Free Study Material from

All Lab Experiments

Solid-State Physics Notes
for NET/GATE Physical Sciences
Magnetism in Solids

Magnetism in Solids

There are 3 principle sources of magnetic moment in an atom

If Magnetisation M is anti ‖ to applied mag. field then there, it is diamagnetic

& If M is ‖ to applied mag field para or ferro.

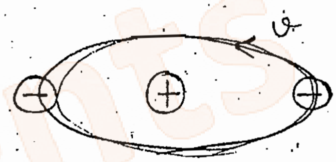
(i) Orbital motion of e^-

(ii) e^- spin motion

(iii) Induce mag. mom. due to change in the orbital motion of e^- on the application of external mag. field

In an atom, there is a nucleus about which e^- do orbital motion & current produce & this constitute a current dipole

$$I = \frac{e}{T} = \frac{ev}{2\pi R}$$



Magnetic Moment. $m = I \cdot A$

$$= \frac{ev}{2\pi R} \cdot \pi R^2$$

$$\boxed{m = \frac{evR}{2}}$$

This m is due to orbital motion of e^-

Dir^n of m will be decided by motion of e^-.

for Orbital Motion, $\frac{m_l}{L} = \frac{e}{2m}$

spin ", $\frac{m_s}{S} = \frac{2e}{2m}$

Orbital & spin motion of e^- both give paramagnetic contribution.

Contribution of spin motion is larger ~~tho~~ to paramag. effect (paramag. effect → attractive effect)

If we apply external mag. field then mag. mom. will be align in the dir^n of field to minimize the energy.

→ When e^-s are unpaired in atom then there will be spin contribution.

When ~~atom is~~ e^-s are not paired then atom 'll have spin mag. mom. & this mom. is aligned in the dir^n of externally applied mag. field.

→ ~~Contribution~~ It is hard to tilt the orbit as compare to flip the spin so spin contribution is large to paramag. effect.

→ When we apply external mag. field on the orbit then there will be change in speed Δv. It will speed up or slow down depending on the dir^n.

→ Due to externally applied field, mag. flux change & emf produced s.t. it will oppose the flux.

So Induce Current will oppose the change in flux.

→ Induced mag. mom. is very small so diamagnetic effect is weak effect,

Diamagnetic effect is a universal effect. It is found in all the solids.

It is universal but not dominent due to weakness.

i.e. Paramag. Effect dominent over diamagnetic effect if there is both these effects in a solid.

Types of Magnetic Materials :-

(i) Diamagnetic

1) Diamagnetic effect arises due to change in the orbital motion of e^- around the nucleus on the application of external mag. field.

2). These materials reppel in the mag. field.

3) It is a very weak effect.

(4) Mag. field $\boxed{\vec{B} = \mu_0(\vec{H} + \vec{M})}$ — (A) $H \rightarrow$ Mag. field intensity

for diamagnet, M is antiparallel to H. $M \rightarrow$ Magnetisation

$$\vec{B} = \mu \vec{H}$$

$\mu \rightarrow$ suceptability

Magnetic Suceptability is defined as

$$\chi_m = \frac{M}{H}$$

If M & H behave linearly then $\chi_m = \frac{M}{H}$ (for diamagnet)

" " are not linear then $\chi_m = \frac{\partial M}{\partial H}$ (for ferro)

(A) $\Rightarrow$

$$\mu H = \mu_0 (H + \chi_m H)$$

$$\Rightarrow \frac{\mu}{\mu_0} = 1 + \chi_m \Rightarrow \boxed{\mu_r = 1 + \chi_m}$$

$$\boxed{\vec{B} = \mu_0 \mu_r \vec{H}}$$

for diamagnet, $\boxed{\mu_r < 1}$

then $\boxed{\chi_m < 0}$ i.e. -ve.

So Suceptability of diamagnetic material is negative.

Diamagnetic suceptability of a solid according to Quantum mechanics is given by

$$\boxed{\chi_{dia} = \frac{-\mu_0 N Z}{6 m_e} e^2 \langle r^2 \rangle}$$

where, N $\rightarrow$ No. of atoms per unit volume

Z $\rightarrow$ Atomic No.

$m_e \rightarrow$ mass of the e^-

$\langle r^2 \rangle \rightarrow$ Average value of r^2 over the perticular orbit.

for Hydrogen atom ground state $\langle r^2 \rangle = \int \psi^* r^2 \psi \, d\tau$

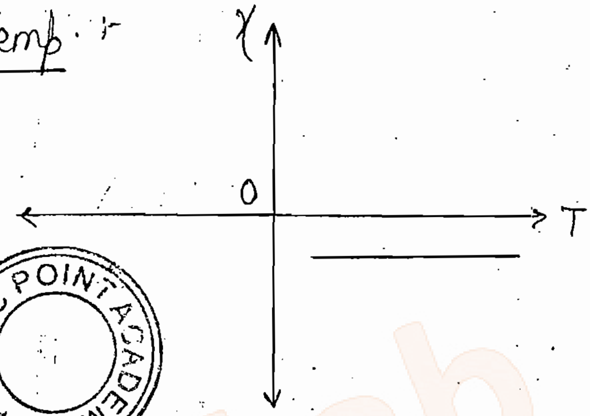
$= 3 a_0^2$

So we can find out diamag. suceptability of H_2.

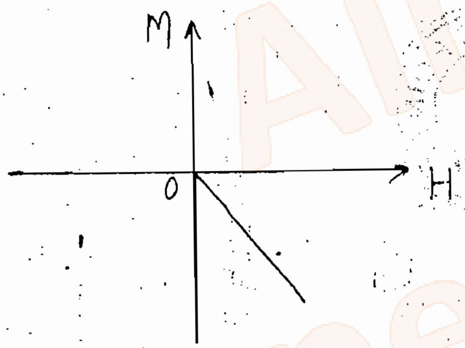
So diamagnetic susceptability is negative & Independent of temprature.

Examples → Helium, Ne, Ar, Cu, Au, Si, Ge etc.

Pilot of χ v/s Temp :-

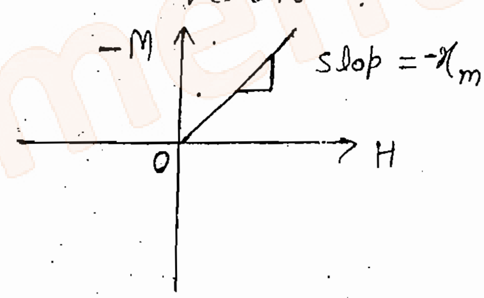


M-H Curve :-



As H ↑, M ↑ in reverse i.e. -ve dir^n.

OR



(ii) Paramagnetic Materials :-

(1) Paramagnetism occur in those solids which have parmanent magnetic moment due to spins of unpaired e^-s, on the application of magnetic field, These magnetic moment alligned in the dir^n of applied magnetic field. Hence effect is attractive type. This is strong effect as compare to diamagnetic effect.

(2) Molecular Oxygen(O_2) has even no. of e^-s but shows paramagnetic effect (exception)

(3) Paramag susceptability of ions or atoms is given by

$$\boxed{\chi_{ions} = \frac{C}{T}} \Rightarrow \text{Curie law}$$

where $C \rightarrow$ Curie Const.,

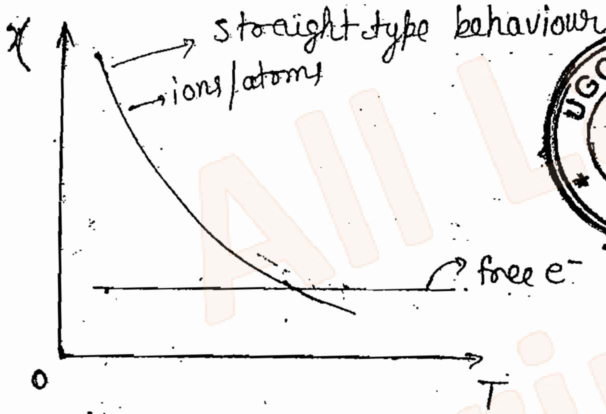
i.e. χ_{ions} is $\propto \frac{1}{T}$ $\Rightarrow$ $\boxed{\chi \propto \frac{1}{T}}$

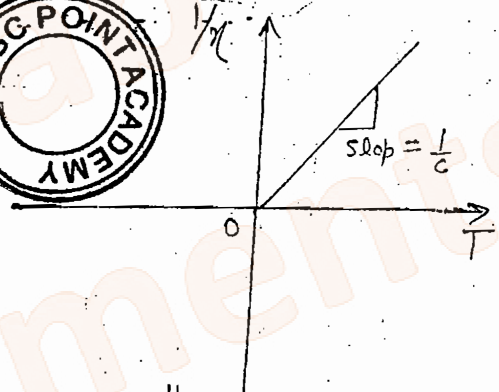
(4) Paramag. susceptability of free e⁻'s (Pauli Paramagnetism)

$$\boxed{\chi_{free\ e^-} = \text{independent of temp. \& +ve}}$$

This can't be explain by classical theory. This is explained by Quantum free e⁻ theory.

Examples :- O_2, $MnSO_4$, Mn^{+2}, Gd^{3+} etc.





$\chi = \frac{C}{T}$

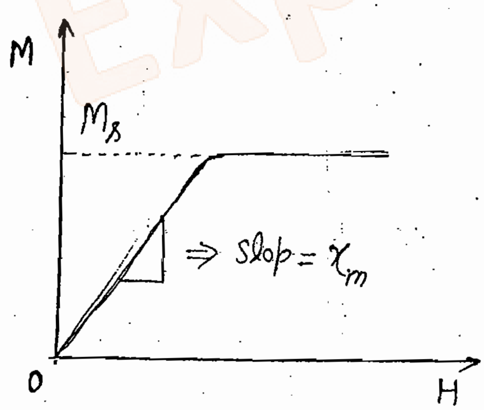
$\frac{1}{\chi} = \frac{T}{C}$

$\frac{(1/\chi)}{T} = \frac{1}{C}$

⇓

(for ions/atoms)

M-H Curve :-



Initially M↑ as H↑ linearly. After a certain value of H, all the spin will alligned in the dirⁿ of applied mag. field & M will saturate & that value of M is called M_s i.e. Saturation Magnetisation.

Magnetisation for a paramagnetic solid :-

$$\boxed{M = N g J \mu_B B_J(x)}$$

where; $N \rightarrow$ No. of atoms per unit volume

$g \rightarrow$ Lande g-factor

$J \rightarrow$ Total angular momentum
$\mu_B \rightarrow$ Bohr magneton
$B_J(x) \rightarrow$ Brillouin func^n

& $\boxed{x = \frac{g J \mu_B B}{K_B T}}$

Special Case :- for Spin-$\frac{1}{2}$ particles,

$$\boxed{M = N\mu_B \tanh\left(\frac{\mu_B B}{K_B T}\right)}$$

(i) $\mu_B B << K_B T$
i.e. temp. is high or strength of mag. field is less.
then Let $\frac{\mu_B B}{K_B T} = x$ so $x \ll 1$
$\tanh x \simeq x \simeq \frac{\mu_B B}{K_B T}$

$$M = N\mu_B \frac{\mu_B B}{K_B T}$$

$$\boxed{M = \frac{N \mu_B^2 B}{K_B T}}$$

Put $B = \mu_0 H \Rightarrow M = \frac{N\mu_B^2 \mu_0 H}{K_B T}$

So $\chi = \frac{M}{H} \rightarrow \boxed{\chi = \frac{\mu_0 N \mu_B^2}{K_B T}}$

i.e. $\boxed{\chi \propto \frac{1}{T}}$

(ii) $\mu_B B >> K_B T$ (temp. is low)
i.e. $x >> 1$ then $\tanh x \simeq 1$

$\boxed{M = N\mu_B} \Rightarrow$ This is called Saturation Magnetisation when all the spins are alligned in the dir^n of applied mag. field.

i.e. $\boxed{M_s = N\mu_B}$

$$\boxed{\chi_s = \frac{M_s}{H} = \frac{N\mu_B}{H}}$$

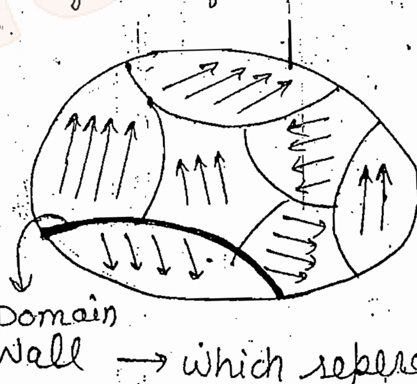
χ is independent of temp. when temp. is low or B is high.

(iii) Ferromagnetic Materials :-

~~Fear~~ Like paramagnetism, ferromagnetism is also associated with permanent magnetic moment but ferromag. materials are characterise by spontaneous magnetisation. (i.e. Non zero magnetisation even in the absence of mag. field)

→ Explanation of ferromagnetism can be given in terms of formation of small regions inside the solid in which magnetisation is saturated. These small regions are called domains.
⇒ Magnetic domains. Inside these domains, spins are parallel alligned & gives rise to a very large magnetic field i.e. called "Exchange field or Weiss field. (B_E)

Reason of formation of domain :-
Every physical process is to achieve the stability & minimize the energy.

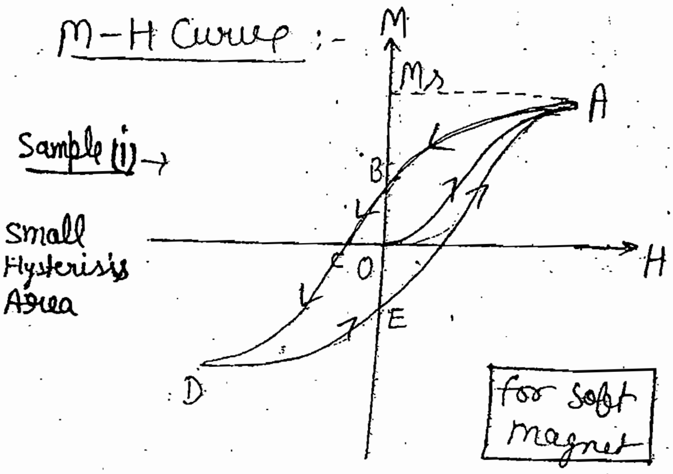


Domain Wall → which seperate the domains.

→ Thickness of domain wall → few Å.

→ ~~Spin~~ energy quanta of spin wave is Magnon.

M-H Curve :-



On appling H, M ↑ upto saturation. When H removed then ~~H~~ M ↓ but not follow the same path.

OB → ~~Remenent~~ retentivity magnetisation

OC → ~~Coerci~~ coercive field

OC → amount of mag. field required to distroy the coercive field.

OABCDEA → is the Hysteresis loop.

This M-H Curve gives the useful information about choosing the magnet for a particular type of application.

(i) paramagnet magnet (Hard magnet)

(ii) Soft Magnet

<u>SQUID</u> (Super Conducting Quantum Interference device)

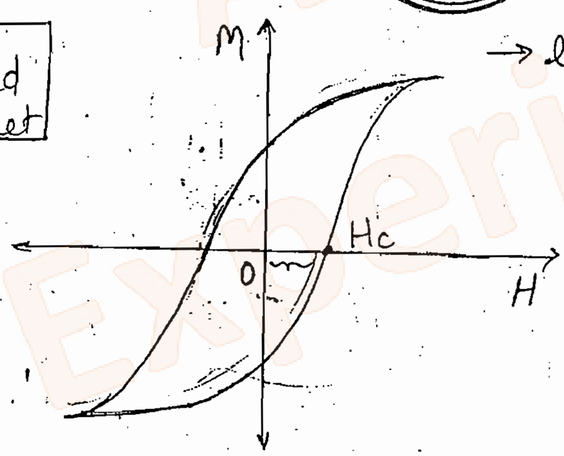
SQUID is used to measure to magnetic ~~magnet~~ mom. It is very sensitive.

<u>VSM</u> → Vibrating

Not much sensetive.

for Hard Magnet

sample (II) →



→ large Hysteresis Area

Area of Hysterisis loop = Hysterisis loss.

If we want frequent magnetisation or demagnetisation, Hysteresis loss should be small → Small area magnet will ~~be require~~ be preferred as in sample (i)

for parmanent magnetisation, hysteresis area should be large as in sample (ii)

H_c → Coercivity

It will decide the magnet is hard or soft.

- Hard magnet & or paramagnet are characterise by high coresive field or by high Ramenent magnetisation e.g. Steel
- Soft Magnets are used in the applications where we need frequent magnetisation & demagnetisation like in Transformer cores. Soft magnets are characterise by low coresive field high Ramenent magnetisation & high permiability. $H_c \downarrow \rightarrow$ easy will be demag./magneti.

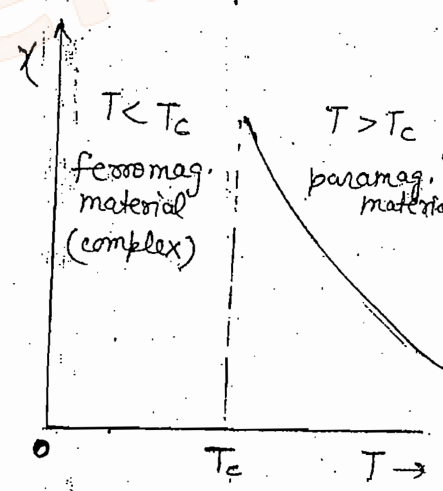
Ex – Soft Iron & ferrits

Suceptability of ferromagnetic material :-

$$\boxed{\chi = \frac{C}{(T-T_c)}}$$ Curie - Weiss law for $T > T_c$

$T_c \rightarrow$ ferromagnetic Curie temp.

for $T < T_c$, $\boxed{\chi = \text{Complex}}$



Any ferromag. material converts into paramagnetic material at $\boxed{T = T_c}$

$$\frac{1}{\chi} = \frac{T-T_c}{C}$$

$$\frac{1}{\chi} = \frac{T}{C} - \frac{T_c}{C}$$

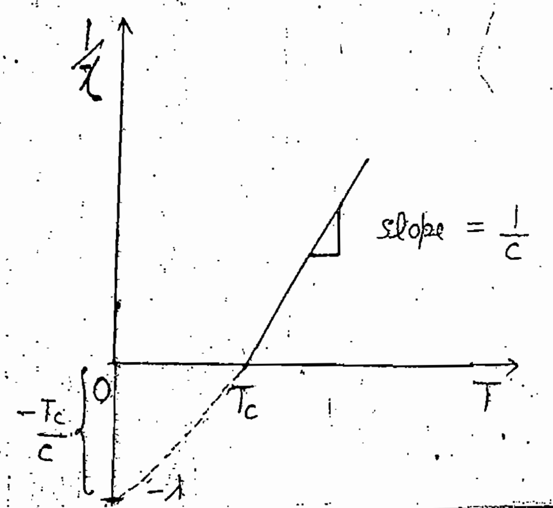
$C \rightarrow$ Curie const.

$\boxed{T_c = C\lambda}$ in C.G.S. unit

$T_c \rightarrow$ Curie temp.

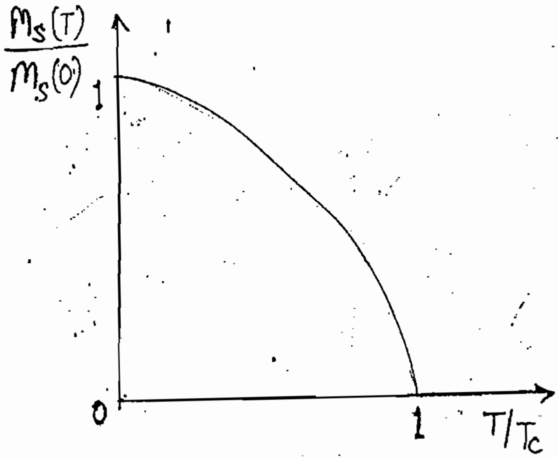
$\lambda \rightarrow$ Weiss const.

So $-\frac{T_c}{C} = -\lambda$



Imp

Temperature dependance of Spontaneous Magnetisation :-



$$\frac{\Delta M_s}{\Delta M_s(0)} = \frac{M_s(0) - M_s(T)}{M_s(0)}$$

$$\boxed{\frac{\Delta M_s}{\Delta M_s(0)} = A\,T^{3/2}} \Rightarrow \text{Bloch} - T^{3/2} \text{ law}$$

As $T \uparrow$, $M_s(T) \downarrow$

So $\frac{\Delta M_s}{\Delta M_s(0)} \uparrow$

→ As $T \to T_c$ (from below) then

$$\boxed{M \propto (T_c - T)^{\beta}} \qquad \boxed{\beta = \frac{1}{2}}$$

Imp Near critical or Curie temp. T_c, Saturation magnetisation or spontaneous magnetisation ~~ha~~shows the behaviour $M \propto (T_c - T)^{1/2}$

→ As $T \to T_c$ (from above) then

$$\boxed{\chi \propto (T - T_c)^{\gamma}} \qquad \boxed{\gamma = -1}$$

$$\chi \propto (T - T_c)^{-1}$$

→ At $T = T_c$ there is a discontinuity i.e. susceptability sufferse a discontinuty.

Magnon :-

It is a energy quanta of spin wave.

Magnon is a boson.

Magnon dispersion relation ω v/s K is determined by the neutron diffraction.

Magnon in ferromagnetic Solid, follows $\boxed{\omega \propto k^2}$ relationship

Not linear, parabolic with k

→ Specific heat for a boson following quadratic relationship then in 3-D $\boxed{C_v \propto T^{3/2}}$

→ If boson follow linear relationship then $\boxed{C_v \propto T^3}$

Magnon in ferromag, $C_v \propto T^{3/2}$

" " Antiferromag., $C_v \propto T^3$

Magnon follow quadratic rel^n inside the solid in ferromag

Density of States, $\boxed{D(E) \propto E^{1/2}}$

Magnetisation, $\boxed{M \propto T^{3/2}}$

ANTI-FERROMAGNETISM:-

Anti-ferromagnetism is characterised by antiparallel spin on neighbouring sides. Hence No spontaneous magnetisation. & spin mag. moment at side A & at side B are equal

A B A B

↑ ↓ ↑ ↓

$$m_A = m_B$$

So they cancel each other. So No net magnetisation in the absence of magnetic field.

Here energy is minimized due to antiparallel spin.

Suceptability of Antiferromag. material is given by

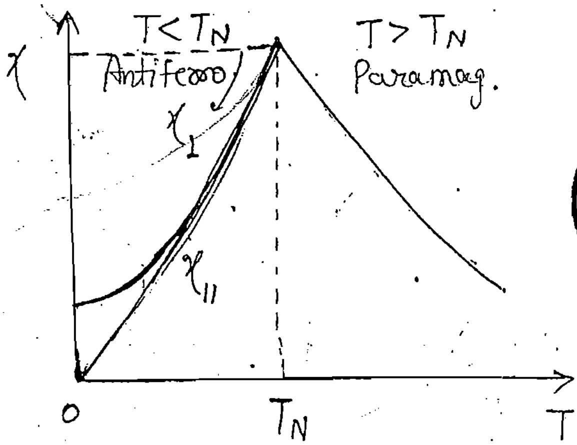
$$\boxed{\chi = \frac{2C}{T+T_N}} \quad \boxed{T > T_N}$$

where T_N → Neel temperature

for $T < T_N$ → behaviour is Antiferromagnetic

$T > T_N$ → " " paramagnetic

At $T = T_N$, behaviour of χ changes. So T_N is reference temp.



χ has two comp. $\rightarrow$ $\chi_\perp$ & $\chi_\parallel$

B is applied $\perp$ to the spin

$$\uparrow\downarrow \longrightarrow B \Rightarrow \chi_\perp$$

B is applied $\parallel$ to the spin.

$$\uparrow\downarrow \quad \uparrow B \Rightarrow \chi_\parallel$$

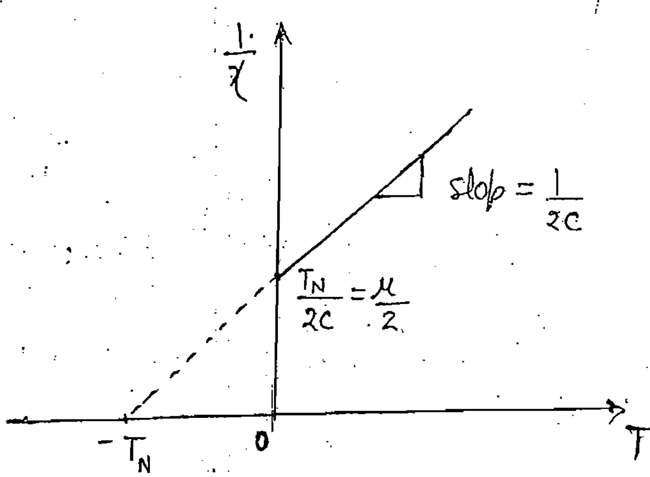
- $\chi_\perp$ is the suceptability is mag. field is applied $\perp$ to the axis of the spin.
- $\chi_\parallel$, if mag. field is applied $\parallel$ to the axis of the spin.
- $\chi_\perp$ is independent on temp.
- $\chi_\parallel$ $\uparrow$ increases linearly with the temp.

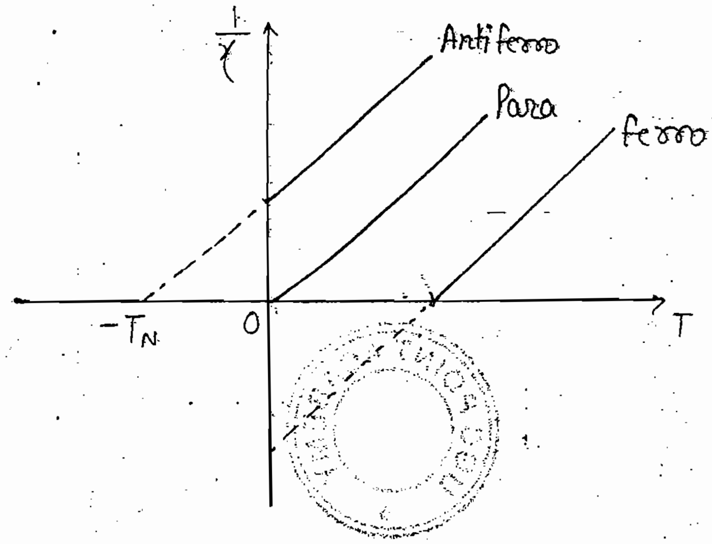
$$\boxed{T_N = \mu C} \qquad \{T_c = C\lambda\}$$

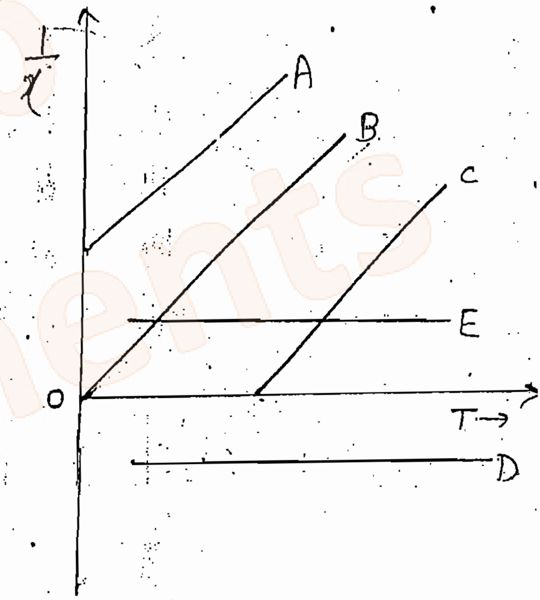
where, $C \rightarrow$ Curie Constant
$\mu \rightarrow$ Mean field constant

- In antiferomag. material, at very-very low temp. (& at very high mag. field) there exist a very thin Hysteresis loop.

At very low temp. ; if mag. field is applied & ~~just~~ removed then spin will not remain same, that's why magnetisation is non-zero in the absence of mag. field.

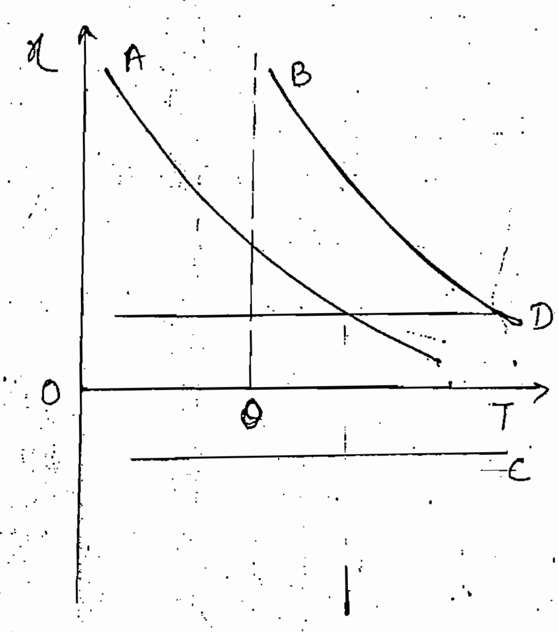






A → Antiferro
B → Para
C → Ferro
D → Dia
E → Polypara

If $\chi \to -ve$ then $1/\chi \to$ large $-ve$
if $\chi \to +ve$ then $1/\chi \to$ large $+ve$



A → Para
B → Ferro (diverge $\theta = \theta_c$)
C → Dia
D → Pauli

{In Antiferro → Not diverge}
{$\theta = \theta_N$}

Examples: MnO, NiO, MnS etc.

Magnon Dispersion Relationship for Antiferro mag material :-

is linear

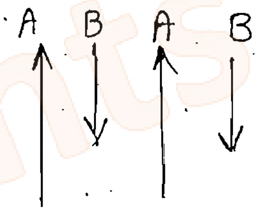
$\boxed{\omega \propto K}$ for magnon.

$$\boxed{\begin{array}{l} D(E) \propto E^2 \\ C_V \propto T^3 \\ M \propto T^3 \end{array}}$$

FERRI-MAGNETISM :-

Ferri-magnetism is similar to antiferromagnetism except spin magnetic moment of 2 neighbouring spin are not equal. Hence spontaneous magnetisation is observed for ferrimagnetism. $\boxed{M_s \neq 0}$

Spins can not cancel each other.



Magnetisation at both sides A & B follows the curie laws.

Example :- Fe_3O_4 ⇒ ferrites.

PHASE TRANSITION :-

First Order Phase Transition :-

If Ist order derivative of free energy changes discontinously at the transition point. This is called First Order phase transition.

for Example :- $U \rightarrow$ Internal Energy

$G \rightarrow$ Gibbs free energy

$H \rightarrow$ Enthalpy

$F \rightarrow$ Helmaltz free energy

First-Order derivatives of Energy are defined as,

(i) Entropy $S = -\left(\frac{\partial G}{\partial T}\right)_P$

(ii) Volume $V = \left(\frac{\partial G}{\partial P}\right)_T$

(iii) Magnetisation $M = \left(\frac{\partial F}{\partial B}\right)$

Second-Order Phase Transition :- In IInd order phase Trans. second order derivative of free energy changes discontinously at the transition point,

(i.) Susceptability $\chi = \left(\frac{\partial M}{\partial H}\right)$

$\frac{\partial M}{\partial H}$ → IInd order derivati

& M is continous i.e. Ist order derivative is conti

(ii) specific heat $C_V = T\left(\frac{\partial S}{\partial T}\right)$

(iii) Isothermal Compressability $= \frac{1}{V}\left(\frac{\partial V}{\partial P}\right)$

Examples of IInd Order Phase Transition -

(i) paramagnetic to ferromagnetic or ferro ↔ Para

at $\boxed{T = T_c}$ T_c → Curie temp.

(ii) Normal state ⟷ Super conducting state

at $T = T_c$ T_c → Critical temp.

(iii) liquid He-I ⟷ liquid He-II

- In IInd order Phase Transition, No requirement of latent heat at the transition point.

If 100°C water is converting into 100°C vapour

⇒ Ist order phase transition

Water & Vapour both have same energy but energy is large for vapour this comes from latent heat.

∴ S & M changes continously.
χ & C_V " dis "

- Antiferro. to para ⇒ I order phase Transition.

Que:- No. of Maxima or Minima for a ferromagnet.

Energy of a ferromagnet as a func^n of magnetisation is given by $\boxed{F(M) = F_0 + 2(T-T_c)M^2 + M^4}$ $\boxed{F_0 > 0}$

No. of Minima in the func^n for $\boxed{T > T_c}$

(a) 0 ✓(b) 1 (c) 3 (d) 4

For Maxima or Minima, $\dfrac{\partial F}{\partial M} = 0$

& for Minima, $\dfrac{\partial^2 F}{\partial M^2} = +ve$ i.e. $\dfrac{\partial^2 F}{\partial M^2} > 0$

$$\frac{\partial F}{\partial M} = 4(T-T_c)M + 4M^3 = 0$$

$$\Rightarrow 4M(T-T_c) = 4M^3$$

$$\Rightarrow M[M^2 + (T-T_c)] = 0$$

$$\Rightarrow \boxed{M = 0} \quad \text{OR} \quad M^2 + (T-T_c) = 0$$

$$\Rightarrow \boxed{M = \pm(T_c - T)^{1/2}}$$

$$\frac{\partial^2 F}{\partial M^2} = 4(T-T_c) + 12M^2$$

For $M=0$, $\dfrac{\partial^2 F}{\partial M^2} = 4(T-T_c)$

Here $T > T_c$ so $4(T-T_c) = +ve$

So we'll get Minima for $M=0$

for $M = \pm(T_c - T)^{1/2}$, $\dfrac{\partial^2 F}{\partial M^2} = 4(T-T_c) + 12(T_c - T)$

$$= 8T_c - 8T = -ve$$

So No. of Minima = 1 (for $M=0$)

Q. Free Energy of a ferromagnet,

$$F(M,T) = (T-T_c)M^2 + bM^4 + cM^6$$

Find Depandency of M on temp. near T_c, $(T \simeq T_c)$

$\{$ Behaviour of M near T_c is $M \propto (T_c - T)^{1/2}$ $\}$

for Minima or Maxima, $\dfrac{\partial F}{\partial M} = 0$

$$\Rightarrow \; 2(T-T_c)M + 4bM^3 + 6cM^5 = 0$$

$$M\left[2(T-T_c) + 4bM^2 + 6cM^4\right] = 0$$

$\boxed{M=0}$ OR

$$\cancel{2(T-}\; 6cM^4 + 4bM^2 + 2(T-T_c) = 0$$

$$M^2 = \frac{-4b \pm \sqrt{16b^2 - 48c(T-T_c)}}{12c}$$

$$M^2 = \frac{-b \pm \sqrt{b^2 - 3c(T-T_c)}}{3c}$$

$$M^2 = \frac{-b}{3c}\left[1 \mp \left\{1 - \frac{3c}{b^2}(T-T_c)\right\}^{1/2}\right]$$

$$= \frac{-b}{3c}\left[1 \mp \left(1 - \frac{3c}{2b^2}(T-T_c)\right)\right]$$

Taking + sign, $M^2 = \dfrac{-b}{3c}\left[\cancel{1} - \cancel{1} + \dfrac{3c}{2b^2}(T-T_c)\right]$

$$M^2 = \frac{1}{2b}(T_c - T)$$

$$\boxed{M = \pm\frac{1}{\sqrt{2b}}(T_c - T)^{1/2}}$$

Taking − sign, $M^2 = \dfrac{-b}{3c}\left[1 + 1 - \dfrac{3c}{2b^2}(T-T_c)\right]$

$$M^2 = -\frac{b}{3c}\left[2 - \frac{3c}{2b^2}(T-T_c)\right]$$

$$M = \pm\sqrt{\frac{b}{3c}}\left[\frac{3c}{2b^2}(T-T_c) - 2\right]^{1/2}$$

So there are 5 solutions,

Now check, for $\left.\begin{matrix} T > T_c \\ \& \ T < T_c \end{matrix}\right\}$ which sol^n gives the minima.