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B.Sc. (Physical Science)

Chapter - 5
Gravitation

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Chapter-05

Gravitation: Newton’s Law of Gravitation. Motion of a particle in a central force field (motion is in a plane, angular momentum is conserved, areal velocity is constant). Kepler’s Laws (statement only). Satellite in circular orbit and applications

Q. write a short-note on:

(i). Conservation of Energy

(ii). Law of Gravitation.

Ans. (i). Conservation of Energy:

The law of conservation of energy is a physical law which states energy cannot be created or destroyed, but may be changed from one form to another. Another way of stating the law is to say the total energy of an isolated system remains constant or is conserved within a given frame of reference.

In classical mechanics, conservation of mass and conversation of energy are considered to be two separate laws.

However, in special relativity, matter may be converted into energy and vice versa, according to the famous equation $E = mc^2$. Thus, it's more appropriate to say mass-energy is conserved.

For example, if a stick of dynamite explodes, the chemical energy contained within the dynamite changes into kinetic energy, heat, and light. If all of this energy is added together, it will equal the starting chemical energy value.

Consequence of Conservation of Energy

One interesting consequence of the law of conservation of energy is that it means perpetual motion machines of the first kind are not possible. In other words, a system must have an external power supply in order to continuously deliver unlimited energy to its surroundings.

It's also worth noting, it's not always possible to define conservation of energy because not all systems have time translation symmetry.

For example, conservation of energy may not be defined for time crystals or for curved space-times.

(ii). LAW OF GRAVITATION

While an apple might not have struck Sir Isaac Newton’s head as myth suggests, the falling of one did inspire Newton to one of the great discoveries in mechanics: *The Law of Universal Gravitation*. Pondering why the apple never drops sideways or upwards or any other direction except perpendicular to the ground, Newton realized that the Earth itself must be responsible for the apple's downward motion.

Theorizing that this force must be proportional to the masses of the two objects involved, and using previous intuition about the inverse-square relationship of the force between the earth and the moon, Newton was able to formulate a general physical law by induction.

The Law of Universal Gravitation states that every point mass attracts every other point mass in the universe by a force pointing in a straight line between the centers-of-mass of both points, and this force is proportional to the masses of the objects and inversely proportional to their separation This attractive force always points inward, from one point to the other. The Law applies to all objects with masses, big or small. Two big objects can be considered as point-like masses, if the distance between them is very large compared to their sizes or if they are spherically symmetric. For these cases the mass of each object can be represented as a point mass located at its center-of-mass.

While Newton was able to articulate his Law of Universal Gravitation and verify it experimentally, he could only calculate the relative gravitational force in comparison to another force. It wasn't until Henry Cavendish's verification of the gravitational constant that the Law of Universal Gravitation received its final algebraic form:

$$F = G\frac{Mm}{r^2}$$

where F represents the force in Newton, M and m represent the two masses in kilograms, and r represents the separation in meters. G represents the gravitational constant, which has a value of 6.674×10^{-11} N-(m/kg^2). Because of the magnitude of G, gravitational force is very small unless large masses are involved.

Q. What are Central Force? What will be the Motion of a particle under central force? Drive the equation for motion.

Ans.

Motion in a Central Force Field

We now study the properties of a particle of (constant) mass m moving in a particular type of force field, a *central force field.* Central forces are very important in physics and engineering. For example, the gravitional force of attraction between two point masses is a central force. The Coulomb force of attraction and repulsion between charged particles is a central force. Because of their importance they deserve special consideration. We begin by giving a precise definition of *central force, or central force field.*

Central Forces: The Definition. Suppose that a force acting on a particle of mass m has the properties that:

- the force is always directed from m toward, or away, from a fixed point O,
- the magnitude of the force only depends on the distance r from O.

Forces having these properties are called *central forces.* The particle is said to move in a *central force field.* The point O is referred to as the *center of force.*

Mathematically, $\mathbf{F}$ is a central force if and only if:

$$\mathbf{F} = f(r)\mathbf{r}_1 = f(r)\frac{\mathbf{r}}{r}, \tag{1}$$

where $\mathbf{r}_1 = \frac{\mathbf{r}}{r}$ is a unit vector in the direction of $\mathbf{r}$.

If $f(r) < 0$ the force is said to be *attractive towards* O. If $f(r) > 0$ the force is said to be *repulsive from* O. We give a geometrical illustration in Fig. 1.

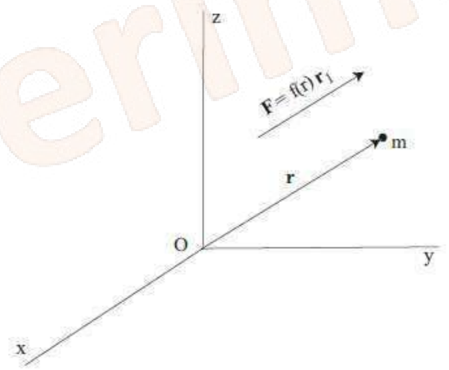


Figure 1: Geometrical illustration of a central force.

Properties of a Particle Moving under the Influence of a Central Force. If a particle moves in a central force field then the following properties hold:

1. The path of the particle must be a *plane curve,* i.e., it must lie in a plane.
2. The angular momentum of the particle is conserved, i.e., it is constant in time.
3. The particle moves in such a way that the position vector (from the point O) sweeps out equal areas in equal times. In other words, the time rate of change in area is constant. This is referred to as the *Law of Areas.* We will describe this in more detail, and prove it, shortly.

Basic Equations of Motion for a Particle in a Central Force Field.

Recall the basic equations of motion as they will be our starting point:

$$
\begin{aligned}
m(\ddot{r} - r\dot{\theta}^2) &= f(r), && (8)\\
m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) &= 0. && (9)
\end{aligned}
$$

we derived the following *constant of the motion*:

$$
r^2\dot{\theta} = h = \text{constant}. \tag{10}
$$

This constant of the motion will allow you to determine the θ component of motion, provided you know the r component of motion. However, (8) and (9) are coupled (nonlinear) equations for the r and θ components of the motion. How could you solve them without solving for both the r and θ components? This is where alternative forms of the equations of motion are useful.

Let us rewrite (8) in the following form (by dividing through by the mass m):

$$
\ddot{r} - r\dot{\theta}^2 = \frac{f(r)}{m}. \tag{11}
$$

Now, using (10), (11) can be written entirely in terms of r:

$$
\ddot{r} - \frac{h^2}{r^3} = \frac{f(r)}{m}. \tag{12}
$$

We can use (12) to solve for $r(t)$, and the use (10) to solve for $\theta(t)$.

Equation (12) is a *nonlinear* differential equation. There is a useful change of variables, which for certain important central forces, turns the equation into a *linear* differential equation with constant coefficients, and these can always be solved analytically. Here we describe this coordinate transformation.

Let

$$
r = \frac{1}{u}.
$$

This is part of the coordinate transformation. We will also use θ as a new "time" variable. Coordinate transformation are effected by the chain rule, since this allows us to express derivatives of "old" coordinates in terms of the "new" coordinates. We have:

$$
\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \frac{h}{r^2}\frac{dr}{d\theta} = \frac{h}{r^2}\frac{dr}{du}\frac{du}{d\theta} = -h\frac{du}{d\theta}, \tag{13}
$$

and

$$
\ddot{r} = \frac{d\dot{r}}{dt} = \frac{d}{dt}\left(-h\frac{du}{d\theta}\right) = \frac{d}{d\theta}\left(-h\frac{du}{d\theta}\right)\frac{d\theta}{dt} = -h^2u^2\frac{d^2u}{d\theta^2}, \tag{14}
$$

where, in both expressions, we have used the relation $r^2\dot{\theta} = h$ at strategic points.

Now

$$
r\dot{\theta}^2 = r\frac{h^2}{r^4} = h^2u^3. \tag{15}
$$

Substituting this relation, along with (14) into (8), gives:

$$
m\left(-h^2u^2\frac{d^2u}{d\theta^2} - h^2u^3\right) = f\left(\frac{1}{u}\right),
$$

or

$$
\frac{d^2u}{d\theta^2} + u = -\frac{f\left(\frac{1}{u}\right)}{mh^2u^2}. \tag{16}
$$

Now if $f(r) = \frac{K}{r^2}$, where K is some constant, (16) becomes a linear, constant coefficient equation.

Q. Write the three Kepler's Law of the Planetary Motion.

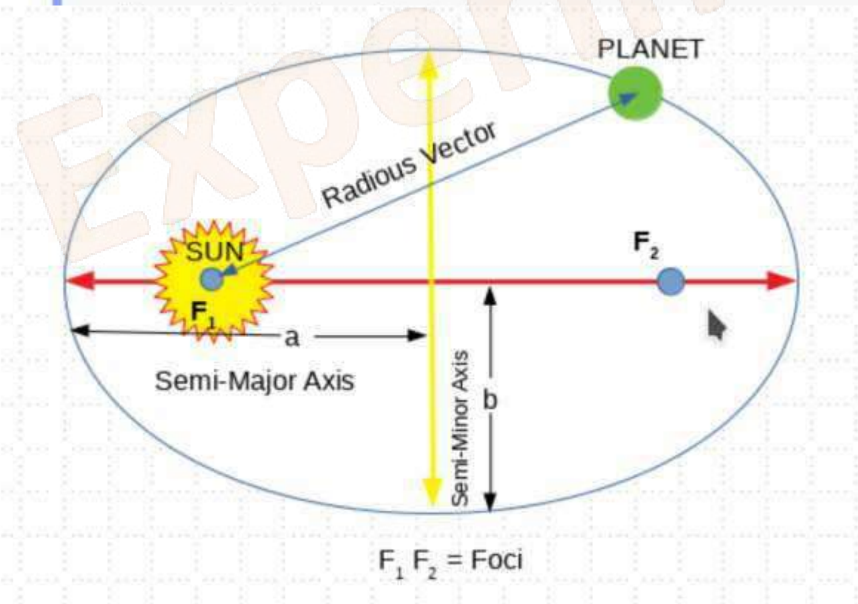
Ans.

Kepler was a great astronomer and physicist from Germany. His theory about planetary motion open a new revolutionary direction in the field of physics.

He formulated three famous laws on the motion of any planetary objects.

The laws are –

> 1.> All planets in our solar system revolve around the sun in elliptical orbits. The sun will remain rest at one of the foci of the elliptical orbit.
>
> 2.> The line joining a planet to the sun i.e., the position vector or radius vector traces out equal areas in equal times. In other words the areal velocity of the radial vector is constant.
>
> 3.> The square of the period of revolution of the planet round the sun is directly proportional to the cube of the major axis of the



From the above figure we can see that the red line (a) is semi-major axis and the yellow (b) line is semi-minor axis. F_1 and F_2 are two foci. The sun is situated on one of its focus (F_1).

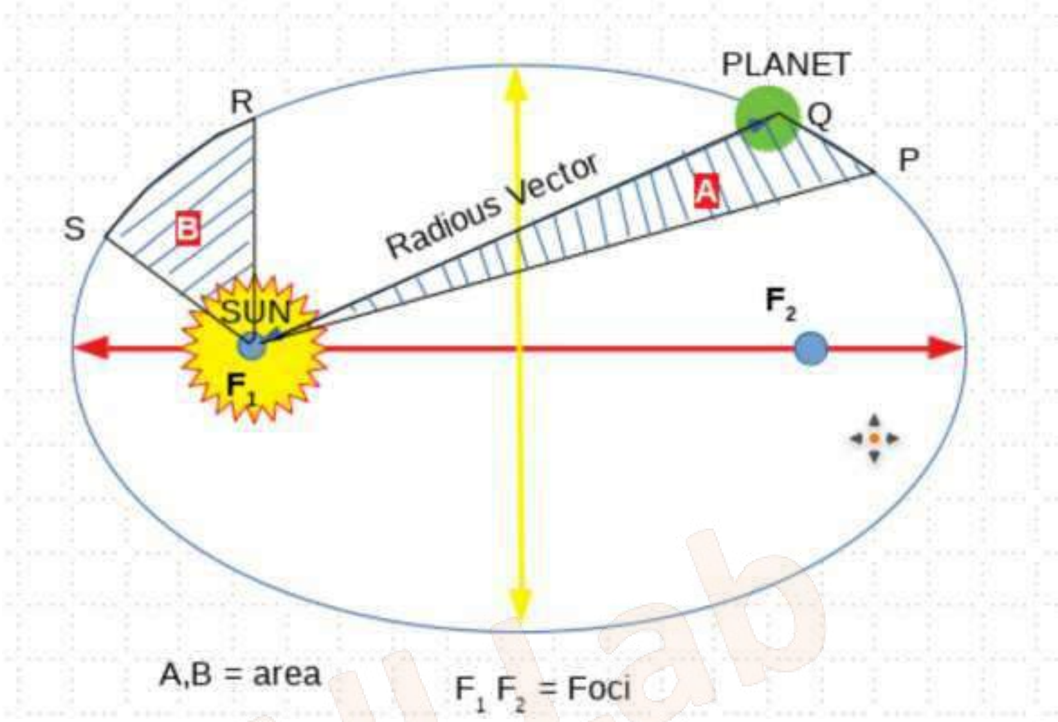


Figure for IInd Law

Q. Write a Short-note on satellite circular orbit.

Ans.

Though no orbit is perfectly circular, the general name for any orbit that is not highly elliptical (egg-shaped) is circular. Circular orbits have an eccentricity of 0. There are several types of circular orbits and they include:

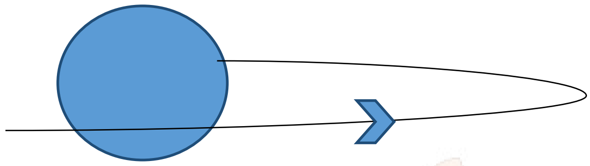
1. geostationary
2. polar
3. sun-synchronous
4. equatorial

1. Geostationary:

A satellite in geostationary orbit appears to remain in the same spot in the sky all the time. Really, it is simply travelling at exactly the same speed as the Earth is rotating below it, but it looks like it is staying still regardless of the direction in which it travels, east or west. A satellite in geostationary orbit is very high up, at 35, 850 km above the Earth. Geostationary orbits, therefore, are also known as high orbits.

Geostationary satellites are always located directly above the equator with a zero angle of inclination. Geostationary orbit, therefore, is really just a special type of equatorial orbit.

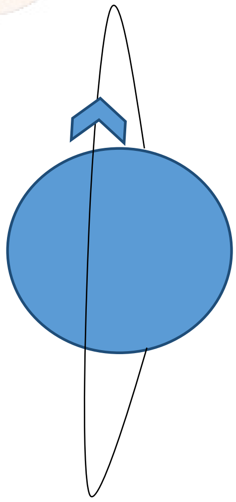
When a satellite is in geostationary orbit, its instruments are looking at a certain part of the Earth. That part of the Earth is called a footprint. A footprint can be pretty big. For example, the footprint for most Canadian communications satellites is almost the whole of Canada.



2. Polar:

A polar orbit usually has an inclination of 90 degrees to the equator. On every pass around the Earth, it passes over both the north and south poles. Therefore, as the Earth rotates to the east underneath the satellite which is travelling north and south, it can cover the entire Earth's surface. A polar orbiting satellite covers the entire globe every 14 days.

Polar orbits are usually medium or low orbits.



3. Sun-synchronous Orbit:

This orbit is a special case of the polar orbit. Like a polar orbit, the satellite travels from the north to the south poles as the Earth turns below it. In a sun-synchronous orbit, though, the satellite passes over the same part of the Earth at roughly the same local time each day. This can make communication and various forms of data collection very convenient. For example, a satellite in a sun-synchronous orbit could measure the air quality of Ottawa at noon.

There is a special kind of sun-synchronous orbit called a dawn-to-dusk orbit. In a dawn-to-dusk orbit, the satellite trails the Earth's shadow. When the sun shines on one side of the Earth, it casts a shadow on the opposite side of the Earth. (This shadow is night-time.) Because the satellite never moves into this shadow, the sun's light is always on it (sort of like perpetual daytime). Since the satellite is close to the shadow, the part of the Earth the satellite is directly above is always at sunset or sunrise. That is why this kind of orbit is called a dawn-dusk orbit. This allows the satellite to always have its solar panels in the sun.

Generally, sun-synchronous orbits are medium or low orbits.

Radarsat is an example of a satellite in a low sun-synchronous orbit. Radarsat is in orbit 798 kilometres above the Earth, at an angle of inclination of 98.6 degrees to the equator as it circles the globe from pole to pole. Radarsat relies on its dawn-to-dusk orbit to keep its solar panels facing the sun almost constantly. Radarsat can therefore rely mostly on solar power and not on batteries.

4. Equatorial Orbit:

A satellite in equatorial orbit flies along the line of the Earth's **equator**. To get into equatorial orbit, a satellite must be launched from a place on Earth close to the equator. NASA often launches satellites aboard an Ariane rocket into equatorial orbit from French Guyana.

Equatorial orbits can be useful for satellites observing tropical weather patterns, as they can monitor cloud conditions around the globe.

Equatorial orbits are usually medium or low orbits.