
6. Repeat above steps by increasing load on the test wire to obtain more values of stresses and strains.
7. Plot the above values on stress strain graph; it should be a straight line. Determine the value of the slope Y(young’s modulus).

7(c) Ans: In particular, it means that inertial observers in different frames of reference measure different intervals of time between events, and different spatial separations between events. As we’ll see in a moment, clocks that appear to be moving run slow; rulers that appear to be moving are shrunk. Consider a pulse of light emitted from a bulb in the center of a train. The train is moving at speed v in the x direction with respect to a station. The pulse of light is directed toward a photosensor on the floor of the train, directly under the bulb. Two events are of interest to observers on the train and in the station: the emission of light from the bulb, and the reception of this light by the photodetector.



How do things look to observers in the station, watching the train go by? They agree that the pulse of light moves vertically through a distance h. However, they claim it also moves through *horizontal* distance because the train is moving with speed v. If the elapsed time between emission and reception as seen in the station frame is $\Delta t_{\text{station}}$, then the pulse moves through a total distance $h' = \sqrt{h^2 + (v\Delta t_{\text{station}})^2}$. Since the speed of light is the same in all frames of reference, the time interval $\Delta t_{\text{station}} = h'/c$:

$$\Delta t_{\text{station}} = \frac{1}{c}\sqrt{h^2 + (v\Delta t_{\text{station}})^2}$$

Substitute $h = c\Delta t_{\text{train}}$:

$$(\Delta t_{\text{station}})^2 = (\Delta t_{\text{train}})^2 + \frac{v^2}{c^2}(\Delta t_{\text{station}})^2$$

$$\begin{aligned} \Delta t_{\text{station}} &= \frac{\Delta t_{\text{train}}}{\sqrt{1 - v^2/c^2}} \\ &\equiv \gamma \Delta t_{\text{train}} \end{aligned}$$

where we have defined $\gamma = 1/\sqrt{1 - v^2/c^2}$; note that $\gamma \geq 1$. This relation tells us that *the moving clock runs slow*: the time interval measured by observers who see the clock moving (those in the station) is larger than the time interval measured by observers who are at rest with respect to it.