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UNIT-6 Oscillations

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UNIT-6
OSCILLATIONS

Q 1. What is periodic motion? Explain harmonic and simple harmonic motion.

Ans. The repeated motion moving the particle along a definite path on regular intervals of time is called periodic Motion and interval of time after which the particle repeat its path is called time period (T) of the periodic motion.

In general, there are two types of periodic motions

(i) Harmonic motion- If the particle undergoing periodic motion covers the same path to and fro about a mean position, the motion is known as vibratory or oscillatory motion. This motion is bounded on both sides within a well-defined limit. This motion is also known as harmonic motion. Such motion is associated with musical instruments.

(ii) Simple harmonic motion-If the displacement of the particle is equal on both sides, the motion is known as Simple Harmonic Motion (SHM). This is the most fundamental type of motion and all other periodic motions, whether harmonic or non-harmonic, can be achieved by a suitable combination of two or more simple harmonic motions

Q.2. Why a system of particle oscillates.

Ans. There are basically two main reasons which are responsible for the oscillation in the system and two basic properties of the system. These are elasticity and inertia.

When a body is in equilibrium, the net forces acting on the body are balanced. When we apply a force on the body, the system is displaced from its position in direction of force w.r.t. body through a distance ψ.

When the force is removed from the body, a restoring force comes into play which tries to restore the original position of equilibrium of the body. It' tries to restore ψ to zero. This is possible by imparting a suitable negative velocity $d\psi/dt$ to the body. The elastic properties of the system determine the magnitude of this restoring force.

The inertia of the body tries to oppose any change in velocity. When $\psi = 0$, i.e., the body reaches the equilibrium position, the negative velocity is maximum and a negative displacement is produced. The body moves to the other side of the equilibrium position. Now the restoring force becomes positive. Now it increases w. The restoring force has to overcome the inertia of the negative velocity. The velocity goes on decreasing and becomes zero. But by this time the displacement becomes large and negative. The process is now reversed.

Thus elasticity and inertia are two responsible causes for the oscillation of a body. let's suppose a particle free to move on the x-axis, is being acted upon by a force yen by,

$$\mathbf{F = - kx^n}$$

Here, k is a positive constant. Now, following cases are possible depending on the value of n.

(a) If n is an even integer (0,2,4....etc), force is always along negative x-axis, whether x is positive or negative. Hence, the motion of the particle is not oscillatory. If the particle is released from any position on the x-axis (except at x= 0) a force in the negative direction of X-axis acts on it and it moves rectilinearly along negative x-axis.

(b) If n is an odd integer (1,3,5, ...etc), force is along negative x-axis for x **0,** along positive x-axis for x <**0** and zero for x = 0. Thus, the particle Will oscillate about stable equilibrium position, x = 0. The force in this case is called the restoring force. Of these, if n = 1, i.e., F= -kx the motion is said to be SHM.

Q. 3. Prove that the velocity of a particle in SHM is ahead of its displacement by $\pi/2$. Also draw graphs for dispacement, velocity and acceleration for the particle executing SHM.

Ans. Let the displacement equation of the particle executing SHM be given by,

$$y = A \sin(\omega t + \phi_0) \qquad ...(i)$$

Differentiating (i) w.r.t. t, we have,

$$V = \frac{dy}{dt} A\,\omega \cos(\omega t + \phi_0) = V_0 \cos(\omega t + \phi_0)$$

where $V_0 = A\,\omega$ is called velocity amplitude of the particle.

$$V = V_0 \sin\left[(\omega + \phi_0) + \frac{\pi}{2}\right] \qquad ...(ii)$$

Thus, velocity is ahead of displacement by $\frac{\pi}{2}$ or $\frac{1}{4}$ th of a cycle.

Differenting (*ii*) w.r.t. t we have, acceleration given by.

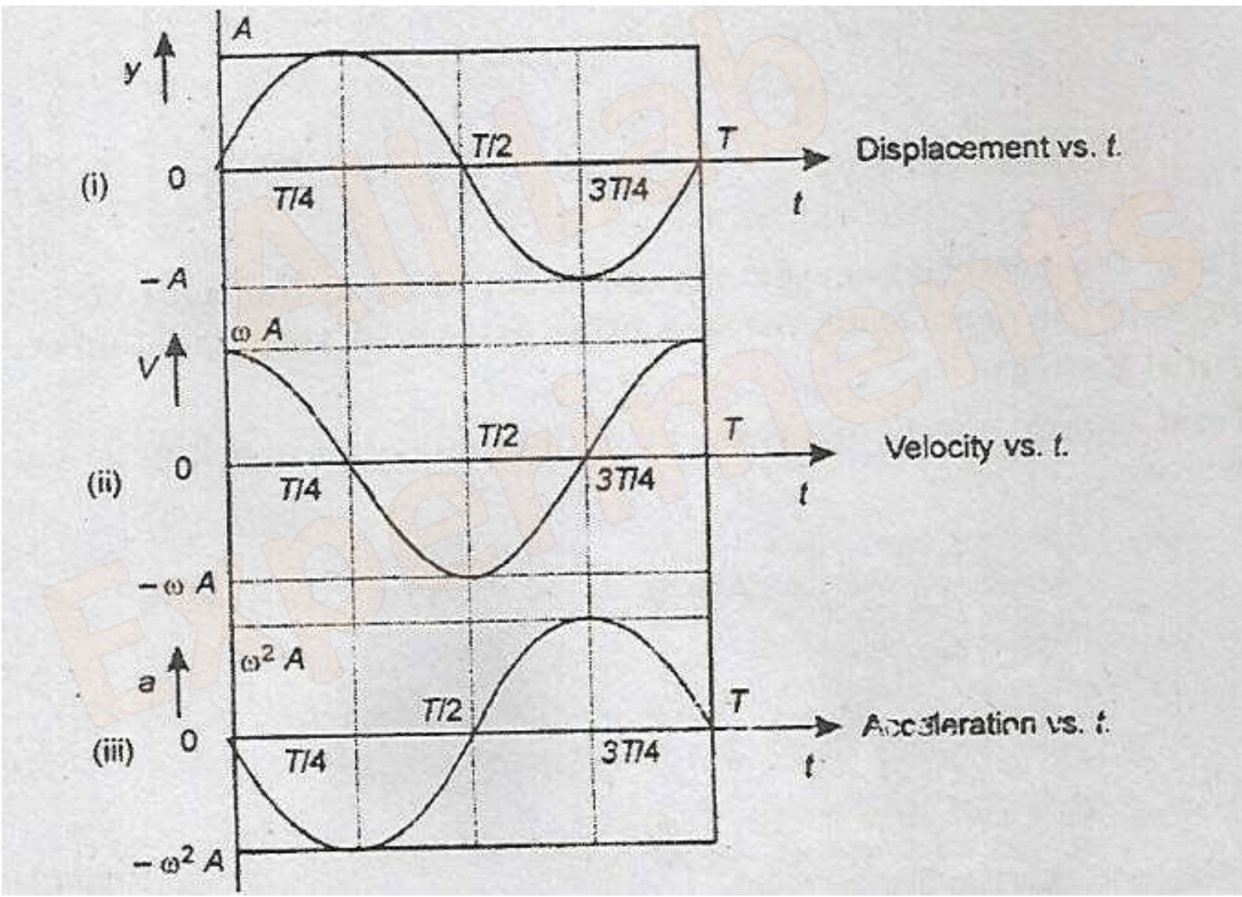
$$a = \frac{dV}{dT} = -A\omega^2 \sin(\omega t + \phi_0)$$

$$= (A\omega^2) \sin[(\omega t + \phi_0) + \pi]$$

where $(A\omega^2)$ is acceleration amplitude.

Thus, acceleration is ahead of displacement by π or half of a cycle; or acceleration is ahead of velocity by or $\frac{\pi}{2}$ or $(1/4)^{th}$ of a cycle.

The graphical variations of displacement, velocity and acceleration of the particle (executing SHM) with time is shown in the Fig. given below.



Q 4. Deriving the expressions for kinetic, potential and total energy of a particle executing S.H.M. plot graph for the same.

Ans. Kinetic Energy. The Kinetic energy of the particle is,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m \ A^2 \sin^2 (\omega t + \phi)$$

Since, $\sin^2\theta = 1 - \cos^2\theta$

and using $x = A\cos(\omega t + \phi)$ for the displacement, we can also express the kinetic energy as,

$$K = \frac{1}{2}m\omega^2 A^2\ [1 - \cos^2(\omega t + \phi)]$$

which can be written as,

$$K = \frac{1}{2}m\omega^2\ (A^2 - x^2) = \frac{1}{2}\ k\ (A^2 - x^2)$$

From this expression we can see that, the kinetic is maximum at the centre ($x = 0$) and zero at the extremes of oscillation ($x = \pm A$).

Potential Energy

To obtain the potential energy we use the relation,

$$F = -\frac{dU}{dx} \text{ or } \frac{dU}{dx} = kx \qquad (\text{as } F = -kx)$$

$$\therefore \quad \int_0^U dU = \int_0^x kx\,dx$$

$$\therefore \quad U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$$

Thus, the potential energy has minimum value at the centre ($x = 0$) and increases as the particle approaches either extreme of the oscillation ($x = \pm A$).

Total Energy

Total energy can be obtained by adding potential and kinetic energies. Therefore,

$$E = K + U = \frac{1}{2}m\omega^2\ (A^2 - x^2) + \frac{1}{2}m\omega^2 x^2$$

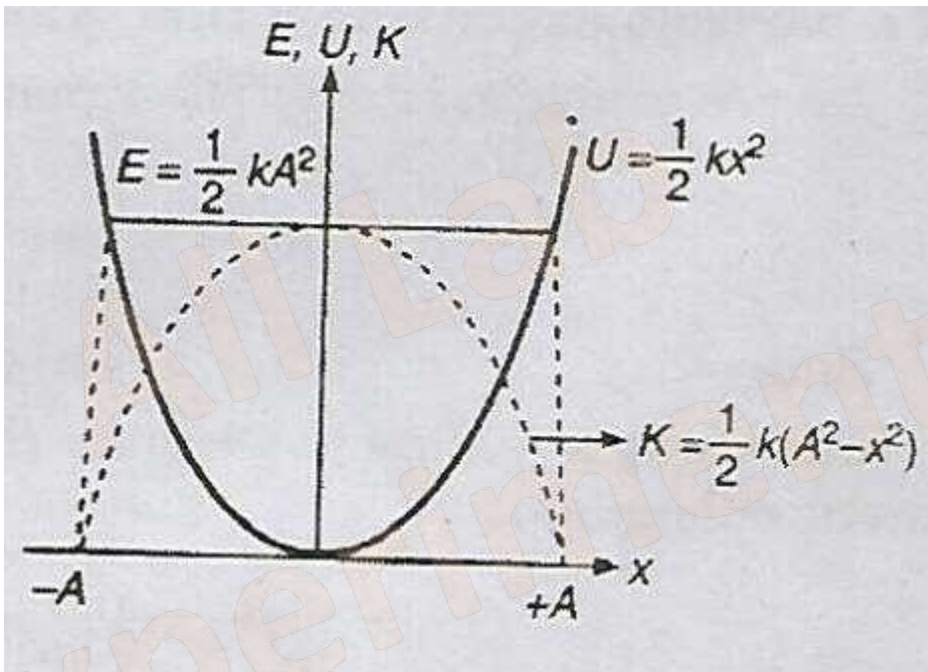
$$E = \frac{1}{2}m\omega^2 A^2$$

or

$$E = \frac{1}{2}KA^2 \qquad (\text{as } m\omega^2 = k)$$

Which is a constant quantity. This was to be expected since the force is conservative.

Therefore, we may conclude that, during the oscillations, there is a continuous exchange of kinetic and potential and kinetic energies. While moving away from the equilibrium position, the potential energy increases while the kinetic energy decreases. When the particle moves towards the equilibrium position, the reverse happens. The figure shows the variation of total energy (E), potential energy (U) and kinetic energy (K) with displacement (x)



Q 5. Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give the period for each case of periodic motion: (ω is any positive constant).

(*i*) $\sin \omega t - \cos \omega t$.

(*ii*) $\sin^3 \omega t$.

(*iii*) $3 \cos \left(\frac{\pi}{4} - 2\omega t\right)$.

(*iv*) $\cos \omega t + \cos 3\omega t + \cos 5\omega t$.

(*v*) $e^{-\omega^2 t^2}$

(*vi*) $1 + \omega t + \omega^2 t^2$

Ans. (*i*) $f(t) = \sin \omega t - \cos \omega t$; Put $1 = r\cos\theta = r\sin\theta$

$\therefore \quad f(t) = r \sin(\omega t - \theta)$

(*i*) $\quad$ as $r\sin\theta = 1,\ r\cos\theta = 1$

$$r^2(\sin^2\theta + \cos^2\theta) = 1 + 1 \text{ or } r = \sqrt{2}$$

From (*i*) we have, $\quad f(t) = \sqrt{2}\sin(\omega t - \theta) \qquad (ii)$

Again, $\quad \tan\theta = 1 \text{ or } \tan^{-1}(1) = \theta$

or $\quad \tan^{-1}\tan(\pi/4) = \theta \qquad \therefore \theta = \pi/4$

Putting in (*ii*) we get,

$$f(t) = \sqrt{2}\sin(\omega t - \pi/4) \qquad (iii)$$

Equation (*iii*) can be written as,

$$f(t) = \sqrt{2}\sin\left[\left(wt - \frac{\pi}{4}\right) + 2\pi\right] = \sqrt{2}\sin\left[w\left(t - \frac{2\pi}{w}\right) - \frac{\pi}{4}\right]$$

Thus, $f(t)$ represents SHM and its time period is givenby,

$$T = \frac{2\pi}{w}$$

(ii) $\quad f(t) = \sin 3\,\omega t = \frac{1}{4}(3\sin 3\,\omega t - \sin 3\,\omega t) \qquad (i)$

It represents two SHMs separately, but their sum does not represent SHM.

The period of function $\frac{3}{4}\sin \omega t$ is given by $T = \frac{2\pi}{w}$, and the period of function

$\sin 3\,\omega t$ is given by, $\quad T' = \frac{2\pi}{w} = \frac{T}{3}$

Thus, the least time taken by the function to repeat the same value is T.

$$T = \frac{2\pi}{w}$$

Hence $\sin^3 wt$ is an example of a periodic function which is not SHM.

(iii) $$f(t) = 3\cos\left(\frac{\pi}{4} - 2wt\right) = 3\cos\left(2wt - \frac{\pi}{4}\right)$$

$$T = \frac{2\pi}{2w} = \frac{\pi}{w}$$

Thus, $3\cos(2\omega t - \pi/4)$ represents SHM, with, $T = \dfrac{\pi}{w}$

(iv) $$f(t) = \cos\omega t + \cos 3\omega t + \cos 5\omega t$$

The given function is a combination of three independent SHMs, but their sum is not SHM.

Time periods are $\dfrac{2\pi}{w}, \dfrac{2\pi}{3w}, \dfrac{2\pi}{5w}$

Time period of $f(t)$ is given by, $T = \dfrac{2\pi}{w}$

Therefore, the above function is periodic, but not SHM.

(v) $f(t) = e^{-\omega^2 t^2}$; when $t \to \infty$, $f(t) \to 0$

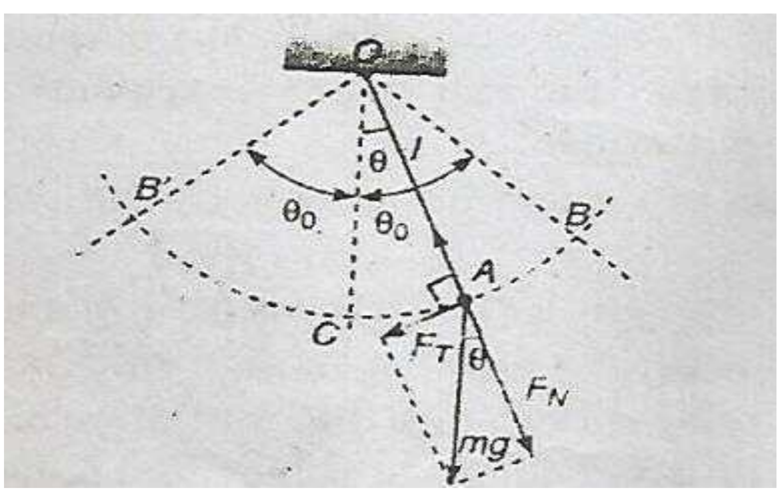
$\therefore$ It is a non-periodic function

(vi) $f(t) = 1 + \omega t + \omega^2 t^2$, when $t \to \infty$, $f(t) \to \infty$

Thus, the above function is a non-periodic function.

Q.6. what is simple pendulum? Explain in detail.

Ans. A simple pendulum is a device consisting of a spherical bob suspended by an inelastic massless 'string which is fixed to a rigid support. Let us suppose the mass of the bob be m and length of the string is l



When the particle is pulled aside to position B so that the string makes an angle θ. with the vertical OC and then released, the pendulum will start to oscillate between B and the symmetric position B'. The oscillatory motion is due to the tangential component F_T of the weight mg of the particle. This force (F_T) is maximum at the point B and B', and zero at C. Thus, we can write,

$$F_T = -mg\sin\theta$$

Here, minus sign appears because it is opposite to the displacement.

$$x = CA$$

$$\therefore \quad ma_T = -mg\sin\theta \qquad \ldots(i)$$

Here, $$a_T = l\alpha \qquad \left(\text{where } \alpha = \frac{d^2\theta}{dt^2}\right)$$

and $$\sin\theta \approx \theta \text{ for small oscillations}$$

$$\therefore \quad ml\alpha = -mg\theta$$

or $$\alpha = -\left(\frac{g}{l}\right)\theta$$

or $$\left|\frac{\theta}{\alpha}\right| = \frac{l}{g}$$

$$\therefore \quad T = 2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|}$$

or $$T = 2\pi\sqrt{\frac{l}{g}}$$

Note that the period is independent of the mass of the pendulum.

Following points should be remembered in case of a simple pendulum.

1. For large amplitudes, the approximation sin θ≈θ is not valid and the calculation of the period is more complex. The time period, in this case, depends on the amplitude q, and is given by,

$$T = 2\pi\sqrt{\frac{l}{g}\left(1+\frac{\theta_0^2}{16}\right)}$$

The angle θ_0 must be expressed in radians. This is sufficient approximation for most practical situations is a simple pendulum.

(2) If the time period of a simple pendulum is 2 seconds, it is called seconds pendulum.

(3) If the length of the pendulum is large, g no longer remains vertical but will be directed towards the center of the earth and, the expression for time can be written as,

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}}$$

Here, R is the radius of earth. From this expression we can see that,

(a) if $l << R, \frac{1}{l} >> \frac{1}{R}$ and $T = 2\pi\sqrt{\frac{1}{g}}$

(b) as $l \to \infty, \frac{1}{l} \to 0$ and $T = 2\pi\sqrt{\frac{R}{g}}$ and substituting the value of R and g we get $T = 84.6$ minutes.

(4) Time period of a simple pendulum depends on acceleration due to gravity 'g' $\left(\text{as} T \propto \frac{1}{\sqrt{g}}\right)$ so take $|\vec{g}_{eff}|$ in $T = 2\pi\sqrt{\frac{l}{g}}$. Following two cases are possible :

(i) If a simple pendulum is in a carriage which is accelerating with acceleration, $\vec{a}$, then

$$\vec{g}_{eff} = \vec{g} - \vec{a}$$

e.g., 'if the acceleration $\vec{a}$ is upwards, then

$$|\vec{g}_{eff}| = g + a \text{ and } T = 2\pi\sqrt{\frac{l}{g+a}}$$

If the acceleration $\vec{a}$ is downwards, then ($g>a$)

$$|\vec{g}_{eff}| = g - a \text{ and } T = 2\pi\sqrt{\frac{l}{g-a}}$$

If the acceleration $\vec{a}$ is in horozontal direction, then

$$|\vec{g}_{eff}| = \sqrt{a^2+g^2}$$

In a freely falling lift $g_{eff} = 0$ and $T = \infty$, *i.e.*, the pendulum will not oscillate.

(ii) If in addition to gravity one additional force $\vec{F}$, (*e.g.*, electrostatic force $\vec{F}_e$)is also acting on the bob, then in that case.

$$\vec{g}_{eff} = \vec{g} + \frac{\vec{F}}{m}$$

Here, m is the mass of the bob.

(5) Due to change in temperature, length of pendulum and so the time period will change. If $\Delta\theta$ is the increase in temperature then.

$$l' = l\,(1+\alpha\Delta\theta) \text{ or } \frac{l'}{l} = 1+\alpha\Delta\theta$$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{l'}{l}} = (1+\alpha\Delta\,\theta)^{1/2}$$

$$\approx \left(1+\frac{1}{2}\alpha\Delta\theta\right)$$

$$\therefore \quad \frac{T'}{T} - 1 \approx \frac{1}{2}\alpha\Delta\theta$$

$$\text{or} \quad \frac{T'-T}{T} = \frac{1}{2}\alpha\Delta\theta$$

$$\text{or} \quad \Delta T = \frac{1}{2}T\alpha\Delta\theta$$

Note : In case of a pendulum clock, time is lost if T increases and gained if decreases. Time lost or gained in time t is given by,

$$\Delta t = \frac{\Delta T}{T'}.t$$

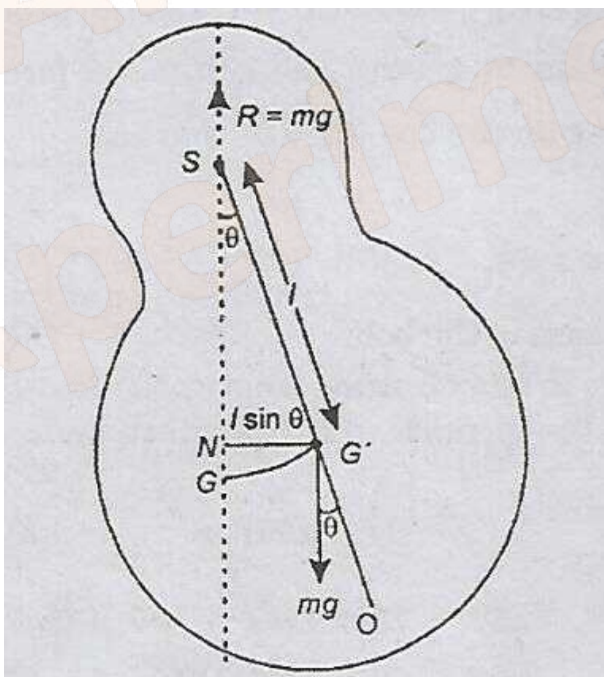
e.g., if $T = 2$ s, $T' = 3$ s, then $\Delta T = 1$ s

$\therefore$ Time lost by the clock in 1 hr. $\Delta t = \frac{1}{3} \times 3600 = 1200$ s

Q.7. Give the theory of compound pendulum. Show that there are four points of the pendulum having the same time period.

Ans. In the case of a compound pendulum, the oscillating mass has dimensions comparable to the distance between the axis of suspension and its center of gravity. As rigid body capable of oscillating in a vertical plane about a horizontal axis passing through the body other than the center of gravity is regarded as a compound pendulum.

Centre of suspension is the point of intersection of the horizontal axis of rotation and the vertical plane through the centre of gravity of the compound pendulum. This is indicated by S.



Let the Fig. represent a compound pendulum of mass m. Let *G* be the center of gravity. The distance between the center of gravity and the center of suspension is known as the length of the compound pendulum and denoted by *1*.

Thus $GS = l$, When in equilibrium, G will be below S.

Let us displace the pendulum slightly from a mean position towards one side through an angle θ. Let G" be the new position of G.

The will cause in restoring couple produced is the couple formed by weight W= mg acting through the center of gravity G vertically downward and the reaction $R = mg$ acting vertically upward at the point of suspension S.

The restoring couple = $—mg\ (G'\ N) = mgl$ sinθ. The negative sign indicates that the couple is oppositely directed to the displacement θ.

If angular displacement θ is small, then sinθ = θ, so that restoring couple = mglθ

If I is the moment of inertia of the body about the horizontal axis passing through S then

$\alpha = d^2\theta/dt^2$ is the angular acceleration.

And we can write,

$$\text{deflecting couple} = I\alpha = I\,\frac{d^2\theta}{dt^2}.$$

Since mg is the weight of the pendulum, its equation of motion can be written as

$$I\frac{d^2\theta}{dt^2} = -mgl\,\theta,$$

or

$$\frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\theta$$

or

$$\frac{d^2\theta}{dt^2} = -\omega^2\,\theta, \quad \text{where } \omega^2 = \frac{mgl}{I}.$$

or

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0.$$

This is the differential equation of motion of the compound pendulum. The solution of the above equation is,

$$\theta = A \sin(\omega t + \phi) \quad ...(i)$$

and $$\frac{d\theta}{dt} = A\, w \cos(\omega t + \phi) \quad ...(ii)$$

The values of constants A and ϕ can be determined from the initial conditions.

At $$t = 0, \ \theta = \theta_0$$

and $$\frac{d\theta}{dt} = 0$$

(i) Gives us, $$\theta_0 = A \sin(\omega t + \phi)$$

(ii) Gives us, $$0 = A\,\text{w} \cos(\omega t + \phi)$$

or $$0 = \cos\phi \qquad [\because t = 0]$$

$\therefore$ $$\phi = \pi/2$$

$\therefore$ $$\theta = A \sin(\omega t + \pi/2)$$

or $$\theta = A \cos \omega t$$

Since $$\theta = \theta_0 \text{ at } t = 0,$$

$$\theta_0 = A \cos 0^{\circ}$$

or $$\theta_0 = A$$

$\therefore$ $$\theta = \theta_0 \cos \omega t$$

Here θ represents the angular amplitude and ω represents the initial phase.

$$\theta = \theta_0 \cos \omega t$$

If we replace t by $\left(t + \frac{2\pi}{\omega}\right)$, the equation does not change.

Therefore, time period, $T = \frac{2\pi}{\omega}$

or $$T = \frac{2\pi}{\sqrt{mgl/I}}$$

or $$T = 2\pi\sqrt{\frac{I}{mgl}} \quad ...(iii)$$

where I is the moment of inertia.

The above relation gives us the time period of a compound pendulum.

If I_G is the moment of inertia of the compound pendulum about an axis passing through G, then

$$\begin{aligned} I &= I_G + ml^2 \qquad \text{[Using axis theorem]} \\ &= mK^2 + ml^2 \\ &= m(K^2 + l^2) \end{aligned}$$

Here K is the redius of gyration of the pendulum about the parallel axis through G.

The relation (*iii*) Gives us,

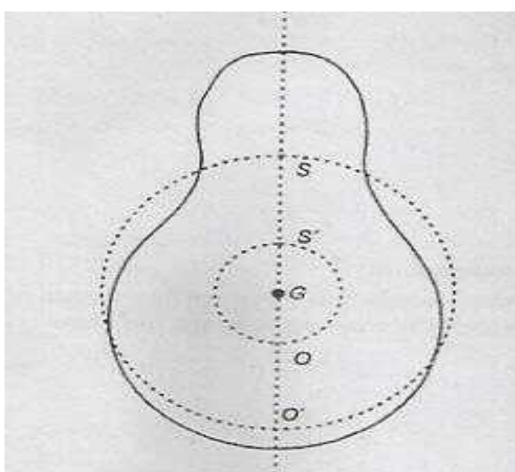
$$T = 2\pi\sqrt{\frac{m(K^2 + l^2)}{mgl}}$$

$$T = 2\pi\sqrt{\frac{l + \frac{K^2}{l}}{g}}$$

If we represent $l + \frac{K^2}{l} = L$

Then, $$T = 2\pi\sqrt{\frac{L}{g}}$$

The length, L, is known as the effective length of the compound pendulum. It is also known as the length of equivalent simple pendulum.



We take a point on the side opposite to S from G at a distance $\frac{k^2}{l}$ from G.

This point is known as centre of oscillation.

A horizontal axis passing through 0 parallel to the axis of suspension is own as axis of oscillation.

Now $$SO = SG' + G'_0$$

$$= 1 + \frac{k^2}{l}$$

Let $$G'_0 = \frac{k^2}{l} = l'$$

Then, $$T = 2\pi\sqrt{\frac{l+l'}{g}}$$

or $$T = 2\pi\sqrt{\frac{L}{g}}$$

It should be noted that the point of suspension and the centre of oscillation interchangeable.

In this case,

$$T' = 2\pi\sqrt{\frac{l' + (k^2/l')}{g}}$$

$$l' = \frac{k^2}{l} \text{ and hence } \frac{k^2}{l'} = l.$$

$$T' = 2\pi\sqrt{\frac{l'+l}{g}} = T$$

This confirms interchangeability.

In the case of a compound pendulum there are four points collinear with the centre of gravity about which the time periods are the same.

We have, $$T = 2\pi\sqrt{\frac{\left(l+\frac{k^2}{l}\right)}{g}}$$

Squaring both sides, $$T^2 = \frac{4\pi^2}{g}\left(l+\frac{k^2}{l}\right)$$

or $$l^2-\left(\frac{gT^2}{4\pi^2}\right)l+k^2 = 0.$$

Let l_1 and l_2 be the roots of this quadratic equation

$$l_1+l_2 = \frac{gT^2}{4\pi^2}$$

and $$l_1 l_2 = k^2$$

[If $ax^2+bx+c=0$, $\alpha_1+\alpha_2=-\frac{b}{a}$ and $\alpha_1\,\alpha_2=\frac{c}{a}$] .

The above relations indicates that both l_1 and l_2 are positive. If $l_1 = l$, then $l_2 = \frac{k^2}{l}$.

Now l_1 and l_2 are both interchangeable. The pendulum will have the same time period whether suspended at a distance of l or $\frac{k^2}{l}$ from G. We will get two such points on either side of G.

Let us draw two circles of radii e and $\frac{k^2}{l}$ with G as centre. Through G draw a vertical line which intersects these circles at four points S, S', O, O' such that $SG=GO'=l$ and $GS'=GO=k^2/l$. Therefore $SO=S'O'=l+k^2/l$. Hence there are four points collinear with the centre of gravity about which the period of oscillation is the same.

Now $$T = 2\pi\sqrt{l+\frac{k^2/l}{g}}$$

$$T = 2\pi\sqrt{\frac{l+k^2/l}{g}}$$

$$T^2 = \frac{4\pi^2}{g}\left(\frac{k^2 + l^2}{l}\right)$$

Differentiat l

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left(-\frac{k^2}{l^2} + 1\right)$$

For T to be maximum or minimum,

$$\frac{dT}{dl} = 0$$

This is possible when $k^2 = l^2$

or $$k = \pm l$$

Now $\frac{d^2T}{dl^2}$ is positive, therefore it is a minima.

$$\therefore \quad T_{\min} = 2\pi\sqrt{\frac{2k}{g}}$$

When $l = 0$ or $l = \infty$, $T = \infty$.

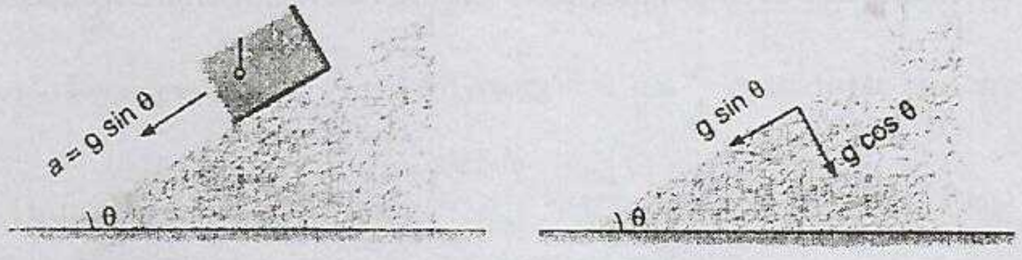
But $l \neq \infty$, $\therefore$ $l = 0$ is the only condition for maximum time period.

And $$T_{\max} = \infty$$

This means that the axis of suspension should pass through G, i.e. G should be the point of suspension. Now the pendulum will be in the state of netural equilibrium. There is no restoring action due to gravity on it.

Q.8. A simple pendulum of length l is suspended from the ceiling which is sliding without friction on an inclined plane of inclination θ. What will be the time period of the pendulum?

Ans. Since the point of suspension of the simple pendulum has an acceleration of **a** = g sinθ, down the plane. We can resolve g into two components g sinθ along the while g cosθ perpendicular to the plane.



$$\vec{g}_{eff} = \vec{g} - \vec{a}$$

$$= g\cos\theta \qquad \text{(perpendicular to plane)}$$

$$T = 2\pi\sqrt{\frac{l}{|\vec{g}_{eff}|}}$$

$$= 2\pi\sqrt{\frac{l}{g\cos\theta}}$$

Note : If $\theta = 0°$, $T = 2\pi\sqrt{\frac{l}{g}}$ which is quiet obvious.

Q.9 (a) A simple pendulum consists of a small sphere of mass m suspended by a thread of length 1. The sphere carries a positive charge θ. The pendulum is placed in a uniform electric field of strength *E* directed vertically upwards. With what period will pendulum oscillate if the electrostatic force acting on the sphere is less than the gravitational force? (b) Discuss the representation of simple harmonic motion by a complex exponential.

Ans. (a) Here, two forces are acting on the bob of pendulum shown in figure and g_{eff} is given by

will be $\frac{w - F_e}{m}$

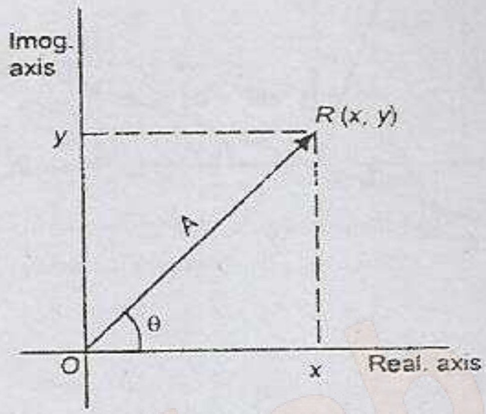
or $$g_{eff} = \frac{mg - qE}{m} = g - \frac{qE}{m}$$

$\therefore$ $$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$$= 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}$$

(b)

Ans. The displacement in simple harmonic motion can be represented by a rotating vector.



Let A complex No. $Z = x + iy$ be represented by a point R in the complex plane. This number Z can also be represented by the vector $\vec{OR}$ directed from origin O to R. Let the plane polar coordinater of vector $\vec{OR}$ be (A, θ), where A is the magnitude of the vector $\vec{OR}$ and θ is the angle it makes with the real axis. Now since,

$$x = A\cos\theta$$

$$y = A\sin\theta$$

we have

$$z = x + iy = A(\cos\theta + i\sin\theta)$$

$$= Ae^{i\theta}$$

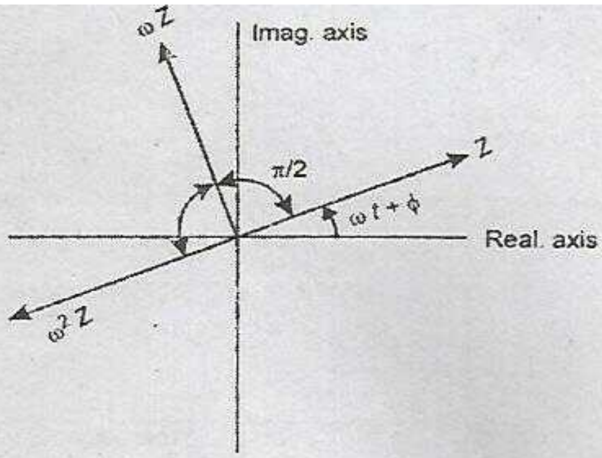
where $\theta = \omega t + \phi$ and A is a constant.

The term $A\,e^{i\theta}$ signifies a vector of constant magnitude A rotating at an angular frequency ω. Either the real or imaginary part represents a quantity varying harmonically with time.

The solution of the equation can be expressed as a combination of sin wt and cos wt by

$$z = A^{e^{i(\omega t+\phi)}} = A\cos(\omega t + \phi) + \sin(\omega t + \phi)$$

The significant property of the exponential function is that the function itself reappears after every operation of differentiation or integration. If we use the exponential function z and adopt the convention to use only the real part of the function to check physical measurements, the real parts of z, z' and z'' represent the displacement ψ, velocity ψ' and acceleration ψ'' of the motion.



Rotating vector representation of displacement, velocity and acceleration

$$z' = i\,w\,z = w\,z\,e^{i\pi/2}$$

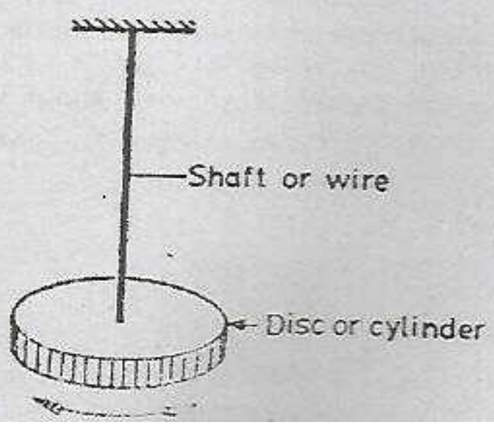
and $$z'' = -w^2 z = w^2 z\,e^{i\pi}$$

the velocity leads the displacement by $\frac{\pi}{2}$ and acceleration leads the displacement by π. Thus, differentiation involves counter-clockwise rotation of the vector by $\frac{\pi}{2}$ and multiplication by w.

Q.10. **explain the torsion pendulum in detail.**

Ans. A device consisting of a disk of large moment of inertia mounted on one end of a torsionally flexible elastic rod and other end is held fixed. When this disk is twisted and unconfined it will undergo simple harmonic motion provided the torque in the rod that is proportional to the angle of twist. This whole arrangement is called as torsion pendulum.

The bob in this case is a heavy cylinderical disc instead of a sphere. The suspending material is a wire which is suspended from a rigid support.



The disc is twisted on the side and released. Now it will execute torsional to and fro motion about the shaft which fixed with the disc as its axis.

If the disc is turned through an angle θ, the shaft is also twisted through the same angle θ. A restoring torsional couple = -τθ comes into play, which tends to bring the pendulum back to its original position. Here τ is the torsional couple per unit twist. If I is the moment of inertia of the disc about the shaft as the axis

and $\frac{d^2\theta}{dt^2}$, its angular acceleration, the couple due to the acceleration is given by $I\frac{d^2\theta}{dt^2}$. In the dynamic equilibrium, both these couples must balance, giving

$$I\frac{d^2\theta}{dt^2} = -\tau\theta$$

or

$$\frac{d^2\theta}{dt^2} = -\frac{\tau}{I}\theta$$

Thus the disc executes SHM and its time period is given by

$$T = 2\pi\sqrt{\frac{I}{\tau}}$$

From mechanics of materials, τ, in case of shaft is given by

$$\tau = \frac{\pi\eta d^4}{32l}$$

Where d is the diameter of the shaft, l its length and h is the modulus of rigidity of its material. In the case of a wire of radius r, length l and modulus of rigidity η, we have

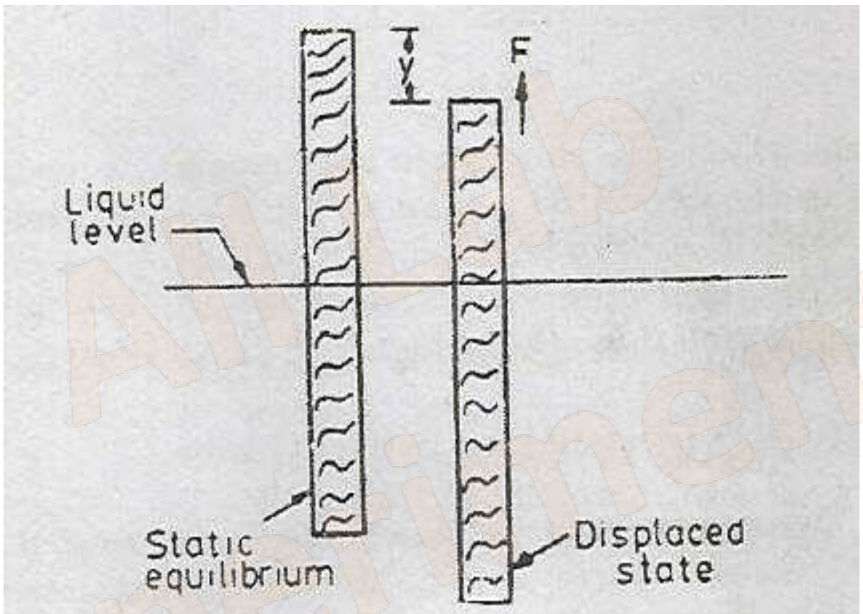
$$\tau = \frac{\pi\eta r^4}{2l}$$

Q.11. **explain the concept of an object floating in a liquid.**

Ans The upward force exerted by a fluid that opposes the downward force due to the weight of an immersed object. If the density of an object in the fluid is greater than the density of the fluid, the object will sink. If the density is less than that of the fluid, the object will float upward due to the buoyancy of the fluid.

An object of lower density will float to the top and only be submerged by an amount according to the ratio of the densities.

Whenever an object floating in a liquid is displaced vertically, say, by pressing it down and then released, it exhibits an up-down motion which is simple harmonic in nature. Consider a pole of cross-sectional area A and mass M floating in a vertical position in a liquid of density ρ. This is the static equilibrium state because the weight of the pole is balanced by the weight of the liquid it displaces. If we displace the pole by a distance y (by dipping it further in the liquid), the buoyant force on the pole increase by ρAgy because ρAy, is the mass of the liquid displaced



by this further dipping; g being the acceleration due to gravity. We have neglected viscous effects. The restoring force F on the pole is given by

$$F = -\rho A g y = -K y$$

where $K = \rho A g$ is the force constant. The angular frequency of the resulting harmonic oscillations is given by

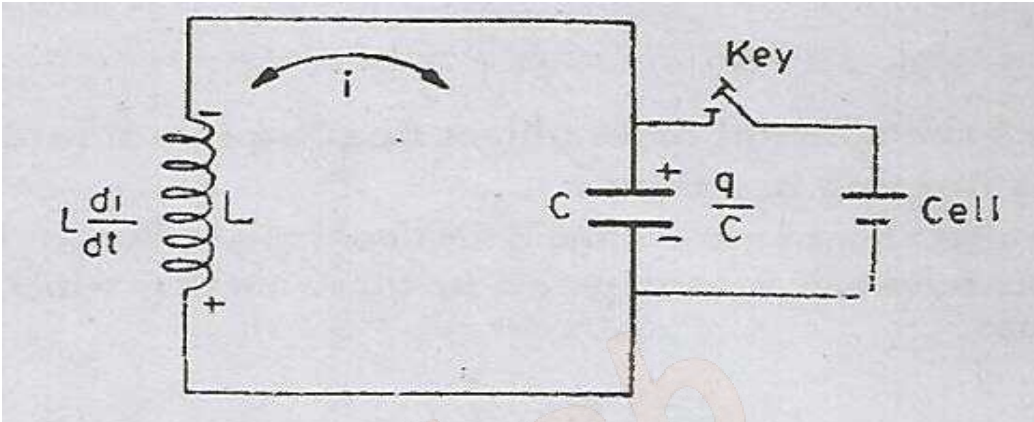
$$\omega = \sqrt{\frac{\rho A g}{M}}$$

and period

$$T = 2\pi \sqrt{\frac{M}{\rho A g}}$$

Q.12. **Discuss Inductor-capacitor electrical circuit as S.H.M.**

Ans. The inductor and capacitor circuit system also exhibit simple harmonic motion. This circuit is also known an oscillatory tank circuit or simply oscillatory circuit in electrical systems.



The oscillatory circuit

The equilibrium state of this circuit is the state when the capacitor is uncharged and no current is flowing in the circuit. This state is disturbed by pressing the key which allows the current to pass through a capacitor, thus charging the capacitor. Let q be the charge on the capacitor so that V = qlC is the voltage across the capacitor plates, When the key is released, the capacitor starts discharging through the inductor, i.e. the charge changes with time and a current $i = dq/dt$ established in the inductor.

In this circuit, the restoring force is due to the force of repulsion between electrons. This force tries to distribute electrons equally on the capacitor plates, Inductance, on the other hand, tends to oppose this redistribution, i.e. it opposes the increase of current. At any instant of time, the voltage across the inductor is given by

$$V = -L\frac{di}{dt} = -L\frac{d^2q}{dt^2}$$

The minus sign indicates that the voltages opposes the increase of current. From Kirchhoff's law this voltage must equal the voltage q/C across the capacitor plates, giving,

$$-L\frac{d^2q}{dt^2} = \frac{q}{C}$$

or
$$\frac{d^2q}{dt^2} = -w^2q$$

with
$$\omega = \frac{1}{\sqrt{LC}}$$

Thus, in an electrical circuit consisting of an inductance L and a capacitance C, the charge oscillates harmonically with an angular frequency $\omega = 1/\sqrt{LC}$ and period $T = 2\pi\sqrt{LC}$. At any instant of time, the charge q is given by

$$q = q_0 \cos(\omega t + \phi)$$

where q_0 is the maximum value of the charge and ϕ is the phase of electron osicllations. The current in the circuit is given by

$$i = \frac{dq}{dt} = -\omega q_0 \sin(\omega t + \phi)$$

or

$$i = -i_0 \sin(\omega t + \phi)$$

where $i_0 = \omega q_0$, is the maximum value of the current. If V_0 is the applied voltage,

$$i_0 = V_0\sqrt{\frac{C}{L}}$$

since $q_0 = CV_0$ and $\omega = 1/\sqrt{LC}$

Let us assume that initially the capacitor C carries a charge q and the current in inductor L is zero. At this instant, the electrostatic energy stored in the capacitor is

$$Ee = \frac{1}{2}\frac{q^2}{C}$$

and that in the inductance is zero since i = 0 initially. As time passes, the capacitor starts to discharge through the inductance and a current i = dq/dt is established in the inductor. As q decreases, Ee decreases and i increases, so that the energy now appears around inductance as the current is building up. When the capacitor is completely discharged, the magnetic energy,

$$Em = \frac{1}{2} Li^2$$

associated with inductance is maximum because the current is maximum and $E_e = 0$ since $q = 0$. Thus, although at this time $q = 0$, $\frac{dq}{dt}$ is not zero, it is, in fact, maximum. The large current flow due to inductor starts transporting charge to the capacitor plates and the capacitor is charged again. The capacitor starts discharging again and the current now flows in the opposite direction. Eventually, the current returns to its initial value and the process continue. The energy exchange occurs between the electric field of the capacitance and the magnetic field of the inductance. The total energy of the system is conserved since the system considered here does not contain any resistive component so that there is no dissipation of energy. Thus

$$E = Ee + Em$$

$$= \frac{1}{2}\frac{q^2}{C} + \frac{1}{2} L\, i^2 = \text{constant}$$

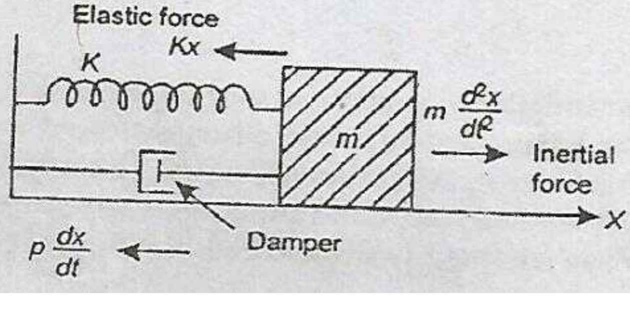
Differentiating and setting $\frac{dE}{dt} = 0$, yields $\left(\text{using } i = \frac{dq}{dt}\right)$

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

Q.13. derive the differential equation for damped harmonic motion Derive its possible solutions. Derive expressions for Relaxation time Logarithmic decrement and quality factor.

Ans. Damping is the decrease in the amplitude or energy due to any kind of dissipation process of an oscillating body and the oscillations are known as damped. A resistive or viscous element is normally the cause of damping in oscillation. When the damping force, or frictional force, is too small to change the amplitude significantly, to the undamped motion of the body, the system is called damped harmonic oscillator. In such a case the damping force is proportional to the velocity of the vibrating body.

Differential equation of damped harmonic motion. The mass-spring system is a common example of damped harmonic oscillator. Here the mass, m, oscillates under a spring force constant, k.



Let us displace the mass from its equilibrium position and then release it. The mass will move under the effect of following forces—

(*i*) a restoring force $-kx$, where k is the restoring force constant, x is the displacement. It acts in the opposite direction to the displacement and

(*ii*) a damping force $-pdx/dt$, where p is the coefficient of the damping or frictional force and dx/dt is the velocity of the moving body. Damping force is in opposite direction to motion. The total force acting on the body is given by

$$F = -kx - p\frac{dx}{dt}$$

If $\frac{d^2x}{dt^2}$ is the instantaneous acceleration of the mass, then according to Newton's 2nd law,

$$m\frac{d^2x}{dt^2} = -kx - p\frac{dx}{dt} \qquad ...(i)$$

The above relation is valid for small displacements and for small velocities. Relation (*i*) can be written as

$$m\frac{d^2x}{dt^2} + p\frac{dx}{dt} = +kx = 0 \qquad ...(ii)$$

$$or \quad \frac{d^2x}{dt^2} + \frac{p}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

Putting $\quad \frac{p}{m} = 2\gamma$

And $\quad \frac{k}{m} = \omega^0$, we have

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = 0$$

2 γ is the damping force per unit mass at an instant when the vibrating body is moving with unit velocity. The constant ω_0 represents the angular frequency in the absence of damping and is called the natural frequency of the oscillator. Relation (*ii*) can be reduced to

$$(D^2 + 2\gamma D + \omega_0^2)x = 0 \text{ or } D^2 + 2\gamma D + \omega_0^2 = 0$$

where D is the differentia operator $\left(\frac{d}{dt}\right)$

Solving the above quadratic, we get

$$D = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

$$\left.\begin{aligned} \alpha &= -\gamma + \sqrt{\gamma^2 - \omega_0^2} \\ \beta &= -\gamma - \sqrt{\gamma^2 - \omega_0^2} \end{aligned}\right\}$$

$$\therefore \qquad \frac{dx}{dt} = \alpha x \qquad \text{...}(iii)$$

$$\text{and} \qquad \frac{dx}{dt} = \beta x \qquad \text{...}(iv)$$

Integrating Relation (*iii*), we have $x = C_1 e^{\alpha t}$

Integrating Relation (*iv*), we have $x = C_2 e^{\beta t}$

The general Solution will be a linear combination of both the above solutions

$$x = C_1 e^{\alpha t} + C_2 e^{\beta t}$$

$$= C_1 \exp\left[\left(-\gamma + \sqrt{\gamma^2 - \omega_0^2}\right)t\right] + C_2 \exp\left[\left(-\gamma - \sqrt{\gamma^2 - \omega_0^2}\right)t\right]$$

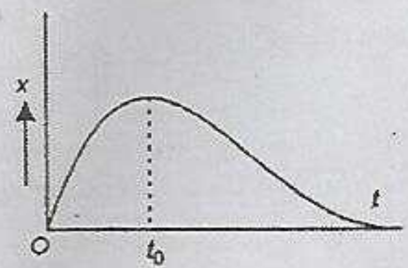
$$\text{or } x = e^{-\gamma t}\left[C_1 \exp\left(\sqrt{\gamma^2 - \omega_0^2}\right)t + C_2 \exp\left(-\sqrt{\gamma^2 - \omega_0^2}\right)t\right]$$

The initial conditions will determine the constants C_l and C_2 There can be three different options to its solutions—

1. Over-damped motion (or dead beat)
2. Critically damped motion
3. Under damped motion.

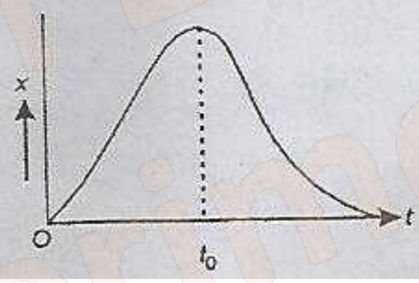
Over-damped motion, $\gamma > \omega_0$. Here the damping term γ dominates the stiffness term ω_0 and the term $\sqrt{\gamma^2 - \omega_0^2}$ is real and positive quantity.

There is exponential decay of x till it becomes zero.



Displacement x decays exponentially with time, eventually becoming zero i.e. the displacement increases until time $t = t_0$, which occurs when $dx/dt = 0$, after which it slowly becomes zero. The displacement x never becomes negative and there is no *oscillation* at all. Such a motion is called dead *beat* or *over damped*. Fig. shows the behaviour of the over-damped motion.

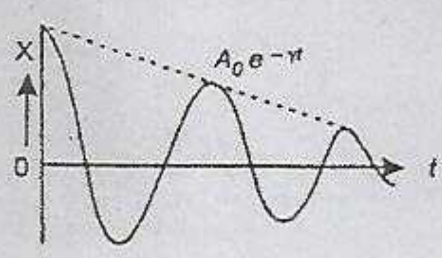
Critically damped motion, $\gamma = \omega_0$. With this condition the roots become $\alpha = \beta = -\gamma$.



t_0

Here the oscillator attains equilibrium position more rapidly than in the 1st case.

Under damped motion. Here $\gamma < \omega_0$



The oscillation is not simple harmonic as 'amplitude' $A_0\, e^{-\gamma t}$ is not constant ...t decreases exponentially with time. The motion is not even periodic as it ...ver repeats itself. The angular frequency of oscillation $w = \sqrt{\omega_0^2 - \gamma^2}$ is less ...an the natural. angular frequency of undamped oscillations.

Relaxation Time. The decay can be characterized by the time constant Γ, ...o called the *damping time* or *relaxation time*. It is defined as the time required ...ergy to drop to $e^{-1} = 0.368$ of its initial value E_o. Thus when $t = \Gamma$, then

$$\frac{E_0}{e} = E_0 e^{-2\gamma\Gamma} \text{ or } 2\gamma\Gamma = 1$$

and
$$\Gamma = \frac{1}{2\gamma}$$

Γ is also called energy decay time and comparing it with amplitud time ...$= 1/\gamma$, we get relation between the two, as $\Gamma = \tau/2$.

Logarithmic Decrement ($\lambda_d = \gamma T = pT/2$ m, is the logarithmic ...crement). This is the measure of the rate of amplitude decay.

We have,
$$x = A_0\, e^{-\gamma T} \sin(\omega t + \delta)$$

$$\sin(\omega t + \delta) = \sin(\omega t + \pi/2)$$
$$= \cos \omega t$$
$$x = A_0\, e^{-\gamma t} \cos \omega t$$

so that at $t = 0$, $x = A_0$. Let A_1, A_2, A_3 ....be the amplitudes at time $t = T, 2T,$ ...T... where T is the period of oscillation. Then

$$A_1 = A_0\, e^{-\gamma T}$$
$$A_2 = A_0\, e^{-2\gamma T}$$
$$A_3 = A_0\, e^{-3\gamma T}$$

or
$$\frac{A_0}{A_1} = \frac{A_1}{A_2} = \frac{A_2}{A_3} = \ldots . e^{\gamma T} = e^{\lambda d}$$

$$\therefore \quad \lambda_d = \log_e\left(\frac{A_0}{A_1}\right) = \log_e\left(\frac{A_1}{A_2}\right) = \log_e\left(\frac{A_2}{A_3}\right) = \ldots ..$$

The *logarithmic decrement is the lograithm of the ratio of two amplitudes of ...llation which are separated by one period.*

Quality Factor (Q). This is a dimensionless parameter and measures the ...e of decay of energy. It also characterizes the degree of damping of an oscillator. ...factor is defined as

$$Q = \frac{\omega}{2\gamma} = \omega\Gamma$$

where $\omega = \sqrt{\omega_0^2 - \gamma^2}$ is the angular frequency of the damped oscillation.

The decay of energy depends on the average energy stored and the average rate of loss of energy.

$$\frac{d}{dt} < E(t) > \;=\; <P(t)> = 2\gamma < E(t) >$$

If we consider the energy dissipated in time period T_d, where, T_d is given by:

$$T_d = \frac{2\pi}{\omega}$$

Then,

$$2\gamma T_d < E(t) \;=\; \frac{2\pi}{\omega} \cdot 2\gamma < E(t) > = \frac{2\pi}{Q} < E(t) >$$

$$= \frac{2\pi}{Q} \cdot \text{(Average energy stored)}$$

$$Q = 2\pi \frac{\text{Average energy stored in one period}}{\text{Average energy dissipated in one period}}$$

Thus the quality factor of a damped harmonic oscillator may be defined as 2π *times the ratio between average energy stored and average energy lost per period*. But we are concerned with the situation in which $\gamma << \omega_0$ and thus as a very close approximation, we can write $\omega \cong \omega_0$, we get

$$Q \cong \frac{\omega_0}{2\gamma}$$

Which is a constant for a damped system.

Now Q is a dimensionless pure number and is greater than unity. The value of damping is inversely proportional to the value of Q.

As $\quad \gamma \to 0, Q \to \infty$.

$$x = A_0 e^{(-\omega_0 t/2Q)} \sin(\omega_0 t + \delta)$$

The average energy of the oscillator is expressed as

$$< E(t) > \;=\; E_0 e^{(-\omega_0 t/2Q)}$$

Thus Q is closely related to number of oscillations over which the energy falls to $(1/e)$ of its original value E_o. This happens in time $t = \Gamma$, where T is given by

$$\frac{\omega_0 \Gamma}{Q} = 1 \text{ or } Q = \omega_0 \Gamma$$

For an undamped oscillator Γ is infinite and Q-factor is infinite.

$$\Gamma = \frac{Q}{\omega_0} = \frac{T}{2\pi}Q$$

where T is the period of oscillator. During the time Γ, the number of complete oscillations is given as

$$v = \frac{\omega_0}{2\pi}\Gamma = \frac{Q}{2\pi}$$

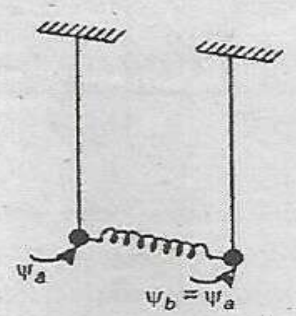
Q 14. Define degrees of freedom and normal coordinates. Explain by giving examples.

Ans. Degrees of freedom in a system is the number of independent coordinates required to completely explain the system. If we consider a system of n. values of $x_1, x_2, \ldots x_n$ then there will be n degree of freedom in the system.

At the same time the system of sun $\sum_{i=1}^{n}(x_i - \bar{x})^2$ of n numbers will have $(n - 1)$ degrees of freedom. This is due to fact that $\bar{x}$ is known and only $(n-1)$ values can be assigned to the system.

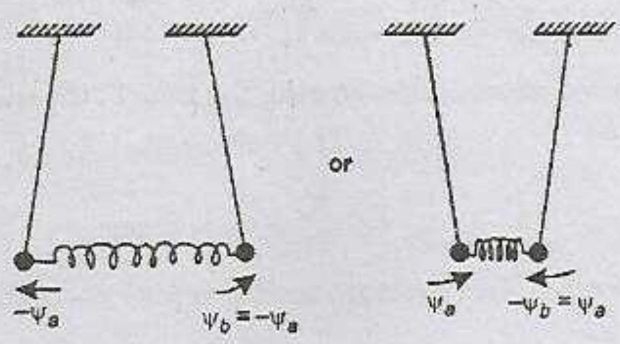
Normal Co-ordinates: The normal coordinates of a coupled system are those parameters in terms of which three equations of motion of the system can be written as a set of linear differential equations along with constant coefficients. Normally these are used in a coupled system.The simple harmonic motion associated with each normal coordinate is called normal mode of the coupled system. Each normal mode has its own characteristic frequency called the normal mode frequency. Suppose we have two simple pendulums coupled by a spring attached to the bobs as shown below. These will be "In phase" mode or "Out of phase" mode. In our example, there are two normal modes, one associated with normal ordinate X and the other with **Y**

In Phase Mode.



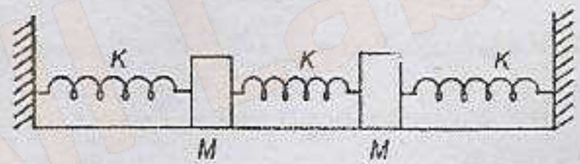
$$X = \Psi_a + \Psi_b \text{ is the normal co-ordinate}$$

"Out of Phase" Mode :



$$Y = \Psi_b - \Psi_a \text{ is the normal co-ordiante}$$

Q 16. Two equal masses M are connected with three springs of same spring constant. Calculate frequencies of oscillation in longitudinal mode.

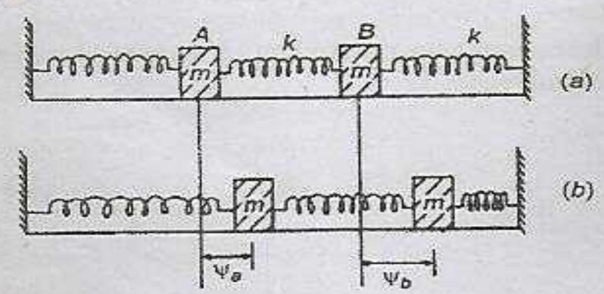


Ans. The Figure shows the equilibrium state of the system. We assume that the springs are massless. Let the length of each spring be a.

Figure (b) depicts the general configuration of the oscillating system.

Let Ψ_a and Ψ_b be the displacements of the masses A and B at any instant of time. Assuming that at this instant $\Psi_b > \Psi_a$ the equation of motion for a general configuration are

$$m\ddot{\Psi}a = k\Psi_a + k(\Psi_b - \Psi_a)$$
$$m\ddot{\Psi}a = k\Psi_b - k(\Psi_b - \Psi_a)$$



(a) *Equilibrium state, and*
(b) *General configuration*
Longitudinal oscillation

\ can find the frequencies of the normal modes by guessing the normal mode configurations from symmetry considerations. Assume the existence of a normal mode at angular frequency ω and phase constant f. This implies that

$$\Psi_a = A\cos(\mathrm{w}t + \mathrm{f})$$
$$\Psi_b = B\cos(\mathrm{w}t + \mathrm{f})$$

Substituting in Eq. (*i*), we have

$$\frac{\Psi_b}{\Psi_a} = \frac{2k/m - \omega^2}{k/m} \quad ...(iii)$$

Similarly Eq. (*ii*) gives

$$\frac{\Psi_b}{\Psi_a} = \frac{k/m}{2k/m - \omega^2} \quad ...(iv)$$

Equating the right-hand sides of Eqs (*iii*) and (*iv*), we have

$$\left(\frac{2k}{m} - \omega^2\right)^2 = \frac{k^2}{m^2}$$

or

$$-\omega^2 + \frac{2k}{m} = \pm\frac{k}{m}$$

$$-\omega^2 = \pm\frac{k}{m} - \frac{2k}{m}$$

1st solution

$$-\omega^2 = \frac{k}{m} - \frac{2k}{m}$$

$$= -\frac{k}{m}$$

$$\omega^2 = \frac{k}{m}$$

2nd solution

$$-\omega^2 = -\frac{k}{m} - \frac{2k}{m}$$

or

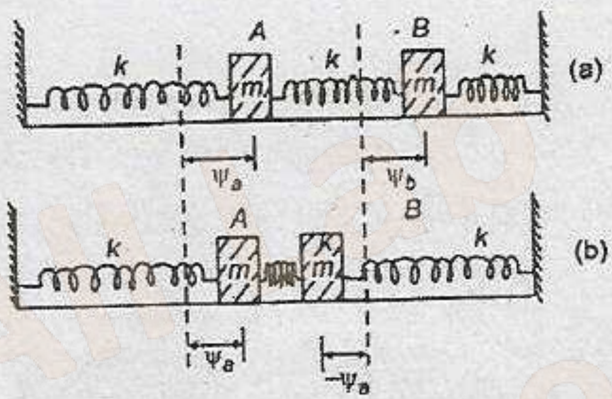
$$-\omega^2 = -\frac{3k}{m}$$

$$\omega^2 = \frac{3k}{m}$$

These are the angular frequencies of the two normal mode. The shape or configuration of mode 1 with frequency ω_1 is obtained from either Eq. (*iii*) or Eq. (*iv*) by setting $\omega^2 = R/m$.

$$\left(\frac{\Psi_b}{\Psi a}\right)_{\text{mod e 1}} = +1 \quad \text{...(v)}$$

$$\left(\frac{\Psi_b}{\Psi a}\right)_{\text{mod e 2}} = -1 \quad \text{...(vi)}$$



(a) Mode with lower frequency

(b) Mode with higher frequency

Normal modes of longitudinal oscillations

The diagram (*a*) above represents (*v*) and (*b*) represents (*vi*) **Hence proved**

UNIT-7
NON-INERTIAL SYSTEMS

Q 1. What is frame of reference? Define Non inertial, fictitious force and inertial frame of reference.

Ans. Frame of reference (or reference frame) is defined as the set of abstract coordinate system used to specify the position, motion and others standardize measurements of any object in space.

1. **Non-Inertial Frame of reference**: The frame of reference that is undergoing acceleration with respect to an inertial frame. If there is an accelerometer at rest in a non-inertial frame then it will be generally detect non-zero acceleration. Therefore an accelerating frame of reference with respect to an inertial frame of reference is non-inertial.

   Consider that a frame of reference S' is accelerating with an acceleration a_0 with respect to the inertial frame S. If no force acts on a particle P, it has zero acceleration as observed by an observer in frame S but the observer in S' will find that the acceleration of P relative to it is $-a_o$.
   It means that the observer in frame S' will observe that a force $F_o = -ma_o$ acts on the particle P. But actually no external force is acting on P. Such a force which does not really act on a particle but appears to act due to the acceleration of the frame is called a fictitious force.

   $$F_o = -ma_o$$

   Where m is the mass of the particle P
   Suppose an external force F_i acts on the particle P. It will experience acceleration, let this be a_i. in the inertial frame.

   $$F_i = m\ a_i.$$

   The apparent force F acting on the particle is observed in the non-inertial reference frame S'.

   $$F = F_i + F_0 = ma_i + (-rna_0)$$

   Therefore fictitious force is that quantity which must be added to the real force acting on the particle in the inertial frame to give the value of apparent force in the non-inertial frame of reference.
   When $ma_i = 0$
   so $F = F_o = -m\ a_o$
   Here F_o is called the fictitious force.

2. **Inertial Frame of reference:** An inertial frame of reference may be defined also be called Galilean reference frame, non accelerating frame. All inertial frames are in a state of constant, rectilinear motion with respect to one another; an accelerometer moving with any of them would detect zero acceleration.

Q.2 Discuss freely falling elevator in the context of non-inertial frame of reference.

Ans. lets assume that that an elevator is falling freely under the action of gravity. It is a non-inertial frame of reference. If earth is the inertial frame of reference,

then $$a_0 = -g\hat{x}$$

Here $\hat{x}$ is measured upwards from the surface of the earth and g is the acceleration due to gravity.

The fictitious force F_0 on a mass m in the falling elevator is given by

$$F_0 = -ma_0 = mg\ \hat{x}$$

the total apparent force $F = F_i + F_o$

$$F_i = -mg\hat{x}$$

$$\therefore \quad F = F_i + F_0$$

$$= -mg\hat{x} + mg\hat{x} = 0$$

It means if the body in the elevator is not accelerated with respect to the falling elevator i.e. non-inertial frame, in that case $F_i = 0$. It shows that the apparent weight of a body in the non-inertial frame of a freely falling elevator is zero. It also means weightlessness.

If a body is in the space of the elevator and has zero velocity with respect to the elevator it will remain suspended in space in the elevator when the elevator is falling freely under the action of gravity.

Special case: If the elevator is moving downward with the acceleration a_o, where $a_o < g$

$$F = F_i + F_0$$

$$F = -mg\hat{x} + ma_0\hat{x}$$

If the body is on the surface of the elevator, its apparent weight,

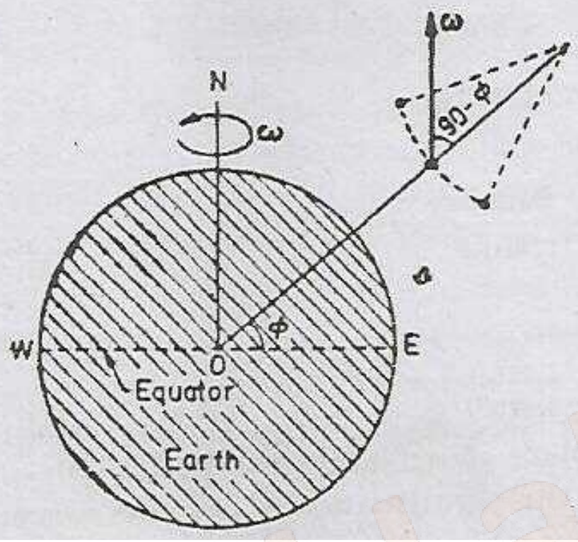
$$R = -F = m(g - a_0)$$

If the elevator is moving up, the apparent weight

$$R = m(g + a_0)$$

Q 3. Discuss the working and application of focault's pendulum.

Ans.



It is just a simple pendulum with a heavy bob of 28kg and having a suspension wire of about 70 meter length. The time period of the pendulum works out to approximately 17 sec.

Let us consider that the pendulum is kept above the place P and observed. For simplicity centrifugal force is neglected, only Coriolis acceleration is taken into account. The horizontal component of the Coriolis acceleration tries to change its plane of oscillation. As shown in figure if the bob is left at A, due to horizontal component of the Coriolis acceleration would not reach at B but it will reach B' and will come back to A. In this way the plane of oscillation goes on changing. The plane of oscillation rotates clockwise in the Northern hemisphere implying that the earth rotates anticlockwise to an observer in the Northern hemisphere i.e., the earth rotates from west to east, If the experiment is performed in the southern hemisphere the plane of pendulum rotates in the anti-clockwise direction to an observer. In the Southern hemisphere, to the observer, the earth appears to rotate in the clockwise direction *i.e,* from west to east. This experiment proves' that the earth rotates from west to east.

If ω is the angular velocity of the rotation of the earth then the plane of oscillation rotates with an angular velocity ωsinΦ above the vertical axis passing through the mean position of the pendulum. If r is the radius of the circle then the time period

$\frac{2\pi r}{v}$ here $v = r\omega \sin\phi$.

$$\therefore \quad T = \frac{2\pi r}{r\omega \sin\phi} = \frac{2\pi}{\omega \sin\phi}$$

for the complete rotation of the plane of oscillation is given by

At the poles $\phi = 90°$ and,

$$\frac{2\pi}{\omega} = 24 \text{ hours}$$

$\therefore$ At the poles $T = 24$ hours

Therefore in general,

$$T = \frac{24\text{hours}}{\sin\phi}, \text{ Here } T > 24 \text{ hours}$$

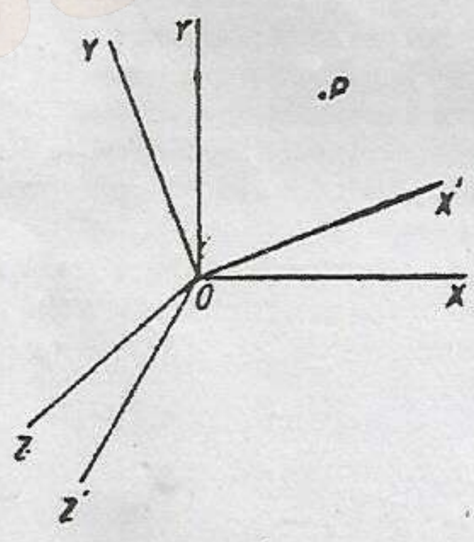
At the equator $\phi = 0$

$$\therefore \quad T = \frac{2\pi}{\omega \sin 0} = \infty$$

and the time period of the plane of oscillation becomes infinite.

Q.4. **Explain fictitious forces like centrifugal force and Coriolis force in the context of uniformly rotating frame of reference.**

Ans. A frame rotating with an angular velocity with respect to an inertial frame of reference is a non-inertial frame.



Consider two frames of reference S (X, Y, Z) and S' (X', Y', Z'). They have common origin and S' is rotating with an angular velocity w about the axis Y' relative to S. Here S is inertial frame of reference and S' is non-inertial frame of reference.

The observer O in frame S observes that the observer O' in frame S' is rotating with an angular velocity ω. The observer O' observes that O is rotating with an angular velocity – ω. Consider a particle P in space.

Position vector r of P in reference frame S is given by

$$r = i\,x + j\,y + k\,z \qquad ...(i)$$

The position vector r' of P in reference frame 5" is given by

$$r' = i'x' + j'y' + k'z' \qquad ...(ii)$$

As both the systems have the same origin O,

$$r = r'$$

$$\therefore \quad r = i'x' + j'y' + k'z' \qquad ...(iii)$$

According to observer 0', the frame of references S' is not rotating and its unit vectors remains constant, therefore differentiating equation (*ii*)

$$\frac{dr'}{dt} = i'\frac{dx'}{dt} + j'\frac{dy'}{dt} + k'\frac{dz'}{dt}$$

$$\therefore \quad V' = i'\frac{dx'}{dt} + j'\frac{dy'}{dt} + k'\frac{dz'}{dt} \qquad ...(iv)$$

V' is the velocity of P measured by $0'$ relative to its own frame of reference S' (x', y', z')

According to observer O, the frame S' is rotating and its unit vectors are also changing in direction. Therefore differentiating equation (*iii*) with respect to time

$$\frac{dr}{dt} = i'\frac{dx'}{dt} + j'\frac{dy'}{dt} + K'\frac{dz'}{dt} + x'\frac{di'}{dt} + y'\frac{dj'}{dt} + z'\frac{dk'}{dt} \qquad ...(v)$$

The end points of unit vectors i', j', k' are in uniform circular motion with angular velocity ω relative to observer O,

$$\therefore \quad \frac{di'}{dt} = w \times i'$$

$$\frac{dj'}{dt} = w \times j'$$

$$\frac{dk'}{dt} = w \times j'$$

Substituting these values in equation (v)

$$\frac{dr}{dt} = \left[i'\frac{dx'}{dt} + j'\frac{dy}{dt} + k'\frac{dz'}{dt}\right]$$

$$+ [(w \times i')\, x' + (w \times j')\, y' + (w \times k')\, z']$$

or $$V = V' + w \times r \qquad ...(vi)$$

Here V is the velocity *of P* as observed by *O* and V' is the velo-city of *P* as observed by *O'*.

Acceleration

The acceleration of *P* as measured by *O* relative to reference frame *S'* is given by

$$a = \frac{dV}{dt} = i\,\frac{dV_x}{di} + j\,\frac{dV_y}{dt} + K\,\frac{dV_z}{dt} \qquad ...(vii)$$

The acceleration of *P* as measured by *O'* relative to reference frame *S'* is given by

$$a' = i'\,\frac{dV_x{}'}{dt} + j'\,\frac{dV_y{}'}{dt} + k'\,\frac{dV_{z'}}{dt} \qquad ...(viii)$$

Differentiating equation (W) with respect to time, taking w to be constant.

$$\frac{dV}{dt} = \frac{dV'}{dt'} + w \times \frac{dr}{dt} \qquad ...(ix)$$

Also from equation (*iv*)

$$V' = i'V_{x'} + j'V_{y_1} + k'Vz' \qquad ...(x)$$

Differentiating with respect to time

$$\frac{dV'}{dt} = \left[i'\,\frac{dV_{x'}}{dt} + j'\,\frac{dV_{y'}}{dt} + k'\,\frac{dV_{z'}}{dt}\right]$$

$$+ \left[V_{x'}\,\frac{di'}{dt} + V_{y'}\,\frac{dj'}{dt} + V_{z'}\,\frac{dk'}{dt}\right]$$

$$\frac{dV'}{dt} = a' + w \times V' \qquad ...(xi)$$

Also $$\frac{dr}{dt} = V = V' + w \times r \qquad ...(xii)$$

Substituting these values in equation (*ix*)

The total force acting on a particle of mass m as observed in rotating frame of reference S' is given by

$$F' = ma' = ma - 2mw \times V' - mw \times (w \times r)$$

$$\therefore \quad F' = F - 2mw \times V' - mw \times (w \times r) \quad ...(xv)$$

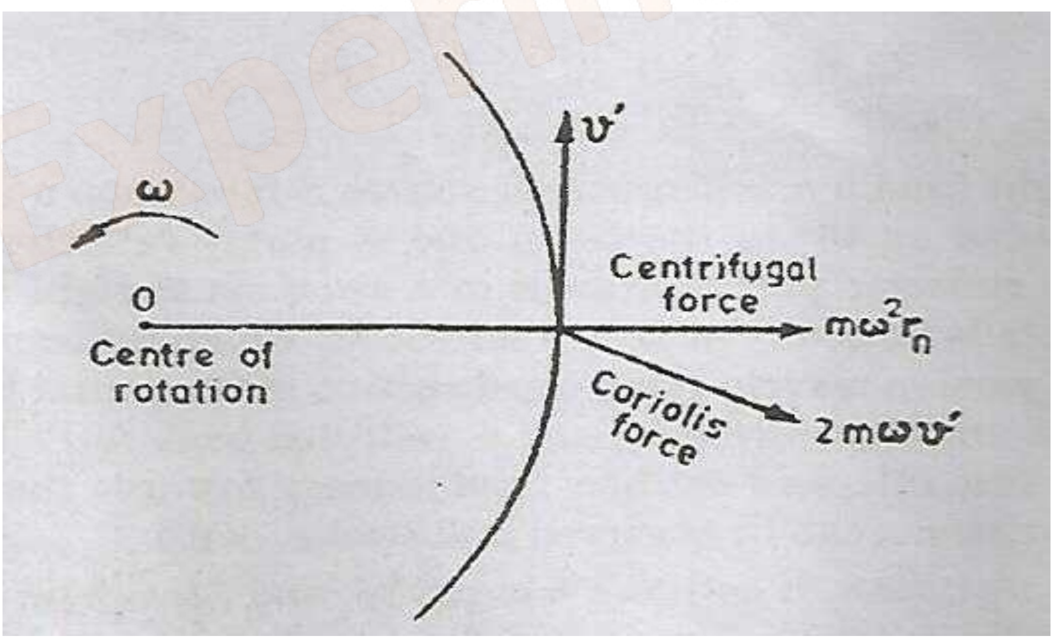
Special Cases.

(1) When the particle is at rest with respect to rotating, frame of reference, $V' = 0$

$$\therefore \quad F' = F - mw \times (w \times r) \quad ...(xvi)$$

The *centrifugal force* is $-mw \times (w \times r)$. It is a fictitious force acting on a particle at rest in a rotating frame of refeence. The effect of centrifugal force is to reduce the value of g on the surface of the earth because the earth rotates about its own axis.

The centrifugal for is a fictitious force which acts on a particle at rest relative to a rotating frame of reference. Being responsible for keeping the particle at rest in the rotating frame, it is numerically equal to the centripetal force, $m\omega^2 r_n$, where r_n is the distance of the particle from the centre of rotation but is oppositely directed, i.e. outwards, away from the axis of rotation, Coriolis force too is a fictitious force, which acts on a particle in motion relative to a.rotating frame of reference. It is proportional to the angular velocity (ω) of the rotating frame and to the velocity v' of the particle relative to it. Its direction is always perpendicular to that of v' and is obtained by a rotation through 90° in the opposite sense to that of ω, and its value is -2m v'



It will obviously be zero if either v' or w is zero, i.e., if the particle is a rest relative to the rotating frame in which case the only fictitious force acting on the particle will be the centrifugal force) or if the reference frame be a non-rotating one.

It will further be seen that particle P will move relative to the rotating frame S' in accordance with Newton's law of motion if we add to the true force F, these two fictitious forces.

Q.5. **Discuss very briefly the significance of Coriolis force.**

Ans. lets us assume that a body on the surface of the earth is in motion relative to the earth then the acceleration measured by the observer is given by the equation

$$a' = a - 2w \times V' - w \times (w \times r)$$

In this equation the term $[-2w \times V']$ is the *coriolis force.* The Coriolis force acts when a body is falling freely under the action of gravity in the vertical direction. It deviates the true vertical path of the body. Due to this the bodies falling freely under the action of gravity are deviated towards east.

The deviation is due to the horizontal component of the Coriolis force.

It has been observed that the cyclonic winds are turned towards their right in the Northern hemisphere and towards left in the Southern hemisphere. They are produced due to a low pressure centre combined with the Coriolis acceleration. This has also been confirmed by the photographs taken by the satellites from the earth's atmosphere.