Free Study Material from
All Lab Experiments

Nuclear & Particle Physics Notes for NET/GATE Physical Sciences
Nuclear Decay & others

Nuclear Decay

α-decay

Occurs when the nucleus is unstable due to its large size.

4 nucleons (2p+2n) emitted at a time.

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X + {}^{4}_{2}He $$

β-decay

Occurs when the nucleus is unstable is due to improper value of $\frac{n}{p}$ ratio

$n \rightarrow p$ or $p \rightarrow n$ with emission of e^- or e^+.

no net change in nucleon no.

γ-decay

α or β decay are generally followed by γ decay. after α or β decay if the daughter nucleus is in excited state then surplus energy is emitted in the form of γ-rays.

$$ {}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma $$

⇒ α-Decay :-

α-particles.

Range of α-particles :-

v_0 - initial velocity

~~E_0 - initial velocity~~

$$ \boxed{R \propto v_0^3} $$

Geiger law → Or $\boxed{R \propto E_0^{3/2}}$ ⇒ Then the range of α-particles depends upon v_0.

for medium range α-particles.

for low range α particle $R \propto v_0^{3/2}$

$\Rightarrow \boxed{R \propto E_0^{3/4}}$

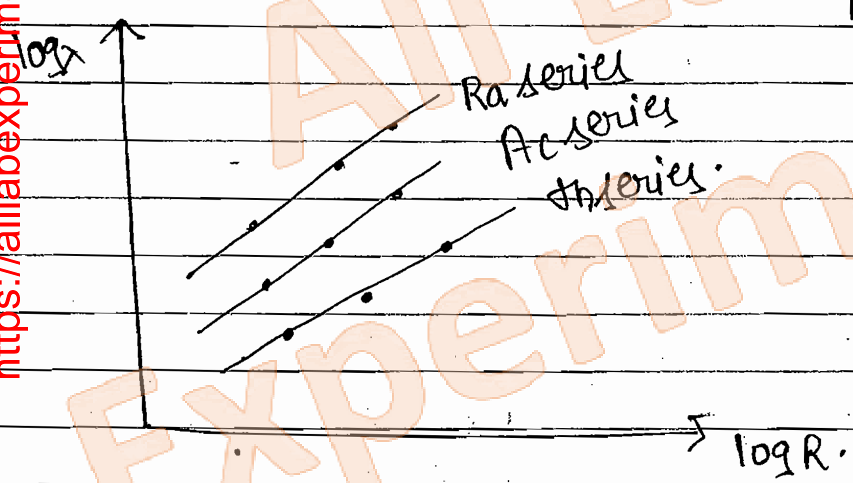
for large range α particle :- $R \propto v_0^4$

or $\boxed{R \propto E_0^2}$

Geiger-Nuttle law :- this is the relationship between range and decay constant.

$$\boxed{\log \lambda = A \log R + B}$$

λ - decay constant.
R - Range.
A - constant for all elements.
B - different for different radioactive series.



Energy and momentum conservation in α decay :-

Disintegration energy =

$${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}X + {}^{4}_{2}He + Q_\alpha$$

Parent, daughter, α-particle

Q_α - disintegration energy - total energy released in the process.

Q_α = final K.E. − initial K.E.

$$\text{Parent} \longrightarrow \text{daughter} + \alpha$$

at rest. $\vec{P_i} = 0$; daughter: $\vec{P_d}$, m_d, v_d ; α: $\vec{P_\alpha}$, m_α, v_α

$$\boxed{m_\alpha v_\alpha = m_d v_d}$$

$$\vec{P_i} = \vec{P_f} = \vec{P_d} + \vec{P_\alpha}$$

$$\Rightarrow \boxed{P_\alpha = P_d} \text{ numerically.}$$

$$Q_\alpha = \frac{1}{2} M_d v_d^2 + \frac{1}{2} M_\alpha v_\alpha^2 - 0$$

$$= \frac{1}{2} M_d \left(\frac{m_\alpha v_\alpha}{m_d}\right)^2 + \frac{1}{2} m_\alpha v_\alpha^2$$

$$\boxed{Q_\alpha = \frac{1}{2} m_\alpha v_\alpha^2 \left[1 + \frac{m_\alpha}{m_d}\right]}$$

$$\boxed{Q_\alpha = K_\alpha \left(1 + \frac{m_\alpha}{m_d}\right)} \qquad K_\alpha = \frac{1}{2} m_\alpha v_\alpha^2 .$$

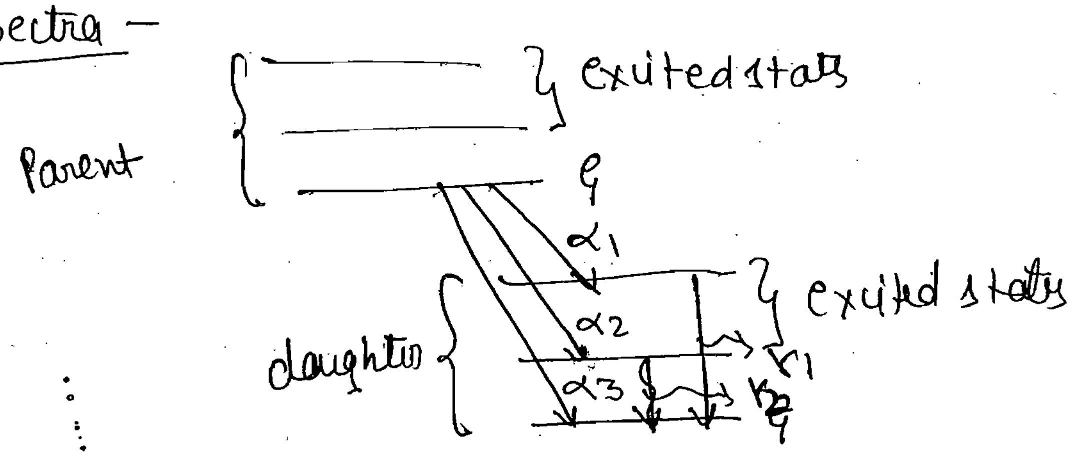
$$\Rightarrow \boxed{E_{dis} > K_\alpha}$$

If we replace mass by mass no's. i.e. $\frac{m_\alpha}{m_d} \simeq \frac{4}{A-4}$

$$\boxed{E_{dis} = Q_\alpha = K_\alpha \left(\frac{A}{A-4}\right)}$$

⇒ most of the energy is carried out by α-particles.

α-ray-spectra –



Features:— 1. α-spectrum is a line spectrum

2. we find a group of lines corresponding different transition energies.

~~Explanation~~ 3. this spectra indicates that nuclear levels are discrete.

theory of α-decay (Gammow theory):—

Gammow explain the α-decay problem on the basis of Quantum mechanical tunnelling. Since, the observed kinetic energy of α-particle is less than the barrier height.

K.E. < P.E.

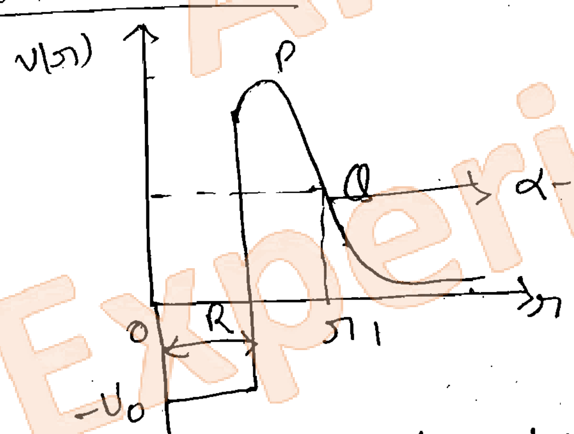
So classically α particle can't be found outside the nucleus.

→ But there is very-very small but finite probability of tunneling if K.E. < P.E.

⇓

we have chance to find α particle outside the nucleus.

Potential barrier—



→ for the r>R, there is repulsive force b/w α-particle and daughter nuclei.

→ for r=R, nuclear force comes to play.

→ for r<R, nuclear forces dominates and we have attractive potential well.

At r=R:— $V_{max} = \frac{1}{4\pi\epsilon_0} \frac{(Z-2)e \times 2e}{R}$

$$U_{max} = \frac{1}{4\pi\epsilon_0} \frac{2(Z-2)e^2}{R}$$

ex:- for U^{238}

$U \simeq 30\,MeV$

$K \simeq 4\,MeV$

$\Rightarrow K << U$

- classically we have no chance to find α particles outside the nucleus.
- But quantum tunneling is possible and we have a small change to find it outside

$\Rightarrow$ Decay Probability :- λ = decay constant.

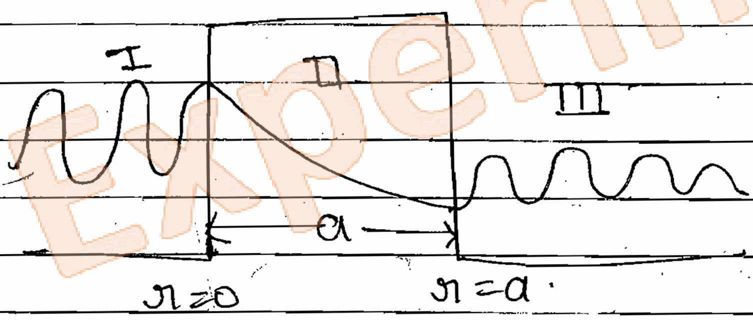
P = prob. of transition in each collision.

ν = collision frequency.

$T = \frac{2R}{v}$ $\quad v$ - velocity of α-particle.

$\nu = \frac{1}{T} = \frac{v}{2R}$

$$\boxed{\lambda = \nu P}$$



transition probability $\Rightarrow P = \frac{16 K_1^2 K_2^2}{(K_1^2 + K_2^2)} e^{-2K_2 a}$.

$K_1 = \sqrt{\frac{2mE}{\hbar^2}}$

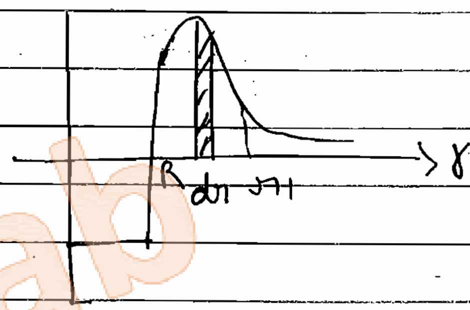
$K_2 = \sqrt{\frac{2m(V-E)}{\hbar^2}}$

for a thick barrier

$$K_2 a \gg 1.$$

then, $\boxed{P \simeq e^{-2K_2 a}}$

in our case the barrier height is not fixed but it varies with distance.



so, $P = e^{-2\int_R^{r_1} K_2 dr}$

imp $\boxed{P = e^{-G}}$

where $G = 2\int_R^{r_1} K_2 dr$.

Gammow factor.

imp. $G = 2\int_R^{r_1} \sqrt{\frac{2m(V-E)}{\hbar^2}}\, dr$

On simplification we get

$$\lambda = \nu P = \frac{v}{2R} \times e^{-G}$$

$$\log\lambda = \log\frac{v}{2R} - G.$$

$$\boxed{\log\lambda = \log\frac{v}{2R} + 2.97 Z_D^{1/2} R^{1/2} - 3.95 Z_D E_\alpha^{-1/2}}$$

$Z_D = (Z-2)$ - daughter atomic no.

this eqn. is of the form

$$\boxed{\log\lambda = C + D E^{-1/2}}$$

which another form of G-T law.

Limitations of Gammow - theory —

Gammow theory is a semiclassical theory which has certain limitations.

1. The theory is applicable only for ground state to ground state transition where the relative orbital angular momentum is $L = 0$
2. The internal nuclear structure is not taken into account
- It is only applicable for even- even nuclei not for odd-even, even-odd and odd-odd nuclei

Effects of various factors in α-decay :—

1. Effect of centrifugal Barrier : — Gammow theory is applicable to only ground state to ground state for decay even-z nuclei where α particle has no orbital angular momentum, but if the decay takes place from excited state $\rightarrow$ G.S.

G.S. $\rightarrow$ E.S.

E.S. $\rightarrow$ E.S.

Then, change in orbital angular momentum takes place and α particle is emitted with $l \neq 0$

then centrifugal potential $= \dfrac{l(l+1)\hbar^2}{2mr^2}$

$$\text{total } V = \underbrace{V(r)}_{\text{coloumbian energy}} + V_{centrifugal}$$

$\therefore$ the transition probability $T = e^{-G}$ will be different

$$T = e^{-2\sqrt{\frac{2m}{\hbar^2}}\int_R^r \left[\frac{2(Z-2)e^2}{4\pi\epsilon_0 r} + \frac{l(l+1)\hbar^2}{2mr^2}\right]^{1/2} dr}$$

for large z:- T is not very much effective
for small z:- T is affected by Centrifugal term.

(ii) Effect of Nuclear structure:- the decay constants for even-odd, odd-even and odd-odd nuclei are considerably smaller than ~~the class~~ those for even-z transition of equal energies. such transitions are called hindered transition.

21.11.2015 –

Q:– A nucleus Po–212 emits α particles with K.E. 10.54 MeV. Find the disintegration energy.

Sol:–
$$E_{dis} = K_\alpha \left(\frac{A}{A-4}\right)$$
$$= 10.54\left(\frac{212}{208}\right)$$

Ans
Q54:– Consider the α-decay reaction.
$$^{210}_{84}Po \rightarrow {}^{206}_{82}Pb + {}^{4}_{2}He$$
$M(Po) = 210.0483$ amu.
$M(Pb) = 206.0386$ amu
$M(^4He) = 4.0039$ amu.
~~M~~ 1 amu = 931.141 meV.
K.E. of α particle is.
(a) 5.4 mev (b) 2.7 MeV
(c) 5.2 MeV (d) 10.8 mev.

Sol:–
$$E_{dis} = Q_\alpha = [\text{initial mass} - \text{final mass}]$$
$$= [210.0483 - 206.0386 - 4.0093]\text{ amu}.$$
$$= .0058\text{ amu}$$
$$= .0058 \times 931 \times 1.41\text{ MeV}.$$
$$= 5.4\text{ mev}.$$
$$E_{dis} = K_\alpha\left(\frac{A}{A-4}\right)$$
$$K_\alpha = E_{dis}\left(\frac{A-4}{A}\right) = 5.4 \times \left(\frac{206}{210}\right)$$
$$= 5.2\text{ MeV}.$$

- if Q value is +ve → unstable.
- if Q value is −ve → stable.

Q:- $^{236}_{94}Pu$ is unstable against α-decay.

Given that
$M(^{236}Pu) = 236.04607$ amu.
$M(^{232}U) = 232.03717$ amu.
$m(\alpha) = 4.0026$ amu.

Sol:- if Q > 0 then α decay will occur.
Q = (initial mass − final mass).
=

Q:- α particles emitted from radioactive nuclei
(i) have definite energy.
(ii) have continuous energy from 0 to ∞.
(iii) have continuous energy from E_{min} to E_{max}, $E_{min} \neq 0$,
$E_{max} \neq \infty$.
(4) several discrete energies.

Sol:- α particles → have discrete energies.

Q:- if 5 MeV α as well as β particles are incident on a potential barrier of 10 MeV height and 2 fm width then the transition probability
(a) higher for α particles.
(b) higher for β particles
(c) equal for α + β particles.
(d) zero for both.

to convert atomic mass into nuclear mass :— $(M - Zm_e)$

• $\boxed{m_\alpha > m_\beta}$

• in weak interaction, no need to conserve parity, β-decay is a weak interaction process.

Sol:- $T \propto e^{-2k_2 a}$

$V = 10 MeV$

$a = 2 fm$

$k_2 = \sqrt{\frac{2M(V-E)}{\hbar^2}}$

since $m_\alpha > m_\beta$

$\Rightarrow \boxed{T_\alpha < T_\beta}$

Beta - Decay

→ β decay occurs to modify $\frac{n}{p}$ ratio. three modes of β decay are

1. e^- emission (β^- decay)

$$n \rightarrow p + e^-$$

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^-$$

2. e^+ emission (β^+ decay)

$$p \rightarrow n + e^+$$

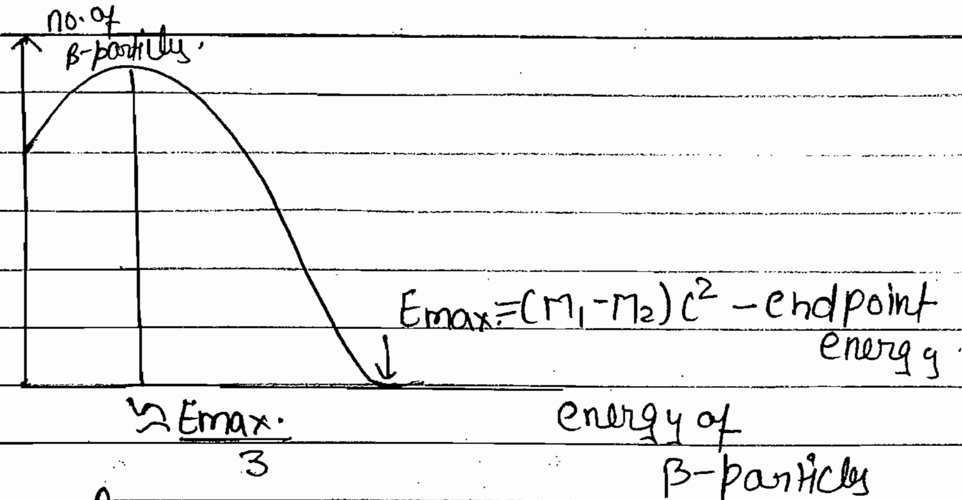
$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}_{1}e^0$$

3. electron capture

$$p + e^- \rightarrow n$$

$$^{A}_{Z}X + {}_{-1}e^0 \rightarrow {}^{A}_{Z-1}Y$$

Continuous β spectrum :—



$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^- + Q$$

M_1 = atomic mass of parent.

M_2 = atomic mass of ~~Parent~~ daughter

m_e

$$Q = [\text{initial mass} - \text{final mass}]c^2$$

$$= [(M_1 - Zm_e) - (M_2 - (Z+1)m_e) - m_e]c^2$$

$(M_1 - Zm_e)$ → nuclear mass of parent

$(M_2 - (Z+1)m_e)$ → nuclear mass of daughter.

$$Q = [M_1 - M_2]c^2 = E_{max}.$$

→ All β particles must be emitted with this energy.

⇔ Difficulties: — 1. Energy conservation problems — the emitted β-particles have energy from 0 to E_{max}. most of the β particles have energy $\simeq \frac{E_{max}}{3}$. therefore, the Question is what happens to remaining energy?

2. Angular momentum Conservation problem:—

$$n \rightarrow p + e^-$$

$$\tfrac{1}{2} \qquad \tfrac{1}{2}\uparrow \ \tfrac{1}{2}\uparrow \qquad S = 1$$

$$\tfrac{1}{2}\downarrow \ \tfrac{1}{2}\uparrow \qquad S = 0$$

⇒ angular momentum is not Conserved.

3. Linear momentum conservation problem:—

$P_i = 0$ (at rest).

so

P_f should be $= 0$.

⇒ $p_d - p_\beta$ and in opposite direction.

but observations show that they are not emitted exactly in opposite direction.

⇒ linear momentum is not conserve.

⇒ Solution to the problems:— Pauli suggested that a chargeless, massless and spin $S = \frac{1}{2}$ particle must be emitted with β particle to remove above difficulties.

⇓

Particle is neutrino

therefore the modified reaction will be

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e \longrightarrow \beta^- \text{ decay}$$

$(n \rightarrow p + e^- + \bar{\nu}_e)$.

Transition Probability: —

- $W = \frac{2\pi}{\hbar} |M_{if}|^2 \rho(E)$ → density of β-particles per unit volume.
- $\rho = \frac{dN}{dE_\beta}$

$(_{-1}e^0 = e^-)$
$(_{+1}e^0 = e^+)$

$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \nu_e$ → β^+ decay.
$(p \rightarrow n + e^+ + \nu_e)$

H_{if} → ~~perturbed term~~ perturbation Hamiltonian.

$^{A}_{Z}X + {}_{-1}e^0 \rightarrow {}^{A}_{Z-1}Y + \nu_e$ → e^- capture.
$[p + \bar{e} \rightarrow n + \nu_e]$.

Here $E_{max} = E_\beta + E_\nu$
if $E_\beta = 0 \Rightarrow E_\nu = E_{max}$
if $E_\beta = E_{max} \Rightarrow E_\nu = 0$.
[the energy is shared between β-particles and neutrino]

Fermi theory of β-decay —

Fermi used the time dependent perturbation theory to calculate the transition prob. from initial state i to final state f.

$\boxed{H_{if} = gM}$ g – weak interaction factor.
$M - \sum_{n=0}^{\infty} M_{if}^{(n)}$

Fermi Golden Rule: —

$$W = \frac{2\pi}{\hbar^2} |H_{if}|^2 \rho(E) \quad \text{--- (1)}$$

W → transition probability per unit time.

where H_{if} = Interaction Hamiltonian causing the transition from i to f state.
Exact form of it is not known.
It can be written as:

$$H_{if} = gM \quad \text{--- (2)}$$

M → matrix element operator.
g → fermi coupling constant $= 0.9 \times 10^{-4}$ MeV/m^3

$$M = \sum_{n=0}^{\infty} M_{if}^{(n)} \quad \text{----} \; (\#)$$

$$\text{So, } M_{if} = g \sum_{n=0}^{\infty} M_{if}^{(n)} \quad \text{----} \; (3)$$

where $M_{if}^{(n)} = \frac{(-i)^n}{n!} \vec{K}^n \langle \psi_f | \vec{r}^n | \psi_i \rangle$

ψ_f – final wave function.
ψ_i – initial wave function.

$$M_{if}^{(n)} = \frac{(-i)^n}{n!} \vec{K}^n \langle \psi_f | \vec{r}^n | \psi_i \rangle$$

$$M_{if}^{(0)} = \langle \psi_f | \psi_i \rangle$$

$$M_{if}^{(1)} = (-i) \vec{K} \langle \psi_f | \vec{r} | \psi_i \rangle$$

$$M_{if}^{(2)} = \frac{(-1)}{2!} \vec{K}^2 \langle \psi_f | \vec{r}^2 | \psi_i \rangle$$

etc.

=> <u>Density of states $\rho(E)$</u>:–

$\rho(E)$ = density of states per unit volume.

$$= \frac{dN}{dE_\beta}$$

$$= \frac{dN_\beta}{dE_\beta} \times dN_\nu$$

$$dN_\beta = \frac{4\pi P_\beta^2 dP_\beta}{h^3}$$

$$dN_\nu = \frac{4\pi P_\nu^2 dP_\nu}{h^3}$$

$$\rho(E) = \frac{16\pi^2}{h^6} p_\beta^2 p_\nu^2 \, dp_\beta \, dp_\nu$$

$$E_{max.} = E_\beta + E_\nu \Rightarrow dE_\beta = -dE_\nu$$

$$\boxed{E^2 = p^2c^2 + m^2c^4}$$

$$E_\nu = p_\nu c \qquad \boxed{E_\nu = p_\nu c}$$

$$\boxed{p_\nu = \frac{E_\nu}{c}}$$

learn.

$$\boxed{\rho(E) = \frac{16\pi^2}{h^6} p_\beta^2 (E_{max.} - E_\beta)^2 dp_\beta \times \frac{1}{c}}$$

So the transition probability from i to f in the Range p_β to $p_\beta + dp_\beta$ is

learn

$$\boxed{dw_{if} = \frac{g^2}{2\pi^3 c^3 \hbar^7} |M_{if}^{(n)}|^2 (E_{max.} - E_\beta)^2 p_\beta^2 dp_\beta}$$

Coulomb Correction:— in the derivation of above expression we have not considered the interaction b/w the e^- (or positron) and the +ive charge on the nucleus. if this is also taken into account then the corrected transition probability becomes.

imp.

$$\boxed{dw_{if} = \frac{g^2}{2\pi^3 c^3 \hbar^7} |M_{if}^{(n)}|^2 F(Z, E_\beta)(E_{max.} - E_\beta)^2 p_\beta^2 dp_\beta}$$

$F(Z, E_\beta)$ — fermi function or coulomb factor.
(depend upon the charge and energy of emitted particle).

$$F(Z, E_\beta) = \frac{2\pi\eta}{1 - e^{-2\pi\eta}}$$

$$\eta = \frac{ze^2}{4\pi\epsilon_0 \hbar v} \text{ for } e^-$$

$$= \frac{-ze^2}{4\pi\epsilon_0 \hbar v} \text{ for } e^+$$

Discussion: —

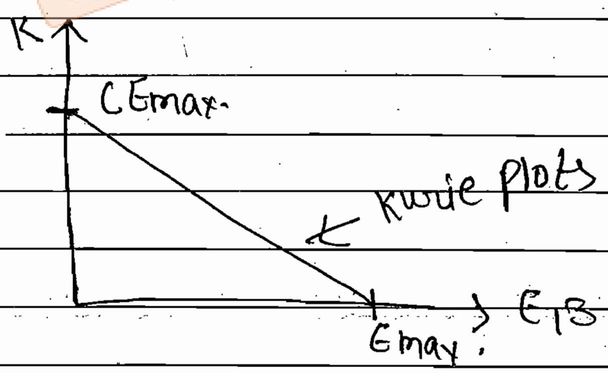
1. Kurie Plots —

$$dw_{if} = \underbrace{\frac{g^2}{2\pi^3 c^3 \hbar^7}}_{\text{constant}} |M_{if}|^2 F(Z, E_\beta)(E_{max.} - E_\beta)^2 P_\beta^2 \, dP_\beta$$

$$dw_{if} = \text{constant} \cdot F(Z, E_\beta)(E_{max.} - E_\beta)^2 P_\beta^2 \, dP_\beta$$

$$\Rightarrow \underbrace{\left[\frac{dw_{if}}{F(Z, E_\beta) P_\beta \, dP_\beta}\right]^{1/2}}_{\text{Kurie function } K} = \underbrace{C}_{\text{constant}} (E_{max.} - E_\beta)$$

$K = C(E_{max} - E_\beta)$: —

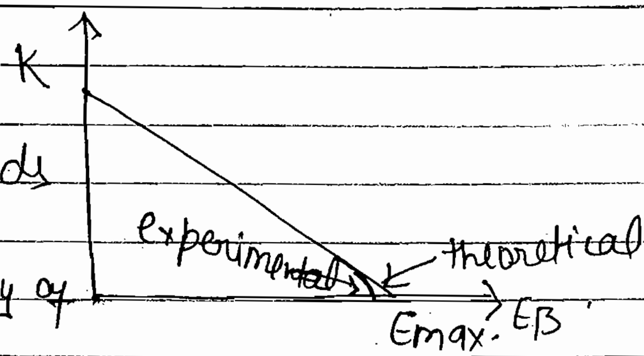


$$K = C(E_{max.} - E_\beta)$$

- thus we get a straight line of -ve slope for K vs E_β graph. this is called kurie Plot

- Experimental curve is slightly different to the predicted one at higher energy.

⇒ Neutrino mass :—



→ the Kurie curve bends towards the energy axis at higher energy of β particle.

→ this deviation can be explained by considering the finite value of neutrino mass.

for $m_{\bar{\nu}} \neq 0$

$$dw_{if} = \frac{g^2}{2\pi^3 c^3 \hbar^7} |M_{if}^{(n)}|^2 \{F(Z,E_\beta)(E_{max}-E_\beta)(E_{max}-E_\beta)^2 + m_{\bar{\nu}}^2 c^4 p_\beta^2 \, dp_\beta$$

if we plot Kurie curve with this eqn. then the agreement b/w theoretical and experimental values is better at higher energy.

→ Experiments have been performed for the estimation of neutrino mass. Initial experiments on the β-decay of $^{1}_{3}H$

$$^{1}_{3}H \longrightarrow {}^{3}_{2}He + e^- + \bar{\nu}_e$$ indicate that $m_{\bar{\nu}} < 250$ eV

some recent experiment, suggest that $m_{\bar{\nu}}$ is around 50-60 eV.

Since $m_e = 0.511$ MeV ≃ 511000 eV. ∴ $m_e \gg m_{\bar{\nu}}$ therefore for practical purposes, the neutrino mass is taken to be zero.

3. Decay Constant (λ) :— $\lambda = \frac{.693}{T_{1/2}}$ $T_{1/2}$ - Half life.

$\lambda = \frac{1}{\tau}$ τ - mean life.

$$\lambda = \int_0^{P_{max}} dw_{if} = \frac{g^2}{2\pi^3 c^3 \hbar^7} |M_{if}^{(n)}|^2 \int_0^{P_{max}} F(Z, E_\beta)(E_{max} - E_\beta)^2 P_\beta^2 \, dP_\beta$$

for large Z and E_β

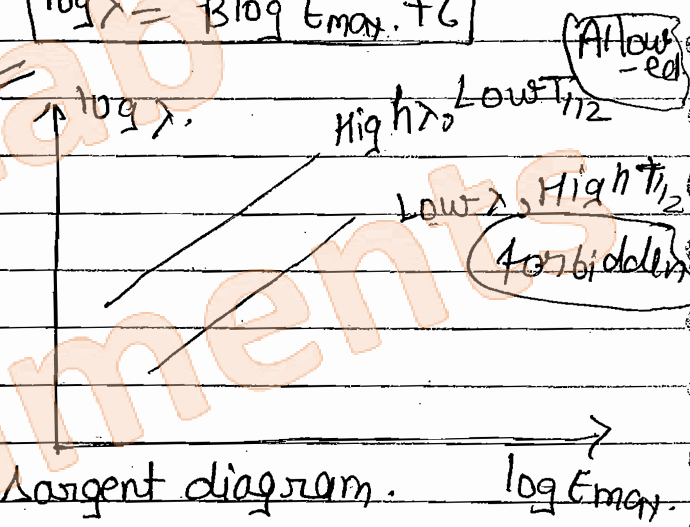
$$F(Z, E_\beta) \simeq 1.$$

On simplification:

$$\boxed{\lambda = \underset{A}{\text{Constant}}\, E_{max}^5} \rightarrow \text{Sargent law.}$$

$$\boxed{\log \lambda = \beta \log E_{max} + C}$$

$$\boxed{\log \lambda = \log A + 5 \log E_{max}} =$$



Sargent diagram.

Selection Rules in β decay:-

Transitions

Allowed	Forbidden.
when e^- and $\bar{\nu}_e$ are emitted with relative orbital angular momentum $l=0$.	when $l \neq 0$ $l=1$ — Ist forbidden $l=2$ — IInd forbidden $l=3$ — IIIrd forbidden $l=4$

Fermi G.T.

$\Delta I = 0$ ... $\Delta S = 1$.

Allowed

Fermi transition | Gammow-teller transitions:

$\Downarrow$ | $\Downarrow$

$\uparrow \downarrow \quad S = 0$ | $\uparrow\uparrow \quad S = 1$.

e^- $\bar{\nu}_e$

e^- and $\bar{\nu}_e$ are emitted with opposite spins | e^- and $\bar{\nu}_e$ are emitted with $\parallel$ spins.

allowed transition is nearly 100 times more probable than the 1st forbidden transition.

similarly Ist forbidden transition is 100 times more probable than IInd forbidden and so on.

$\Rightarrow$ Allowed transitions: — $l=0$ — S wave emissions of e^- and $\bar{\nu}_e$

$${}^{3}_{1}H \longrightarrow {}^{3}_{2}He + e^- + \bar{\nu}_e$$

${}^{3}_{1}H$ $\downarrow$ due to odd proton $(1s_{1/2})^1$

${}^{3}_{2}He$ $\downarrow$ due to odd neutron $(1s_{1/2})^1$

$$\frac{1}{2}^+ \longrightarrow \frac{1}{2}^+ + e^- + \bar{\nu}_e$$

$\uparrow \downarrow$ $l=0$, $S=0$ $\parallel$ $I = 0$ fermi

$1/2$ $\uparrow \uparrow$ $l=0$, $S=1$ $\parallel$ $I = 1$

$\frac{1}{2}, \frac{3}{2}$

$P = (-1)^{l}$

• If Parity is not change then,
$l = 0, 2, 4, ---$

⇒ above transition occurs by both fermi and G.T. transitions.

Ex: $^{6}_{2}He \longrightarrow {}^{6}_{3}Li + e^{-} + \bar{\nu}_e$

$(J_i = 0^+)$ $(J_f = 0^+)$ → observed value.

no parity change.
$\Delta J = 1$.

It is purely Gammow teller transition allowed transition.

$0^+ \longrightarrow 1^+ + e^- + \bar{\nu}_e$

$\uparrow\uparrow$ $S=0$, $l=0$ | $J=0$ F.

✗ $J=1$

→ Allowed by G.T.

$\uparrow\uparrow$ $S=1$, $l=0$ | G.T

✓ $J = 0, 1, 2$. $J=1$

Ex: $^{14}_{8}O \longrightarrow {}^{14}_{7}N^{*} + e^{+} + \nu_e$ N^{*} – excited state.

no parity change

$\Delta J = 0$

$\uparrow\uparrow$ $S=0$, $l=0$ | $J=0$. Fermi

✓ $J=0$

$S=1$, $l=0$ | $J=1$ G.T.

✗ $J=1$

Purely-fermi transition.

· If $I_i \neq I_f \neq K$, then allowed by G.T.

· If $I_i = I_f = 0$, then fermi.

· If $I_i = I_f = K$, then allowed by both (fermi & G.T.)

General Rule:— for allowed transition

$l = 0$ no parity change.

$\Delta I = 0$, $\Delta s = 0$ — fermi

$\Delta I = 0, \pm 1$ $\Delta s = 1$ — G.T.

Ex:— other examples:—

$${}^{35}_{16}S \rightarrow {}^{35}_{17}Cl + e^- + \bar{\nu}_e$$

$$\downarrow$$

$$\left(\frac{3}{2}\right)^+ \rightarrow \left(\frac{3}{2}\right)^+ + e^- + \bar{\nu}_e$$

${}^{35}_{16}S$ $p = 16$

$n = 19$

$(1s_{1/2})^2(1p_{3/2})^4(1p_{1/2})^2$

$(1d_{5/2})^6(2s_{1/2})^2$

$(1d_{3/2})^3$.

$\Delta I = 0$, no parity change.

allowed by both fermi and G.T. transitions.

Gate 2013

Q:— In the β decay process the transitions $2^+ \rightarrow 3^+$ is

(a) allowed by both fermi and G.T.

(b) allowed by fermi but not G.T.

(c) not allowed by fermi but allowed by G.T.

(d) not allowed by both fermi transition.

Sol:— $2^+ \rightarrow 3^+ + e^- + \bar{\nu}_e$

no parity change.

$\Delta I = \cancel{0}\ 1$.

$\Rightarrow$ allowed by G.T.

but not by fermi.

for allowed transition

· $l = 0$

· no parity change

for fermi $\Delta S = 0$

for G.T.— $\Delta S = 1$.

for fermi $\rightarrow$ $\Delta I = 0$

for G.T. $\rightarrow$ $\Delta I = 0, \pm 1$.

in the allowed transition, $\Delta l = 0$ {both in same state}

but in forbidden transition, $\Delta l \neq 0$ {both are not in same state}.

⇒ forbidden Transition :— ($l \neq 0$).

• 1st forbidden —

$l = 1$ e^- and $\bar{\nu}_e$ emitted with $l = 1$

Parity change occurs (P wave emission).

Selection Rule :—

Fermi — $\Delta I = 0, \pm 1$, $\boxed{\Delta S = 0}$

parity change — yes. $0 \nleftrightarrow 0$.

G.T.

$\underbrace{\Delta I = 0, \pm 1, \pm 2,}$ $\boxed{\Delta S = 1}$ $0 \nleftrightarrow 0$ $0 \nrightarrow 1$

Parity change — yes. $\frac{1}{2} \nleftrightarrow \frac{1}{2}$

Ex $^{37}_{16}S \longrightarrow ^{37}_{17}Cl + e^- + \bar{\nu}_e$

$(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^6 (2s_{1/2})^2 (1d_{3/2})^4 (1f_{7/2})^1$

$(1f_{7/2})^1$ $(1d_{3/2})^1$

$l = 3$ $l = 2$

$\frac{7}{2}^- \longrightarrow \frac{3}{2}^+$

Parity change — yes.

$\Delta l = 1$.

so e^- and $\bar{\nu}_e$ will be emitted with $l = 1$.

$\boxed{\Delta I = 2}$

⇒ purely G.T. 1st forbidden transition.

$\frac{3}{2} - 2$ $\quad \frac{1}{2} - \frac{3}{2} \leftarrow \cancel{\frac{-1}{2}}$ $\frac{1}{2}$, $\cancel{\frac{3}{2}+2 \leftarrow \frac{5}{2}}$

check → $\frac{7}{2} \rightarrow \frac{3}{2}$.

fermi ✗ $l=1$, $s=0$, $J=1$

G.T. $l=1$, $S=1$, $0,1,2$. $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$

Ex:— $Kr^{85} \rightarrow Rb^{85} + e^- + \bar{\nu}_e$

$\frac{9}{2}^+$ $\quad \frac{5}{2}^-$ $\quad l=1$, $S=1$) $0,1,2$.

Parity change - yes.

$\Delta J = 2$

Purely G.T. 1st forbidden transition.

IInd forbidden transition:—

$l=2$.

Parity change - no.

fermi

$\Delta J = \pm 1, \pm 2$ $\quad (0 \nleftrightarrow 1)$.

G.T.

$\Delta J = \pm 2, \pm 3$ $\quad (0 \nleftrightarrow 2)$.

ex:— $^{135}Cs \rightarrow {}^{135}Ba + e^- + \bar{\nu}_e$

$\frac{7}{2}^+$ $\quad \frac{3}{2}^+$ $\quad l=2$, $s=0$ | $J=2$ fermi

$\Delta J = 2$ $\quad \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$.

2nd forbidden by both fermi and G.T.

$l=2$, $S=1$ | $J=1,2,3$.

$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$

$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$ $\quad \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$

$\Delta J = 3$

Ex:— ${}^{10}_{4}Be \rightarrow {}^{10}_{5}B + e^- + \bar{\nu}_e$

0^+ $\quad$ 3^+

no parity change.

$\Delta J = 3$

$\Rightarrow$ Purely G.T.

IInd forbidden.

$l = 2$, $S = 0$ | $J = 2$

1, 2, 3, 4, 5

$l = 2$, $S = 1$ | 1, 2, 3

Summary:—

General rule:— $l = 0$ allowed

$l = 1$ Ist forbidden.

$l = 2$ IInd forbidden.

etc. . . .

l = odd $\rightarrow$ parity change occur.

l = even $\rightarrow$ no parity change.

Fermi:—

$\Delta J \leq l$.

G.T.:— $\Delta J \leq l+1$.

Q:— In a β-decay if a $\frac{3}{2}^+$ nuclear state decays by G.T. transition, what are the possible spin-parity states of final nucleus?

Sol:— $J_i = \frac{3}{2}^+$

$J_f = ?$

$\frac{3}{2}^+ \rightarrow J_f^{\,P_f} + e^- + \bar{\nu}_e$ (by G.T.)

for G.T. $l = 0$

$S = 0$.

possible values of I_f are
$$I_f = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+.$$

Q- In a β-decay if a $\frac{3}{2}^+$ nuclear state decays by Ist forbidden transition. then calculate the possible spin-parity states of final nucleus.

Sol:- $I_i = \frac{3}{2}^+$

$I_f = ?$

Ist forbidden. → $l = 1$.

⇒ ~~par~~ final state parity will be −1.

For fermi →

$$I_i^{0^+} \longrightarrow I_f + e^- + \bar{\nu}_e$$
$$\frac{3}{2}^+ \longrightarrow I_f + e^- + \bar{\nu}_e$$

$l = 1$
$s = 0$ | $J = 1$.

Possible values for I :→ $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$.

$$I^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$$

G.T. $\frac{3}{2}^+ \longrightarrow I_f + e^- + \bar{\nu}_e$

$l = 1$
$s = 1$ | $0, 1, 2$.

$I^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-$

Possible values of I are.
$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$.

~~$\frac{7}{2}+1 = \frac{7}{2}+\frac{9}{2}$~~ ~~$\frac{7}{2} - \frac{7}{2}$~~

Ans:– $\frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{5}{2}^-$, $\frac{7}{2}^-$

Q:– for given transition, state whether they are allowed or forbidden. name the transition in each case.

1. ${}^{41}_{21}Sc\left(\frac{7}{2}^-\right) \rightarrow {}^{41}_{20}Ca\left(\frac{7}{2}^-\right)$ (i) parity is not change, $\Delta J = 0$ ⇒ allowed by both fermi and G.T.

2. ${}^{87}_{37}Rb\left(\frac{3}{2}^-\right) \longrightarrow {}^{87}_{38}Sr\left(\frac{7}{2}^-\right)$ (ii) no parity change, $\Delta J = 2$ and forbidden by G.T.

3. ${}^{14}_{8}O\ (0^+) \longrightarrow {}^{14}_{7}N^*\ (0^+)$ (iii) no parity, $\Delta J = 0$. allowed by purely fermi transition.

4. ${}^{14}_{6}C\,(0^+) \longrightarrow {}^{14}_{7}N\ (1^+)$ (iv) no parity change, $\Delta J = 1$. allowed by fermi.

another classification of allowed transition:–

Allowed.

Superallowed (favoured) ↓ {no change in l value}.

the nucleon which change its sign remain in the same sublevel.

$p_{3/2} \rightarrow p_{3/2}$

$d_{5/2} \rightarrow d_{5/2}$

etc.

normall allowed (unfavoured). ↓ {no change in l value}.

the nucleon which changes its sign changes its sublevel

$p_{1/2} \rightarrow p_{3/2}$

$p_{3/2} \rightarrow p_{1/2}$

$d_{5/2} \rightarrow d_{3/2}$.

⇒ Comparative Half life and transitions –

fT or $fT_{1/2}$ – Comparative Half life.

$\log_{10} fT$ Range.	– transition.
2.7 – 3.7	super allowed.
4 – 5	normal allowed.
6 – 10	Ist forbidden.
10 – 14	IInd forbidden.
14 – 17	IIIrd forbidden.
	etc.

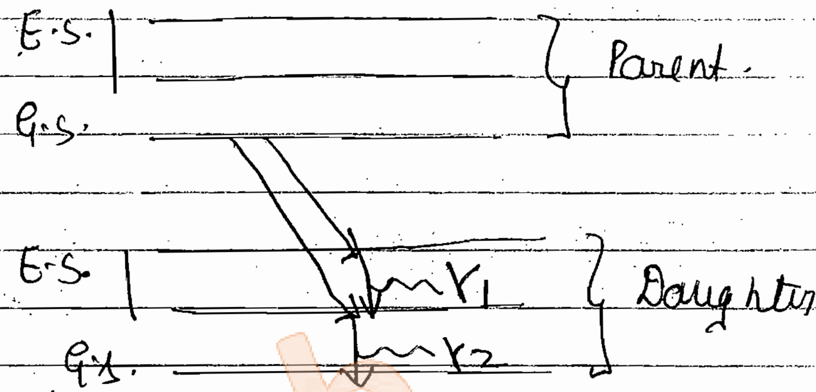
$$fT_{1/2} = \frac{2\pi^3 \hbar^7}{m e^5 g^2 c^4} \frac{\log_e 2}{|M_{if}|^2}$$

$$= \frac{0.693\tau_0}{|M_{if}|^2} \quad \text{– universal time constant.}$$

γ-Decay

→ after α or β decay, if the daughter nucleus is left in the excited state, then the surplus energy is emitted in the form of γ rays.

- The γ-ray spectra is a line spectra



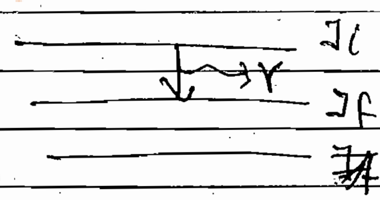
- The γ-rays are of two types.

  - electric multipole
  - magnetic multipole.

- γ ray spectra provides us the ways to measure the spin and parity of nuclear states.

⇒ Multipole Radiation:— L = multipolarity

$L = |J_i - J_f|$ to $(J_i + J_f)$



$L = 0, 1, 2, 3, \ldots$

$L = 0$ — no radiation due to transverse nature of EM waves.

Order of the pole $= 2^L$.

$L = 1 \longrightarrow 2^1 = 2 \rightarrow$ dipole.

$L = 2 \longrightarrow 2^2 = 4 \rightarrow$ quadrupole.

$L = 3 \longrightarrow 2^3 = 8 \rightarrow$ octupole.

$E(L)$ — E_L — electric multipole.

$M(L)$ — M_L — magnetic multipole.

E_1 – electric dipole.
M_1 – magnetic dipole.
E_2 – electric Quadrupole.
M_2 – magnetic Quadruple.
et.

• E_1 and M_1 & M_2 and E_2 can't occur simultaneously.
E_1 and $\cancel{E_2}$, E_2 and M_1 can occur with different transition probability.

⇒ transition probability:–

for electric multipole:–

$$\lambda(E_L) = \frac{(2L+1)}{\hbar L[(2L+1)!!]^2}\left(\frac{\omega}{c}\right)^{2L+1} Q_L^2$$

$$(2L+1)!! = (2L+1)(2L-1)(2L-3)\ldots 2 \text{ or } 1$$

(2 or 1) depend upon whether L is even or odd.

L – multipolarity.
ω – freq.
Q_L – closely related to electric multipole moment.

for magnetic multipole:–

$$\lambda(M_L) = \frac{2(L+1)}{\hbar L[(2L+1)!!]^2}\frac{1}{c^2}\left(\frac{\omega}{c}\right)^{2L+1} Q_M^2$$

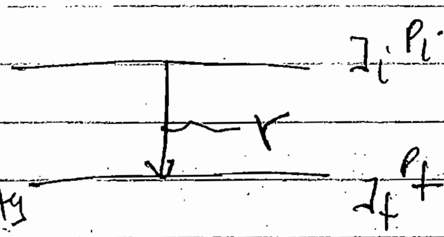
Q_M – closely related to the magnetic multipole moment.

• as multipolarity increases, transition probability decreases.

⇒ Selection Rules in γ decay :—

γ decay Occurs via EM interaction so angular momentum and parity remains conserved.

Parity conservation



$P_i = P_\gamma P_f \rightarrow$ final state parity

P_i → initial state parity ; P_γ → γ-photon parity

⇒ ~~$P_i = (-1)^L P_f$ for electric multipole~~
~~$= (-1)^{L+1} P_\gamma$ for magnetic multipole.~~

Or.

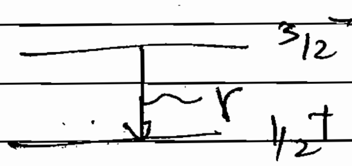
$P_i = (-1)^L P_f$ for electric multipole.

$P_i = (-1)^{L+1} P_f$ for magnetic multipole.

for electric multipole :— $P = (-1)^L$
for magnetic multipole :— $P = (-1)^{L+1}$

Q:— find allowed multipole transitions in $\frac{3}{2}^- \rightarrow \frac{1}{2}^+$

Sol:— multipolarity



$L = |J_i - J_f|$ to $J_i + J_f$.

$\therefore \boxed{L = 1, 2}$

Expected multipoles are E_1, E_2, M_1 and M_2.

$$P_i = P_\gamma P_f$$
$$(-1) = P_\gamma (+1)$$
$$\Rightarrow \boxed{P_\gamma = -1}$$

only multipoles which have -ve polarity will exist.

multipole	parity change
E_1	-1 ✓
E_2	$+1$ ✗
M_1	$+1$ ✗
M_2	-1 ✓

allowed multipoles are E_1 and M_2.

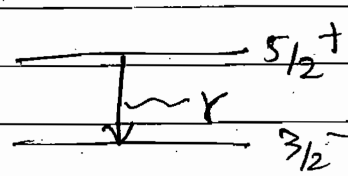
Q:- what are the possible multipoles in the transition.

$$d_{5/2} \rightarrow p_{3/2}.$$

Sol:-

$$d_{5/2} \qquad p_{3/2}.$$
$$\downarrow \qquad\qquad \downarrow$$
$$J^P = \frac{5}{2}^+ \qquad J^P = \frac{3}{2}^-$$

$$\frac{5}{2}^+ \rightarrow \frac{3}{2}^-$$



Multipolarity -

$$L = \left|\frac{5}{2} - \frac{3}{2}\right| \text{ to } \left|\frac{5}{2} + \frac{3}{2}\right|$$

$$L = 1, 2, 3, 4.$$

Expected multipoles :— $E_1, M_1, E_2, M_2, E_3, M_3, E_4, M_4$.

Parity selection Rule :— $P_i = P_\gamma P_f$.

$+1 = P_\gamma (-1)$

$\boxed{P_\gamma = -1}$

multipole	Parity change
E_1	-1 ✓
M_1	$+1$ ✗
E_2	$+1$ ✗
M_2	-1 ✓
E_3	-1 ✓
M_3	$+1$ ✗
E_4	$+1$ ✗
M_4	-1 ✓

for allowed multipoles E_1, M_2, E_3, M_4.

note:— 1. the probability of multipole transition decreases with increase in multipolarity ~~with~~ 'L'.

2. for the same value of 'L', electric multipoles ~~are~~ are ~~most~~ more probable than magnetic probability.

i.e.

$\lambda(E_L) > \lambda(M_L)$

E_1 ~~> M~~, most allowed

after that M_1

then E_2

etc.

3. the weisskopf results for transition prob. are

$\Gamma = \hbar \times$ transition prob. λ

$\Gamma_\gamma(E_1) = 0.068 A^{2/3} E_\gamma^3$

(ii) $\Gamma_\gamma(M_1) = 0.021\, E_\gamma^3$

(iii) $\Gamma_\gamma(E_2) = 4.9\times10^{-8} A^{4/3} E_\gamma^5$

(iv) $\Gamma_\gamma(M_2) = 1.5\times10^{-8} A^{2/3} E_\gamma^5$

E_γ - transition energy.

A → mass no. of decay nucleus.

⇒ **Internal Conversion** –

$$\alpha = \frac{N_e}{N_\gamma}$$

internal → conversion coefficient.

$0^+ \longrightarrow 0^+$

$L = 0, \quad N_\gamma = 0$

$\alpha = \infty$

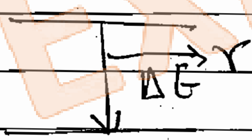
• it is a single step process.

• γ and e^- exist mutually

• $N = N_\gamma + N_e$

N_γ → no. of photon emit by γ-decay.

N_e → no. of photon emit by internal conversion.



$$\bullet \quad \alpha = \frac{N_e}{N_\gamma}$$

$$= \frac{(N_e)_K + (N_e)_L + (N_e)_M}{N_\gamma}$$

$$= \alpha_K + \alpha_L + \alpha_M.$$

⇒ Neutrino Detection (Ghost Particle):—

• Rains and Cowan exp:

inverse β-decay:

β-decay →

$n \rightarrow p + e^- + \bar{\nu}_e$

$p \rightarrow n + e^+ + \nu_e$

$\left.\begin{array}{l} p + \bar{\nu}_e \longrightarrow n + e^+ \\ n + \nu_e \longrightarrow p + e^- \end{array}\right\}$ inverse β-decay.

annihilation process:— $e^- + e^+ \longrightarrow 2\gamma$.

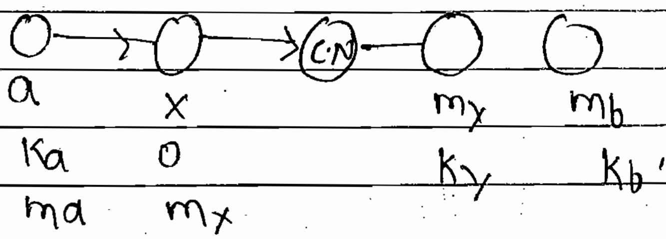
22.11.2015

Nuclear Reactions.

General form: → $a + X \longrightarrow Y + b$

$X(a,b)Y$

Q = final K.E. − Initial K.E.



• magnetic moment and quadrupole moment do not conserve in nuclear reaction.

$$E_{th} = -Q\left[1 + \frac{m_i}{m_t}\right]$$

Reaction Mechanism

Compound nucleus Reactions.

0–15 MeV

$a + X \longrightarrow C^*$ — Ist

$C^* \longrightarrow Y + b$ — II

two step reaction.

Direct Reactions.

15–50 MeV.

• $\tau \sim 10^{-23}$ sec.

• One step reaction.

Pickup   Stripping.

Assig

Q12:— Q value is always +ive.

Q13:— It decays very fast. and it is unstable.

Q14:— fission due to large oscillations.

Q15:— $U^{235} \longrightarrow Q \simeq 200\,MeV.$

for 4.6×10^{10} fission/sec.

total energy in 4.6×10^{10} fission/sec.

$= 4.6\times10^{10}\times200\,MeV.$

$= 4.6\times200\times10^{10}\times1.6\times10^{-13}\ J/sec.$

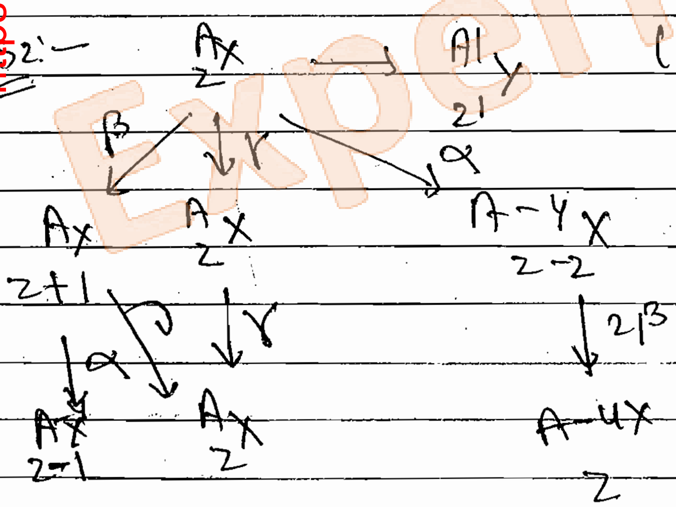
$= 014.72\times10^{-1}\ J/sec.$

$= 1.47$ watt.

$\simeq 1.5$ watt.

Q51:— C

Q52:— (b).



Q53:— $E_{dis.} = K_\alpha \left(\frac{A}{A-4}\right)$