B.Sc. Hons.
Semester-1
mechanics paper 2016

1. Attempt any five of the following questions :

   (i) A galaxy moves away from the earth at 0.2c. What is the natural wavelength of a spectral line whose wavelength measured in a laboratory is 600 nm ?

   (ii) Calculate the minimum velocity with which a body may be projected so that it may become a satellite of the earth assuming it takes a circular orbit around earth.

   (iii) A particle executing a S.H.M. has a maximum displacement of 4 cm and its acceleration at a distance of 1cm from its mean position is 3 cm/s^2. What will its velocity be when it is at a distance of 2 cm from its mean position ?

(iv) Two objects, one initially at rest, undergo a one dimensional elastic collision. If half the kinetic energy of the initially moving object is transferred to the other object, what is the ratio of their masses ?

(v) Show that the force field $F = (y^2z^3 - 6xz^2)i + 2xyz^3j + (3xy^2z^2 - 6x^2z)k$ is a conservative force field. Hence, find the work done in moving a particle from the point A (–2, 1, 3) to (1, –2, –1) in the given force field.

(vi) Find the centre of mass of a homogeneous semi-circular plate of radius R.

(vii) Distinguish between inertial and gravitational mass of a body.

(viii) A hoop of radius 100cm and mass 19 Kg is rolling along a horizontal surface, so that its centre of-mass has a velocity of 20 cms^{-1}. How much work will have to be done to stop it ? (5×3)

2. (i) Setup the differential equation of motion of a damped harmonic oscillator subjected to a sinusoidal force, $F = F_0 \sin\omega t$. Discuss its steady state solution and obtain an expression for its maximum amplitude.

(ii) What is sharpness of resonance ? Explain the effect of damping on sharpness of resonance. (12,3)

3. (i) What is reduced mass ? Reduce two body problem to one body problem and obtain equation of motion for equivalent one body problem for two masses.

(ii) A uniform sphere of mass M and radius R and a uniform cylinder of mass M and radius R are released simultaneously from rest at the top of an inclined plane. Which body reaches the bottom first if both roll without slipping ? Find the velocity of both at the bottom. (9,6)

4. (i) What are Inertial and Non-Inertial frames ? Explain giving an example of each.

(ii) How does the rotation of Earth about its axis affect the acceleration due to gravity experienced by a body at rest at a point on the surface of earth ? Support your answe with a suitable derivation and diagram. (6,9)

5. (i) Deduce the mathematical expression for the law of addition of relativistic velocities. Hence, show that in no case the resultant velocity of a material particle can be greater than c and that the Lorentz velocity transformation equations reduce to Galilean ones for values of $v << c$.

(iii) A spaceship moving away from the earth with velocity 0.6c fires a rocket (whose velocity relative to the spaceship is 0.7c),

(a) away from the earth

(b) towards the earth.

What will be the velocity of the rocket, as observed from the earth in the two cases. (10,5)

6. (i) A projectile launched at an angle θ to the horizontal reaches a maximum height h. Show that its horizontal range is $4h/\tan\theta$.

(ii) Prove that in the Centre of mass frame of reference, magnitude of velocities of the two particles remain unaltered in an elastic collision between them.

(iii) A head-on elastic collision between two particles with equal initial speeds v leaves the more massive particle at rest. Find the ratio of the particle masses. (6,4,5)

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7. (i) Obtain expressions for gravitational potential at a point inside and outside a thin uniform spherical shell of radius R and mass M. Also depict your results graphically.

(ii) (a) Find the moment of inertia of a solid cylinder of length L, radius R and mass M about an axis passing through its centre and perpendicular to its geometrical axis.

(b) Calculate the radius of gyration of the solid cylinder of mirr 34, length 24 cm and radius 8 cm about an axjs through its centre and perpendicular to its geometrical axis. (7,6,2)

Q 1(i) Ans.

Ans. Speed of galaxy = $0.2\,c$

Observed wave length = 600 nm

Let spectral line wave length = A

Then,

$$600 = A\sqrt{\frac{1+\frac{0.2c}{c}}{1-\frac{0.2c}{c}}}$$

$$= A\sqrt{\frac{3}{2}}$$

$$\therefore \quad A = \frac{600\sqrt{2}}{\sqrt{3}}$$

or $\quad A = 490.2$ nm.

Q 1(ii) Ans.

Earth and Its Satellite

Consider a satellite of mass m revolving in a circular orbit around the Earth, which is located at the centre of its orbit. If the satellite is at a height h above the Earth's surface, the radius of its orbit

$r = R_e + h$,

where R_e is the radius of the Earth. The gravitational force between M_e & m provides the centripetal force necessary for circular motion, i.e

$GM_e m/(R_e+h)^2 = mv^2/(R_e+h)$

Or $v^2 = GM_e/(R_e+h)$ or $v = \sqrt{GM_e/(R_e+h)}$

Hence orbital velocity depends on the height of the satellite above Earth's surface.

Time period T of the satellite is the time taken to complete one revolution.

Therefore, $T = 2\Pi r/v = 2\Pi(R_e + h)\sqrt{(R_e+h)/GM_e}$

or $T^2 = 4\Pi^2(R_e+h)^3/GM_e$ where $r = R_e + h$

If time period of a satellite is 24 hrs. Then,

$r = [GM_eT^2/4\Pi^2]^{1/3} = 42400$ km

and $h = 36000$ km.

This gives the height of a satellite above the Earth's surface whose time period is same as that of Earth's. Such a satellite appears to be stationary when observed from the Earth's surface and is hence known as Geostationary satellite.

For a satellite very close to the surface of Earth i.e. $h << R_e$ then

$r \approx R_e$

$v_{orbital} = \sqrt{GM_e/R_e} = \sqrt{gR_e}$

$$V_O = \sqrt{\frac{GM_e}{R_e}} = \sqrt{gR_e}$$

Q 1(iii) Ans.

Q1(iii) Given, max. displacement = 4cm

$A\ x, a = 3\ cm/s^2$

$x = 1$ cm.

$a = \omega^2 x \Rightarrow 3 = \omega^2(1)$

$\omega = \sqrt{3}$ cm/sec.

Now, velocity, $V = \omega\sqrt{A^2 - x^2}$

$x = 2$ cm

$V = \sqrt{3}\sqrt{4^2 - 2^2}$

$= \sqrt{3}\sqrt{16 - 4} = \sqrt{3}\sqrt{12}$

$= 3$ cm/s

Q 1(iv) Ans.

Ans. Let m_1 and m_2 be two masses which undergo an one dimensional elastic collosion. Let the initial velocity of m_1 be u_1 and let m_2 be initially at rest. Therefore $u_2 = 0$

Let their velocities after the collision be v_1 and v_2 respectively. Conservation of momentum gives us,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

But $$u_2 = 0$$

$\therefore$ $$m_1 u_1 = m_1 v_1 + m_2 v_2 \qquad \text{...(i)}$$

Conservation of K.E. gives us

$$\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

But $$u_2 = 0$$

$\therefore$ $$\frac{1}{2}m_1 u_1^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \qquad \text{...(ii)}$$

The K.E. of m_1 is divided equally between m_1 and m_2 after collision.

Q 1(v) Ans.

Ans. $$F = (y^2 z^3 - 6\,xz^2)\,i + 2xyz^3 j + (3xy^2 z^2 - 6\,x^2 z)\,k$$

$$\text{Curl } F = \Delta F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2z^3 - 6xz^2 & 2xyz^3 & 3xy^2z^2 - 6x^2z \end{vmatrix}$$

$$\left[\frac{\partial}{\partial y}\left(3xy^2z^2 - 6x^2z\right) - \frac{\partial}{\partial z}\left(2xyz^3\right)\right]i - \left[\frac{\partial}{\partial z}\left(y^2z^3 - 6xz^2\right) - \frac{\partial}{\partial x}\left(3xy^2z^2 - 6x^2z\right)\right]j$$

$$+\left[\frac{\partial}{\partial y}\left(2xyz^3\right) - \frac{\partial}{\partial y}\left(y^2z^3 - 6xz^2\right)\right]k$$

$$= \left(6xyz^2 - 6xyz^2\right)i - \left(3y^2z^2 - 12xz - 3y^2z^2 + 12xz\right)j + \left[2yz^3 - 2yz^3\right]k$$

$$= [0]i - [0]j + [0]\,k$$

$$= 0$$

Hence the force is conservative.

Work done by force in displacing the particle from A to B is given by:

$$w = \int_A^B \left(F_x dx + F_y dy + F_z dz\right)$$

$$= \int_A^B \left(y^2z^3 - 6xz^2\right)dx + \left(2xyz^3\right)dy + \left(3xy^2z^2 - 6x^2z\right)dz$$

$$= \int_A^B y^2z^3 dx - \int_A^B 6xz^2 dx + \int_A^B 2xyz^3 dy + \int_A^B 3xy^2z^2 dz - \int_A^B 6x^2z\, dz$$

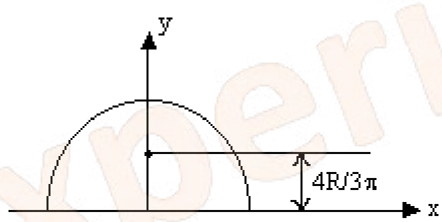
$$= \left[xy^2z^3\right]_A^B - \left[\frac{6x^2}{2}z^2\right]_A^B + \left[\frac{2xy^2}{2}z^3\right]_A^B + \left[\frac{3xy^2z^3}{3}\right]_A^B - \left[\frac{6x^2z^2}{2}\right]_A^B$$

Point A is $(-2, 1, 3)$

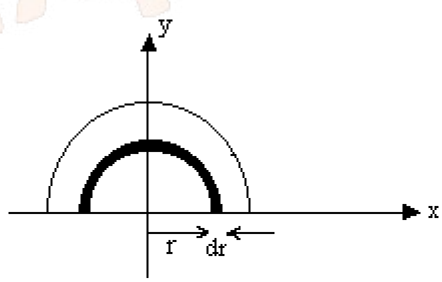
and B is $(1, -2, -1)$

Please put these values and solve.

Q 1 (vi) Ans. Center of Mass of a uniform semi circular plate:



Derivation:



Here the element chosen is a thin wire (semi circular) of radius r. As derived earlier, the for this is at.

$$\text{Now, } dm = \frac{m}{\frac{\pi R^2}{2}} \times \pi r dr$$

$$= \frac{2mrdr}{R^2}$$

$$\text{So, } Y_{COM} = \int \frac{ydm}{m} = \frac{1}{m}\int_0^R \frac{2r}{\pi} \times \frac{2mr}{R^2} dr$$

$$= \frac{4}{\pi R^2} \cdot \left[\frac{r^3}{3}\right]_0^R$$

$$= \frac{4R}{3\pi}$$

Q 1(vii) Ans.

1) Inertial mass. This is mainly defined by Newton's law, the all-too-famous F = ma, which states that when a force F is applied to an object, it will accelerate proportionally, and that constant of proportion is the mass of that object. In very concrete terms, to determine the inertial mass, you apply a force of F Newtons to an object, measure the acceleration in m/s^2, and F/a will give you the inertial mass m in kilograms.

2) Gravitational mass. This is defined by the force of gravitation, which states that there is a gravitational force between any pair of objects, which is given by

$F = G\, m_1\, m_2/r^2$

where G is the universal gravitational constant, m_1 and m_2 are the masses of the two objects, and r is the distance between them. This, in effect defines the gravitational mass of an object.

Q 1(viii) Ans.

work done = KE = Rotational energy + translation Energy

$= \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$

$I_{ring} = mR^2$

work done $= \frac{1}{2} MR^2 \frac{v^2}{R^2} + \frac{1}{2} mv^2$

$= Mv^2$

$M = 19\,kg \qquad V = 20\,cm/s$

workdone $= 19 \times 20 \times 20 \times 10^{-4}$

$= 7600 \times 10^{-4}\, J$

$= 0.76\, J$

Q 2(i) Ans.

Ans. $$F = F_0 \text{ Sin } \omega t$$

This is a sinusoidal force of amplitude F_0 and frequency $\frac{w}{2\pi}$

The damping force acting on the oscillator is $-\gamma \frac{dx}{dt}$ and the restoring force is $-$ C x respectively. Its equation of motion becomes

$$m\frac{d^2x}{dt^2} = -\gamma\frac{dx}{dt} - Cx + F$$

or $$m\frac{d^2x}{dt^2} + \frac{\gamma}{m}\frac{dx}{dt} + \frac{C}{m}x = \frac{F_0}{m}\sin\omega t$$

Now $$\frac{\gamma}{m} = 2k \text{ and } \frac{C}{m} = w_0^2,$$

where w_0 is the natural angular frequency of the oscillator.

we put $$\frac{F_0}{m}\sin\omega t = f_0 \sin \omega t$$

Hence we have,

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + w_0^2 x = f_0 \sin \omega t \quad \text{...(I)}$$

When the steady state has been reached, let us try $x = A \sin(\omega t - \theta)$ as a particular solution of the equation of motion, where θ is the possible phase difference between the applied force and the displacement of the oscillator.

$$\frac{dx}{dt} = A\, w \cos(\omega t - \theta)$$

and $$\frac{d^2x}{dt^2} = -A\, w^2 \sin(\omega t - \theta)$$

Substituting these values in I, we get

$$-Aw^2 \sin(\omega t - \theta) + 2k\, A\, \omega \cos(\omega t - \theta) + w_0^2 A \sin(\omega t - \theta)$$

$$= f_0 \sin [\ (\omega t - \theta) + \theta] = f_0 \sin\ (\omega t - \theta) \cos\theta + f_0 \cos\ (\omega t - \theta) \sin\theta$$

or $$A\ (w_0^2 - w^2)\sin\ (\omega t - \theta) + 2k\,A\,w\cos\ (\omega t - \theta)$$

$$= f_0 \cos\theta \sin\ (\omega t - \theta) + f_0 \sin\theta \cos\ (\omega t - \theta) \qquad \text{...(II)}$$

If this solution is to hold good for all values of t, the co-efficients of sin $(\omega t - \theta)$ and cos $(\omega t - \theta)$ on either side of equation II must respectvely be equal, i.e.,

$$A\,(w_0^2 - w^2) = f_0 \cos\theta \qquad \text{...(III)}$$

and $$2kAw = f_0 \sin\theta \qquad \text{...(IV)}$$

Squaring and adding (III) and (IV),

$$A\,(w_0^2 - w^2) + 4k^2 A^2 w^2 = f_0^2 \cos^2\theta + f_0^2 \sin^2\theta$$

or $$A^2\,[(w_0^2 - w^2)^2 + 4k^2 w^2] = f_0^2$$

or $$A^2 = \frac{f_0^2}{\left(w_0^2 - w^2\right)^2 + 4k^2w^2}$$

∴ Amplitude of forced oscillator,

$$A = \frac{f_0}{\sqrt{\left(w_0^2 - w^2\right)^2 + 4k^2w^2}}$$

Taking only the positive value of the square root. Its negative value will mean opposite phase but then θ too will change by π and there would, therefore, be no effect on the value of A.

$$\tan\theta = \frac{f_0 \sin\theta}{f_0 \cos\theta} = \frac{2kw}{\left(w_0^2 - w^2\right)}$$

The amplitude A will be maximum

when $$w^2 + 2k^2 - w_0^2 = 0$$

or $$w = \sqrt{w_0^2 - 2k^2}$$

$$A_{max} = \frac{f_0}{2k\left(w_0^2 - k^2\right)^{1/2}}$$

or $$A_{max} = \frac{f_0}{2k\left(w^2 + k^2\right)^{1/2}}$$

Q 2(ii) Ans

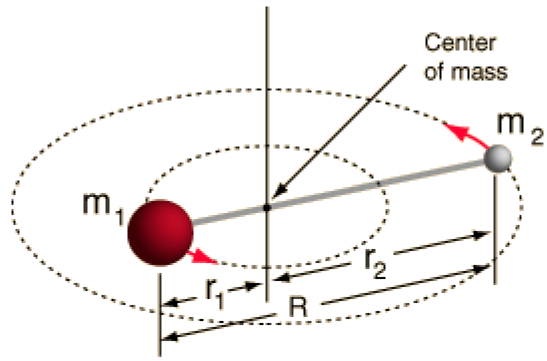
(12,3)

Ans. The sharpness of resonance is a measure of the rate of fall of amplitude from its maximum value at resonant frequency on either side of it. The sharper the fall in amplitude the sharper the resonance.

The smaller the damping , the sharper the resonance. The sharpness of resonance is inversely proportional to the square of the samping constant.

Q 3(i) Ans.

let two particles of mass m1 and m2 situated at r1 and r2. The force on two particle is equivalent to the force on one particle having reduced mass.



The moment of inertia about the center of mass is

$$I = m_1 r_1^2 + m_2 r_2^2$$

From the center of mass definition

$$m_1 r_1 = m_2 r_2$$

and

$$r_1 = \frac{m_2 R}{m_1 + m_2}$$

Substituting to eliminate r_1 and r_2 gives

$$I = \frac{m_1 m_2 R^2}{m_1 + m_2} = \mu R^2$$

where μ is called the "reduced mass."

Q 3(ii) Ans.

Ans. Suppose a body of mass m, radius R and radius of gyration k is placed at the top of an inclined plance of length l and height h. The angle of inclination of the inclined plane is θ.

If the inclined plane is smooth, the body will slide down. But if there is sufficient friction, the mass will roll down.

The acceleration down the inclined is given by:

$$a_r = \frac{g \sin\theta}{1 + \frac{k^2}{R^2}}$$

The velocity at the bottom of the inclined plane is given by :

$$v_r = \frac{\sqrt{2gh}}{\sqrt{1 + \frac{k^2}{R^2}}}$$

The time taken to reach the bottom is given by :

$$t_s = \left[\frac{2l}{a}\right]^{1/2}$$

$$= \left[\frac{2l}{g \sin\theta}\left(1 + \frac{k^2}{R^2}\right)\right]^{1/2}$$

The values of $\frac{k^2}{R^2}$ are as follows—

cylinder — $\frac{1}{2}$

Solid sphere — $\frac{2}{5}$

$$v_r \text{ for cylinder} = \frac{\sqrt{2gh}}{\sqrt{1+\frac{1}{2}}} = \frac{\sqrt{2}\sqrt{gh}\,\sqrt{2}}{\sqrt{3}}$$

$$= \sqrt{\frac{4gh}{3}}$$

$$v_r \text{ for sphere} = \frac{\sqrt{2gh}}{\sqrt{1+\frac{2}{5}}}$$

$$= \sqrt{\frac{10gh}{7}}$$

Now $$\frac{10}{7} - \frac{4}{3} = \frac{30-28}{21} = \frac{2}{4} > 0$$

$\therefore$ v_r for sphere is more than that of cylinder. Therefore sphere will reach the bottom before the cylinder reaches.

Q 4(i)

Ans.

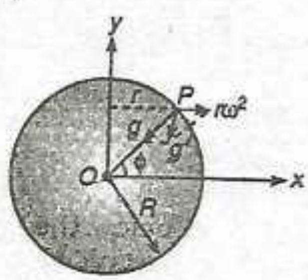
An **inertial frame of reference** in classical physics and special relativity is a frame of reference in which a body with zero net force acting upon it is not accelerating; that is, such a body is at rest or it is moving at a constant speed in a straight line

if your frame of reference has a non-uniform, or accelerated motion, then the Law of Inertia will appear to be wrong, and you must be in a non-inertial frame of reference. A **non-inertial reference frame** is a frame of reference that is undergoing acceleration with respect to an inertial frame.

the Coriolis force or the centrifugal force, as derived from the acceleration of the non-inertial frame.

Q 4(ii)

Ans. Consider a particle P at rest on the surface of the earth, in latitude ϕ. Then the pseuoo force acting on the particle is mrw^2 in outwrds direction.



The true acceleration g is acting towards the centre O of the earth. Thus, the effective acceleration g' is the resultant of g and rw^2, or

$$g' = \sqrt{g^2 + \left(rw^2\right)^2 + 2grw^2\cos(180-\phi)}$$

or $$g' = \sqrt{g^2 + r^2w^4 - 2grw^2 \cos\phi}$$

Here, the term r^2w^4 comes out to be too small as $w = \frac{2\pi}{T} = \frac{2\pi}{24 \times 3600}$ rad/s is small. Hence this can be ignored.

Also $$r = R\cos\phi$$

$\therefore$ $$g' = (g^2 - 2gRw^2\cos^2\phi)^{1/2}$$

$$= g\left(1 - \frac{2Rw^2\cos^2\phi}{g}\right)^{1/2}$$

$$= g\left(1 - \frac{Rw^2\cos^2\phi}{g}\right)$$

or $$g' = g - Rw^2\cos^2\phi$$

Following can be concluded:

(i) The effective value of g is not vertical.

(ii) The effect of centrifugal force due to rotation of earth is to reduce the effective value of g.

(iii) At equator, $\phi = 0°$

$\therefore$ $g' = g - Rw^2$

At poles, $\phi = 90°$

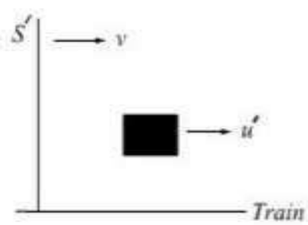
$\therefore$ $g' = g$

At equator, g' is minimum while it is maximum at poles.

Q 5(i)

Ans.

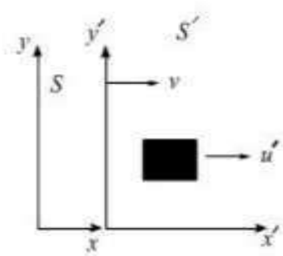
In classical physics, if we consider a case of a train moving with velocity v with respect to ground and a passenger on the train moving with a velocity u' with respect to train, Then the velocity of passanger with respect to the ground is,



$$u = u' + v \quad \ldots\ldots\ldots\ldots\ldots (1)$$

This is simply Galilean or classical velocity addition theorem. Let us discuss how the velocities are added according to special theory of relativity.

Consider for a special case, all velocities are along $x - x'$ direction of two inertial frames S and S', S - is the laboratory frame and S' - is moving frame with constant velocity v. A body moves in S' frame with a velocity u' and its position can be written as $x' = u't'$. The speed of the body with respect to S frame can be calculated as follows,



$$x' = \gamma(x - vt) \quad \ldots\ldots\ldots (2)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \quad \ldots\ldots\ldots (3)$$

$$x' = u't' = u'\gamma\left(t - \frac{vx}{c^2}\right) = \gamma(x - vt)$$

$$x - vt = u'\left(t - \frac{vx}{c^2}\right)$$

$$x\left(1 + \frac{u'v}{c^2}\right) = t(u' + v)$$

$$u = \frac{x}{t} = \frac{u' + v}{\left(1 + \frac{u'v}{c^2}\right)}$$

$$u = \frac{u' + v}{\left(1 + \frac{u'v}{c^2}\right)} \quad \ldots\ldots\ldots\ldots (4)$$

where $u = x/t$ is the velocity of body with respect to S frame. This is the relativistic or Einstien velocity addition theorem.

If u' and v are small compared to c, $u = u' + v$, which is the classical result. On the other hand, if $u' = c$, u would also be equal to c, irrespective of the value of v.

Hence, when $u' \to c$

$$u = \frac{c + v}{1 + \frac{cv}{c^2}} = \frac{c + v}{c(c + v)}c^2 = c$$

Thus any velocity relativistically added to *c* gives a resultant value *c*. In this sense, c plays the same role in relativity that an infinite velocity in classical case.

Let us calculate the velocities perpendicular to the relative motion of frames. Let y_1' and y_2' are the positions of the body at t_1' and t_2' in S' system. So the velocity in S' system is,

$$u_y' = \frac{y_2' - y_1'}{t_2' - t_1'} \quad \ldots\ldots\ldots\ldots (4)$$

We can use Lorentz transformation for the calculation of corresponding velocities in S frame.

$$y_2 - y_1 = y_2' - y_1' \quad \ldots\ldots\ldots\ldots (5)$$

$$t_2' = \gamma\left(t_2 - \frac{vx_2}{c^2}\right), \quad t_1' = \gamma\left(t_1 - \frac{vx_1}{c^2}\right)$$

$$t_2' - t_1' = \Delta t' = \gamma\left[(t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)\right]$$

$$\therefore \quad u'_y = \frac{y'_2 - y'_1}{\Delta t'} = \frac{y_2 - y_1}{\gamma\left[\Delta t - \frac{v}{c^2}\Delta x\right]} = \frac{\frac{\Delta y}{\Delta t}}{\gamma\left[1 - \frac{v}{c^2}\frac{\Delta x}{\Delta t}\right]}$$

$$or, \; u'_y = \frac{u_y}{\gamma\left[1 - \frac{u_x v}{c^2}\right]} = \frac{u_y\sqrt{1 - \frac{v^2}{c^2}}}{\left(1 - \frac{u_x v}{c^2}\right)} \qquad \ldots\ldots\ldots\ldots \quad (7)$$

We can write the corresponding inverse transformation by changing v to $-v$,

$$u_y = \frac{u'_y\sqrt{1 - \frac{v^2}{c^2}}}{\left(1 + \frac{u_x v}{c^2}\right)} \qquad \ldots\ldots\ldots\ldots \quad (8)$$

Similarly,
$$u_z = \frac{u'_z\sqrt{1 - \frac{v^2}{c^2}}}{\left(1 + \frac{u_x v}{c^2}\right)} \qquad \ldots\ldots\ldots\ldots \quad (9)$$

From equation (3),

$$u_x = \frac{u'_x + v}{\left(1 + \frac{u_x v}{c^2}\right)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

Consider the case when both U_x' and v are c

Then,
$$U_x = \frac{U_x' + v}{1 + \frac{U_x' \cdot v}{c^2}} = \frac{c + c}{1 + \frac{c \cdot c}{c^2}} = \frac{2c}{c} = c$$

Thus, the resultant is still c. It means that in no case the resultant velocity be greater then c.

Q 5(ii)
Ans.

Ans. Let the space ship be denoted by A and the rocket by B.

The velocity of A = $0.6\,c$

The velocity of B = $0.7\,c$

Case I. $U_x' = 0.6\,c$

$v = 0.7\,c$

$$U_x = \frac{U_x' + v}{1 + \frac{U_x' \cdot v}{c^2}} = \frac{0.6\,c + 0.7c}{1 + \frac{0.6c \times 0.7c}{c^2}}$$

$$= \frac{1.3c}{1.42}$$

$$= 0.915\,c$$

Case II. $U_x' = 0.6\,c$

$v = -0.7\,c$

Q 6(i) Ans.

6 (i) we know, max. height, $H = \frac{u^2 \sin^2\theta}{2g}$

$$\text{Range} = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{u^2\, 2\sin\theta\cos\theta}{g}$$

$$\frac{\text{Range}}{\text{Height}} = \frac{4\cos\theta}{\sin\theta}$$

$$\boxed{\text{Range} = \frac{4H}{\tan\theta}}$$

Q 6(ii) Ans.

What we'll do is change to a moving coordinate system, which goes with the velocity of the center of mass! That's sounds weird. It makes the problem sound evem harder. But actually you'll see how simple things look in this frame.

Remember we said that if momentum is conserved, the center of mass velocity of the system v_c is also. As the collision is taking place, it doesn't alter the motion of the center of mass a bit. It just plods along at a constant velocity. If we were coasting along on a bike at this center of mass velocity, watching the collision, what would we see?

Well in this reference frame, the center of mass velocity, by definition, is zero. And therefore by eqn. 1.37 the total momentum is also zero. I'll notate all variables in this new system the same as the old, but just to remind ourselves that we're in this new frame I'll also add " ' " to them. So for example, the initial momentum of thr first particle is denoted p'_{i1}.

So let's play before and after again, but this time in the center of mass reference frame.

Before:

The total momentum $p'_{tot} = 0 = p'_{i1} + p'_{i2}$. The total energy is $E' = \frac{1}{2m_1}p'^2_{i1} + \frac{1}{2m_2}p'^2_{i2}$.

After:

The total momentum $p'_{tot} = 0 = p'_{f1} + p'_{f2}$. The total energy is $E' = \frac{1}{2m_1}p'^2_{f1} + \frac{1}{2m_2}p'^2_{f2}$.

This looks a lot simpler. The momentum equations say that the particles have equal and opposite momenta, $p'_{i1} = -p'_{i2}$ and $p'_{f1} = -p'_{f2}$ Using this, equating energy is almost as easy

$$p'^2_{i1}/m_1 + p'^2_{i1}/m_2 = p'^2_{f1}/m_1 + \frac{1}{2}p'^2_{f1}/m_2 \qquad (1.55)$$

Factoring the masses and cancelling gives $p'^2_{i1} = p'^2_{f1}$. There are two solutions to this. One is kind of boring, $p'_{i1} = p'_{f1}$. It means that before and after, nothing changes. This certainly obeys conservation of energy and momentum, but means that the particles haven't bounced off each other. So what's the other more interesting solution? It's $p'_{i1} = -p'_{f1}$. From conservation of momentum, that means $p'_{i2} = -p'_{f2}$. In terms of velocity this gives

$$v'_{i1} = -v'_{f1} \qquad (1.56)$$

$$v'_{i2} = -v'_{f2} \qquad (1.57)$$

This says that after the collision, the two balls have reversed their initial velocities. That's it. This satisfies both momentum and energy conservation.

Q 6(iii) Ans.

(6,4,5)

Ans. A collision is said to be head on collision if the directions of the velocity of colliding objects are along the line of action of the impulses, acting at the instant of collision. If both the objects are moving with the same speed, these can never collide when moving in the same direction.

Therefore the objects must be moving in the opposite direction.

We have,

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

where the symbols have their usual meanings.

Here $\qquad v_2 = -v_1$

$$\therefore \qquad v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)(-v_1)$$

$$\text{or} \qquad v_1' = \left(\frac{v_1}{m_1 + m_2}\right)(m_1 - m_2 - 2m_2)$$

$$\text{or} \qquad v_1' = \left(\frac{v_1}{m_1 + m_2}\right)(m_1 - 3m_2)$$

But $\qquad v_1' = 0$

$\therefore \qquad m_1 - 3m_2 = 0$

$\Rightarrow \qquad m_1 = 3m_2$

Q 7(i) Ans.

We will consider the gravitational attraction that such a shell exerts on a particle of mass *m*, a distance *r* from the center of the shell. The total mass of the shell is *M* and its radius is *R*.

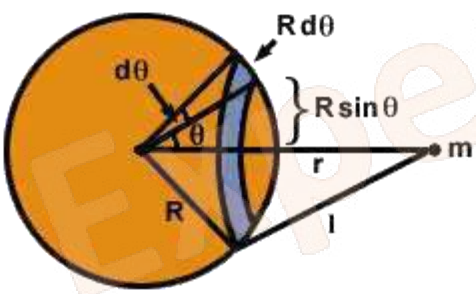


Figure %: A thin spherical shell.

The principle of superposition (see Newton's Law) tells us that we need to add up the vector sum of all the forces on *m* from the particles in the shell. It turns out that it is easier to calculate the sum of the gravitational potentials (since this is a scalar, not a vector) and take derivatives to find the force. We can do this by using $U = \frac{-GMm}{r}$ and summing over all the masses.

To do this, consider cutting the shell into rings as shown in . Every point on the ring is a distance *l* from *m*, and the ring has width $Rd\vartheta$ and radius $R\sin\vartheta$. The surface area of the ring equals $2\Pi\times$ the area × the width = $2\Pi R^2\sin\vartheta d\vartheta$. The total mass of the shell, *M*, is evenly distributed over the surface, so the mass of the ring is given by the fraction of the total surface area ($4\Pi R^2$):

$$M_i = M\times\frac{2\pi R^2\sin\theta d\theta}{4\pi R^2} = \frac{M\sin\theta d\theta}{2}$$

For infinitely thin rings, we can take the integral to find the total potential:

$$U = -\int \frac{GmM\sin\theta d\theta}{2l}$$

But applying the law of cosines to the triangle with sides R, r, and l in we find $l^2 = R^2 + r^2 \pm 2rR\cos\vartheta$ and taking the differential of both sides: $2l dl = 2rR\sin\vartheta d\vartheta$. This last expression implies that: $\frac{\sin\theta d\theta}{l} = \frac{dl}{rR}$. We can now rewrite our integral as:

$$U = -\int \frac{GMm dl}{2rR} = \frac{-GMm}{2rR}\int dl$$

For the ring closest to m, the value of l is $r - R$ and for the ring farthest from m it is $R + r$. So we can now perform the integral:

$$U = \frac{-GMm}{2rR}\int_{r-R}^{R+r} dl = \frac{-GMm}{2Rr}(2R) = \frac{-GMm}{r}$$

This result mirrors the result we would receive if all the mass had been concentrated at the center of the shell. This similarity holds true for all shells, and since a sphere is composed of such shells, it must be true for a sphere too. The phenomenon holds even if the different shells are not of equal mass density--that is, if the density is a function of the radius. We can conclude that the gravitational force exerted by one planet on another acts as if all the mass of each planet were concentrated at its center.

Mass within a gravitating shell

Now let us consider the potential for a particle inside such a shell.

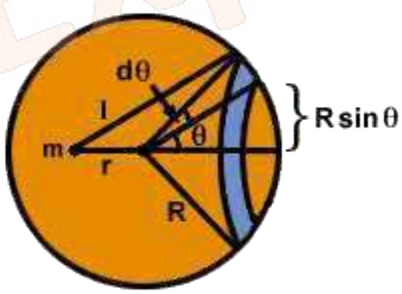


Figure %: Particle m inside thin shell.

The only change in the mathematics is now that l extends from $R - r$ to $R + r$ and hence:

$$U = \frac{-GMm}{2rR}\int_{R-r}^{R+r} dl = \frac{-GMm}{2rR}(2r) = \frac{-GMm}{R}$$

Thus the potential inside the sphere is independent of position--that is it is constant in *r*. Since $F = \frac{-dV}{dr}$ we can infer that the shell exerts *no force* on the particle inside it. For a solid sphere this means that for a particle, the only gravitational force it feels will be due to the matter closer to center of the sphere (below it). The matter above it (since it is inside its shell) exerts no influence on it. clearly illustrates this fact.

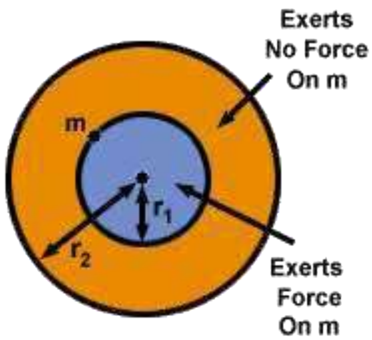
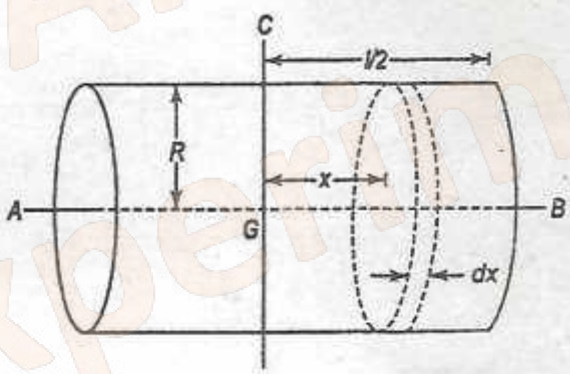


Figure %: Forces exerted on a particle inside a solid sphere.

Q 7(ii)a Ans.

Ans. Let CD be the axis of rotation of the given cylinder of mass M and length = *l*, so that CD is perpendicular to the axis AB of the cylinder and passes through its centre of mass , G.



M.I. of a solid cylinder about an axis perpendicular to its length

Imagine the whole cylinder to be divided into a large number of thin circular discs at right angle to the length of the cylinder and consider one such elementary disc of thickness *dx* and distant *x* from CD.

Volume of the disc= $\pi R^2\,dx$

Volume of the cylinder = $\pi R^2 l$

$$\text{Mass per unit volume} = \frac{M}{\pi R^2 l}$$

$$\text{Mass of disc} = \frac{M}{\pi R^2 l} \times \pi R^2 dx$$

$$= \frac{M}{l} dx$$

M.I. of disc about a diameter which is parallel to the axis CD

$$= \frac{M}{l} dx \cdot \frac{R^2}{4}$$

M.I. of disc about CD, by the principle of parallel axes is :

$$= \frac{M}{l} dx \frac{R^2}{4} + \frac{M}{l} dx \cdot x^2$$

M.I. of the whole cylinder is obtained by integrating the above expression within $x = -\frac{l}{2}$ to $x = +\frac{l}{2}$

$$\therefore \qquad I = \int_{-l/2}^{+l/2} \left(\frac{MR^2}{4l} dx + \frac{M}{l} x^2 dx \right)$$

$$= \frac{MR^2}{4l} \int_{-l/2}^{+l/2} dx + \frac{M}{l} \int_{-l/2}^{+l/2} x^2 dx$$

$$= \frac{MR^2}{4l} \cdot l + \frac{M}{l} \left[\frac{x^3}{3} \right]_{-l/2}^{+l/2} x^2 dx$$

$$= \frac{MR^2}{4} + \frac{Ml^2}{l}$$

$$= M \left[\frac{R^2}{4} + \frac{l^2}{12} \right]$$

This is the required result.

Q 7(ii)b Ans.

(7,6,2)

Ans. There is a term "min 34" in the equation. This is ambigous as there is no such dimension in a cylinder. we take it to be " moment of inertia."

$$I = 34 \text{ units}$$
$$R = 8 \text{ cm}$$
$$L = 24 \text{ cm}$$

Now I for a cylinder about an axis through its centre and perpendicular to its geometrical axis is given by :

$$I = m\left(\frac{R^2}{4}+\frac{l^2}{12}\right)$$

But $\quad I = Mk^2$, where k is the radius of gyration

$$\therefore \quad Mk^2 = m\left(\frac{R^2}{4}+\frac{l^2}{12}\right)$$

or $$k^2 = \frac{R^2}{4}+\frac{l^2}{12}$$

$$\therefore \quad k^2 = \frac{8\times 8}{4}+\frac{24\times 24}{12}$$
$$= 16+48 = 64$$

$$\therefore \quad k = \sqrt{64}$$

or $$k = 8 \text{ cm.}$$