Free Study Material from

All Lab Experiments

UNIT-1 Fundamentals of Dynamics
UNIT-2 Work and Energy
UNIT-3 Collision

UNIT – 1
Fundamentals of Dynamics

Q 1. State and Explain Newton's three laws of motion.

Ans. Newton's Laws of motion

First Law: A body must continue in the state of rest or of uniform motion along a straight line unless acted upon by an external unbalanced force.

This law is also known as the law of inertia.

The first law can be stated mathematically when the mass is a non-zero constant, as,

$$\sum \mathbf{F} = 0 \Leftrightarrow \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = 0.$$

Second Law: The rate of change of momentum is proportional to the impressed force and takes place in the directions of the force.

$$\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \frac{\mathrm{d}(m\mathbf{v})}{\mathrm{d}t}.$$

$$\mathbf{F} = m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = m\mathbf{a},$$

If it does not remain constant, the second law does not hold good.

This law gives us the definition of force. The force will be unity if it produces an acceleration of 1m/sec' in a mass of 1kg. This unit force is 1 newton(N). This is S.I unit, in CGS units, the unit of force is dyne.

1 newton = 10^5 dynes.

Third Law: To every action, there is an equal and opposite reaction. if one object *A* exerts a force $\mathbf{F}_A$ on a second object *B*, then *B* simultaneously exerts a force $\mathbf{F}_B$ on *A*, and the two forces are equal in magnitude and opposite in direction: $\mathbf{F}_A = -\mathbf{F}_B$.

The above three laws do not hold good in non-inertial frame of reference.

Q 2. Explain the inertial and noninertial frames of reference.

Ans. An inertial frame of reference: It is a frame of reference in which bodies move with constant velocity only if there is no net force on it and obey Newton law of inertia and other laws of Newtonian mechanics.

The non-inertial frame of references are the frames in which bodies are accelerating or rotating. Newton's laws of motion are not applicable and pseudo forces exist.

Suppose the body has coordinates (x,y,z) at any instant. Since there is no net force on the body, according to Newton's laws of motion,

(ii) **Reference frame starts moving with constant velocity vector-v** :-

The acceleration of frame = $\vec{a} = 0$

Thus, acceleration of mass m relative to frame is given by

$$\vec{a}_{inertial} = \vec{a}_{rest} - \vec{a} = \vec{a}_{rest}$$

Force on it will be $\vec{F}$ inertial and we will reason that

$$\vec{F}_{inertial} = m\vec{a}_{inertial} = m\vec{a}_{rest} = \vec{F}_{rest}$$

(iii) **Reference frame moves with constant acceleration**:-

Let the acceleration of frame be $\vec{a}_{frame}$.

Thus, acceleration of mass relative to frame will be $\vec{a}_{rel}$.

$$\vec{a}_{rel} = \vec{a}_{inertial} - \vec{a}_{frame} = \vec{a}_{rest} - m\vec{a}_{frame}$$

Let there be force $\vec{F}$ frame on mass we will reason, that

$$\vec{F}_{frame} = m\vec{a}_{rel} = m\vec{a}_{rest} - m\vec{a}_{frame}$$

$$= \vec{F}_{rest} + m(-\vec{a}_{frame}) = \vec{f}_{rest} + \vec{F}_{pseudo}$$

All the symbols have their usual meanings.

Thus, we can conclude, that all frames of reference moving with a constant velocity with reference to an inertial frame, are also inertial frames of reference. The inertial frames are non-accelerating and non-rotating.

Q 3. What is a Galilean transformation? Which of the following are Galilean Invariant: Length, Velocity, Acceleration and Linear momentum? Explain with reason

Ans.

If an event happens at (x, y, z, t) as measured in *S*, what are its coordinates (x', y', z', t') in *S'*? It's easy to see $t' = t$ —we synchronized the clocks when *O'* passed *O*. Also, evidently, $y' = y$ and $z' = z$, from the figure. We can also see that $x = x' + vt$. Thus (x, y, z, t) in *S* corresponds to (x', y', z', t') in *S'*, where

$$\begin{aligned} x' &= x - vt, \\ y' &= y, \\ z' &= z, \\ t' &= t. \end{aligned}$$

That's how *positions* transform; these are known as the *Galilean* transformations.

What about *velocities* ? The velocity in *S'* in the x' direction

$$u'_x = \frac{dx'}{dt'} = \frac{dx'}{dt} = \frac{d}{dt}(x - vt) = \frac{dx}{dt} - v = u_x - v.$$

This is obvious anyway: it's just the addition of velocities formula

$$u_x = u'_x + v.$$

How does *acceleration* transform?

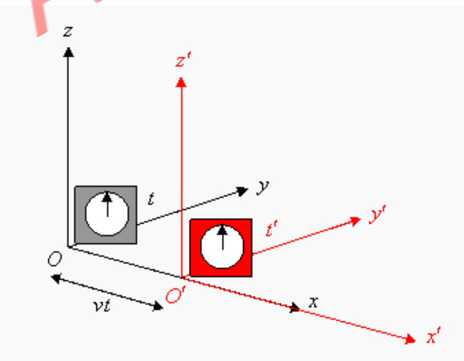
$$\frac{du'_x}{dt'} = \frac{du'_x}{dt} = \frac{d}{dt}(u_x - v) = \frac{du_x}{dt}$$

since *v* is constant.

That is to say,

$$a'_x = a_x,$$

the *acceleration is the same* in both frames. This again is obvious—the acceleration is the rate of change of velocity, and the velocities of the same particle measured in the two frames differ by a *constant* factor-the relative velocity of the two frames.



Velocity: since motion is only along X-direction. The above shows velocity is variant under Galilean transformations.

Acceleration: Acceleration is the rate of change of velocity. Hence the acceleration is invariant in the Galilean transformations.

Since movement is along X-axis only,

Length: These are Galilean coordinate transformations. Length transformation: Let there be a rod of length *L*

Then in frame *S*

$$l' = x_2 - x_l$$

When observed in frame *S'* at time *t'*, the length appears to be *l'* According⁻ to Galilean transformations. Hence length is invariant in Galilean transformations.

Linear momentum: Linear momentum is the product of mass and velocity. Mass is unaltered in the Galilean transformations but velocity is a variant. Hence the product my will be a variant in the Galilean. transformations.

Q.4 Explain the law of conservation of linear momentum?

Ans. Law of conservation of linear momentum

The product of mass and the velocity of a particle is defined as its linear momentum (represented as 'p'). So,

$$P = mv$$

In physics, the term *conservation* refers to something which doesn't change. This means that the variable in an equation which represents a conserved quantity is constant over time. It has the same value both before and after an event. If the subscripts iiii and ffff denote the initial and final momenta of objects in a system, then the principle of conservation of momentum says

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} + \ldots = \mathbf{p}_{1f} + \mathbf{p}_{2f} + \ldots$$

Conservation of momentum is actually a direct consequence of Newton's third law. Consider a collision between two objects, object A and object B. When the two objects collide, there is a force on A due to B—F_{AB}—but because of Newton's third law, there is an equal force in the opposite direction, on B due to A—F_{BA}.

$$F_{AB} = -F_{BA}$$

The forces act between the objects when they are in contact. The length of time for which the objects are in contact—t_{AB} and t_{BA}—depends on the specifics of the situation. For example, it would be longer for two squishy balls than for two billiard balls. However, the time must be equal for both balls.

$$t_{AB} = t_{BA}$$

Consequently, the impulse experienced by objects A and B must be equal in magnitude and opposite in direction.

If we recall that impulse is equivalent to change in momentum, it follows that the change in momenta of the objects is equal but in the opposite directions. This can be equivalently expressed as the sum of the change in momenta being zero.

$$m_A \cdot \Delta v_A = -m_B \cdot \Delta v_B$$

$$m_A \cdot \Delta v_A + m_B \cdot \Delta v_B = 0$$

Hence, this relation proves the conservation of momentum.

Example:

Two bodies of mass M and m are moving in opposite directions with the velocities v. If they collide and move together after collision, we have to find the velocity of the system.

Since there is no external force acting on the system of two bodies, momentum will be conserved.

Initial momentum = Final momentum

$(Mv - mv) = (M+m)V_{Final}$

From this equation we can easily find the final velocity of the system.

Q. 5 Explain the concept of impulse with suitable examples.
Ans. Impulse:

The formula for impulse looks like this:

$$\text{Impulse} = \text{Force} \times \text{time} = \vec{F}\Delta t$$

$$\Delta t = t_{final} - t_{initial}$$

Because impulse is a measure of how much the momentum changes as a result of force acting on it for a period of time, an alternative formula for impulse looks like this:

$$\text{Impulse} = \Delta\vec{p} = \vec{p}_{final} - \vec{p}_{initial}$$

This formula relates impulse to the change in the momentum of the object. Impulse has two different units, either kilogram times meters per second (kg m/s) or Newton times seconds (Ns).

Examples of Impulse

Let's take a look at a few examples.

In this first example, we'll look at the impulse for an object that collides with a wall and stops after the collision. If the 2.0 kg object travels with a velocity of 10 m/s before it hits the wall, then the impulse can be calculated using the formula.

Instantaneous Impulse: There are many occasions when a force acts for such a short time that the effect is instantaneous, e.g., a bat striking a ball. In such cases, although the magnitude of the force and the time for which is acts may each be unknown the value of their product (i.e., impulse) can be known by measuring the initial and final momenta. Thus, we can write

Impulse **J** produced from time t_1 to t_2 is defined to be[4]

$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F}\,\mathrm{d}t$$

where **F** is the resultant force applied from t_1 to t_2.

From Newton's second law, force is related to momentum **p** by

$$\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$$

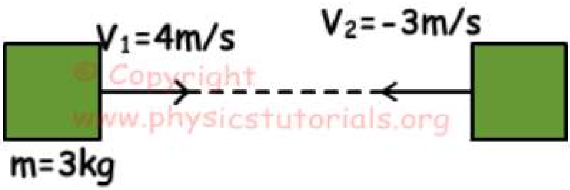
Therefore,

$$\mathbf{J} = \int_{t_1}^{t_2} \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}\,\mathrm{d}t = \int_{\mathbf{p}_1}^{\mathbf{p}_2} \mathrm{d}\mathbf{p} = \mathbf{p}_2 - \mathbf{p}_1 = \Delta\mathbf{p}$$

where $\Delta\mathbf{p}$ is the change in linear momentum from time t_1 to t_2. This is often called the impulse-momentum theorem.

$$\text{Impulse} = \Delta\vec{p} = \vec{p}_{final} - \vec{p}_{initial}$$

1. An object travels with a velocity 4m/s to the east. Then, its direction of motion and magnitude of velocity are changed. Picture given below shows the directions and magnitudes of velocities. Find the impulse given to this object.



$I = F.\Delta t = \Delta p = m.\Delta V$

where $\Delta V = V_2 - V_1 = -3 - 4 = -7 m/s$

$I = m.\Delta V = 3.(-7) = -21 kg.m/s$

Q.6 Explain the motion of the centre of mass. Explain that in absence of an external force center of mass remains at rest or at constant velocity.

Ans. The Co-ordinates of center of mass of a mass distribution is -

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots\dots\dots + m_nx_n}{m_1 + m_2 + m_3 + \dots\dots\dots + m_n} = \frac{1}{m}\sum_{i=1}^{n} m_ix_i$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \dots\dots\dots + m_ny_n}{m_1 + m_2 + m_3 + \dots\dots\dots + m_n} = \frac{1}{m}\sum_{i=1}^{n} m_iy_i$$

$$Z_{cm} = \frac{m_1z_1 + m_2z_2 + m_3z_3 + \dots\dots\dots + m_nz_n}{m_1 + m_2 + m_3 + \dots\dots\dots + m_n} = \frac{1}{m}\sum_{i=1}^{n} m_iz_i$$

So, the general co-ordinates of the center of mass are -

$$\vec{R}_{CM} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i}$$

to find the velocity of center of mass, differentiate it and we get-

$$\frac{d\boldsymbol{R}_{cm}}{dt} = \frac{d}{dt}\left(\frac{\sum_{i=1}^{n} m_i \boldsymbol{r}_i}{\sum_{i=1}^{n} m_i}\right) = \frac{\sum_{i=1}^{n} m_i \frac{d\boldsymbol{r}_i}{dt}}{\sum_{i=1}^{n} m_i}$$

but, $\frac{d\boldsymbol{R}_{cm}}{dt} = \boldsymbol{V}_{cm}$ which is the velocity of centre of mass

$\frac{d\boldsymbol{r}_i}{dt} = \boldsymbol{v}_i$ is the velocity of i'th particle of the system

Therefore

$$\boldsymbol{V}_{cm} = \frac{\sum_{i=1}^{n} m_i \boldsymbol{v}_i}{\sum_{i=1}^{n} m_i} = \frac{1}{M}\sum_{i=1}^{n} m_i \boldsymbol{v}_i$$

but, $m_i \boldsymbol{v}_i = \boldsymbol{p}_i$ which is the linear momentum of the i'th particle of the system

Therefore $\boldsymbol{V}_{cm} = \frac{1}{M}\sum_{i=1}^{n} \boldsymbol{p}_i = \frac{\boldsymbol{p}}{M}$ (18)

Or, $M\boldsymbol{V}_{cm} = \boldsymbol{p}$

where $\boldsymbol{p} = \sum_{i=1}^{n} \boldsymbol{p}_i$

p is the vector sum of linear momentum of various particles of the system or it is the total linear momentum of the system.

- If no external force is acting on the system then its linear momentum remains constant. Hence in absence of external force

$$\left.\begin{aligned} &M\boldsymbol{V}_{cm} = \text{constant} \\ &\text{or,} \\ &\boldsymbol{V}_{cm} = \text{constant} \end{aligned}\right\} \quad (19)$$

- In the absence of external force velocity of centre of mass of the system remains constant or we can say that centre of mass moves with the constant velocity in absence of external force.

example - If a projectile explodes in the air in different parts, the path of the center of mass remains unchanged. This is because during explosion no external force (except gravity) acts on the center of mass. Then it follows the same parabolic path as it should be.

Q. 7 Explain the position of center of mass for different bodies?

Ans. For a uniform massive body the center of mass can be calculated by the given formulas -

$$\mathbf{R}_{cm} = \frac{1}{M}\int \mathbf{r}\,dm$$

and the values of its co-ordinates

$$X_{cm} = \frac{1}{M}\int x\,dm$$

$$Y_{cm} = \frac{1}{M}\int y\,dm$$

$$Z_{cm} = \frac{1}{M}\int z\,dm$$

- For symmetric bodies like rectangle, square, circle, sphere etc., the center of mass is at its body center.
- the centroid of a triangle is its center of mass.

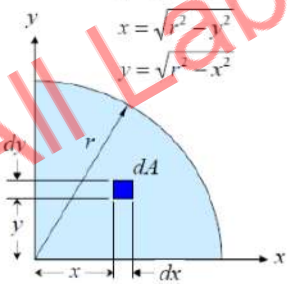
Q. 8.

Find the centroid of a quarter circle by double integration in rectangular coordinates

$$x^2 + y^2 = r^2$$

$$x = \sqrt{r^2 - y^2}$$

$$y = \sqrt{r^2 - x^2}$$



$$M_x = \int_A y\,dA = \int_0^r \left(\int_0^{\sqrt{r^2-x^2}} y\,dy \right) dx$$

$$= \int_0^r \left(\left[\frac{y^2}{2} \right]_0^{\sqrt{r^2-x^2}} \right) dx = \int_0^r \frac{r^2 - x^2}{2}\,dx = \left[\frac{r^2 x}{2} - \frac{x^3}{6} \right]_0^r = \frac{r^3}{3}$$

$$A = \int_A dA = \frac{\pi r^2}{4}$$

$$y_c = \frac{M_x}{A} = \frac{\int_A y\,dA}{A} = \frac{r^3/3}{\pi r^2/4} = \frac{4r}{3\pi}$$

$$x_c = \frac{M_y}{A} = \frac{\int_A x\,dA}{A} = \frac{4r}{3\pi}$$

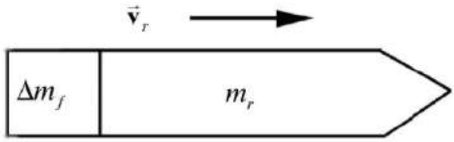
Q. 9. explain the motion of a variable mass rocket-

Ans.

Rocket Propulsion

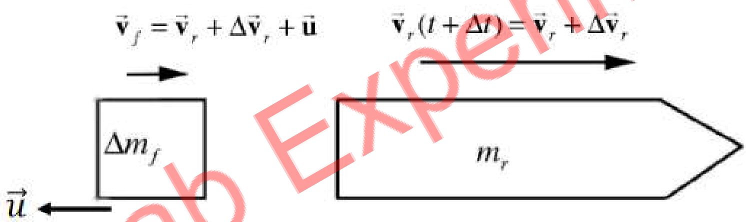
A rocket at time $t = 0$ is moving with speed $v_{r,0}$ in the positive x-direction in empty space. The rocket burns fuel that is then ejected backward with velocity $\vec{\mathbf{u}}$ relative to the rocket. The rocket velocity is a function of time, $\vec{\mathbf{v}}_r(t)$, and increases at a rate $d\vec{\mathbf{v}}_r/dt$. Because fuel is leaving the rocket, the mass of the rocket is also a function of time, $m_r(t)$, and is decreasing at a rate dm_r/dt. Determine a differential equation that relates $d\vec{\mathbf{v}}_r/dt$, dm_r/dt, $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}_r(t)$, and $\vec{\mathbf{F}}_{ext}^{total}$, an equation to be called as the rocket equation.

system at time t



$$\vec{\mathbf{p}}_{system}(t) = (m_r + \Delta m_f)\vec{\mathbf{v}}_r$$

system at time $t + \Delta t$



$$\vec{\mathbf{p}}_{system}(t + \Delta t) = m_r(\vec{\mathbf{v}}_r + \Delta\vec{\mathbf{v}}_r) + \Delta m_f(\vec{\mathbf{v}}_r + \Delta\vec{\mathbf{v}}_r + \vec{\mathbf{u}})$$

$$\vec{\mathbf{F}}_{ext}^{total} = \lim_{\Delta t \to 0} \frac{\vec{\mathbf{p}}_{system}(t+\Delta t) - \vec{\mathbf{p}}_{system}(t)}{\Delta t} = \lim_{\Delta t \to 0} m_r \frac{\Delta\vec{\mathbf{v}}_r}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta m_f \Delta\vec{\mathbf{v}}_r}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta m_f}{\Delta t}\vec{\mathbf{u}} \approx m_r \frac{d\vec{\mathbf{v}}_r}{dt} + \frac{dm_f}{dt}\vec{\mathbf{u}}.$$

$$\frac{dm_r}{dt} = -\frac{dm_f}{dt}$$

$$\vec{\mathbf{F}}_{ext}^{total} = m_r \frac{d\vec{\mathbf{v}}_r}{dt} - \frac{dm_r}{dt}\vec{\mathbf{u}}$$

is called the *rocket equation*.

Rocket in Free Space

If there is no external force on a rocket, ***F*** = 0 and its motion is given by

$$M\frac{d\mathbf{v}}{dt} = \mathbf{u}\frac{dM}{dt} \qquad \text{or} \qquad \frac{d\mathbf{v}}{dt} = \frac{\mathbf{u}}{M}\frac{dM}{dt}.$$

Generally the exhaust velocity ***u*** is constant,

$$\int_{t_0}^{t_f}\frac{d\mathbf{v}}{dt}\,dt = \mathbf{u}\int_{t_0}^{t_f}\frac{1}{M}\frac{dM}{dt}\,dt = \mathbf{u}\int_{M_0}^{M_f}\frac{dM}{M}$$

$$\mathbf{v}_f - \mathbf{v}_0 = \mathbf{u}\ln\frac{M_f}{M_0} = -\mathbf{u}\ln\frac{M_0}{M_f}.$$

If $\mathbf{v}_o = 0$, then $\qquad \mathbf{v}_f = -\mathbf{u}\ln\frac{M_0}{M_f}.$

The final velocity is independent of how the mass is released-the fuel can be expended rapidly or slowly without affecting $\mathbf{v}_f$. The only important quantities are the exhaust velocity and the ratio of initial to final mass.

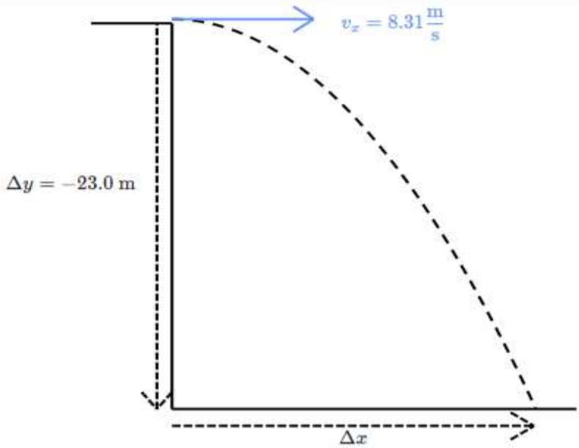
Q10. What is the projectile motion? explain in detail-

Ans. Projectile motion is a form of motion experienced by an object or particle (a projectile) that is thrown near the Earth's surface and moves along a curved path under the action of gravity only.

A water balloon is thrown horizontally with a speed of $v_0 = 8.31\frac{\mathrm{m}}{\mathrm{s}}$ from the roof of a building of height $H = 23.0\ \mathrm{m}$.

How far does the balloon travel horizontally before striking the ground?

We can start by drawing a diagram that includes the given variables.



The vertical component of the initial velocity is zero ($v_{0y} = 0$) since the balloon was **thrown horizontally to start.** In order to have a vertical component of initial velocity the balloon would have to be thrown diagonally at an angle upward or downward to start.

The vertical displacement is -23 m since the balloon **fell downward** through the entire height of the building.

The acceleration in the vertical direction for a projectile is always the acceleration due to gravity $a_y = -g = -9.8\frac{m}{s^2}$.

So we'll use a kinematic formula in the vertical direction to solve for time t. We don't know the final velocity v_y, and we aren't asked for the final velocity v_y so we'll use the vertical kinematic formula that doesn't include final velocity.

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 \quad \text{(use the vertical kinematic formula that does}$$

$$-H = (0)t + \frac{1}{2}(-g)t^2 \quad \text{(plug in known vertical values)}$$

$$t = \sqrt{\frac{2H}{g}} \quad \text{(solve symbolically for time t)}$$

$$t = \sqrt{\frac{2(-23.0 \text{ m})}{-9.8\frac{\text{m}}{\text{s}^2}}} = 2.17 \text{ s} \quad \text{(plug in numerical values and find th}$$

$$\Delta x = v_x t \quad \text{(use the equation for the horizontal displacement)}$$

$$\Delta x = (8.31\frac{\text{m}}{\text{s}})(2.17 \text{ s}) \quad \text{(plug in the time of flight and } v_x\text{)}$$

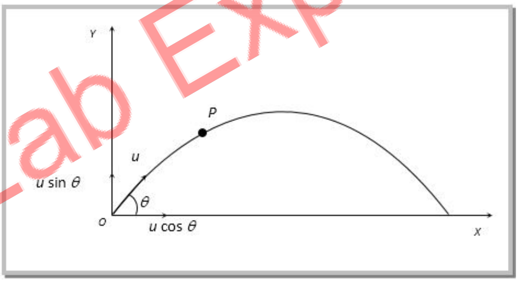
$$\Delta x = 18.0 \text{ m} \quad \text{(calculate and celebrate)}$$

So the water balloon struck the ground 18.0 m horizontally from the edge of the building. *[Could we have solved this problem symbolically?]*

Q.11. Give the formulae for oblique projectile?

Ans.

The total time spent in the air is $T = \frac{2u \sin\theta}{g}$.



The maximum height a projectile reaches above its release point is $H_{max} = \frac{u^2 \sin^2\theta}{2g}$.

The net displacement in the horizontal direction for an object with no vertical displacement is $R = \frac{u^2 \sin 2\theta}{g}$

UNIT – 2
WORK and ENERGY

Q. 1 What are the conservative and non-conservative forces? Explain the related terms like path independent, Kinetic and potential energy, work and power?

Ans. Force can be divided into two broad classes-

1. Conservative forces
2. Non-conservative forces.

The work done by a conservative force in displacing a particle form one point to another depends only on the position of the two given points and it is quite independent of the actual path taken between them. On the other hand, the work done by a non-conservative force does depend upon the actual path taken. work done in a close path under a conservative force is zero.

Energy - The ability to do work.

Kinetic Energy - The energy of motion.

Path independence - Property of conservative forces which states that the work done on any path between two given points is the same.

Potential energy - The energy of configuration of a conservative system. For formulae, see Definition of potential energy, gravitational potential energy, and Definition of potential energy given a position-dependent force.

Total mechanical energy - The sum of the kinetic and potential energy of a conservative system. See definition of total mechanical energy.

Work - A force applied over a distance. For formulas, see work done by a constant force parallel to displacement and work done by any constant force, and work done by a position-dependent force.

Power - Work done per unit time. For formulas, see Formula for average power, Definition of instantaneous power, and formula for instantaneous power.

Q. 2 Discuss the concepts of potential energy.

Ans. Potential Energy :

The energy possessed by a body or system by virtue of its position or configuration is known as the potential energy. For example, a block attached to a compressed or elongated spring possesses some energy called elastic potential energy. This block has a capacity to do work. Similarly, a stone when released from a certain height also has energy in the form of gravitational potential energy.

Regarding the potential energy it is important to note that it is defined for a conservative force field only. For non-conservative forces it has no meaning. The change in potential energy *(dU)* of a system corresponding to a conservative internal force is given *by*

We generally choose the reference point at infinity and assume potential energy to be zero there, if we take r_i= *00* (infinite) and U = 0^- then we can write

or potential energy of a body or system is the negative of work done by the *conservative* forces in bringing it from infinity to the present position.
Regarding the potential energy it is worth noting that:

1. Potential energy can be defined only for conservative forces and it should be considered to be a property of the entire system rather than assigning it to any specific particle.
2. Potential energy depends on frame of reference.

In a conservative force field, the potential energy of a particle is, in general, a function of space and, therefore, changes from point to point. A curve, showing variation of the potential energy of a participant its position in the field is spoken as a *potential energy curve* and supplies a great deal of information about the motion of the particle without having to solve any equations of motion.

Q 3 Discuss the concept of electric potential as potential energy.

Ans. The intensity E of an electric field at a point is defined as the force experienced by a unit positive charge when placed at that point in the field. It follows, therefore, that the force experienced by a charge Q placed at the point will be qE.

Again, the potential (V) at a point in an electric field is defined as the amount of work done in moving a unit positive charge from 00 *(where the potential is assumed to be zero)* to that point. It is thus, in other words, the *electrostatic potential energy of a unit positive charge placed at that point.* So that, if potential energy of a charge of + *q* units be *U* at a point r in an electric field, we have

V= *U/q* = *E.dr,*

where**E** is the intesity of the field at the point.
Hence, the potential difference between two points r, and r_0 is

given by V_2 —V_1 =

where V, and V_c are the potentials at the two points respectively. This is obviously equal to the change in the electrostatic potential.

Q 4 Explain the Law of conservation of mechanical energy.

Ans. Law of Conservation of Mechanical Energy

Mechanical energy, EME_MEME, start subscript, M, end subscript, is the sum of the potential energy and kinetic energy in a system.
*E*M=*E*P+*E*K
Only conservative forces like gravity and the spring force that have potential energy associated with them. Nonconservative forces like friction and drag do not. We can always get back the energy that

we put into a system via a conservative force. Energy transferred by nonconservative forces however is difficult to recover. It often ends up as heat or some other form which is typically outside the system—in other words, lost to the environment.

Consider a golfer on the moon—gravitational acceleration 1.625 m/s^2 striking a golf ball. The ball leaves the club at an angle of 45 degree to the lunar surface traveling at 20 m/s both horizontally and vertically—total velocity 28.28 m/s. *How high would the golf ball go?*

We begin by writing down the mechanical energy:

$$E_{\mathrm{M}} = \tfrac{1}{2}mv^2 + mgh$$

Applying the principle of conservation of mechanical energy, we can solve for the height h—note that the mass cancels out.

$$\tfrac{1}{2}mv_i^2 = mgh_f + \tfrac{1}{2}mv_f^2$$

$$\begin{aligned} h &= \frac{\frac{1}{2}v_i^2 - \frac{1}{2}v_f^2}{g} \\ &= \frac{\frac{1}{2}(28.28\ \mathrm{m/s})^2 - \frac{1}{2}(20\ \mathrm{m/s})^2}{1.625\ \mathrm{m/s^2}} \\ &= 123\ \mathrm{m} \end{aligned}$$

Sometimes it is convenient to call $E = K + U$, the sum of kinetic and potential energy contributions, the *energy function*. The kinetic energy contribution K is equal to $\frac{1}{2}Mv^2$. The potential energy depends on the field force acting, and it has the essential property that $U = -\int F\,dy$, which is an expression for work where the field force F may be a function of position y. Then

$$F = -\frac{dU}{dy} \qquad (5.10)$$

where F, the force acting on the particle, results from interactions intrinsic to the problem, such as electrical or gravitational interactions, and is what we have called the *force of the field*, or the *force of the problem*. (In the above example, $U = Mgy$, so that $F = F_G = -Mg$.)

Q. 5
Ans.

What is thermal energy?

Thermal energy refers to the energy contained within a system that is responsible for its temperature. Heat is the flow of thermal energy. A whole branch of physics, *thermodynamics*, deals with how heat is transferred between different systems and how work is done in the process (see the 1st law of thermodynamics).

In the context of mechanics problems, we are usually interested in the role thermal energy plays in ensuring conservation of energy. Almost every transfer of energy that takes place in real-world physical systems does so with efficiency less than 100% and results in some thermal energy. This energy is usually in the form of *low-level* thermal energy. Here, low-level means that the temperature associated with the thermal energy is close to that of the environment. It is only possible to extract work when there is a temperature difference, so low-level thermal energy represents 'the end of the road' of energy transfer. No further useful work is possible; the energy is now 'lost to the environment'.

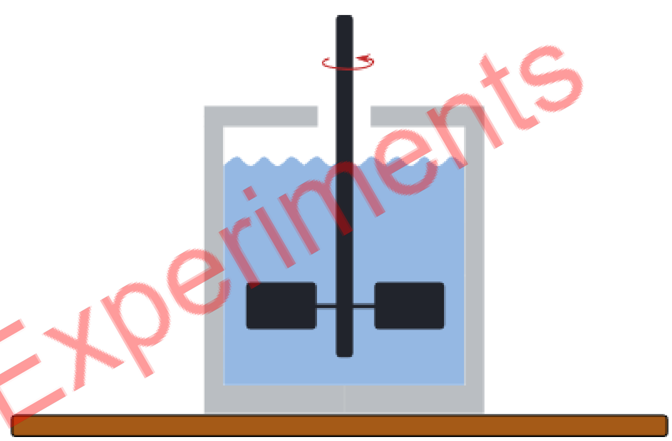


Figure 2: A paddle wheel rotating in a water tank.

Exercise 2a: Suppose the paddle wheel depicted in Figure 2 is rotated by an electric motor which is rated at 10 W output power for 30 minutes. How much thermal energy is transferred to the water? *[Hide solution]*

> In this system, all the energy eventually is transferred to thermal energy of the water (assuming the heat capacity of the paddles is negligible). Therefore, we can use the definition of power to find the total thermal energy:

$$\begin{aligned} E_T &= \text{Power} \cdot \text{Duration} \\ &= 10\ \text{W} \cdot (30 \cdot 60\ \text{s}) \\ &= 18\ \text{kJ} \end{aligned}$$

Q 6. What is Gravitational and electrostatic potential energy?
Ans.

Gravitational Potential Energy

From the work done against the gravity force in bringing a mass in from infinity where the potential energy is assigned the value zero, the expression for gravitational potential energy is

$$U = \frac{-GMm}{r}$$

This expression is useful for the calculation of escape velocity, energy to remove from orbit, etc. However, for objects near the earth the acceleration of gravity g can be considered to be approximately constant and the expression for potential energy relative to the Earth's surface becomes

$$U = mgh$$

where h is the height above the surface and g is the surface value of the acceleration of gravity.

Electric potential energy, or **electrostatic potential energy**, is a potential energy (measured in joules) that results from conservative Coulomb forces and is associated with the configuration of a particular set of point charges within a defined system.

The electric potential energy of a system of point charges is defined as the work required assembling this system of charges by bringing them close together, as in the system from an infinite distance.

The electrostatic potential energy, U_E, of one point charge q at position $\mathbf{r}$ in the presence of an electric field $\mathbf{E}$ is defined as the negative of the work W done by the electrostatic force to bring it from the reference position $\mathbf{r}_{ref}$[note 1] to that position $\mathbf{r}$

$$U_E(\mathbf{r}) = -W_{r_{ref}\to r} = -\int_{\mathbf{r}_{ref}}^{\mathbf{r}} q\mathbf{E}(\mathbf{r}')\cdot d\mathbf{r}',$$

where **E** is the electrostatic field and **dr'** is the displacement vector in a curve from the reference position $\mathbf{r}_{ref}$ to the final position **r**.

e electrostatic potential energy can also be defined from the electric potential as follows:

The electrostatic potential energy, U_E, of one point charge q at position **r** in the presence of an electric potential Φ is defined as the

$$U_E(\mathbf{r}) = q\Phi(\mathbf{r}),$$

where Φ is the electric potential generated by the charges, which is a function of position **r**.

Q. 7. The potential energy of a conservative system is given by $U = ax^2 - bx$

Where a and b are positive constants. Find the equilibrium position and discuss whether the equilibrium is stable,

Ans. In a conservative field

$$F \quad \frac{dU}{dx}$$

$$F = \frac{dU}{dx}(ax^2 - bx) = b\ 2a\ x$$

For equilibrium $F = 0$.

Therefore, x = 2a is the stable equilibrium position.

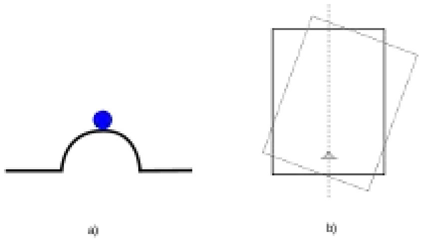
Q.8. Explain the concept of stable and unstable equilibrium?

Ans.

Kinds of Equilibrium

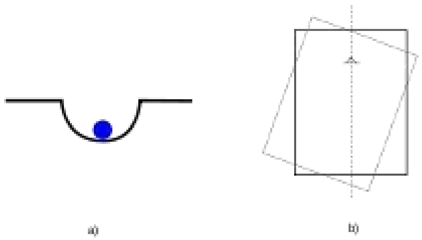
There are three types of equilibrium, namely stable, neutral and unstable equilibrium. Prof. Schumpeter explains the three positions with a simple illustration of a ball placed in three different states. According to Schumpeter, "A ball that rests at the bottom of a bowl illustrates the first case; a ball that rests on a billiard table, the second case, and a ball that is perched on the top of an inverted bowl, the third case."

If we have a function which describes the system's potential energy, we can determine the system's equilibria using calculus. A system is in mechanical equilibrium at the critical points of the function describing the system's potential energy. We can locate these points using the fact that the derivative of the function is zero at these points. To determine whether or not the system is stable or unstable, we apply the second derivative test:



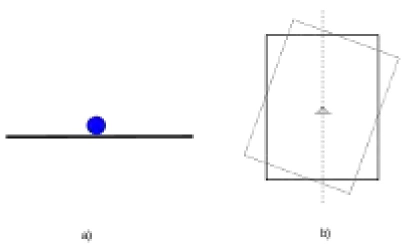
Unstable equilibrium
Second derivative < 0

The potential energy is at a local maximum, which means that the system is in an unstable equilibrium state. If the system is displaced an arbitrarily small distance from the equilibrium state, the forces of the system cause it to move even farther away.



Stable equilibrium
Second derivative > 0

The potential energy is at a local minimum. This is a stable equilibrium. The response to a small perturbation is forces that tend to restore the equilibrium. If more than one stable equilibrium state is possible for a system, any equilibria whose potential energy is higher than the absolute minimum represent metastable states.



Neutral equilibrium
Second derivative = 0 or does not exist

The state is neutral to the lowest order and nearly remains in equilibrium if displaced a small amount. To investigate the precise stability of the system, higher order derivatives must be examined. The state is unstable if the lowest nonzero derivative is of odd order or has a negative value, stable if the lowest nonzero derivative is both of even order and has a positive value, and neutral if all higher order derivatives are zero. In a truly neutral state the energy does not vary and the state of equilibrium has a finite width. This is sometimes referred to as state that is marginally stable or in a state of indifference.

When considering more than one dimension, it is possible to get different results in different directions, for example stability with respect to displacements in the *x*-direction but instability in the *y*-direction, a case known as a saddle point. Generally, an equilibrium is only referred to as stable if it is stable in all directions

Q 9. Explain the concept of power and its mathematical relation with force and work?
Ans.

POWER

The *power* P is the time rate of transfer of energy. We have defined the work done on the particle in a displacement $\Delta\mathbf{r}$ by an applied force as

$$\Delta W = \mathbf{F} \cdot \Delta\mathbf{r}$$

The rate at which work is done by the force is

$$\frac{\Delta W}{\Delta t} = \mathbf{F} \cdot \frac{\Delta\mathbf{r}}{\Delta t}$$

In the limit $\Delta t \to 0$ we have the power

$$\boxed{P = \frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} = \mathbf{F} \cdot \mathbf{v}} \qquad (5.41)$$

From the power $P(t)$ as a function of time we can write the work input as

$$W(t_1 \to t_2) = \int_{t_1}^{t_2} P(t)\, dt$$

Q 10.

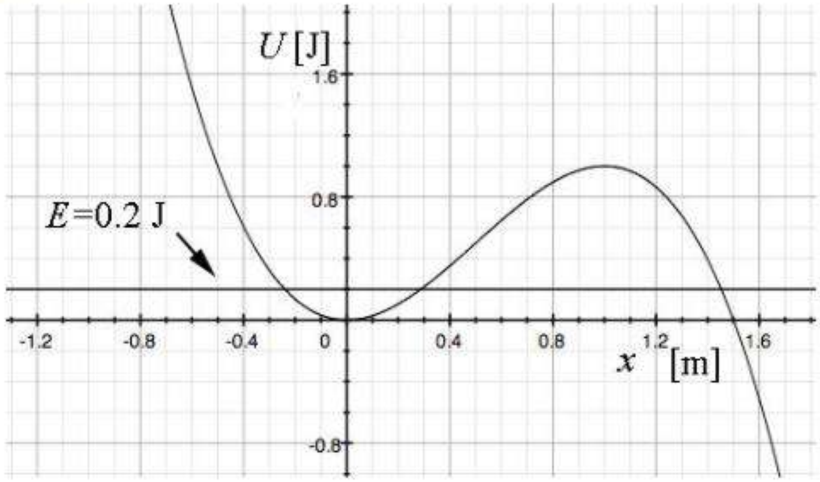
The potential energy function for a particle of mass m, moving in the x-direction is given by

$$U(x) = -U_1\left(\left(\frac{x}{x_1}\right)^3 - \left(\frac{x}{x_1}\right)^2\right), \qquad (14.5.10)$$

where U_1 and x_1 are positive constants and $U(0) = 0$. (a) Sketch $U(x)/U_1$ as a function of x/x_1. (b) Find the points where the force on the particle is zero. Classify them as stable or unstable. Calculate the value of $U(x)/U_1$ at these equilibrium points. (c) For energies E that lies in $0 < E < (4/27)U_1$ find an equation whose solution yields the turning points along the x-axis about which the particle will undergo periodic motion. (d) Suppose $E = (4/27)U_1$ and that the particle starts at $x = 0$ with speed v_0. Find v_0.

Solution: a) Figure 14.11 shows a graph of $U(x)$ vs. x, with the choice of values $x_1 = 1.5\,\text{m}$, $U_1 = 27/4\,\text{J}$, and $E = 0.2\,\text{J}$.

Solution: a) Figure 14.11 shows a graph of $U(x)$ vs. x, with the choice of values $x_1 = 1.5\text{ m}$, $U_1 = 27/4\text{ J}$, and $E = 0.2\text{ J}$.



b) The force on the particle is zero at the minimum of the potential which occurs at

$$F_x(x) = -\frac{dU}{dx}(x) = U_1\left(\left(\frac{3}{x_1^{\,3}}\right)x^2 - \left(\frac{2}{x_1^{\,2}}\right)x\right) = 0$$

which becomes

$$x^2 = (2x_1/3)x\,.$$

We can solve Eq. (14.5.12) for the extrema. This has two solutions

$$x = (2x_1/3) \quad \text{and} \quad x = 0\,.$$

The second derivative is given by

$$\frac{d^2U}{dx^2}(x) = -U_1\left(\left(\frac{6}{x_1^{\,3}}\right)x - \left(\frac{2}{x_1^{\,2}}\right)\right).$$

Evaluating the second derivative at $x = (2x_1/3)$ yields a negative quantity

$$\frac{d^2U}{dx^2}(x = (2x_1/3)) = -U_1\left(\left(\frac{6}{x_1^{\,3}}\right)\frac{2x_1}{3} - \left(\frac{2}{x_1^{\,2}}\right)\right) = -\frac{2U_1}{x_1^{\,2}} < 0\,,$$

indicating the solution $x=(2x_1/3)$ represents a local maximum and hence is an unstable point. At $x=(2x_1/3)$, the potential energy is given by the value $U((2x_1/3))=(4/27)U_1$. Evaluating the second derivative at $x=0$ yields a positive quantity

$$\frac{d^2U}{dx^2}(x=0)=-U_1\left(\left(\frac{6}{x_1^{\;3}}\right)0-\left(\frac{2}{x_1^{\;2}}\right)\right)=\frac{2U_1}{x_1^{\;2}}>0, \qquad (14.5.16)$$

indicating the solution $x=0$ represents a local minimum and is a stable point. At the local minimum $x=0$, the potential energy $U(0)=0$.

c) Consider a fixed value of the energy of the particle within the range

$$U(0)=0<E<U(2x_1/3)=\frac{4U_1}{27}\;. \qquad (14.5.17)$$

If the particle at any time is found in the region $x_a<x<x_b<2x_1/3$, where x_a and x_b are the turning points and are solutions to the equation then the particle will undergo periodic motion between the values $x_a < x < x_b$. Within this region $x_a < x < x_b$, the kinetic energy is always positive because $K(x) = E - U(x)$. There is another solution x_c to Eq. (14.5.18) somewhere in the region $x_c > 2x_1/3$. If the particle at any time is in the region $x > x_c$ then it at any later time it is restricted to the region $x_c < x < +\infty$.

For $E > U(2x_1/3) = (4/27)U_1$, Eq. (14.5.18) has only one solution x_d. For all values of $x > x_d$, the kinetic energy is positive, which means that the particle can "escape" to infinity but can never enter the region $x < x_d$.

For $E < U(0) = 0$, the kinetic energy is negative for the range $-\infty < x < x_e$ where x_e satisfies Eq. (14.5.18) and therefore this region of space is forbidden.

(d) If the particle has speed v_0 at $x = 0$ where the potential energy is zero, $U(0) = 0$, the energy of the particle is constant and equal to kinetic energy

$$E = K(0) = \frac{1}{2}mv_0^2. \tag{14.5.19}$$

Therefore

$$(4/27)U_1 = \frac{1}{2}mv_0^2, \tag{14.5.20}$$

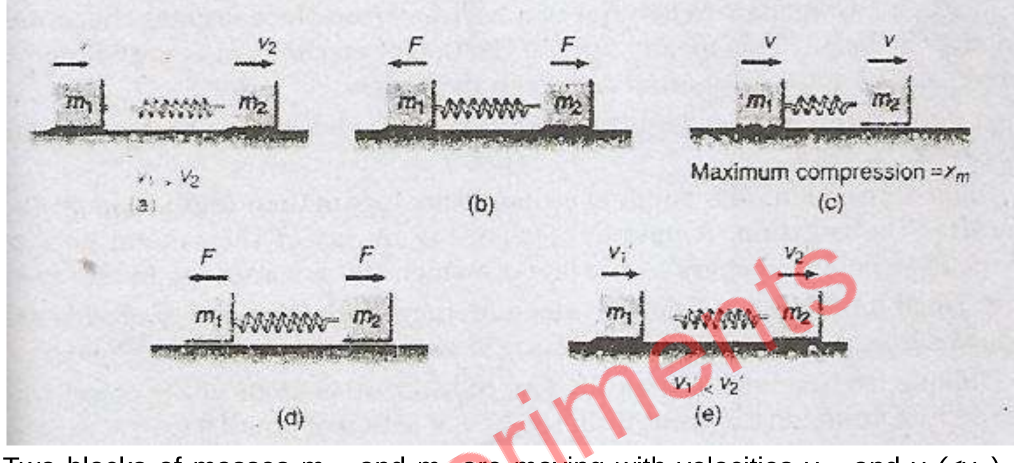
which we can solve for the speed

$$v_0 = \sqrt{8U_1/27m}. \tag{14.5.21}$$

UNIT-3
COLLISION

Q.1 What is collision explain its various types?

Ans. A collision is an event in which two or more bodies exert forces on each other for a relatively short time leading to transferred momentum or energy from one object to another according to law of conservation .It is not necessary that particles striking against each other. Although the most common colloquial use of the word "collision" refers to incidents in which two or more objects strikes with great force, the scientific use of the word "collision" here in physics implies nothing about the magnitude of the force



Two blocks of masses m_1, and m_2 are moving with velocities v_1, and $v_2(<v_1)$ along the same straight line in a smooth horizontal surface. A spring is attached to the block of mass m_2 . Now, let us see what happens during the collision between two particles.

Figure (a) Block of mass m_1 is behind m_2. Since, $v_1 > v_2$ the blocks will collide after some time.

Figure (b) The spring is compressed. The spring force *F (=kx)* acts on the blocks in the directions shown in figure. This force decreases the velocity of m_1 and increases the velocity of m_2.

Figure (c) The spring will compress till velocity of both the blocks become equal. So,at maximum compression (say x_m velocities of both the blocks are equal (say v).

Figure (d) Spring force is still in the directions shown in figure, *i.e.,* velocity of block m1 is further decreased and that of m2 is increased. The spring now starts relaxing.

Figure (e) The two blocks are separated from one another. Velocity of block m_2 becomes more than the velocity of block m_1, *i.e.,* $v_2' > v_1$

Assuming spring to be perfectly elastic following two equations can be applied in the above 5 situation.

(i) In the absence of any external force on the system the linear momentum of the system will remain conserved before, during and after collision, *i.e.*,

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v = m_1v_1' + m_2v_2'$$

(ii) In the absence of any sissipative forces, the mechanical energy of the system will also remain consered, *i.e.*,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2$$

$$= \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

The Spring is supposed to be perfectly elastic

Collision: Collision between two bodies may be classified in three ways:

1. Elastic Collision.
2. Inelastic collision.
3. Head on collision.
4. Oblique collision.

1. Elastic Collision: When the two bodies regain their orginal shape and size after collision. This means that no fraction of mechanical energy remains stored as deformation potential energy in the bodies.In an elastic collision both linear momentum and knietic energy remain conserved. In reality, examples of perfectly elastic collisions are not part of our everyday experience. There are some examples of collisions in mechanics where the energy lost can be negligible. These collisions can be considered elastic, even though they are not perfectly elastic. Collisions of rigid billiard balls or the balls in a Newton's cradle are two such examples.

2. Inelastic colision: The colliding bodies do not regain their orginal shape and size after the collision. A part of mechanical energy of the system goes to defromation potential energy. only linear momentum is conserved in this case. This is because some kinetic energy had been transferred to something else. Thermal energy, sound energy, and material deformation are likely culprits. The ballistic pendulum is a practical device in which an inelastic collision takes place. Until the advent of modern instrumentation, the ballistic pendulum was widely used to measure the speed of projectiles.

3. Head on (direct) collsion : If the directions of the velocity of objects are along the line of action of impulse, acting at the instant of collision.

4. Oblique (Indirect) collision : If before collsion, atleast one of the object was moving in a direction different from the line of action of impulses.

Q. 2 Explain the head on elastic collision.

Ans. Consider the two balls of mass m_1 and m, while moving collide each other elastically with velocities v1 and v_2 in the directions shown in Fig (a). Their velocities become v_1' and v_2' after the colliding along the same line. By the law of conservation of linear momentum, we get

(a) Before collision (b) After collision

In an elastic collision kinetic energy ... conserved. Hence,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + m_2v_2'^2 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii) for v_1', we get

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2 \quad \text{...(iii)}$$

$$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1 \quad \text{...(iv)}$$

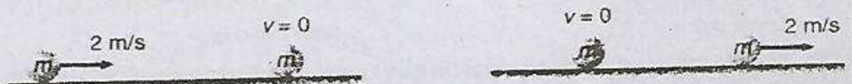
Special Cases

1. If m1 = m2 then from Eqs. (iii) and (iv) we can see that

$$v_1' = v_2 \text{ and } v_2' = v_1$$

i.e. when two particles of equal mass collide elastically and the collision is head on, they exchange their velocities., e.g.,

Before collision After collision



Before collision After collision

2. if $m_1 >> m_2$ and $v_1 = 0$

Before collision After collision

Then $\frac{m_2}{m_1} = 0$

with these two substitutions $\left(v_1 = 0 \text{ and } \frac{m_2}{m_1} = 0\right)$

we get the following two results:

$v_1' = 0$ and $v_2' = -v_2$

i.e., the particle of mass m_1 remains at rest while the particle of mass m_2 bounces back with same speed v_2.

3. If $m_2 >> m_1$ and $v_1 = 0$

Before collision After collision

with the substitution $\frac{m_1}{m_2} = 0$ and $v_1 = 0$, we get the results

$v_1' \approx 2v_2$ and $v_2' \approx v_2$

i.e., the mass m1 moves with velocity $2v_2$ while the velocity of mass m_2 remains unchanged.

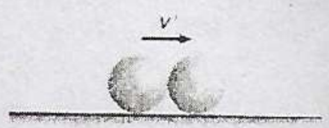
Q. 3 Write a short note on head on inelastic collision.

Ans. We know that in an inelastic collision, the particles do-not regain their original shape and size completely after collision. Some fraction of mechanical energy is retained by the colliding particles in the form of deformation potential energy. Thus, the kinetic energy of the particles no

Before collision After collision

longer remains -conserved. However, in the absence of external forces, law of conservation of linear momentum still holds

Consider the velocities of two particles of mass m_l and m, before collision be v_1 and v_2 in the directions shown in figure. Let v_1' and v_2' be their velocities after collision. The law of conservation of linear momentum gives



$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \qquad \text{...(v)}$$

The collision is said to be perfectly inelastic if both the particles stick together after collision and move with same velocity, say v' as shown in figure. In this case, Eq. (v) can be written as

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

or

$$v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \qquad \text{...(vi)}$$

Q. 4 Summaries the Newton's law of restitution.

Ans. Newton's Law of Restitution state that When two objects are in direct (head on) impact, the speed with which they separate after impact is usually less than or equal to their speed of approach before impact. Experimental evidence suggests that the ratio of these relative speeds is, constant for two given set of objects. This property formulated by Newton, is known as the law of restitution and can be written in form

$$\frac{\text{separation speed}}{\text{approach speed}} = e$$

The ratio *e* is called the coefficient of restitution and is constant for two particular objects. In general $0 \le e \le 1$

If e = 0, collision is for completely inelastic, as both the objects stick together. So, their separation speed is zero or *e* = 0 from Eq. (vii).

If e =1, collision is for elastic, as we can show from Eq. (iii) and (iv), that

Let us now find the velocities of two particles after collision if they collide

$$v_1' - v_2' = v_2 - v_1$$

or separation speed = approach speed

or

$$e = 1$$

Before collision

After collision

directly and the coefficient of restitution between them is given as e. Applying 'conservation of linear momentum

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \qquad \text{...(viii)}$$

Further, separation speed = e (approach speed)

or $$v_1' - v_2' = e(v_2 - v_1) \qquad \text{...(ix)}$$

Solving Eqs. (viii) and (ix), we get

$$v_1' = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)v_1 + \left(\frac{m_2 + em_2}{m_1 + m_2}\right)v_2$$

and $$v_2' = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)v_2 + \left(\frac{m_1 + em_1}{m_1 + m_2}\right)v_1$$

Special Cases

1. If collision is elastic, i.e., e = 1, then

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

and $$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

which are same as Eqs. (iii) and (iv).

2. If collision is perfectly inelastic, i.e., e = 0 then

$$v_1' = v_2' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = v' \text{ (say)}$$

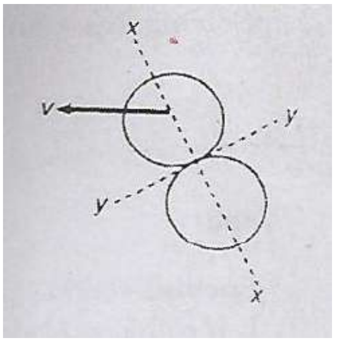
which is same as Eq. (vi).

3. If $m_1 = m_2$ and $v_1 = 0$, then

$$v_1' = \left(\frac{1+e}{2}\right)v_2 \quad \text{and} \quad v_2' = \left(\frac{1+e}{2}\right)v_2$$

Q. 5 Explain the concept of oblique collision.

Ans. **Oblique Collision:** When two object collides with one another then a pair of equal and opposite impulses act at the moment of impact. If just before impact at least one of the objects was moving in a



direction different from the line of action of these impulses the collision is said to be oblique.

In the above figure, two balls collide obliquely. During collision impulses act in the direction *xx.* Hence, we will call this direction as common normal direction and a direction perpendicular to it *(i.e.,yy)* as common tangent. Following four points are important regarding an oblique collision.

A pair of equal and opposite impulses act along common normal direction. Hence, linear momentum of individual particles do change along common normal direction. If mass of the colliding particles remain constant during collision, then we can say that linear velocity of the individual particles change during collision in this direction.

No component of impulse act along common tangent direction. Hence, linear momentum or linear velocity of individual particles (if mass is constant) remain unchanged along this direction.

Net impulse on both the particles is zero during collision. Hence, net momentum of both the panicles remain conserved before and after collision in any direction.

Definition of coefficient of restitution can be applied along common normal direction, *i.e.,* along common normal direction we can apply
Relative speed of separation = *e* (relative speed of approach)
Here, *e* is the coefficient of restitution between the particles.

Q. 6 Explain elastic collision in the centre-of-mass reference frame.
Ans. **Centre of mass frame of reference**
The system which is not experiencing any external force, is an inertial frame of reference. This frame of reference carries the centre of mass of an isolated system of particles.
Let R be the position vector of centre of mass. In.this frame of reference R= o

Since there R is zero dr/dt=0, this means that the velocity of centre of mass is also zero, i.e., v = 0

Further, since R = 0, the liner momentum p=mv is also zero.
Since there is no external force, this frameof reference moves with a constant velocity. This also implies that the centre of mass moves with a constant velocity.
This frame of reference does not impose any restrictions on the **angle** of scattering in collisions. Further, if we use the centre of frame of reference, since the centre of mass is at rest, we need only three co-ordinates for solving a problem.

Let the c.o.m. frame of reference move with a velocity v relative to laboratory frame of reference. The c.o.m. remains stationary in this frame of reference

then $$v = \frac{m_1 u_1}{m_1 + m_2}$$

The initial velocity of $m_1, u_1' = u_1 - v$

The initial velocity of $m_2, u_2' = -v$

final velocity of $m_1, v_1' = v_1 - v$

final velocity of $m_2, v_2' = v_2 - v$

The centre of mass is stationary in this frame of reference. Therefore the net change of linear momentum should be zero. This implies that the momentum of two particles must be equal and in opposite direction. This means that v_1' and v_2'must be in opposite directions after colliding with each other. Further, these should be inclined at the same angle to the initial direction of motion of the particles.
Since the momentum change must be zero.

$$m_1 u_1' + m_2 u_2' = 0$$

or $$m_1 u_1' = -m_2 u_2'$$

similarly $$m_1 v_1' = -m_2 u_2'$$

We can write,

$$m_1 u_1' = m_1 (u_1 - v_1)$$

$$= m_1 \left[u_1 - \frac{m_1 u_1}{m_1 + m_2} \right]$$

or $$m_1 u_1' = \frac{m_1 m_2 u_1}{m_1 + m_2}$$

we can also write,

$$-m_2 u_2' = m_2 v$$

$$= m_2 \left[\frac{m_1 u_1}{m_1 + m_2} \right]$$

$$\frac{1}{2}m_1u_1'^2+\frac{1}{2}m_2\left(\frac{m_1^2u_1'^2}{m_2^2}\right)=\frac{1}{2}m_1v_1'^2+\frac{1}{2}m_2\left(\frac{m_1^2v_1'^2}{m_2^2}\right)$$

This gives us, $\frac{1}{2}m_1u_1'^2\left(1+\frac{m_1}{m_2}\right)=\frac{1}{2}m_1v_1'^2\left(1+\frac{m_1}{m_2}\right)$

or $\quad \frac{1}{2}m_1u_1'^2 = \frac{1}{2}m_1v_1'^2$

o r $\quad u_1' = v_1'$

similarly, $\quad u_2' = v_2'$

when we consider c.o.m. frame of reference with c.o.m. as atationary magnitudes of the velocities of the particles do not change due to an elastic collision.

Q.7 What is impulse? Write a brief note.

Ans. Impulse of a force is defined as the product of force and the time for

$$= \frac{m_1m_2u_1}{m_1+m_2}$$

also, we have

$$m_1v_1' = -m_2v_2'$$

$$\therefore \quad v_2' = -\frac{m_1}{m_2}v_1'$$

and $\quad u_2' = \frac{m_1}{m_2}v_1'$

In a collsion, total energy is conserved

$$\therefore \frac{1}{2}m_1v_1'^2+\frac{1}{2}m_2v_2'^2=\frac{1}{2}m_1v_1'^2+\frac{1}{2}m_2v_2'^2$$

If we substitute the values of u_2' and v_2' in the above equation, we get

which it acts". Therefore, large force acting for a short time to produce a finite change in momentum which is called impulse of this force and the force acted is called impulsive force or force of impulse.

According to Newton's second law of motion

$$\vec{F} = m\vec{a} = m\frac{(\vec{v}-\vec{u})}{t}$$

or,

$$\vec{F}t = m\vec{v} - m\vec{u} = \vec{J}$$

So, Impulse of a force = change in momentum.

If the force acts for a small duration of time, the force is called impulsive force.

As force is a variable quantity, thus impulse will be,

$$J = \int_{t_1}^{t_2} F dt$$

The area under F - t curve gives the magnitude of impulse.

Impulse is a vector quantity and its direction is same as the direction of $\vec{F}$ **Q.8**

Unit of Impulse:- The unit in S.I. system is kgm/sec or newton -second.

Dimension of Impulse:- MLT^{-1}

Define coefficient of Coefficient of Restitution (e)
Ans. Coefficient of Restitution (e)
It is defined as the ratio between magnitudes of impulse during period of restitution to that during period of deformation.

e = relative velocity after collision / relative velocity before collision

$= v_2 – v_1/u_1 – u_2$

Thus, coefficient of restitution (of a collision), is also defined as the ratio between relative velocities of two bodies after collision to that before collision

Case (i): For perfectly elastic collision, e = 1. Thus, $v_2 – v_1 = u_1 – u_2$. This signifies the relative velocities of two bodies before and after collision are same.
Case (ii):-For inelastic collision, e<1. Thus $v_2 – v_1 < u_1 – u_2$. This signifies, the value of e shall depend upon the extent of loss of kinetic energy during collision.

Case (iii):- For perfectly inelastic collision, e = 0. Thus, $v_2 – v_1 = 0$, or $v_2 = v_1$. This signifies the two bodies shall move together with same velocity. Therefore, there shall be no separation between them.

On the basis of coefficient of restitution, collision is divided into three types.

If e = 1, collision is called elastic if in this collision mechanical energy of the whole system is conserved.

If 0 < e < 1, collision is called inelastic.
If e = 0, collision is called completely inelastic.

Q.9 A 70 g ball collides with another ball of mass 140 g. the initial velocity of the first ball is 9 ms-1 to the right while the second ball is at rest. If the collision were perfectly elastic what would be the velocity of the two balls after the collision?

Solution: lets the mass and velocities of first and second ball's are m_1=70g , v_1=9 ms^{-1} and m_2=140g, v_2=0 respectively . Let after collision velocities of two ball will $v_1^!$ and $v_2^!$.

$$m_1 = 70\,g \qquad v_1 = 9\,ms^-1 \qquad v_2 = 0$$

$$m_2 = 140\,g \qquad v'_1 = ? \qquad v'_2 = ?$$

We know that $$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

$$= \frac{70\,g - 140\,g}{70\,g + 140\,g}\ x\ 9ms^{-1} = -3\,ms^{-1}$$

$$v'_2 = \frac{2m_1}{m_1 + m_2} v_1$$

$$= \frac{2\,x 70\,g}{70\,g + 140\,g}\ x\ 9ms^{-1} = -6\,ms^{-1}$$

Q.10. A 100 g stone ball is moving to the right with a velocity of 20 ms^{-1}. It makes a head on collision with an 8 kg iron ball, initially at rest. Compute velocities of the balls after collision.

Solution: lets the mass and velocities of first and second ball's are m_1=100g, v_1=20 ms^{-1} and m_2=8g, v_2=0 respectively. Let after collision velocities of two ball will $v_1^!$ and $v_2^!$

$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 \quad and \quad v'_2 = \frac{2m_1}{m_1 + m_2} v_1$$

Hence

$$v'_1 = \frac{0.1\,kg - 8\,kg}{0.1\,kg\ + 8\,kg}\ x\ 20ms^{-1} = -19.5\,ms^{-1}$$

$$v'_2 = \frac{2\ x\ 0.1\,kg}{0.1\,kg\ + 8\,kg}\ x\ 20ms^{-1} = 0.5\,ms^{-1}$$

Q.11. A 10 kg mass traveling 2 m/s meets and collides elastically with a 2 kg mass traveling 4 m/s in the opposite direction. Find the final velocities of both objects.

Solution: lets the mass and velocities of first and second ball's are m_A=100g, v_{Ai}=20 ms^{-1} and m_B=8g, v_{Bi}=0 espectively . Let after collision velocities of two ball will v_{Af} and v_{Bf}

m_A = 10 kg

V_{Ai} 2 m/s

m_B = 2 kg

V_{Bi} = −4 m/s. The negative sign is because the velocity is in the negative direction.

Now we need to find V_{Af} and V_{Bf}. Use the equations from above. Let's start with V_{Af}.

$$V_{Af} = \frac{m_A - m_B}{m_A + m_B} V_{Ai} + \frac{2m_B}{m_A + m_B} V_{Bi}$$

Plug in our known values.

$$V_{Af} = \frac{(10 - 2)\text{kg}}{(10 + 2)\text{kg}} \cdot (2\text{ m/s}) + \frac{2(2\text{ kg})}{(10 + 2)\text{kg}} \cdot (-4\text{ m/s})$$

$$V_{Af} = \frac{8}{12} \cdot (2\text{ m/s}) + \frac{4}{12} \cdot (-4\text{ m/s})$$

$$V_{Af} = \frac{16}{12}\text{ m/s} + \frac{-16}{12}\text{ m/s}$$

V_{Af} = 0 m/s

The final velocity of the larger mass is zero. The collision completely stopped this mass.

Now for V_{Bf}

$$V_{Bf} = \frac{2m_A}{(m_A + m_B)} V_{Ai} + \frac{(m_B - m_A)}{(m_A + m_B)} V_{Bi}$$

Plug in our known values

$$V_{Bf} = \frac{2(10\text{ kg})}{(10 + 2)\text{ kg}} \cdot 2\text{ m/s} + \frac{(2 - 10)\text{ kg}}{(10 + 2)\text{ kg}} \cdot -4\text{ m/s}$$

$$V_{Bf} = \frac{20\text{ kg}}{12\text{ kg}} \cdot 2\text{ m/s} + \frac{-8\text{ kg}}{12\text{ kg}} \cdot -4\text{ m/s}$$

$$V_{Bf} = \frac{40}{12}\text{ m/s} + \frac{32}{12}\text{ m/s}$$

$$V_{Bf} = \frac{72}{12}\text{ m/s}$$

V_{Bf} = 6 m/s

The second, smaller mass shoots off to the right (positive sign on the answer) at 6 m/s while the first, larger mass is stopped dead in space by the elastic collision.