Free Study Material from

All Lab Experiments

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B.Sc. (Physical Science)

Chapter - 8

Special Theory of Relativity

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Speed Theory of Relativity: Constancy of speed of light. Postulates of special theory of Relativity. Length contraction. Time dilation. Relativistic addition of velocities.

Q. Write a Shortnote on Constancy of speed of light.
Ans.

The speed of light is measured to be about 1,079,253,000 km/hr, and presumably this is its speed relative to the ether. Presumably the ether is stationary with respect to the fixed stars. This section investigates these two presumptions.

Galileo attempted to measure the speed of light around 1600. He and a colleague each had a lantern with a shutter, and they went up on neighboring mountains. Galileo opened the shutter on his lantern and when his colleague saw the light from Galileo's lantern he opened the shutter on this lantern. The time delay between when Galileo opened the shutter on his lantern and when he saw the answering light from his colleague's lantern would allow him to calculate the speed of light. This is absolutely correct experimental procedure in principle. However, because of our human reaction times the lag between when the colleague saw the light from Galileo's lantern until when he could get the shutter of his lantern open is so long that the light could have circled the globe many many times.

In 1676 Römer successfully measured the speed of light, although his results differed from the accepted value today by about 30%

Q. What are the basic postulates of Special Theory of Relativity? Explain Length contraction and Time Dilation.
Ans.

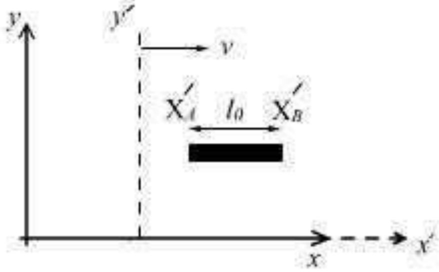
Einstein in 1905, formulated this special theory of relativity on the basis of following postulates:

- The laws of physics are same in all inertial frames. This is the principle of relativity.
- The speed of light in vacuum is constant and same as observed from all inertial frames. This is known as the principle of constant of speed of light.

The first postulate implies that not only the laws of mechanics but also that of electrodynamics and optics should be same for all inertial reference systems. There is no privileged frame like ether or absolute space.

Length Contraction:

Two dramatic results of the special theory of relativity are that a meter stick is shorter when moving than at rest and the moving clock runs slow. These results are quite real.



Consider a stick at rest in the S' system lying along the x' axis with its ends at x'_A and x'_B. The length of the stick is,

$$l_0 = x'_B - x'_A \quad \ldots\ldots\ (1)$$

l_0 is called the rest or proper length of the system.

Now let us determine the length of the stick in S system, where the observer is at rest. According to the observer in the S system, the stick moves with velocity v towards right. The length of the stick is distance between its ends at the same instant of time. According to Lorentz transformation

$$x'_B = \gamma(x_B - vt) \quad \ldots\ldots\ldots\ (2)$$

$$x'_A = \gamma(x_A - vt) \quad \ldots\ldots\ldots\ (3)$$

from(2)and(3),

$$x'_B - x'_A = \gamma(x_B - x_A)$$

$$l = (x_B - x_A) = \frac{l_0}{\gamma}$$

$$\text{or} \quad l = l_0\sqrt{1 - \frac{v^2}{c}}$$

l is shorter than l_0 ; as $v \to c,\ l \to 0$

This shortening is known as length contraction or Lorentz contraction and it occurs only along the direction of motion. If the stick lay along the y -axis we would use $y = y'$ and $l = l_0$

Time Dilation :

Let us investigate the effect of motion on time. Consider a clock at rest in the S' system and two events A and B both occurs at the same point x'_0.

$$A = x'_0,\ t'_A$$

$$B = x'_0,\ t'_B$$

the time interval $\tau = t'_B - t'_A$ is the time interval between two events in S' system or rest system. It is called the proper time. The corresponding time interval in laboratory system can be calculated as follows:

$$t'_A = \gamma\left(t_A - \frac{v\,x_A}{c^2}\right)$$

$$t'_B = \gamma\left(t_B - \frac{v\,x_B}{c^2}\right)$$

or $$t_A = \gamma\left(t'_A + \frac{v\,x'_0}{c^2}\right) \qquad \because x'_A = x'_B = x'_0$$

$$t_B = \gamma\left(t'_B + \frac{v\,x'_0}{c^2}\right)$$

The last two equations are written based on the argument that for an observer at S' system, it would appear that S system moves with velocity -v.

$$(t_B - t_A) = \gamma(t'_B - t'_A)$$

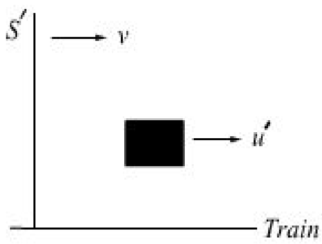
$$T = \gamma T_0 = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Therefore, the time interval in the laboratory system is greater than that of rest system (S'). The moving clock runs slower and this effect is known as time dilation.

Q. How to Add Relative Velocities?

Ans.

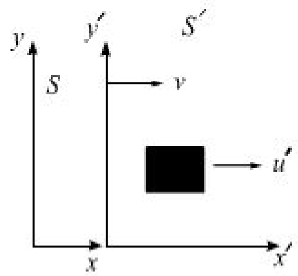
5. The Relativistic Addition of Velocity :



In classical physics, if we consider a case of a train moving with velocity v with respect to ground and a passenger on the train moving with a velocity u' with respect to train, Then the velocity of passanger with respect to the ground is,

$$u = u' + v \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This is simply Galilean or classical velocity addition theorem. Let us discuss how the velocities are added according to special theory of relativity.



Consider for a special case, all velocities are along x - x' direction of two inertial frames S and S', S - is the laboratory frame and S' - is moving frame with constant velocity v. A body moves in S' frame with a velocity u' and its position can be written as $x' = u't'$. The speed of the body with respect to S frame can be calculated as follows,

$$x' = \gamma(x - vt) \qquad \ldots\ldots\ldots (2)$$

$$t' = \gamma\left(t - \frac{v\,x}{c^2}\right) \qquad \ldots\ldots\ldots (3)$$

$$x' = u't' = u'\gamma\left(t - \frac{\mathrm{v}\,x}{c^2}\right) = \gamma(x - \mathrm{v}t)$$

$$x - \mathrm{v}t = u'\left(t - \frac{\mathrm{v}\,x}{c^2}\right)$$

$$x\left(1 + \frac{u'\mathrm{v}}{c^2}\right) = t(u' + \mathrm{v})$$

$$u = \frac{x}{t} = \frac{u' + \mathrm{v}}{\left(1 + \frac{u'\mathrm{v}}{c^2}\right)}$$

$$u = \frac{u' + \mathrm{v}}{\left(1 + \frac{u'\mathrm{v}}{c^2}\right)} \qquad \ldots\ldots\ldots\ldots \quad (4)$$

where $u = x / t$ is the velocity of body with respect to S frame. This is the relativistic or Einstien velocity addition theorem.

If u' and v are small compared to c, $u = u' + \mathrm{v}$, which is the classical result. On the other hand, if $u' = c$, u would also be equal to c, irrespective of the value of v.

Hence, when $u' \to c$

$$u = \frac{c + \mathrm{v}}{1 + \frac{c\,\mathrm{v}}{c^2}} = \frac{c + \mathrm{v}}{c(c + \mathrm{v})} . c^2 = c$$

Thus any velocity relativistically added to c gives a resultant value c. In this sense, c plays the same role in relativity that an infinite velocity in classical case.

Let us calculate the velocities perpendicular to the relative motion of frames. Let y_1' and y_2' are the positions of the body at t_1' and t_2' in S' system. So the velocity in S' system is,

$$u_y' = \frac{y_2' - y_1'}{t_2' - t_1'} \qquad \ldots\ldots\ldots\ldots \quad (4)$$

We can use Lorentz transformation for the calculation of corresponding velocities in S frame.

$$y_2 - y_1 = y_2' - y_1' \qquad \ldots\ldots\ldots\ldots \quad (5)$$

$$t_2' = \gamma\left(t_2 - \frac{\mathrm{v}\,x_2}{c^2}\right), \quad t_1' = \gamma\left(t_1 - \frac{\mathrm{v}\,x_1}{c^2}\right)$$

$$t_2' - t_1' = \Delta t' = \gamma\left[(t_2 - t_1) - \frac{\mathrm{v}}{c^2}(x_2 - x_1)\right]$$

$$\therefore \quad u'_y = \frac{y'_2 - y'_1}{\Delta t'} = \frac{y_2 - y_1}{\gamma\left[\Delta t - \frac{v}{c^2}\Delta x\right]} = \frac{\frac{\Delta y}{\Delta t}}{\gamma\left[1 - \frac{v}{c^2}\frac{\Delta x}{\Delta t}\right]}$$

$$or,\ u'_y = \frac{u_y}{\gamma\left[1 - \frac{u_x v}{c^2}\right]} = \frac{u_y\sqrt{1 - \frac{v^2}{c^2}}}{\left(1 - \frac{u_x v}{c^2}\right)} \qquad \ldots\ldots\ldots\ldots (7)$$

We can write the corresponding inverse transformation by changing v to $-v$,

$$u_y = \frac{u'_y\sqrt{1 - \frac{v^2}{c^2}}}{\left(1 + \frac{u_x v}{c^2}\right)} \qquad \ldots\ldots\ldots\ldots (8)$$

$$\text{Similarly,} \quad u_z = \frac{u'_z\sqrt{1 - \frac{v^2}{c^2}}}{\left(1 + \frac{u_x v}{c^2}\right)} \qquad \ldots\ldots\ldots\ldots (9)$$

From equation (3),

$$u_x = \frac{u'_x + v}{\left(1 + \frac{u_x v}{c^2}\right)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Example : A 2*m* long stick, when it is at rest, moves past an observer on the ground with a speed of 0.5*c*.
(a) What is the length measured by the observer ?
(b) If the same stick moves with the velocity of 0.05*c* what would be its length measured by the observer ?

Solution:

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$(a) \quad L = L_0\sqrt{1 - 0.25} = 2 \times 0.866$$
$$L = 1.732\ m$$

$$(b) \quad L = L_0\sqrt{1 - \left(25 \times 10^{-4}\right)}$$
$$= 2 \times 0.9987 = 1.997\ m$$

Thus if the velocity of the stick is very small compared to *c*, the result is close to that of classical theory.

Example : A cubical shape of body with 1m length on each side when it is at rest, moves with a velocity 0.6*c* along x - direction. What is the shape and dimension of the body noted by an observer on ground.

Solution:

$$x_0 = y_0 = z_0 = 1m$$

$$v = 0.6c$$

$$x = x_0\sqrt{1-\frac{v^2}{c_2}}, \quad y = y_0, \quad z = z_0$$

$$x = 1\sqrt{1-0.36} = 0.8m$$

It would be a parallelepiped shape with dimension $0.8 \times 1 \times 1m^2$

Example : Calculate the momentum and kinetic energy of α - particle moving with a velocity of

(a) 0.01 *c*
(b) 0.1 *c* and
(c) 0.4 *c*

Solution: α -particle is bassically a He ion with mass number A=4.

$$\text{Momentum} \quad p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m_0 = 4 \times 1.67 \times 10^{-27}\,\text{kg} = 6.68 \times 10^{-27}\,\text{kg}$$

$$(a) \quad v = 0.01c$$

$$p = \frac{m_0 \times 0.01c}{\sqrt{1-10^{-4}}} = 2.0\times10^{-20}\ kg\,m/s^2$$

$$(b) \quad v = 0.1c, \quad p = 2.01\times10^{-19}\ Kgm/s^2$$

$$(c) \quad v = 0.4c, \quad p = \frac{m_0 \times 0.4c}{\sqrt{1-0.16}} = 8.75\times10^{-19}\ kgm/s^2$$