Name of the Department: PHYSICS DEPARTMENT

Name of Course: B.Sc. Hons.–CBCS_Core (NC)

Semester: V- Semester

Name of the Paper: Quantum Mechanics and Applications

Unique Paper Code: 32221501

Time Duration: 3 Hours **Maximum Marks: 75**

Attempt four questions out of six. Each question carries equal marks.

1.

i. A particle is represented at (t=0) by the wave function:

$$\psi(x,0) = \begin{cases} A(a^2 - x^2) & -a \le x \le a \\ 0 & otherwise \end{cases}$$

Find A and expectation value of $x, x^2, p\ and\ p^2$. Find uncertainties in position and momentum.

ii. Show that Divergence of J (probability current density) is zero for stationary states.

iii. Find the Fourier transform of the wave function e^{-ax^2}.

(14.75+2+2)

2.

i. State Heisenberg's Uncertainty principle. What is the origin of concept of uncertainty in position and momentum? Derive $\Delta x \Delta p \ge \hbar/2$.

ii. Verify whether the following operators are linear:

a. $\widehat{f(x)} = \frac{d}{dx} f(x)$

b. $\widehat{f(x)} = \sqrt{f(x)}$

iii. What is uncertainty in the location of a photon of wavelength 5000 Angstrom which is known to an accuracy of one part in 10^7?

(12.75+3+3)

3.

i. Solve Schrodinger's equation for the potential energy $V = (1/2)kx^2$ and show that the energy eigenvalue are $E_n = (n + \frac{1}{2})\hbar\omega$.

ii. Which of the following wave functions

(i) e^{-ax^2} (ii) $\sin(kx)$

are eigenvalues of operator (a) p and (b) p^2.

iii. Find the locations of classical turning points for a One Dimensional Harmonic Oscillator in its ground state.

(12.75+4+2)

4.

i. Describe the Stern-Gerlach Experiment and its theory. Discuss the significance of the experiment. Why is an inhomogeneous magnetic field required?

ii. A beam of silver atoms moving with a velocity of $10^5 cm/s$ passes through a magnetic field of gradient 0.50Wb/m^2/cm for a distance of 10cm. Determine acceleration of Ag atoms, time spent by atoms in the field and displacement of Ag atoms along z-direction as it comes out of the magnetic field (along z-axis).

iii. Show that $\frac{d}{dt}\int_{-\infty}^{\infty} \psi_1^* \psi_2 dx = 0$ for any two (normalizable) solutions to Schrodinger's equation, ψ_1 *and* ψ_2.

(12.75+3+3)

5.

i. Derive an expression for energy difference ΔE between doublets due to Spin-Orbit coupling. How does ΔE depend on quantum numbers n and l?

ii. Show that the angle between angular momentum (**L**) and z-axis is given by $\theta_{m_l} = cos^{-1}(\frac{m_l}{\sqrt{l(l+1)}})$. Find the values of angle θ_{m_l} for l=2.

iii. Calculate the probability of finding the electron in the region $\frac{a_0}{2} < r < 2a_0$ in a hydrogen atom in ground state given that wave function for the ground state of Hydrogen atom is $\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{\frac{-r}{a_0}}$, where $a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2}$.

(12.75+3+3)

6.

i. Solve 1-D time independent Schrodinger's equation for a particle having energy E for a square well of finite depth V_0 ($E<V_0$). Show graphically existence of bound states.

ii. An electron moves in 1-D potential well of width 8 Angstrom and depth 12eV. Find the number of bound states?

iii. Assuming LS coupling scheme, list the possible total angular momentum and spectral terms for three electron having configuration 2p 3p 4d.

(12.75+3+3)

Physical Constants:

Mass of Ag atom = 107.87amu,

Charge of electron = 1.6×10^{-19}C,

h=6.626×10^{-34}Js

Mass of electron = 9.1×10^{-31}Kg.