Free Study Material from

All Lab Experiments

All Lab Experiments

B.Sc. (Physical Science)

Chapter - 6, 7

6. Oscillations

7. Elasticity

Chapter - 6

Oscillations: Simple harmonic motion. Differential equation of SHM and its solutions. Kinetic and Potential Energy, Total Energy and their time averages. Damped oscillations.

Q. What is Simple Harmonic Motion? Write the solution for the equation of Simple Harmonic Motion.

Ans. In mechanics and physics, **simple harmonic motion** is a special type of periodic motion or oscillation motion where the restoring force is directly proportional to the displacement and acts in the direction opposite to that of displacement.

In Newtonian mechanics, for one-dimensional simple harmonic motion, the equation of motion, which is a second-order linear ordinary differential equation with constant coefficients, can be obtained by means of Newton's 2nd law and Hooke's law for a mass on a spring.

$$F_{\text{net}} = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -kx,$$

where m is the inertial mass of the oscillating body, x is its displacement from the equilibrium (or mean) position, and k is a constant (the spring constant for a mass on a spring). (Note that in reality this is in fact an approximation, only valid for speeds that are small compared to the speed of light.)

Therefore,

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{k}{m}x,$$

Solving the differential equation above produces a solution that is a sinusoidal function.

$$x(t) = c_1\cos(\omega t) + c_2\sin(\omega t)$$

This equation can be written in the form:

$$x(t) = A\cos(\omega t - \varphi),$$

Where,

$$\omega = \sqrt{\frac{k}{m}}, \qquad A = \sqrt{{c_1}^2 + {c_2}^2}, \qquad \tan\varphi = \frac{c_2}{c_1},$$

In the solution, c_1 and c_2 are two constants determined by the initial conditions, and the origin is set to be the equilibrium position. Each of these constants carries a physical meaning of the motion: A is the amplitude (maximum displacement from the equilibrium position), $\omega = 2\pi f$ is the angular frequency, and φ is the phase.

Using the techniques of calculus, the velocity and acceleration as a function of time can be found:

$$v(t) = \frac{\mathrm{d}x}{\mathrm{d}t} = -A\omega \sin(\omega t - \varphi),$$

Speed:

$$\omega\sqrt{A^2 - x^2}$$

Maximum speed:(at equilibrium point)

$$a(t) = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -A\omega^2 \cos(\omega t - \varphi).$$

Maximum acceleration: $A\omega^2$ (at extreme points)

By definition, if a mass m is under SHM its acceleration is directly proportional to displacement.

$$a(x) = -\omega^2 x.$$

Where

$$\omega^2 = \frac{k}{m}$$

Since $\omega = 2\pi f$,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}},$$

and, since $T = 1/f$ where T is the time period,

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

These equations demonstrate that the simple harmonic motion is isochronous (the period and frequency are independent of the amplitude and the initial phase of the motion).

Energy:

Substituting ω^2 with k/m, the kinetic energy K of the system at time t is

$$K(t) = \tfrac{1}{2}mv^2(t) = \tfrac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \varphi) = \tfrac{1}{2}kA^2 \sin^2(\omega t + \varphi),$$

and the potential energy is

$$U(t) = \tfrac{1}{2}kx^2(t) = \tfrac{1}{2}kA^2 \cos^2(\omega t + \varphi).$$

In the absence of friction and other energy loss, the total mechanical energy has a constant value

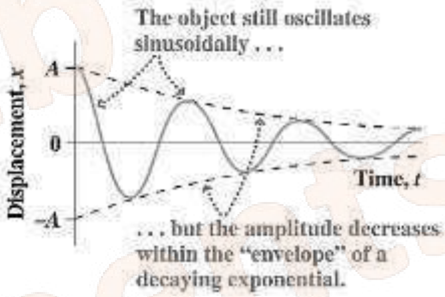
$$E = K + U = \tfrac{1}{2}kA^2.$$

Q. What are Damped Harmonic Oscillation?

Ans. SHM has assumed that the motion is frictionless, the total energy (kinetic plus potential) remains constant and the motion will continue forever. Of course in real world situations this is not the case, frictional forces are always present such that, without external intervention, oscillating systems will always come to rest. The frictional (damping) force is often proportional (but opposite in direction) to the velocity of the oscillating body such that

$$m\frac{d^2x}{dt^2} = F_R - F_f \qquad \Rightarrow \qquad m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$



where b is the damping constant.

This differential equation has solutions

$$x = A'e^{-bt/2m}\cos(\omega' t + \phi)$$

where $\omega' \approx \omega$ when the damping is small (small b). Notice that this solution represents oscillatory motion with an exponentially decreasing amplitude.

Chapter - 7

Elasticity: Hooke's law- Stress-strain diagram - Elastic moduli-Relation between elastic constants- Poisson's Ratio-Expression for Poisson's ratio in terms of elastic constants- Work done in stretching & work done in twisting a wire- Twisting couple on a cylinder- Determination of Rigidity modulus by static torsion- Torsional pendulum-Determination of Rigidity modulus and moment of inertia - q, η & □ by Searles method.

Q. What is Hook's Law?

Ans. An interesting application of work combined with the Force and Displacement graph is examining the force applied by a spring. The more you stretch a spring, the greater the force of the spring… similarly, the more you compress a spring, the greater the force. This can be modeled as a linear relationship, where the force applied by the spring is equal to some constant time the displacement of the spring. Written mathematically:

$$F_s = kx$$

F is the force of the spring in newtons, x is the displacement of the spring from its equilibrium (or rest) position, in meters, and k is the ***spring constant*** which tells you how stiff or powerful a spring is, in Newtons per meter. The larger the spring constant, k, the more force the spring applies per amount of displacement. Hence we can say that, The **Hooke's law** is a principle of physics that states that the force (F) needed to extend or compress a spring by some distance x scales linearly with respect to that distance. That is

$$F_s = kx$$

where k is a constant factor characteristic of the spring: its stiffness, and x is small compared to the total possible deformation of the spring.

Q. Give the Definition of Stress and Strain. What is Young's Modulus?

Ans.

1. Stress:

Stress is defined as the force per unit area of a material, i.e. Stress = force / cross sectional area:

$$\sigma = \frac{F}{A}$$

Where,

σ = stress,

F = force applied, and

A= cross sectional area of the object.

Units of Stress: Nm^{-2} or Pa.

This is of two types

i. Tensile or Compressive Stress - Normal Stress:

Tensile or compressive stress normal to the plane is usually denoted "**normal stress**" or "**direct stress**" and can be expressed as

$$\sigma = F_n / A$$

where
σ = *normal stress (Pa (N/m²), psi (lbf/in²))*
F_n = *normal force acting perpendicular to the area (N, lbf)*
A = *area (m², in²)*

A normal force acts perpendicular to area and is developed whenever external loads tends to push or pull the two segments of a body.

ii. Shear Stress

Stress parallel to a plane is usually denoted as "**shear stress**" and can be expressed as

$$\tau = F_p / A$$

where
τ = *shear stress (Pa (N/m²), psi (lbf/in²))*
F_p = *shear force in the plane of the area (N, lbf)*
A = *area (m², in²)*

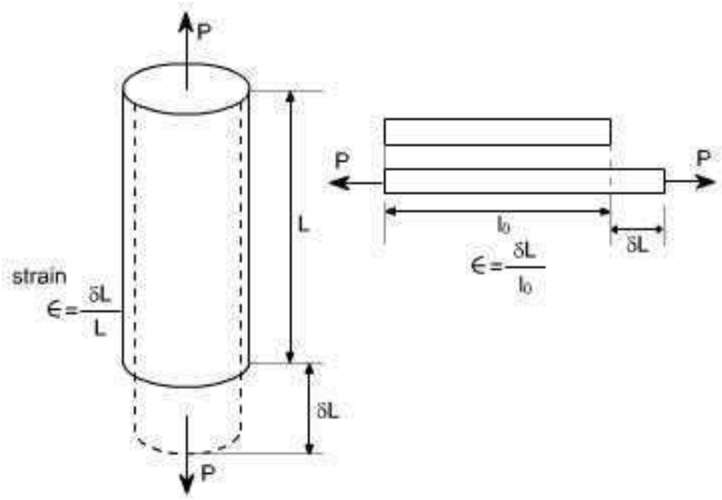
A shear force lies in the plane of an area and is developed when external loads tend to cause the two segments of a body to slide over one another.

2. Strain

Strain is defined as extension per unit length. i.e. **Strain = extension / original length**

$$\text{strain}(\epsilon) = \frac{\text{change in length}}{\text{orginal length}} = \frac{\delta L}{L}$$

Strain has no units because it is a ratio of lengths. This is also of two types

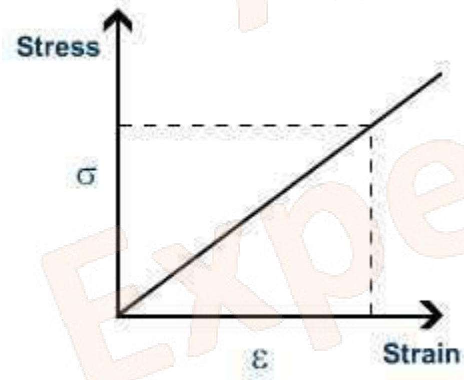


Normal strain - elongation or contraction of a line segment

Shear strain - change in angle between two line segments originally perpendicular.

Young’s Modulus:

If we plot stress against strain for an object showing (linear) elastic behaviour, you get a straight line.



This is because stress is proportional to strain. The gradient of the straight-line graph is the Young's modulus, E

$$E = \frac{stress}{strain} = \frac{\sigma}{\varepsilon}$$

E is constant and does not change for a given material. It in fact represents 'stiffness' property of the material. Values of the young modulus of different materials are often listed in the form of a table in reference books so scientists and engineers can look them up.

$$E = \frac{\sigma}{\varepsilon} \qquad \text{and} \qquad \sigma = \frac{F}{A}$$

$$\text{So, } E = \frac{F}{A\varepsilon} \qquad \text{and} \qquad \varepsilon = \frac{e}{lo} \text{ so } \frac{1}{\varepsilon} = \frac{lo}{e}$$

$$\text{So, } E = \frac{F\,lo}{A\varepsilon}$$

Units of the Young modulus E: Nm^{-2} or Pa.

Note: The value of E in Pa can turn out to be a very large number. Therefore sometimes the value of E may be given **MNm^{-2}**.

Q. Give the definition of Elasticity Modulus.

Ans.

Young's Modulus - Modulus of Elasticity:

Most metals deforms proportional to imposed load over a range of loads. Stress is proportional to load and strain is proportional to deformation as expressed with **Hooke's Law.**

E = stress / strain

$= \sigma / \varepsilon$

$= (F_n / A) / (dl / l_o)$

where

E = Young's Modulus (N/m^2) (lb/in^2, psi)

Modulus of Elasticity, or Young's Modulus, is commonly used for metals and metal alloys and expressed in terms $10^6\ lb_f/in^2$, N/m^2 *or Pa*. Tensile modulus is often used for plastics and is expressed in terms $10^5\ lb_f/in^2$ *or GPa*.

Shear Modulus of Elasticity or Modulus of Rigidity

G = stress / strain

$= \tau / \gamma$

$= (F_p / A) / (s / d)$

where

G = Shear Modulus of Elasticity - or Modulus of Rigidity (N/m^2) (lb/in^2, psi)

τ = shear stress ((Pa) N/m^2, psi)

γ = unit less measure of shear strain

F_p = force parallel to the faces which they act

A = area (m^2, in^2)

s = displacement of the faces (m, in)

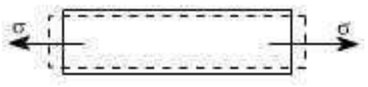
d = distance between the faces displaced (m, in)

Bulk Modulus Elasticity

The Bulk Modulus Elasticity - or Volume Modulus - is a measure of the substance's resistance to uniform compression. Bulk Modulus of Elasticity is the ratio of stress to change in volume of a material subjected to axial loading.

Q. What is Poisson's Ratio? Use Poisson Ratio to Explain Strain in various Dimension.
Ans.

Poisson's ratio: If a bar is subjected to a longitudinal stress there will be a strain in this direction equal to σ / E. There will also be a strain in all directions at right angles to σ. The final shape being shown by the dotted lines.

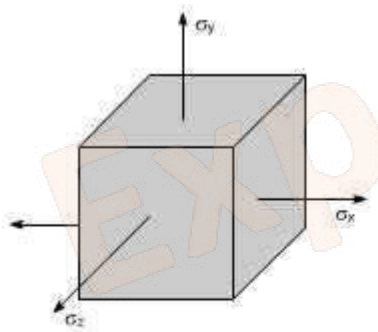


It has been observed that for an elastic materials, the lateral strain is proportional to the longitudinal strain. The ratio of the lateral strain to longitudinal strain is known as the poison's ratio.

Poison's ratio (μ) = – lateral strain / longitudinal strain

For most engineering materials the value of μ is between 0.25 and 0.33.

Three – dimensional state of strain: Consider an element subjected to three mutually perpendicular tensile stresses σ_x , σ_y and σ_z as shown in the figure below.



If σ_y and σ_z were not present the strain in the x direction from the basic definition of Young's modulus of Elasticity E would be equal to

$$\in_x = \sigma_x / E$$

The effects of σ_y and σ_z in x direction are given by the definition of Poisson's ratio 'μ ' to be equal as $-\mu \sigma_y / E$ and $-\mu \sigma_z / E$

The negative sign indicating that if σ_y and σ_z are positive i.e. tensile, these they tend to reduce the strain in x direction thus the total linear strain is x direction is given by

$$\epsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}$$

Principal strains in terms of stress:

In the absence of shear stresses on the faces of the elements let us say that σ_x, σ_y, σ_z are in fact the principal stress. The resulting strain in the three directions would be the principal strains.

i.e. We will have the following relation.

$$\epsilon_1 = \frac{1}{E}\left[\sigma_1 - \mu\sigma_2 - \mu\sigma_3\right]$$

$$\epsilon_2 = \frac{1}{E}\left[\sigma_2 - \mu\sigma_1 - \mu\sigma_3\right]$$

$$\epsilon_3 = \frac{1}{E}\left[\sigma_3 - \mu\sigma_1 - \mu\sigma_2\right]$$

For Two dimensional strain: system, the stress in the third direction becomes zero i.e. $\sigma_z = 0$ or $\sigma_3 = 0$

Although we will have a strain in this direction owing to stresses σ_1& σ_2

Hence the set of equation as described earlier reduces to

$$\epsilon_1 = \frac{1}{E}\left[\sigma_1 - \mu\,\sigma_2\right]$$

$$\epsilon_2 = \frac{1}{E}\left[\sigma_2 - \mu\,\sigma_1\right]$$

$$\epsilon_3 = \frac{1}{E}\left[-\mu\,\sigma_1 - \mu\,\sigma_2\right]$$

Hence a strain can exist without a stress in that direction i.e if $\sigma_3 = 0$; $\epsilon_3 = \frac{1}{E}\left[-\mu\sigma_1 - \mu\sigma_2\right]$

Also

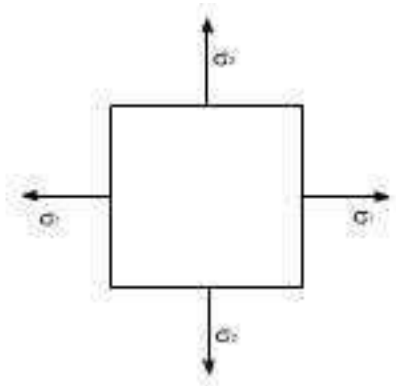
$\epsilon_1 .E = \sigma_1 - \mu\sigma_2$

$\epsilon_2 .E = \sigma_2 - \mu\sigma_1$

so the solution of above two equations yields

$$\sigma_1 = \frac{E}{(1-\mu^2)}\left[\epsilon_1 + \mu\epsilon_2\right]$$

$$\sigma_2 = \frac{E}{(1-\mu^2)}\left[\epsilon_2 + \mu\epsilon_1\right]$$



Q. What is The Work Done in Stretching a wire? Get an expression for Energy stored in stretched wire.

Ans. **Work done in stretching a wire**

In stretching a wire, work is done against internal restoring forces. This work is stored in the wire as elastic potential energy or strain energy.

If a force *F* acts along the length *L* of the wire of cross section *A* and stretches it by *x*, then

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{x/L} = \frac{FL}{Ax} \Rightarrow F = \frac{YA}{L}\cdot x$$

So, the work done for an additional small increase *dx* in length,

$$dw = F\,dx = \frac{YA}{L}x.dx$$

Hence, the total work done in increasing the length by *l*,

$$W = \int_0^l dW = \int_0^l F dx = \int_0^l \frac{YA}{L}.x\,dx = \frac{1}{2}\frac{YA}{L}l^2$$

This work done is stored in the wire.

$\therefore$ Energy stored in wire, $U = \frac{1}{2}\frac{YAl^2}{L} = \frac{1}{2}Fl$

$$\left[\text{as } F = \frac{YAl}{L}\right]$$

Dividing both sides by volume of the wire, we get energy stored in per unit volume of wire.

$$U_V = \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L}$$

$$= \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

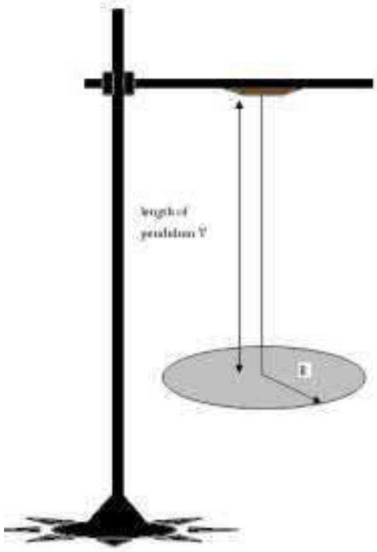
$$= \frac{1}{2} \times Y \times (\text{Strain})^2 = \frac{1}{2Y}(\text{Stress})^2$$

[as AL = volume of wire]

Q. What is Torsional Oscillation?
Ans.

A body suspended by a thread or wire which twists first in one direction and then in the reverse direction, in the horizontal plane is called a torsional pendulum. The first torsion pendulum was developed by Robert Leslie in 1793.

A simple schematic representation of a torsion pendulum is given below,



The period of oscillation of torsion pendulum is given as,

$$T = 2\pi\sqrt{\frac{I}{C}} \,..........\,(1)$$

Where   I=moment of inertia of the suspended body; C=couple/unit twist

But we have an expression for couple per unit twist C as,

$$C = \frac{1}{2}\frac{\pi n r^4}{l} \,..........\,[2]$$

Where l =length of the suspension wire; r=radius of the wire; n=rigidity modulus of the suspension wire

Substituting (2) in (1) and squaring,we get an expression for rigidity modulus for the suspension wire as,

$$n = \frac{8\pi I l}{r^4 T^2} \,............\,(A)$$

We can use the above formula directly if we calculate the moment of inertia of the disc,I as (1/2)MR^2.

Now,let I_0 be the moment of inertia of the disc alone and I_1 & I_2 be the moment of inertia of the disc with identical masses at distances d_1&d_2 respectively.If I_1 is the moment of inertia of each identical mass about the vertical axis passing through its centre of gravity, then

$$I_1 = I_0 + 2\ I^1 + 2md_1{}^2 \,..........\,(3)$$

$$I_2 = I_0 + 2I^1 + 2md_2{}^2 \,...........\,(4)$$

$$I_2 - I_1 = 2m(d_2{}^2 - d_1{}^2) \,.........\,(5)$$

But from equation (1) ,

$$T_0{}^2 = 4\pi^2\frac{I_0}{C} \,....................\,(6)$$

$$T_1{}^2 = 4\pi^2\frac{I_1}{C} \,....................\,(7)$$

$$T_2{}^2 = 4\pi^2\frac{I_2}{C} \,....................\,(8)$$

$$T_2{}^2 - T_1{}^2 = \frac{4\pi^2}{C}(I_2 - I_1) \,..........\,(9)$$

Where T_0,T_1,T_2 are the periods of torsional oscillation without identical mass,with identical pass at position d_1,d_2 respectively.

Dividing equation (6) by (9) and using (5),

$$\frac{{T_0}^2}{\left({T_2}^2-{T_1}^2\right)}=\frac{I_0}{[I_2-I_1]}=\frac{I_0}{2m\left({d_2}^2-{d_1}^2\right)}\text{.........}(10)$$

Therefore,The moment of inertia of the disc,

$$I_0=2m\left({d_2}^2-{d_1}^2\right)\frac{{T_0}^2}{\left({T_2}^2-{T_1}^2\right)}\text{..................}(11)$$

Now substituting equation (2) and (5) in (9),we get the expression for rigidity modulus 'n' as,

$$\mathrm{n}=\frac{16\pi m\left({d_2}^2-{d_1}^2\right)}{r^4}\left(\frac{l}{{T_2}^2-{T_1}^2}\right)\text{............}(12)$$

Applications of Torsional Pendulum:

1.The working of "Torsion pendulum clocks " (shortly torsion clocks or pendulum clocks), is based on torsional oscillation.

2.The freely decaying oscillation of Torsion pendulum in medium(like polymers),helps to determine their characteristic properties.

3.New researches, promising the determination of frictional forces between solid surfaces and flowing liquid environments using forced torsion pendulums.

Q. Drive an Expression for Young's Modulus by Searle's Method.

Ans.

Consider a wire of length L and diameter d. Let its length L increases by an amount l when the wire is pulled by a longitudinal external force F. Young's modulus of the material of the wire is given by,

$$Y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}} = \frac{F/A}{l/L} = \frac{4FL}{\pi d^2 l}$$

The units of Young's modulus are the same as that of stress (note that strain is dimensionless) which is same as the units of pressure i.e., Pa or N/m^2. Graphically, Young's modulus is generally determined from the slope of stress-strain curve.

Figure 1: Wire extension due to pulling force

Normally, we use Searle's method to measure the Young's modulus of a material. As Young's modulus is independent of the shape of the material, we can utilize any shape for its calculation. In particular, a thin circular wire fulfills our requirement. In this method, the length L of the wire is measured by a scale, diameter d of the wire is measured by a screw gauge, length l of the wire is measured by a Micrometer or Vernier scale, and F is specified external force.

Differentiate the expression for Y to get the relative error in the measured value of Y,

$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + 2\frac{\Delta d}{d} + \frac{\Delta l}{l},$$

If a wire of length L and radius r be loaded by a weight Mg and if l be the increase in length.

$$\text{Then, Normal stress} = \frac{Mg}{\pi r^2}$$

$$\text{and} \quad \text{Longitudinal strain} = \frac{l}{L}$$

$$\text{Hence, Young's modulus} = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

or

$$Y = \frac{Mg / \pi r^2}{l / L}$$

or

$$Y = \frac{MgL}{\pi r^2 l}$$

Knowing L and r, and finding l for known Mg, Y can be calculated.

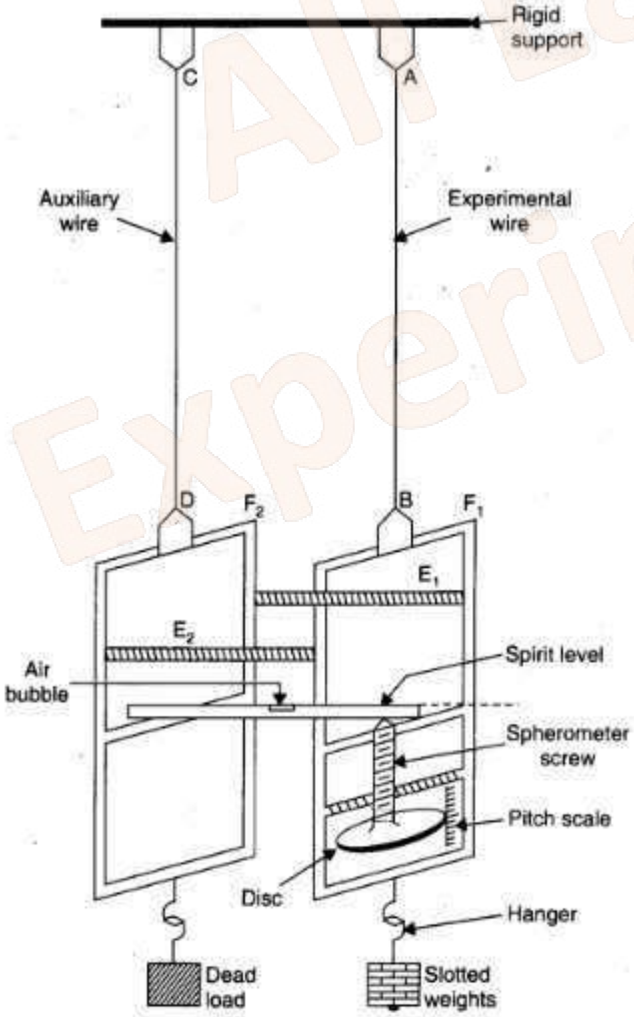


Fig. Searle's apparatus.

Problem : A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ± 0.01 mm. Take $g = 9.8\ \mathrm{m/s^2}$ (exact). The Young's modulus obtained from the reading is

A. $(2.0 \pm 0.3) \times 10^{11}\ \mathrm{N/m^2}$
B. $(2.0 \pm 0.2) \times 10^{11}\ \mathrm{N/m^2}$
C. $(2.0 \pm 0.1) \times 10^{11}\ \mathrm{N/m^2}$
D. $(2.0 \pm 0.05) \times 10^{11}\ \mathrm{N/m^2}$

Solution: Young's modulus of wire material is given by,

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{l/L} = \frac{4FL}{\pi d^2 l},$$

From given data, $F = mg = 9.8\ \mathrm{N}$, $L = 2.0\ \mathrm{m}$, $l = 0.8 \times 10^{-3}\ \mathrm{m}$, and $d = 0.4 \times 10^{-3}\ \mathrm{m}$. Substitute the values in equation to get $Y = 1.95 \times 10^{11} \approx 2.0 \times 10^{11}\ \mathrm{N/m^2}$.

Differentiate the expression for Y and simplify to get error in Y,

$$\Delta Y/Y = 2\,\Delta d/d + \Delta l/l.$$

Substitute $\Delta d = 0.01$ mm and $\Delta l = 0.05$ mm to get

$$\Delta Y = Y(2 \times 0.01/0.4 + 0.05/0.8) = Y(0.1125) \approx 0.2 \times 10^{11}\ \mathrm{N/m^2}.$$

Problem : In a Searle's experiment, the diameter of the wire as measured by a screw gauge of least count 0.001 cm is 0.050 cm. The length, measured by a scale of least count of 0.1 cm, is 110.0 cm. When a weight of 50 N is suspended from the wire, the extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the maximum error in the measurement of Young's modulus of the material of wire from these data.

Solution: The Young's modulus is given by,

$$Y = \frac{4FL}{\pi d^2 l} = \frac{4(50)(110.0)}{3.14(0.050)^2(0.125)} = 2.24 \times 10^7\ \mathrm{N/cm^2} = 2.24 \times 10^{11}\ N/m^2.$$

Differentiate the expression for Y and simplify to get,

$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + 2\frac{\Delta d}{d} + \frac{\Delta l}{l} = \frac{0.1}{110.0} + \frac{2 \times 0.001}{0.050} + \frac{0.001}{0.125} = 0.049.$$

Thus, $\Delta Y = 1.09 \times 10^{10}\ \mathrm{N/m^2}$

Problem : During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is $8 \times 10^{-7}\ \mathrm{m^2}$. The least count of the Vernier scale is 1.0×10^{-5} m. Find the maximum percentage error in the Young's modulus?

Solution: The difference between two measurements by Vernier scale gives elongation of the wire caused by additional load of 2 kg. In first measurement, main scale reading is $\mathrm{MSR} = 3.20 \times 10^{-2}\ \mathrm{m}$ and Vernier scale reading is $\mathrm{VSR} = 20$. The least count of Vernier scale is $\mathrm{LC} = 1 \times 10^{-5}\ \mathrm{m}$. Thus, first measurement by Vernier scale is,

$$\begin{aligned} L_1 &= \mathrm{MSR} + \mathrm{VSR} \times \mathrm{LC} \\ &= 3.20 \times 10^{-2} + 20(1 \times 10^{-5}) = 3.220 \times 10^{-2}\ \mathrm{m}. \end{aligned}$$

In second measurement, $\mathrm{MSR} = 3.20 \times 10^{-2}\ \mathrm{m}$ and $\mathrm{VSR} = 45$. Thus, second measurement by Vernier scale is,

$$L_2 = 3.20 \times 10^{-2} + 45(1 \times 10^{-5}) = 3.245 \times 10^{-2}\ \mathrm{m}.$$

The elongation of the wire due to force $F = 2g$ is,

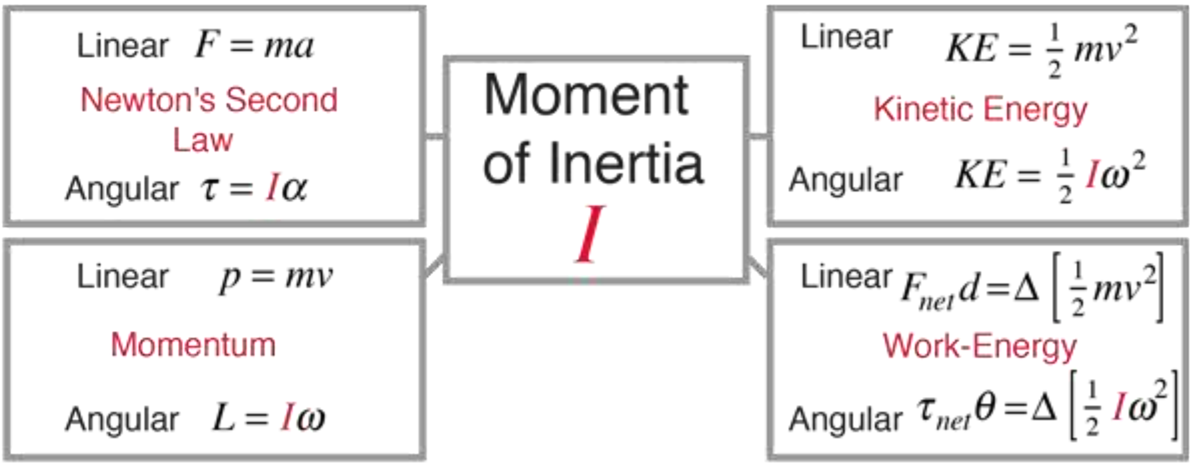
$$l = L_2 - L_1 = 0.025 \times 10^{-2}\ \mathrm{m}.$$

The maximum error in measurement of l is $\Delta l = \mathrm{LC} = 1 \times 10^{-5}\ \mathrm{m}$. Young's modulus is given by $Y = \frac{FL}{lA}$. The maximum percentage error in measurement of Y is,

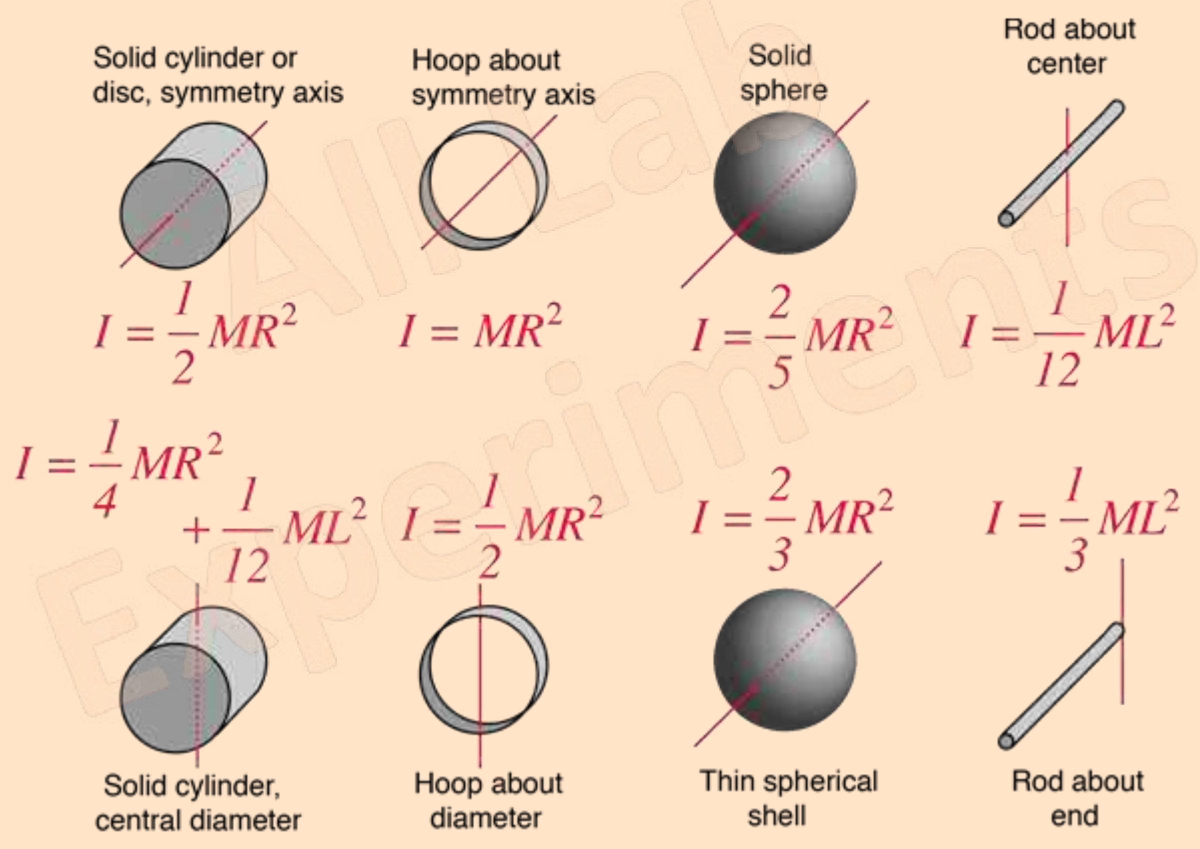
$$\frac{\Delta Y}{Y} \times 100 = \frac{\Delta l}{l} \times 100 = \frac{1 \times 10^{-5}}{0.025 \times 10^{-2}} \times 100 = 4\%.$$

Q. What is Moment of Inertia? Explain.

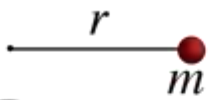
Moment of inertia is the name given to rotational inertia, the rotational analog of mass for linear motion. It appears in the relationships for the dynamics of rotational motion. The moment of inertia must be specified with respect to a chosen axis of rotation. For a point mass the moment of inertia is just the mass times the square of perpendicular distance to the rotation axis, $I = mr^2$. That point mass relationship becomes the basis for all other moments of inertia since any object can be built up from a collection of point masses.



Common Moments of Inertia

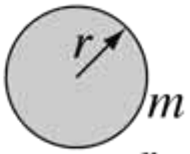


Moment of inertia is defined with respect to a specific rotation axis. The moment of inertia of a point mass with respect to an axis is defined as the product of the mass times the distance from the axis squared. The moment of inertia of any extended object is built up from that basic definition. The general form of the moment of inertia involves an integral.



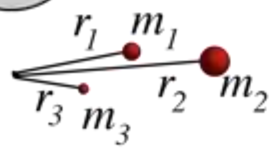
$$I = mr^2$$

For a point mass the moment of inertia is just the mass times the radius from the axis squared. For a collection of point masses (below) the moment of inertia is just the sum for the masses.



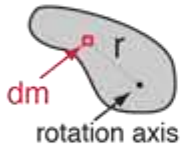
$$I = kmr^2$$

For an object with an axis of symmetry, the moment of inertia is some fraction of that which it would have if all the mass were at the radius r.



$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots$$

Sum of the point mass moments of inertia.



$$I = \int_0^M r^2 dm$$

Continuous mass distributions require an infinite sum of all the point mass moments which make up the whole. This is accomplished by an integration over all the mass.