Sl. No. of Ques. Paper	:	Set B
Unique Paper Code	:	32221102
Name of the Paper	:	Mechanics
Name of the Course	:	B.Sc. Hons. – CBCS_Core
Semester	:	I
Duration	:	3 hours
Maximum Marks	:	75

*Attempt any **four** questions.*

1. (a) Deduce the equation of motion for a rocket as : (10.75)

$$\vec{F} = M\frac{d\vec{V}}{dt} - \vec{v}\frac{dM}{dt}$$

where M and $\vec{V}$ are the instantaneous mass and velocity of the rocket and $\vec{v}$ is the velocity of the gas with respect to the rocket. Hence, find the expression for the final velocity of a rocket launched from the surface of earth.

(b) A proton makes a head-on collision with an unknown particle at rest. The proton rebounds straight back with 4/9 of its kinetic energy. Find the ratio of the mass of the unknown particle to the mass of proton, assuming that the collision is elastic. (8)

2. (a) Calculate the moment of inertia of a solid sphere (mass M, radius R) about its diameter and tangent. (8.75)

(b) A particle of mass 10 g has position and velocity vectors

$\vec{r} = 10\hat{i} + 6\hat{j}$ meter, $\vec{v} = 5\hat{i}$ m/sec.

Find the angular momentum of this particle about the origin. (5)

(c) Find the centre of mass of thin rod of length ℓ whose density ρ varies with distance x from one end as: $\rho = \frac{\rho_0 x}{\ell}$, where ρ_0 is a constant. (5)

3. (a) Show that the areal velocity remains constant, when the particle moves under the influence of a central force. (6)

(b) A planet revolves around a star in an elliptical orbit. The ratio of the farthest distance to the closest one of the planet from the star is 4. Find the ratio of the kinetic energies of the planet at the farthest to the closest positions. (6)

(c) A satellite of mass m is fired from the surface of a stationary planet of mass M and radius R, with speed u at 30° with the vertical direction. The satellite reaches a maximum distance of $5R/2$ from the centre of the planet. Show that

$$u = \sqrt{5GM/4R}$$ (6.75)

4. (a) Write down the equation of motion of a damped harmonic oscillator and solve it for lightly damped case. Also, calculate the rate of energy dissipated in the system. (11.75)

(b) A particle executing SHM has speeds v_1 and v_2 corresponding to the displacements x_1 and x_2, respectively. Show that the period of SHM is given by

$$T = 2\pi\sqrt{\frac{x_2^{\,2} - x_1^{\,2}}{v_1^{\,2} - v_2^{\,2}}} \qquad (7)$$

5. (a) Prove that acceleration of any particle in frame S' rotating with uniform angular speed ω with respect to another frame S given by

$$\vec{a}_{S'} = \vec{a}_S - 2\vec{\omega}\times\vec{v}_{S'} - \vec{\omega}\times\left(\vec{\omega}\times\vec{R}\right), \qquad (8.75)$$

where the symbols have their usual meanings.

(b) Cartesian coordinate system xyz is rotating with angular velocity $\vec{\omega}$ w. r. t. fixed cartesian coordinate system XYZ. If origins of both the systems coincide, prove that angular acceleration is same in both the systems. (4)

(c) A spaceship is receding from earth at a speed of $0.21c$. A light from the spaceship appears as yellow (λ = 589.3 nm) to an observer on earth. What would be its color as seen by the passenger of the spaceship? (6)

6. (a) On the basis of Lorentz transformation equations, derive the expression for time dilation. (6.75)

(b) Frame S' is moving w. r. t. frame S along common x-x' axis with speed 0.8c. If a rod of length 2 m is at rest in frame S, making an angle 45° with the x-axis, find the length and orientation (w. r. t. x'-axis) of rod in frame S'. (6)

(c) A particle at rest decays into two particles of rest mass m_o and $2m_o$. If the lighter particle moves with a speed of 0.8c, find the speed of other particle in lab frame and hence find the rest mass of the original particle. (6)