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UNIT-8
Special Theory of Relativity

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UNIT-8
Special Theory of Relativity

Q 1. Write postulates of the special theory of relativity.

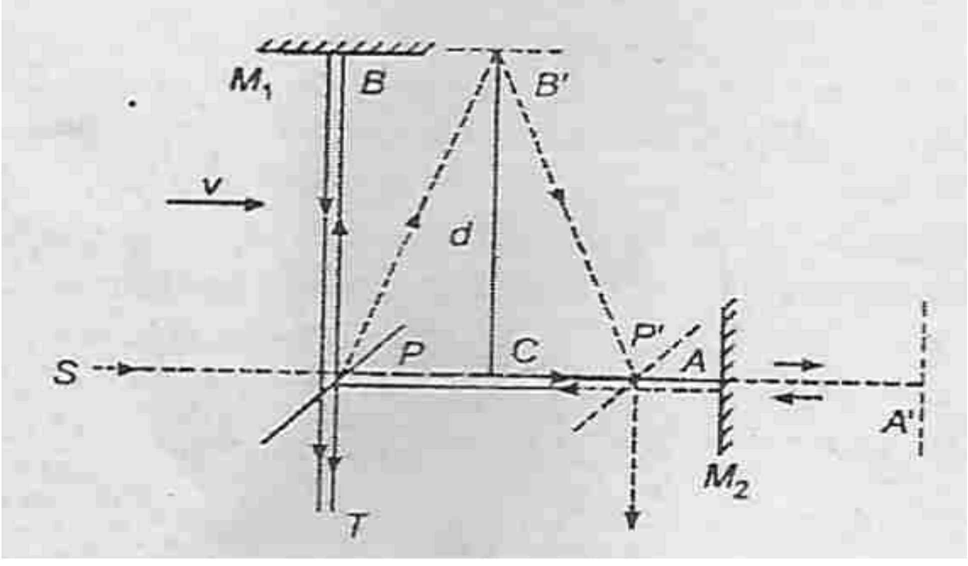
Ans. Postulates of Special Theory of relativity: There are two main postulates of the special theory of relativity

1. The fundamental laws of physics are the same even when started in different inertial frames of reference in uniform motion relative to each other. These are covariant all inertial frames are equivalent. The concept absolute rest does not exist.

2. The speed of light in a vacuum is constant. It is the maximum attainable speed for any object. It neither depends on the direction of propagation or on the relative motion between the source and intervening medium observer. The speed of light in free space has the same value *c* in all inertial frames of reference.

Q.2 Describe the Michelson-Morley experiment and discuss the importance of its negative results.

Ans. Michelson-Morley Experiment: The objective of the experiment was to detect the presence of hypothetical fluid ether by detecting the motion of the earth relative to ether at rest. Let the velocity of light in ether be 'C', the speed of light would be different in different directions when we view it from the earth. According to this consideration, the light should take different time to cover equal distances in different directions. By measuring this time difference, the relative velocity between ether and earth can be calculated.



A monochromatic beam of light from a source S falls upon a semi-silvered glass plate P placed at 45° to the beam and is partly reflected and partly transmitted. The reflected portion travels in a direction at right angles to the initial beam and falls normally at B on a mirror M_1 which reflect it back to P. The transmitted portion travels along the direction of the initial beam and falls normally at A on another mirror M_2 and is reflected back to P again. The two rays thus returned to point P to interfere while coming finally towards the telescope T. The interference pattern can be observed and studied with T.

The effective distances of both the mirrors M_1 and M_2 can be equalized by using a compensating plate. The two rays would take equal time to return to P if this arrangement had been at rest in the ether. But as the earth moves, this complete setup moves. Let us suppose that the direction of motion of the earth coincides with the direction of the initial beam of light. Due to the motion of apparatus along with the earth, the paths of the two rays and the positions of their reflections from the mirrors will be as shown by the dotted lines. The time taken by the two rays on their journey to their respective mirrors and back to the plate will no longer be equal.

Let us assume the following :

Vel. of light thro' ether = c

Vel. of earth = v

$$PA = PB = d$$

The ray reflected from P and moving transversely will strike the mirror M_1 not at B but at B' due to the motion of the earth. If t be the time taken by the ray starting from P to reach B', then

$$PB' = ct \text{ and } BB' = vt$$

$$PP' = 2\text{Pc} = 2BB' \qquad \text{(by laws of reflection)}$$

But, $$PB'P' = PB' + B'P' = 2PB' \qquad \text{(Since PB' = B'P')}$$

$$PB'^2 = PC^2 + CB'^2 = BB'^2 + PB'^2 \qquad \text{(But } CB' = PB\text{)}$$

$\therefore$ $$c^2t^2 = v^2t^2 + d^2$$

or $$t = \frac{d}{(c^2 - v^2)^{1/2}}$$

If t_1 is the total time taken by the ray to travel the whole path $PB'P'$, then

$$t_1 = 2t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$

The ray transmitted through P and moving longitudinally towards M_2 has a velocity $(c - v)$ relative to the apparatus from P to A and $(c + v)$ on the return journey from A' to P' assuming ether to be at rest. If t_2 is the total time taken by this ray to get back to the platet.

$$t_2 = \frac{d}{c-v} + \frac{d}{c+v} = \frac{d(c+v) + d(c-v)}{(c-v)(c+v)}$$

$$= \frac{2dc}{(c^2 - v^2)}$$

$$= \frac{2d}{c} \frac{1}{(1 - v^2/c^2)}$$

Let time difference be Δt.

Then, $$\Delta t = (t_2 - t_1) = \frac{2d}{c}\left[\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right]$$

If we rotate the system thro' 90°, mirrors M_1 and M_2 will interchange their positions. Let the new times be t_1' and t_2'.

$$t_1' = \frac{2d}{c} \frac{1}{(1 - v^2/c^2)}$$

Since $$v <<< c$$

$$\frac{v}{c} << 1$$

The experimental arrangement of the experiment was capable of measuring one-hundredth of the fringe and as such the above shift was capable of accurate measurability. However, no effect was observed. The experiment was repeated again and again with width d = 11 m when $\delta n = 0.4$. But the result was always null. It indicates that ether and earth have no relative motion. According to Einstein's theory of relativity, the velocity of light is same in all directions. This leads us to $\delta n = 0$. Thus according to the principle of constancy of light, no shift in fringes should observe. Hence postulate of the constancy of the speed of light is capable of explaining negative results of Michelson-Merely, experiment.

Negative results give us the following conclusions:

1. Light has the same speed in all directions. The motion of the source, observer are both does not affect it

2. The medium ether has no properties which can be observed. Therefore, there are no such things as absolute space or a fixed fundamental same of reference with respect to which absolute motion of the bodies can be determined.There is no absolute motion.

Q.3 Explain Lorentz transformation in detail.

Ans. The **Lorentz transformations** (or **transformation**) are coordinate transformations between two coordinate's frames which move at constant velocity relative to each other. These are concerned with the conditions occurring when an event is observed from two different inertial frames which are having a relative motion between them.

Let there be two inertial frames S and S'. Let ox, oy and oz be the co-ordinate axes in inertial frame S. The corresponding axes in S' be $o'x$, $o'y$ and $o'z$. Let these axes in both the frames be parallel. If we assume that S' is moving away from S only along x-axis. Then the x-axes of both the frames will coincide.

Let O and O' represent the two observers. Suppose observer o' be moving along x-axis with a velocity v. There would be certainly an instant when O' would have coincided with 0. Let us take measurements from this instant. At this instant, we have

$$x = y = z = t = 0$$

$$x' = y' = z' = t' = 0$$

If at this instant a light signal is emitted. Spherical waves will move out. If, observer from, this passes though a point p(x,y,z) at time $t = t$, we have

$$x^2 + y^2 + z^2 = c^2 t^2$$

From S[1],

$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$

we have taken c as constant rerefering to the second postulate.

$$x^2 + y^2 + z^2 - c^2 t^2 = x'^2 + y'^2 + z'^2 - c^2 t'^2$$

Since the motion is assumed only along x-axis, we have

$$y = y' \text{ and } z = z'$$

$$\therefore \quad x^2 - c^2 t^2 = x'^2 - c^2 t'^2$$

Now $v <<<< c$, therefore we can express $x' = \lambda\,(x - vt)$, where λ is a constant and does not depend on x or t.

we have assumed that S' is moving towards right with a velocity v. This is equivalent to S moving towards left with a velocity $-v$.

$$\therefore \quad x = \lambda\,(x' + vt')$$

$$\text{or} \quad x = \lambda\,[\lambda\,(x - vt) + vt']$$

$$\text{or} \quad x = \lambda^2 x - \lambda^2 vt + \lambda vt'$$

$$\text{or} \quad \lambda vt' = x - \lambda^2 x + \lambda^2 vt$$

$$\text{or} \quad t' = \lambda\left[t - \frac{x}{v}\left(1 - \frac{1}{\lambda^2}\right)\right]$$

$$\therefore \quad x^2 - c^2t^2 = \lambda^2\,(x - vt)^2 - c^2\lambda^2\left[t - \frac{x}{v}\left(1 - \frac{1}{\lambda^2}\right)\right]^2$$

for this relation to hold good the terms of x on both the sides should cancel out. This leaves us with

These are known as Lorentz transformations.

when $\quad v <<<< c,$

we get $\quad \frac{v}{c} <<<< 1$

$$\therefore \quad x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$\text{And} \quad t' = t$$

Now the Lorentz transformations get reduced to Galilean transformations.

Q.4. Explain the concept of Lorentz contraction.

Ans. **Length Contraction**: The length of any object in a moving frame will appear shortened in the direction of motion, or contracted. The amount of contraction can be calculated from the Lorentz transformation. The length is maximum when the frame in which the object is at rest. The shortening or contraction in the length of an object along its direction of motion is known as the Lorentz contraction. When the rod or the object is at rest with respect to the observer, its length is known as the proper length. It will be the same in all stationary frames of reference. But it will be different in a moving reference frame, depending upon their relative velocities with respect to each other and the observer. If the rod is moving in x-direction, there will be no shortening of the length of the rod if it lies along the axis of y or z. However, the rod is inclined to the direction of its velocity, there will some component in the latter direction. There could be some shortening of length due to this component

Let the rod be kept along the axis of x in frames if the co-ordinates of the two ends of the rod are x_2 and x_1. Then the length, L, of the rod is given by–

$$L_0 = x_2 - x_1$$

If the x-coordinates of the ends of the rod in the reference frame S, moving with a uniform velocity v with respect to frame S be x_1 and x'_2, as noted simultaneously at the same instant t', we have length of the rod in the moving reference frame S' given by $L = (x'_2 - x'_1)$. To corelate L_0 and L, we note that inverse Lorentz transformation gives

$$x_1 = \frac{x'_1 + vt'}{\sqrt{1 - v^2/c^2}} \text{ and } x_2 = \frac{x'_2 + vt'}{\sqrt{1 - v^2/c^2}}$$

So that,

$$L_0 = x_2 - x_1 = \frac{x'_2 + vt'}{\sqrt{1 - v^2/c^2}} - \frac{x'_1 + vt'}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{x'_2 - x'_1}{\sqrt{1 - v^2/c^2}} = \frac{L}{\sqrt{1 - v^2/c^2}},$$

whence,

$$L = L_0 \sqrt{1 - v^2/c^2},$$

which is clearly shorter than its proper length Lo.

The greater the velocity, v, greater will be the shortening. If the $v = c$, the length of the rod will become zero.

Q 6. Establish the expression for the law of addition of relativistic velocities. Under what conditions do the Lorentz relativistic velocity transformation equations reduce to Galilean transformation equations?

Ans. The law for the addition of relativistic velocities can be stated as :

If a particle moves with velocity U' in a reference frame S' and if S' has a velocity v relative to a frame S, the velocity of the particle relative to S is given by :

$$\frac{U'+v}{\left(1+U'v/c^2\right)}$$

Let the velocity of the particle in S at rest be U.

Let the components of U and U' along x, y and z axes be U_x, U_y, U_z and $U_{x'}$, $U_{y'}$, $U_{z'}$ respectively.

$$U_x = \frac{dx}{dt}$$

$$U_{x'} = \frac{dx'}{dt}$$

From Lorentz transformations,

$$dx' = \frac{dx-vdt}{\sqrt{1-v^2/c^2}};\ dt' = \frac{dt-\frac{v}{c^2}dx}{\sqrt{1-v^2/c^2}}$$

$$dy' = dy$$

$$dz' = dz$$

As the transverse dimensions do not change because of motion. Dividing dx', dy' and dz' by dt' separately, we get

$$\frac{dx'}{dt'} = \frac{dx - vdt}{dt - \frac{v}{c^2}dx}; \frac{dy'}{dt'} = \frac{dy\sqrt{1 - v^2/c^2}}{dt - \frac{v}{c^2}dx}; \frac{dz'}{dt'} = \frac{dz\sqrt{1 - v^2/c^2}}{dt - \frac{v}{c^2}dx}$$

Divide the numerator and denominator of the right hand side of the above three equations,

$$u'_x = \frac{u_x - v}{1 - \frac{v}{c^2}u_x}; u'_y = \frac{u_y\sqrt{1 - v^2/c^2}}{1 - \frac{v}{c^2}u_x}; u'_z = \frac{u_z\sqrt{1 - v^2/c^2}}{1 - \frac{v}{c^2}u_z}$$

The inverse transformations are

$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x} \quad \text{... (1)}$$

$$u_y = \frac{u'_y + \sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c^2}u'_z}$$

$$u_z = \frac{u_z + \sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c^2}u'_x}$$

(I) Above represents the required addition.
In the non-relativistic approximation, if $v <<< c$

i.e., $\frac{v}{c} <<<< 1$

Then

$$u_x = u'_x + v$$
$$u_y = u'_y$$
$$u_z = u_z$$

These are Galilean velocity transformations.

If we limit the motion along X-axis only,

$$u_x = u$$
$$u_y = 0$$
$$u_z = 0$$
$$u'_y = 0$$
$$u'_z = 0$$

If $u = c$

$$u'_x = \frac{u - v}{1 - \frac{vu}{c^2}} = \frac{c - v}{1 - \frac{v}{c}} = c$$

If a particle moves with velocity c with respect to frame S, which is possible only with zero rest mass, its velocity when observed from frame S' will still be c.

Thus transformations of velocity are consistent with velocity of light. Lorentz transformations are also based on the same.

Q 7. Show that a four dimensional volume element dx dy dz dt is invariant to Lorentz transformation.

Ans. Let S be a stationary frame and S' be a frame moving with a constant velocity relative to S along the X-axis.

Now let us consider frame S'

$$dx' = A\,dx$$
$$dy' = dy$$
$$dz' = dz$$
$$dt' = \frac{1}{A}\,.\,dt$$

$\therefore$ Four dimensional volume element in frame S'

$$dx'\,dy'\,dz'\,dt' = A dx\,dy\,dz\,\frac{1}{A}\,dt$$

The above indicates that the four dimensional volume $dx\,dy\,dz\,dt$ is invariant to Lorentz transformation.

Q. 9. If kinetic energy of a particle is three times its rest mass energy, what is its velocity?

Ans. Rest mass energy = m_0c^2

$$\therefore \quad E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

But $$E = 3\,m_0c^2$$

$$\therefore \quad \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = 3\,m_0c^2$$

or $$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{3}$$

Squaring both sides,

$$1-\frac{v^2}{c^2} = \frac{1}{9}$$

$$\frac{v^2}{c^2} = 1-\frac{1}{9} = \frac{8}{9} = \left(\frac{2\sqrt{2}}{3}\right)^2$$

$$\therefore \quad \frac{v}{c} = \frac{2\sqrt{2}}{3}$$

or $$v = \frac{2\sqrt{2}}{3} \times c = \frac{2\sqrt{2}}{3} \times 3 \times 10^8 m/\text{sec}$$

$$\therefore \quad v = 2\sqrt{2} \times 10^8 m/\text{sec}$$

$$= 2(1.414) \times 10^8 m/\text{sec}$$

$$= 2.828 \times 10^8 m/\text{sec}$$

Q 10. What are mass less particles? Show that they can exist only if they move with the speed of light.

Ans. According to Einstein's momentum-energy relation,

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

where the symbols have their usual meanings. For particles of rest mass zero, energy is given by

$$E = pc$$

or $$p = \frac{E}{c} \quad \text{...(i)}$$

$$p = mv$$

and $$E = mc^2 \Rightarrow m = \frac{E}{c^2}$$

$\therefore$ $$p = \frac{E}{c^2} \cdot v \quad \text{...(ii)}$$

From (*i*) and (*ii*)

$$\frac{E}{c} = \frac{Ev}{c^2}$$

$\therefore$ $$v = \frac{E}{c} \times \frac{c^2}{E} = c$$

Hence $$v = c$$

i.e., The velocity of a particle of zero rest mass is always equal to velocity of light and is invariant.

Q 11. Draw the graph between relativistic mass of a body and velocity of a particle having finite rest mass.

Ans. According to the theory of relativity, the mass of a moving body varies with velocity, it appears to increase with velocity, becoming infinite when velocity approaches the velocity of light.

$$m = m_0\left[1 + \frac{1}{2}\left(\frac{v^2}{c^2}\right) + \frac{3}{8}\left[\frac{v^2}{c^2}\right]^2 + \ldots\right]$$

where m_0 = rest mass

m = mass at velocity, v

v = velocity at any instant

c = speed of light