

Free Study Material from All Lab Experiments



**Solid-State Physics Notes
for NET/GATE Physical Sciences
Hall Effect + Liquid Crystals #**

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Questions

Q.1- B.E. of a Cooper-pair at absolute zero acc. to B.C.S. theory is about $3.5 K_B T_c$, where K_B is Boltzman const., T_c critical temp. If Critical temp. of type -I SC is 10 K.

- What is the Energy gap E_g in eV.
- Calculate the Wavelength of photons whose energy is just sufficient to break the cooper-pair in the SC.
- In which region of EM spectrum these photons will lie.

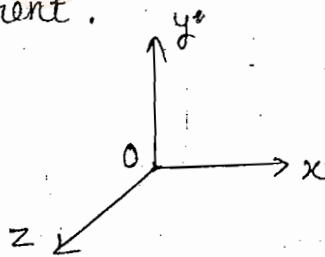
Q.2- For a B.C.S. SC, $T_c = 5 K$, $E_F = 5 eV$ & density of conduc. $e^-s = 10^{29} / m^3$. Calculate

- Magnitude of energy gap in eV
- density of super e^-s n_s at 0K



Hall Effect in metals:- When a current carrying conductor placed in a transverse magnetic field, a voltage is induced b/w the 2 faces of the conductor which are perpendicular to both magnetic field & current.

$V_H \rightarrow$ Hall Voltage
 $E_y \rightarrow$ Hall field



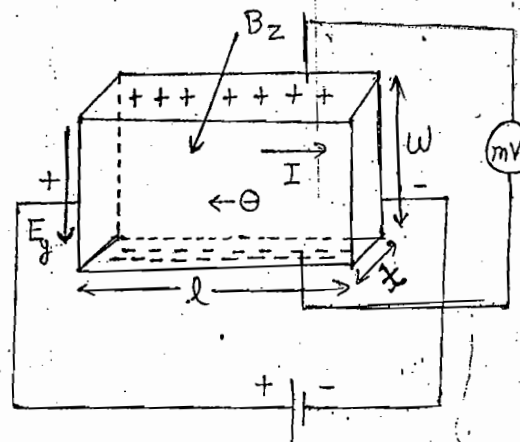
$$\boxed{V_H = W E_y} \quad \text{--- (1)}$$

$$F = -e(E_y + v_x B_z) = 0$$

$$\Rightarrow \boxed{E_y = -v_x B_z} \quad \text{--- (2)}$$

$$\therefore J_x = ne v_x$$

$$\therefore \boxed{E_y = -\frac{J_x B_z}{ne}} \quad \text{--- (3)}$$



We define a new parameter,

$$R_H = \frac{E_y}{J_x B_z} = \frac{-1}{ne} \quad (4)$$

$R_H \rightarrow$ Hall coefficient

It tells about the type of majority charge carriers.

If R_H is -ve then majority charge carriers are e^- s.

In metals, \rightarrow " " " " "

R_H depends on e^- concentration & charge for metals.

$$V_H = W E_y$$

$$J_x = \frac{I_x}{\text{area}} = \frac{I_x}{wt}$$

$$V_H = -\frac{I_x B_z W}{wne t}$$

$$V_H = \frac{-I_x B_z}{ne t} \text{ volt}$$



https://allabexperiments.com

Q. If electric field along x-dirⁿ E_x is doubled then Hall coefficient R_H will become

- (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 4

R_H depend only on e^- conc. & charge, does not depend on applied electric field.

So No change in R_H .

Effect on Hall field :- E_y

$$J = \sigma E$$

$$J_x = \sigma E_x \text{ if } E_x \text{ double} \Rightarrow J_x \Rightarrow \text{double}$$

$$R_H = \frac{2E_y}{2J_x B_z}$$

$$\neq \frac{-1}{ne}$$

So E_y will be doubled to compensate R_H

& also hall voltage $V_H \rightarrow$ doubled

then (c)

→ To observe the Hall effect, length of the sample must be at least 3 times of the thickness.

→ Thinner is the sample, more pronounced is the Hall effect.

Uses of Hall Effect :-

- (1) To find the majority charge carrier type & their concentration.
- (2) For the measurement of magnetic field.
- (3) To find the mobility of charge carriers.

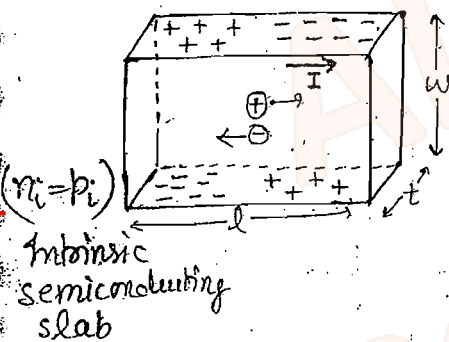
Hall Effect in Semiconductors :-

In semiconductor, there are 2 types of charge carriers.

Hall effect in semiconductor, not only depend on charge carrier concentration but also depend on mobility of charge carrier.

And mobility of e^- & hole are different.

$$\mu_e > \mu_h$$



Mobility is diff. due to the difference in their atomic masses.

$$R_H = \frac{E_y}{J_x B_z}$$

$$R_H = \frac{(p\mu_h^2 - n\mu_e^2)}{e(p\mu_h + n\mu_e)^2}$$

Limiting Cases :-

(i) for Intrinsic Semiconductor :-

$$p = n = n_i$$

$$R_H = \frac{n_i(\mu_h^2 - \mu_e^2)}{en_i^2(\mu_h + \mu_e)^2} = \frac{(\mu_h + \mu_e)(\mu_h - \mu_e)}{en_i(\mu_h + \mu_e)^2}$$

$$R_H = \frac{(\mu_h - \mu_e)}{en_i(\mu_h + \mu_e)}$$

https://www.experiments.com



In general, $\mu_e > \mu_h$

$$\text{So } R_H = -ve$$

i) for N-Type $n \gg p$, neglect $p\mu_h$.

$$R_H = \frac{-n\mu_e^2}{e n^2 \mu_e^2}$$

$$R_H = -\frac{1}{ne} \text{ like metal}$$

ii) for p-type $p \gg n$

$$R_H = \frac{p\mu_h^2}{ep^2\mu_h^2}$$

$$R_H = +\frac{1}{pe}$$



Q. Calculate the Hall coefficient for sodium on the basis of free e^- theory that sodium is monovalent bcc with lattice parameter $a = 4.28 \text{ \AA}$.

Q. An organic conductor has recently been found by optical studies to have $\omega_p = 1.80 \times 10^{15} / \text{sec}$ for plasma freq. and $\tau = 2.83 \times 10^{-15} \text{ sec}$ for the e^- relaxation time for room temp. Calculate (a) σ of the conductor (b) from the crystal & chemical structure conduction e^- concentration is $4.7 \times 10^{21} / \text{m}^3$. Calculate the effective mass of e^- .

Q. Intrinsic Silicon has e^- & hole conc. as $n_i = p_i = 1.5 \times 10^{10} / \text{cm}^3$. Mobilities are $\mu_e = 1350 \text{ cm}^2 / \text{V-sec}$, $\mu_h = 450 \text{ cm}^2 / \text{V-sec}$. Calculate the Hall's coefficient.

Q. A Si bar of length 10 mm & cross-sectional area $a^2 = 0.5 \times 0.5 \text{ mm}^2$ carries a current $I = 5 \text{ mA}$ along its

length in x-dirⁿ. Now it is placed b/w pole pieces of electro-magnet $B_z = 1.6 \text{ T}$ along z-dirⁿ. Now a pair of voltage probes across the cross-section of the sample and I^r to both current & the field & this measures a voltage $V_y = -5 \text{ mV}$. Find the dopar density also find the type of the semiconductor.

Q.5 :- Hall coefficient for a Monovalent fcc lattice metal is given by $R_H = -2.36 \times 10^{-10} \text{ m}^3/\text{C}$. Calculate the lattice parameter of the sample.

- $|R_H(\text{metal})| < |R_H(\text{semiconductor})|$
- $R_H \propto \frac{1}{n}$
- $n \rightarrow$ large in metal so R_H will be smaller.



CRYSTAL DEFECTS

Ideal crystal :- Every crystal is made up of lattice + basis. Ideally we assume that lattice is infinite in dimensions. Lattice point should not be missing.

In real world, these properties does not follow i.e. real crystal are finite in dimension.

Ideal crystal can be defined in terms of lattice + basis.

In ideal crystal, chain of atoms in all the 3-dim must be continuous upto infinity. Deviation of the real crystal from ideality is due to crystal defects. They can be classified as;

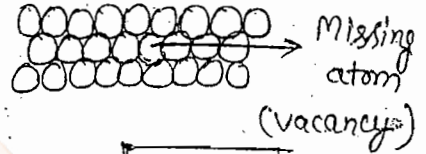
1. Point Defect :- Vacancies, interstitial impurity, substitutional impurity etc. (0-D defect)
2. Line Defect :- Dislocations (1-D defect)
3. Surface Defect :- Grain Boundary (2-D defect)
4. Volume defect :- (3-D defect)

The two most imp defect are point defect and line defect.

1. Point Defect :-

(i) Vacancy :- If some of the atoms are missing from their lattice site - vacancies in the real crystal minimize the free energy of the crystal. Hence, vacancies are necessary in the real crystal. Concentration of vacancies (No. of vacancies per unit volume) are given by

$$n = N e^{-E_v/k_B T}$$

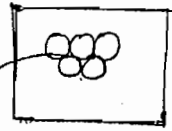


N → Total no. of atoms per unit volume.

E_v → Energy required to create the vacancy.

k_B → Thermal Energy.

interstitial site



(ii) Impurity :- Impurity are of two types :-

Substitutional impurity.

Interstitial impurity.



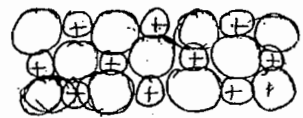
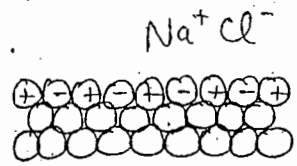
Substitutional impurity :- If impurity occupies Lattice site.

Interstitial impurity :- If impurity occupies interstitial site.

Point Defect in ionic solids :-

In ionic solids, vacancies occurs in pairs. To maintain the charge neutrality, vacancies arises in pairs.

If some cation acquired the interstitial site then cation vacancy + cation interstitial is called Frankel defect.



No. of Frankel defect,

$$n = (N N')^{1/2} e^{-E_i/2 k_B T}$$

where, $N' \rightarrow$ No. of interstitial sites

$N \rightarrow$ No. of lattice sites

$E_i \rightarrow$ Energy required to move cation from lattice site to interstitial site.

If both anion & cation both missing from their lattice sites then the defect is Schottky defect.

Cation vacancy + Anion vacancy = Schottky defect.

No. of Schottky defect,

$$n = N e^{-E_p/2k_B T}$$

$N \rightarrow$ Total No. of atoms

$E_p \rightarrow$ Energy of vacancy pair



- In alkali halides (NaCl, KCl etc) \rightarrow Most common vacancy are Schottky defect and in silver halides (AgBr, AgCl) \rightarrow Most common vacancies are Frenkel defect.

\rightarrow Electrical conductivity of alkali halides & silver halides is due to the motion of ions ^{in the} vacancies, not due to the motion of e^- s.

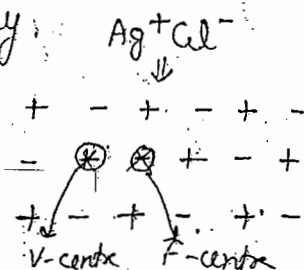
Colour Centres :- Pure alkali halides are transparent to visible light ($E_g > h\nu$). But due to the presence of a lattice defect these crystals absorb visible light. This crystal lattice defect is called Color centre.

(i) F-Centre :- F-centre is a -ve ion vacancy.

ie. if anion is missing from its site then

there form a vacancy called F-centre &

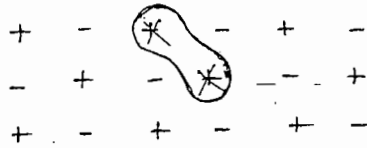
this vacancy is e^- trapped.



(ii) V-Centre :- V-centre is a +ve ion vacancy if cation is missing from its lattice site then there will be a vacancy.

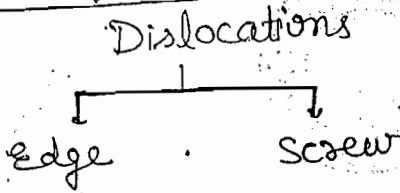
called V-centre & this vacancy is hole trapped.

M-centre :- M-centre is the combination of 2 adjacent F-centres.



R-centre :- R-centre is the combination of 3 adjacent F-centres.

Line Defects :-



Edge dislocations :-

It is "xp" by T or I.

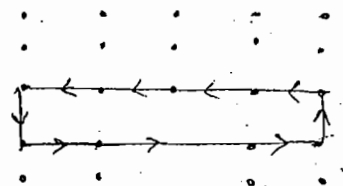
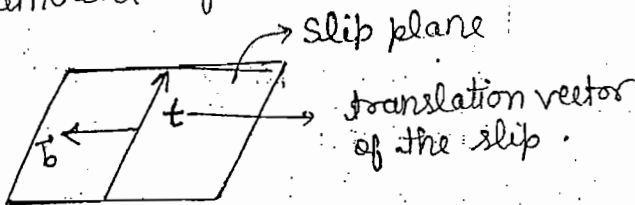
If extra planes are above the slip plane \rightarrow I
 " below " " \rightarrow T

If No. of planes are more above to the slip plane I & below the slip plane T

If some planes terminate abruptly inside the crystal then whole plane is not a defect but its edge is considered as a defect, This is called Edge dislocation.



Burger Vector (\vec{b}) :- Burger Vector \vec{b} gives the magnitude & direction of the slip. Burger vector gives the minimum amount of the slip.



Burger vector is \perp to translation vector in edge dislocation.
 i.e. " " " " dislocation line in " "

$\vec{b} \perp t$

Screw Dislocation :- Burger vector \vec{b} is parallel to dislocation line in screw dislocation.

Mixed Dislocation :- In Mixed dislocation, $\vec{b} \nparallel t$ & $\vec{b} \nperp t$

Line Energy of Dislocation :- Elastic energy per unit length of a dislocation line is given by

$$E = \frac{1}{2} \mu b^2$$

where, $\mu \rightarrow$ Shear Modulus
 $b \rightarrow$ length of the burger vector.

\rightarrow Slip plane is the closest packed plane of the crystal.

fcc \rightarrow (111)

bcc \rightarrow (110)

sc \rightarrow (100)

hcp \rightarrow (100) or (110) (10 $\bar{1}$ 0)
 hkl hki l



$h, k, l \rightarrow$ Miller indices

$i \rightarrow$ bravias index

\Leftrightarrow Miller-Bravias indices

$$i = -(h+k)$$

(hkl) \rightarrow (100) $\Rightarrow k=0, h=1$ so $i = -(1+0) = -1$

when a, b, c are not \perp^r to each other so i is used (It is used in hexagonal cubic plane)

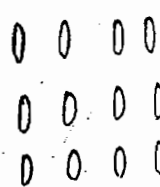
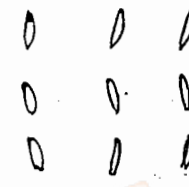

\rightarrow Burger vector in a crystal is in the dirⁿ of close packing.

	slip planes	burger vector (\vec{b})
fcc	(111)	$\langle 110 \rangle$
bcc	(110)	$\langle 111 \rangle$
sc	(100)	$\langle 100 \rangle$
hcp	(100)/(10 $\bar{1}$ 0)	$\langle 110 \rangle / \langle 11\bar{2}0 \rangle$
dc	(111)	$\langle 110 \rangle$
bc	(111)	$\langle 110 \rangle$

Liquid Crystal

Liquid crystal is a new phase of matter, they are unique in their properties & different from isotropic liquids & solids. Properties of liquid crystals lie b/w liquid & solids.

The characteristic property of a liquid crystal is the tendency of a molecule to point along a common axis called Director.

<u>Solid</u>	<u>Liquid Crystal</u>	<u>Liquid</u>
		
Molecules are highly ordered & have very little translation freedom.	Orientalional order & translational order of L.C. lies b/w solid phase & liquid phase.	No particular orientation, but have the translation freedom.

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Orientalional order & translational order of L.C. lies b/w solid phase & liquid phase.

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Order Parameter of Liquid Crystal :-

$$S = \frac{1}{2} \langle 3 \cos^2 \theta - 1 \rangle$$

Where, θ is the angle b/w common director & long axis of the molecule.

This (S) tells about the degree of orientational order.

For solids, $S = 1$ because all the molecules alligned in a particular dirⁿ so $\theta = 0$

For liquids, $S = 0$ because, molecules are order s.t. S is 0, molecules are randomly oriented.

for L.C., $0.3 < S < 0.9$

Characterisation of Liquid Crystals :-

Liquid crystals can be characterised by on the basis of

1. Positional Order (translational order) :- The extent to which position of an average molecule or group of molecules show translational symmetry.

2. Orientalional Order :- Measure of tendency of molecule to align along the director.



PHASES OF L.C.
1. Nematic phase of L.C. (N) :-

It is characterised by orientational order but No positional order.

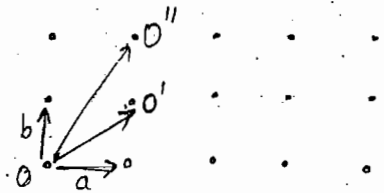
2. Smectic phase (S) :- It is characterised by both position & orientational order. It is more solid like than Nematic.

(Positional Order \Rightarrow Translation order)

Symmetry Operations :- Symmetry operations are those operations in which crystal transform into itself.

There are 4 type of symmetry operations :-

- (i) Translation Symmetry
- (iii) Reflection "
- (ii) Rotational "
- (iv) Inversion "



(i) Translational Vector (OO')

$$\vec{T} = \vec{a} + \vec{b}$$

(O & O' must be identical)

(OO'') $\Rightarrow \vec{T} = \vec{a} + 2\vec{b}$

^{m.m.p}
(ii) Rotational Symmetry :- After rotating by an angle θ , crystal is transforming into itself.

$$n = \frac{2\pi}{\theta}$$

$n \rightarrow$ fold rotation

generally, for naturally occurred crystals

$$n = 1, 2, 3, 4, 6 \Rightarrow \theta = 2\pi, \pi, \frac{2\pi}{3}, \frac{\pi}{2}, \frac{\pi}{3}$$

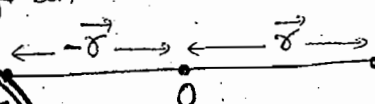
In naturally occurred crystal, five fold symmetry does not exist. (means $\theta = \frac{2\pi}{5} = 72^\circ$ is not possible)

Quasi Crystals :- Quasi crystal is the newly discovered state of matter. Most crystals in nature like diamond, salt (NaCl) & Quartz etc. have same orientation throughout the entire crystal. Hence they can never have 5 fold rotation symmetry, But in Quasi Crystals 5 fold rotation symmetry is possible. (Q.C. \rightarrow Non natural \rightarrow Man made)

Ex of Quasi Crystals :- Aluminium magnese alloy (Al_5Mn)

Reflection Symmetry :- If there exist a plane in the crystal which exactly divide the crystal in two half. And these two halves are mirror images of each other then crystal is said to have reflection symmetry.

Inversion :- If in a crystal with its centre O, there exist a lattice point at a distance \vec{r} & same lattice point also exist at $-\vec{r}$, then O is called centre of inversion and crystal is said to have Inversion Symmetry.



Questions

Q.1 :- $a = 4.28 \text{ \AA} = 4.28 \times 10^{-10} \text{ m}$

$n = \frac{2 \times 1}{a^3} / \text{m}^3$

$R_H = \frac{-1}{ne} \text{ m}^3/\text{c}$

$= - \frac{1}{e} \text{ m}^3/\text{c}$

$R_H = -2.45 \times 10^{-9} \text{ m}^3/\text{c}$ Ap

https://www.experiments.com

Q.2: (a) $\sigma = ?$

$$\omega_p = 1.80 \times 10^{15} \text{ rad/sec}$$

$$\tau = 2.83 \times 10^{-15} \text{ sec}$$

$$\sigma = \frac{ne^2\tau}{m}$$

$$\therefore \omega_p^2 = \frac{ne^2}{\epsilon_0 m} \Rightarrow \frac{ne^2}{m} = \omega_p^2 \epsilon_0$$

So $\sigma = \omega_p^2 \epsilon_0 \tau$

$$= (1.8)^2 \times 10^{30} \times 8.85 \times 10^{-12} \times 2.83 \times 10^{-15}$$

$$= 81.14 \times 10^3$$

$$= 8.114 \times 10^4 \text{ } \underline{\underline{\Omega^{-1}m^{-1}}}$$

Q.2 (b) Effective Mass = ?

$$n = 4.7 \times 10^{21} / m^3$$

$$\sigma = \frac{ne^2\tau}{m^*}$$

$$m^* = \frac{ne^2\tau}{\sigma}$$

$$= \frac{4.7 \times 10^{21} \times (1.6)^2 \times 10^{-38} \times 2.83 \times 10^{-15}}{8.11 \times 10^4}$$

$$m^* = 4.12 \times 10^{-36} \text{ kg}$$

slightly lighter than free e^- .

Q.3: Intrinsic Si: $n_i = p_i = 1.5 \times 10^{10} / cm^3$

$$R_H = \frac{1}{en_i} \frac{(\mu_h - \mu_e)}{(\mu_h + \mu_e)} = \frac{1}{en_i} \frac{(1350 - 450)}{(1350 + 450)}$$

$$= \frac{1}{1.6 \times 10^{-19} \times 1.5 \times 10^{10}} \frac{(-900)}{1800}$$

$$R_H = \frac{-10}{4.8} \times 10^8 \text{ cm}^3/c$$

$$R_H = -2.083 \times 10^9 \text{ cm}^3/c$$



4 Si : $l = 10_{mm} = 10^{-2} m$
 $A = 2.5 \times 10^{-7} m^2 = 0.5 mm \times 0.5 mm$
 $I_x = 5 \times 10^{-3} A$
 $B_z = 1.6 T$
 $V_y = V_H = -5 \times 10^{-3} V$

$$V_H = \frac{-I_x B_z}{net} \Rightarrow n = \frac{-I_x B_z}{V_H e t}$$

$$n = \frac{5 \times 10^{-3} \times 1.6}{5 \times 10^{-3} \times 1.6 \times 10^{-19} \times 0.5 \times 10^{-3}}$$

$$n = \frac{2 \times 10^{22}}{1} = 2 \times 10^{22} / m^3$$



$V_H = -ve$ i.e. n-type

$$R_H = -2.36 \times 10^{-10}$$

$$R_H = \frac{1}{ne} = -2.36 \times 10^{-10}$$

$$n = \frac{4 \times 1}{a^3} \Rightarrow \frac{4}{4 \times 1.6 \times 10^{-19}} = -2.36 \times 10^{-10}$$

$$\Rightarrow a^3 = -2.36 \times 10^{-10} \times 4 \times 1.6 \times 10^{-20}$$

$$\Rightarrow a^3 = 64 \times 2.36 \times 10^{-30}$$

$$\Rightarrow a = 4 (2.36)^{1/3} \times 10^{-10} m$$

$$a = 5.32 \times 10^{-10} m$$

$$\boxed{a = 5.32 \text{ \AA}} \quad \underline{\underline{A_e}}$$

B.C.S. (Questions)

Q.1:- $E_B = 3.5 K_B T_c$

$$K_B = 0.026 \text{ at } 300K$$

(a) $E_g = 3.5 \times K_B \times 10 K$

$$K_B = \frac{0.026}{300} \times 10 = \frac{26 \times 10^{-3}}{30}$$

$$E_g = 3.5 \times \frac{26 \times 10^{-3}}{30} \text{ eV}$$

$$\boxed{E_g = 3.03 \text{ meV}} \quad \underline{\underline{A_e}}$$

$$\begin{aligned}
 (b) \quad \lambda &= \frac{1240}{E_g \text{ (eV)}} \text{ nm} = \frac{1240}{3.03 \times 10^{-3}} \\
 &= 413000 \text{ nm} = 4.13 \times 10^6 \text{ \AA} \\
 &= 413 \text{ \mu m} \rightarrow \text{Microwave region;}
 \end{aligned}$$

Q.2:- $T_c = 5\text{K}$, $E_f = 5\text{eV}$, Z_p



$$\begin{aligned}
 (a) \quad E_g &= 3.53 K_B T_c \\
 &= 3.53 \times \frac{26}{30} \times 10^{-3} \times 5 \\
 E_g &= 15.29 \times 10^{-3} = \underline{15.29 \text{ meV}} \underline{\underline{A}}
 \end{aligned}$$

$$(b) \quad n_s = n_0 \left[1 - \left(\frac{T}{T_c} \right)^4 \right]$$

$$\text{At } T=0\text{K}, \quad n_s = n_0 = \underline{10^{29} / \text{m}^3} \underline{\underline{A}}$$