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Solid-State Physics Notes for NET/GATE Physical Sciences

Superconductivity

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SUPER CONDUCTIVITY

→ discovered in 19[illegible] by → K-~~Onne~~ Onnes

There are certain materials, when cooled below a certain te[illegible] their electrical resistivity is zero. And they show perfect diamagnetism. These are called Superconductors.

Perfect diamagnetic means induced mag. field cancels exact[ly] the applied M.F.

$$\boxed{B_{in} = 0}$$

$$\Rightarrow \mu_0 (H + M) = 0$$

$$\boxed{M = -H}$$

$$\boxed{\chi = \frac{M}{H} = -1} \quad \chi < 0$$

$B = \mu H$
$= \mu_0 \mu_r$

$\mu_r = 1 + \chi_m$

$$\boxed{\mu_r = 0}$$

<u>Superconductivity is Characterised by</u>

→ Zero Electrical Resistance

→ Perfect diamagnetism

By ohm's law, $J = \sigma E$

$E = \rho J$

for perfect conductors, $\sigma \to \infty$

$\rho = 0$

$\because \boxed{\rho = 0} \therefore \boxed{E = 0}$

$\boxed{B = 0}$

- Superconductors are good conductors (perfect)
- " " not good thermal conductor.
- " " good conductor of electricity but not good conductors of heat.

(1) Superconductors are perfect conductors with the additional property of perfect diamagnetism

If we plot a graph b/w electrical resistivity & temp. then E. resistivity ↓ & then drops to zero at $T = T_c$.

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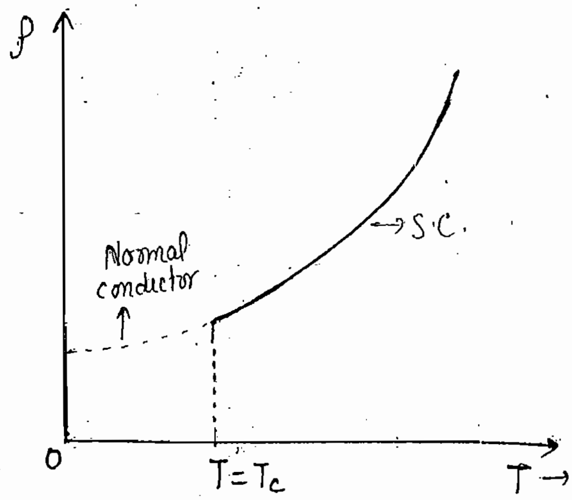
$T_c \rightarrow$ Critical Temp.

First it was found in Hg (Mercury) whose resistivity became zero at $T = T_c = \underline{4.2\,K}$

$\boxed{\rho = 0}$ $(\sigma = \infty)$

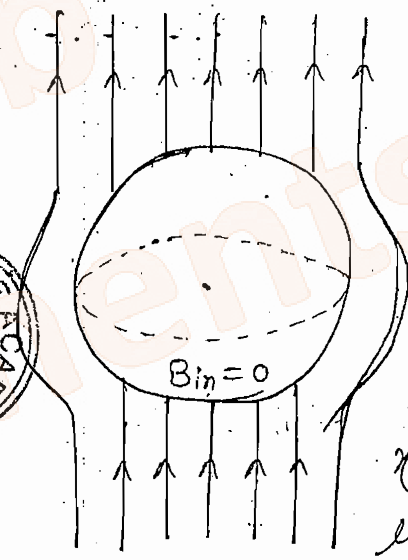
$\boxed{E = 0}$



(2) If we apply a magnetic field B from outside, it will expelled the mag. flux from its interior.

$\boxed{B_{in} = 0}$

And act as a perfect diamagnetic



$\chi = -1$

$\mu_r = 0$

Expulsion of mag. flux from the interior of Super conductor is called '<u>Meissner effect</u>'.

In Super Conductor, $B = 0$
$E = 0$

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In Normal Conductor, $E = 0$
$B \neq 0$

→ In the SC, Current flowing without any resistance, due to the change in magnetic flux called <u>Persistent Current</u>.
It's life time → Millions of years.

1) → Superconductivity can be destroyed by the application of <u>temp.</u> i.e. if $T > T_c$ then SC superconductivity destroy.

(i) $\boxed{T < T_c} \Rightarrow$ SC is in Superconducting State.

(ii) $T > T_c \Rightarrow$ " " Normal "

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2 → Superconductivity can also be destroyed by the applicati of magnetic field.

$H_c \rightarrow$ Critical mag. field

$\boxed{H < H_c}$ for Superconductivity applied m.f. < Critical m.f.

- H_c is the minimum amount of M.F. required to destroy the superconductivity.

3 → Superconductivity can be destroyed by the application of Current density.

$\boxed{J < J_c}$

$J_c \rightarrow$ Maximum amount of current dens that can be carried by the SC.

Current density of SC (due to current flowing in a SC) ~~sh~~ should be less than critical current density. Otherwise Superconductivity will destroy.

for Superconductivity,

$T < T_c$
$H < H_c$
$J < J_c$

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4 → If $h\nu \rightarrow$ optical energy fall on SC

$\boxed{h\nu < 3.5\, K_B T_c}$ only then superconductivity exist.

Temprature depandancy of $H_c(T)$:-

$$\boxed{H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]}$$

$\boxed{H_c(0) \longrightarrow \text{Maximum}}$

i.e. At T = 0K, Critical M.F. will be maximum.

$\boxed{H_c(T_c) = 0}$ if $T = T_c$ then SC is already destroyed by the temp., No need of mag. field.

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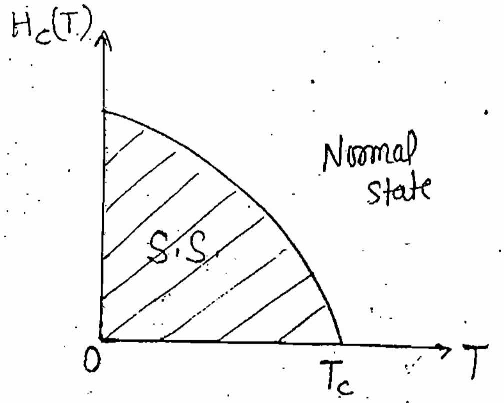
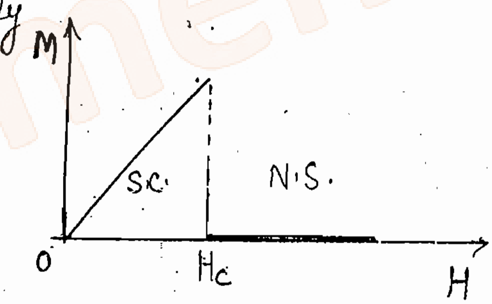


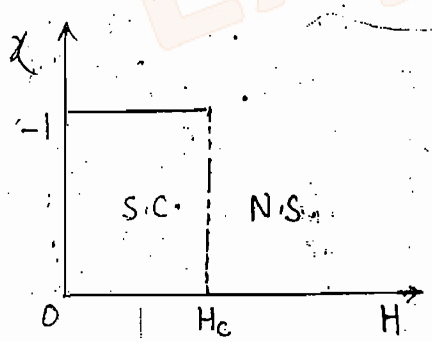
Types of Superconductor :-

(i) type – I
(ii) type – II

Type – I SC :- are those Super-conductors which follow Meissner effect strictely. i.e. $\boxed{B_{in} = 0}$
Change from Super-conducting to Normal state is sharp.
Type – I super-conductors are normally soft super-conductors. i.e. their H_c is low.
Examples → Pure Hg, Pb etc.

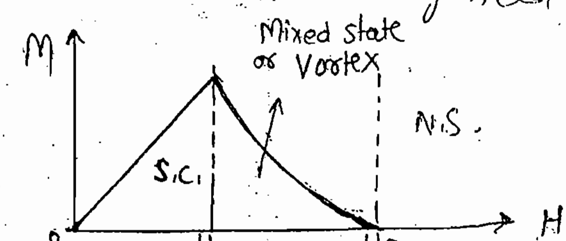


Susceptability behaviour :-



below Hc, $\chi = -1$
above Hc, $\chi = 0$

Type – II SC :- are those super-conductor which do not follow Meissner effect strictely. They have 2 critical mag. field H_{c_1} & H_{c_2}.
Superconducting state → Perfect diamagnetic
$\Rightarrow (H < H_{c_1})$



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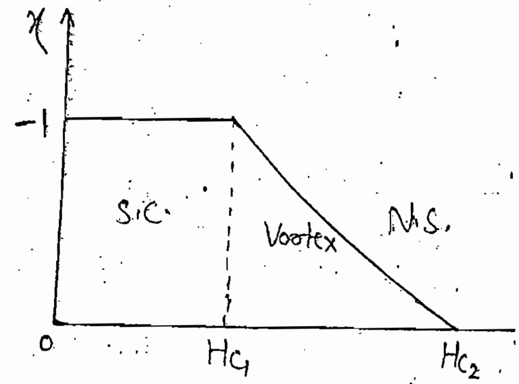
Vortex → diamagnetic

i.e. $H < H_{c_1}$ ⟶ perfect dia.

$H_{c_1} < H < H_{c_2}$ ⟶ diamagnetic

$H > H_{c_2}$ ⟶ Normal



⇒ Type-I & II differentiation is based upon H, not upon T.

Their are 2 fields H_{c_1} & H_{c_2} but only one T_c.

H_{c_2} is very large. So Type-II SC are very useful & corresponding J_{c_2} is " " & then $\rho = 0$

So they carry large amount of Current. → Also called Hard SC

→ First super conductor whose temp. $T_c = 90\,k$ (above than 77k → liquid nitrogen)

$YBa_2Cu_3O_7$ (YBCO → Yettrium Barium Copper oxide)

Also $LaBa_2Cu_3O_7$ (LBCO → Lethium " ")

1 : 2 : 3 : 7

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Thermodynamics of Superconductor :-

Transition b/w super-conducting to Normal state or vice-versa is thermodynamically reversible. Now free energy (Helmoltz) of super-conducting state is small as compare to Normal state. This shows that Super-conducting state is more stable as compare to normal state.

If $F_s(0)$ is the free energy of SC in the absence of M.F. → in Super conducting state is → $F_s(0)$ $(T < T_c)$

Normal " " → $F_N(0)$ $(T > T_c)$

$$\boxed{F_s(0) < F_N(0)}$$

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→ By the application of mag. field,
free energy of Super-conductor changes from -ve to +ve.

→ Difference of free energy b/w Super-conducting & Normal state is given by

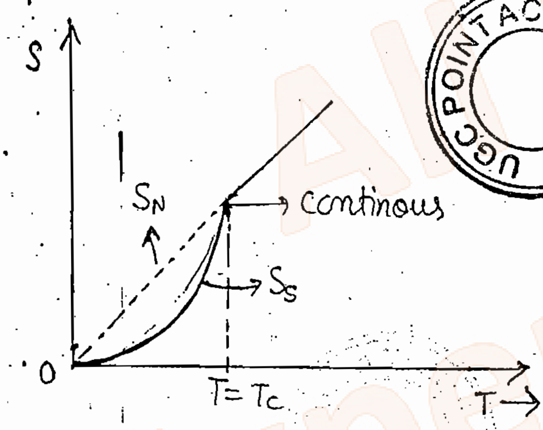
$$\Delta F = F_N(0) - F_S(0)$$

$$\boxed{\Delta F = \frac{\mu_0 H_c^2(T)}{2}}$$

At $T = T_c$, $\Delta F = 0$

$\Delta F \Rightarrow$ Stablization Energy of Super-conductor.

Transition from $SC \longleftrightarrow$ Normal $\Rightarrow$ IInd order



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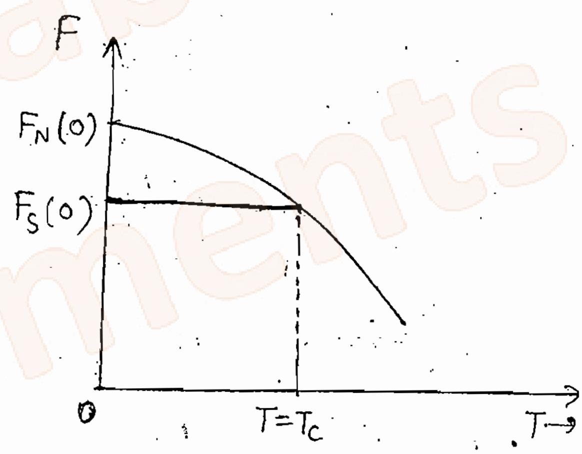

Entropy of SC state is < the N.S. because SC state is more ordered.

Entropy (S) → Measure of Randomness.

as $T \to 0$, $S \to 0$ (for NS)

& $S \to 0$ before $T \to 0$ (for SC)

$\Delta S = S_N - S_S$

Specific heat :- (C_v)

For Normal Metals,

$$\boxed{C_V = AT + BT^3}$$

AT ⇓ for free e^- ; BT^3 ⇓ for lattice [phonon]

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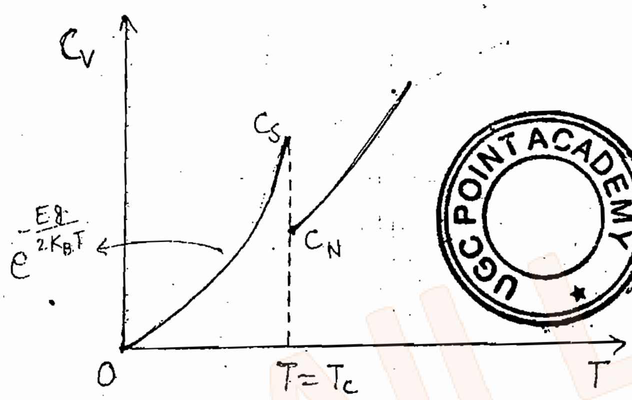
For Super Conductors,

$$\boxed{C_V = A e^{-\frac{E_g}{2K_BT}} + BT^3}$$

free e^- phonon

where, $E_g \rightarrow$ Superconducting band gap

At $T = T_c$, behaviour is discontinuous.



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According to, BCS Theory (BCS → Bardeen, Cooper, Schrieffer)

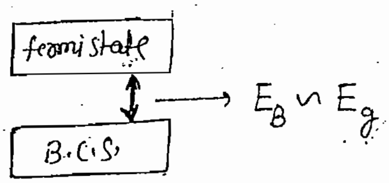
$$\boxed{\left.\frac{C_S - C_N}{C_S}\right|_{T=T_c} = 1.43}$$ for all SC.

Thermal Conductivity :- (K)

Generally Good electrical Conductors are also good thermal conductors. Since e^-'s can transport charge (electrical charge) as well as entropy, Hence good electrical conductors are good thermal conductors.

Electrical conduction takes place due to transport of charge & ~~heat~~ i.e.
Heat " " " " " entropy
transport of randomness.

Superconductors are good conductors of electricity but poor thermal conductors because those e^-'s which participates in the electrical conduction carry no entropy. Poor thermal conductivity indicates that only a fraction of e^-'s are now capable of transporting entropy as most of the e^-s are now coupled together to form 'cooper-pairs'. And cooper-pair carries No entropy.



This band gap makes e^- of fermi state can not go in B.C.S. state untill they are bound by cooper pair. The energy reqi. to bind the e^-s in cooper pair is B.E. $\rightarrow E_B$. This energy is equal to the band gap E_g in superconductor.

Typical order of band gap in SC is

$$\boxed{E_g \sim E_B \sim 1\ meV}$$

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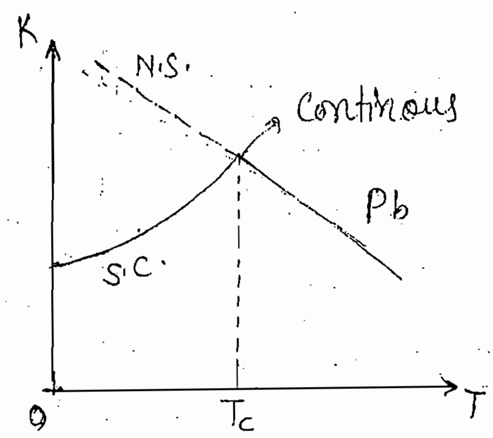
If energy is greater than 1 meV then cooper pair will break.

Acc. to B.C.S. theory, $T_c < 30K$

→ So this theory is succesful for low temp. SC but fails for high temp. SC.

Efficiency of phonons in transporting the heat remains unchanged but that contribution is very small at low temp.



<u>LONDON's Equations</u> :- Dynamics of magnetic field or current in a super-conductor can be explained by London equations.

Meissner effect in super-conductor can not be described by Maxwell's eq^n alonge. This effect can be described by

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'Two fluid model': Acc. to this model super-conductor consis of 2 types of e^-s

(i) Normal e^-s

(ii) Super e^-s

→ Current due to normal e^-s flows with resistance while due to super e^- flows without resistance.

→ Normal e^-s follow 'Ohm's law' while super e^-s does not.

Eqn of motion of super-e^-s is given by

$$m \frac{d\vec{v}_s}{dt} = -e\vec{E}$$

$v_s \rightarrow$ velocity of super e^-

Super Current density, $\vec{J}_s = -n e \vec{v}_s$

$$\Rightarrow \vec{v}_s = \frac{-\vec{J}_s}{n_s e}$$

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so $\frac{m}{n_s e} \frac{d\vec{J}_s}{dt} = e\vec{E}$

$$\boxed{\frac{d\vec{J}_s}{dt} = \frac{n_s e^2}{m}\vec{E}} \quad (1)$$

This is 'First London Eqn'

if $E = 0$, $\Rightarrow$ $\boxed{J_s = \text{Constant}}$

Normal Current density, $\vec{J}_n = \sigma\vec{E}$ (ohm's law)

$$E = 0 \Rightarrow \boxed{J_n = 0}$$

. Maxwell's eqn $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

ohm's law is not valid in super-conductor

Maxwell's eqn hold good ie. it is valid inside a SC.

$$\Rightarrow \frac{m}{n_s e^2} \frac{d}{dt}(\vec{\nabla} \times \vec{J}_s) = -\frac{\partial \vec{B}}{\partial t}$$

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$$\vec{\nabla} \times \vec{J_s} = \frac{-n_s e^2}{m} \vec{B} \qquad \text{---------(2)}$$

This is 'IInd London eqn'

$\because \vec{B} = \nabla \times \vec{A} \quad \therefore \quad \vec{\nabla} \times \vec{J_s} = -\frac{n_s e^2}{m}(\vec{\nabla} \times \vec{A})$

$$\boxed{\vec{J_s} = -\frac{n_s e^2}{m} \vec{A}}$$

This is another form of IInd London eqn.

$\boxed{\vec{J_s} \propto \vec{A}}$ ⇒ Postulate of SC

Total electrons, $n = n_s + n_n$

$n_s \rightarrow$ super e^- per unit volume

$n_n \rightarrow$ normal " " "

$$\boxed{n_s = n\left[1 - \left(\frac{T}{T_c}\right)^4\right]}$$

When $T = 0$, $\boxed{n_s = n}$

$T = T_c$, $\boxed{n_s = 0}$

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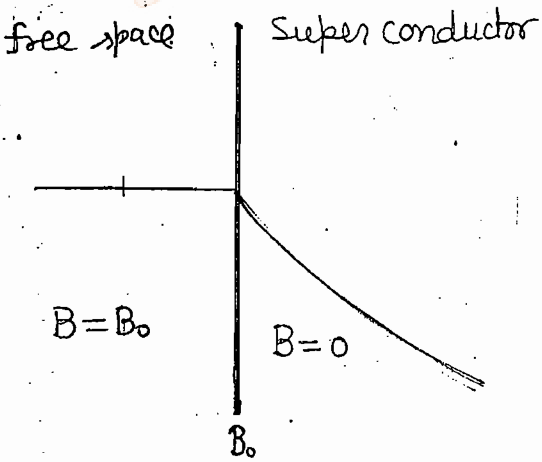
London Penetration Depth :- (λ). Magnetic field decays exponentially inside the SC.



Penetration depth is the distance at which value of the magnetic field inside the super-conductor becomes $\frac{1}{e}$ value of its value on the surface.

At this distance M.F. is $\frac{B_0}{e}$

$B_0 \rightarrow$ M.F. at surface.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J_s}$$

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• Taking curl, $\vec{\nabla}\times\vec{\nabla}\times\vec{B} = \mu_0(\vec{\nabla}\times\vec{J_s})$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} = -\mu_0 \frac{n_s e^2}{m}\vec{B}$$

$\vec{\nabla}\cdot\vec{B} = 0$

$$\Rightarrow \boxed{\nabla^2\vec{B} - \frac{\mu_0 n_s e^2}{m}\vec{B} = 0}$$

Solution of this eq^n is (in 1-D)

$$B = B_0 e^{-\frac{x}{\lambda}}$$

i.e. $\boxed{B(x) = B_0(0)\, e}$

If $x = \lambda$ then $\boxed{B(\lambda) = \frac{B_0(0)}{e}}$

If $x = 2\lambda$ then $\boxed{B(\lambda) = \frac{B_0(0)}{e^2}}$

where $\boxed{\lambda = \left(\frac{m}{\mu_0 n_s e^2}\right)^{1/2}}$

Penetration depth for free space :- $\boxed{\lambda = \infty}$

$$\boxed{\lambda \propto \frac{1}{\sqrt{n_s}}}$$

More the super e^- density, lesser the penetration depth.

Temperature Variation of λ :-

$$\lambda(T) = \lambda(0)\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-1/2}$$

At $T = 0$, $\boxed{\lambda(T) = \lambda(0)}$

$T = T_c$, $\boxed{\lambda(T_c) = \infty}$

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Coherence Length :- (ξ)

It is another independent parameter of super-conductor. This is the measure of distance within which super-conducting e^- concentration does not changes drastically in the non-uniform magnetic field (This is also called size of the cooper pair).

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For Type-I Super Conductor, $\boxed{\xi > \lambda}$

Type-II " " , $\boxed{\xi < \lambda}$

Its value Quantum mechanically, $$\boxed{\xi = \frac{2\hbar v_F}{E_g}}$$

$v_F \rightarrow$ fermi velocity of e^- in normal state.

$E_g \rightarrow$ Super-conducting band gap.

$$\boxed{\xi \propto \frac{1}{E_g}}$$

In impure superconductor $\boxed{\xi_{imp} < \xi_{pure}}$

$$\boxed{\xi_{imp} = \left(\xi_{pure}\, l\right)^{1/2}}$$

$l \rightarrow$ Mean free path of the e^-.

Penetration depth for impure SC,

$$\boxed{\lambda_{imp} = \lambda_{pure}\left(\frac{\xi_{pure}}{l}\right)^{1/2}}$$

generally, type-II SC are impure & type-I SC are pure.

$$\boxed{\frac{\lambda_{imp}}{\xi_{imp}} = \left(\frac{\lambda_{pure}}{l}\right) = K}$$

K $\rightarrow$ any constant

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H_{c_1} and H_{c_2} for a type-II SC :-

Imp

$$H_{c_1} = \frac{\phi_0}{\pi \lambda^2}$$

$$H_{c_2} = \frac{\phi_0}{\pi \xi^2}$$

where ϕ_0 is Fluxon or Flux Quanta,

$$\phi_0 = \frac{h}{2e}$$

$h \rightarrow$ Plank constant

$2e \rightarrow$ charge of cooper-pair.

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B.C.S Theory :- This is the most sucessful theory of super-conductivity.

Important features of B.C.S. Theory :-

(i) **e^--e^- interaction via lattice deformation** :- When e^--lattice e^- attractive interaction overcomes the e^--e^- repulsion then e^-s are attractively bind together with the help of lattice, This bound pairs of 2 e^-s is called cooper-pair. These cooper-pair are bosons and having charge 2e. And their energy is less than the sum of energy of individual e^-s. As they are behaving as a boson so they condance into a new Quantum state called B.C.S. ground state. and there is a band (energy) gap opens up b/w BCS ground state & lowest fermi state. And value of this band gap is approximately equal to the binding energy of the cooper pair.

$$E_g \sim E_B$$

(ii) **B.C.S. Ground State** :- Cooper-pairs condance into the B.C.S. ground state flows without resistance ~~has~~ ~~do~~ they do not scatter with the lattice. Hence their relaxation time is infinite ideally. These cooper-pairs help in the electrical conduction but do not help in the thermal conduction as they can not carry the entropy (Randomness).

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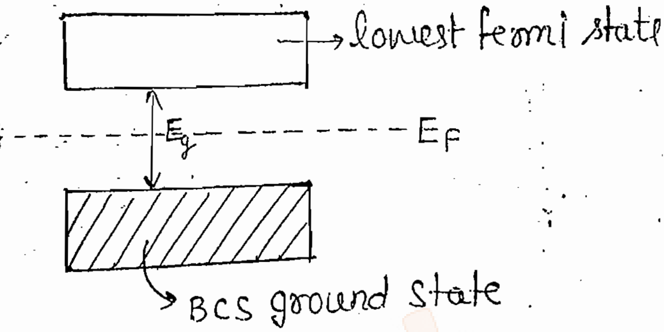

Acc. to B.C.S. theory,

Value of band gap at $T=0$,

$$\boxed{E_g(T=0) = 3.53\, K_B T_c} \quad \text{(more precise)}$$

$$\simeq 4\, K_B T_c$$

Typical order of E_g,

$\boxed{E_g \simeq 1 \text{ meV}}$ for superconductor

$\simeq 1$ eV for semiconductor

At finite temp,

$$\boxed{E_g(T) = E_g(0)\left[1-\left(\frac{T}{T_c}\right)\right]^{1/2}}$$

At $T=0$ K, $\boxed{E_g(T) = E_g(0)}$

$T=T_c$ K, $\boxed{E_g(T_c) = 0}$

This E_g lies in 'Microwave region' of E.M. spectrum.
If band gap is large then lie in I-R region (far infrared)
Presence of this band gap confirmes from the discontinuity in the specific heat at $T=T_c$ and Microwave absorption in SC.

Minimum amount of (E_g) energy required to destroy the superconductivity, $E_g = h\nu$

so $\boxed{E_g < h\nu}$

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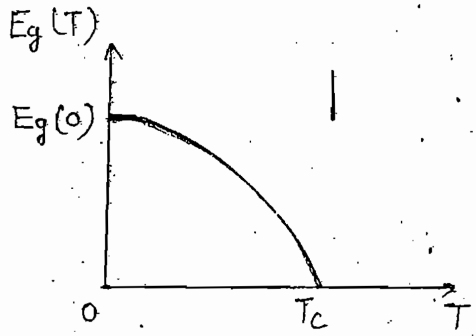
Cooper pair :- charge $= 2e$
Boson
spin $\rightarrow 0$ $(\uparrow\downarrow)$

$E_B \sim 1$ meV

$$\xi = \frac{2\hbar v_F}{E_g}$$



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Isotop Effect :- It is found that superconducting critical temp T_c,

$$\boxed{T_c \propto M^{-1/2}}$$

where, $M \rightarrow$ Isotopic Mass

$$\Rightarrow \boxed{T_c M^{1/2} = \text{Constant}} \quad \text{———} (1)$$

As $M\uparrow$, $T_c\downarrow$. $\Rightarrow$ heigher is the isotopic mass, lower is the critical temp.

Debye temp. of phonon spectrum is also related to isotopic mass as

$$\boxed{\theta_D \propto M^{-1/2}}$$

$$\Rightarrow \boxed{\theta_D M^{1/2} = \text{Constant}} \quad \text{———} (2)$$

For any Super-Conductor,

$$\boxed{\frac{T_c}{\theta_D} = \text{constant}}$$

This confirms that e^-- phonon interaction is responsible for superconductivity.

Flux Quantisation :- Magnetic flux passing through a superconding ring is quantised and given by

$$\boxed{\phi = \frac{nh}{2e} = n\phi_0}$$

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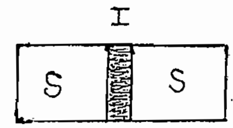
where, $n \rightarrow$ some integer

$\phi_0 \rightarrow$ fluxon or flux Quanta

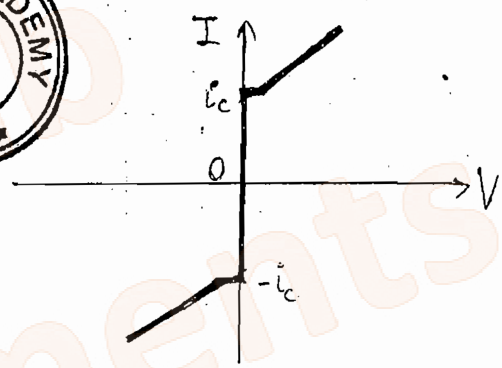
Josephson Effect :- When a thin insulating layer sandwitched between 2 super-conductors such a junction is called Josephson-junction.



(SIS Josephson jucⁿ)

a) dc Josephson Effect :-

Acc. to this effect, a dc current flows across the jucⁿ in the absence of any applied electric or magnetic field.



b) A.C. Josephson effect :-

When a dc bais apply at the jucⁿ a radio freq. a.c. current flows across the jucⁿ. This effect is utilized in the determination of value $\frac{e}{h}$. Freq. of the current is given by

$$\boxed{\omega = \frac{2eV}{\hbar}}$$

where $V \rightarrow$ applied voltage at the jucⁿ.

This gives the ideal defination of 1 volt.

For $1\,\mu V$,	$\omega \simeq 483.6$ MHz
$1\,V$,	$\omega \simeq 483.6$ THz

Switching time of Josephson jucⁿ,

$$t \sim 10^{-15} \text{ sec}$$

i.e. jucⁿ can ON-OFF, 10^{-15} times in 1 sec.

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Questions

Q.1:- B.E. of a cooper-pair at absolute zero acc. to B.C.S. theory is about $3.5 K_B T_c$, where K_B is Boltzman const., T_c critical temp. If critical temp. of type-I SC is 10 K.

(a) What is the energy gap E_g in eV.

(b) Calculate the wavelength of photons whose energy is just sufficient to break the cooper-pair in the SC.

(c) In which region of EM spectrum these photons will lie.

Q.2:- For a B.C.S. SC, $T_c = 5\,K$, $E_F = 5\,eV$ & density of conduc. $e^-s = 10^{29}/m^3$. Calculate

(a) Magnitude of energy gap in eV

(b) density of super e^-s n_s at 0 K

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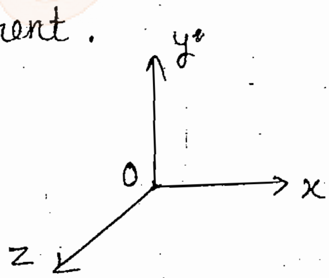
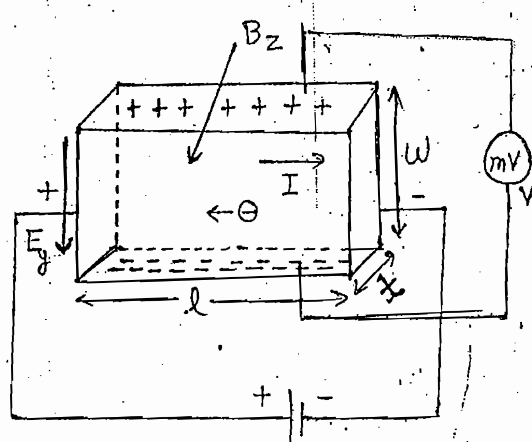
Hall Effect in metals :-

When a current carrying conductor placed in a transverse magnetic field, a voltage is induced b/w the 2 faces of the conductor which are perpendicular to both magnetic field & current.





$V_H \rightarrow$ Hall Voltage

$E_y \rightarrow$ Hall field

$$\boxed{V_H = W E_y} \quad \text{—— (1)}$$

$$F = -e(E_y + v_x B_z) = 0$$

$$\Rightarrow \boxed{E_y = -v_x B_z} \quad \text{—— (2)}$$

$\because$ $J_x = n e v_x$

$$\therefore \boxed{E_y = -\frac{J_x B_z}{ne}} \quad \text{—— (3)}$$

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