Free Study Material from

All Lab Experiments

All Lab Experiments

Nuclear & Particle Physics Notes for NET/GATE Physical Sciences

Shell Model & other topics

SHELL MODEL:— • based on the analogy with atomic system.

- The primary objective of the shell model was to explain the magic no's.

Assumptions • nucleons move inside the nucleus in a potential field which arises due to interaction b/w the nucleons.

- both p and n have separate shells. they are filled at particular no.'s which are expected to be magic no's.
- nucleons have orbital and spin motions.
- the no. of nucleons in each shell is limited by pauli principle.

Mathematical analysis —

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{--- (1)}$$

$V \downarrow$ the potential field in which nucleus are moving.

$\downarrow$ exact form is not known

- the simplest central potentials which were tried are
  1. square well potential.
  2. Harmonic Oscillator potential.
- It has been observed that results are not very much shape sensitive.
- both potentials could not produce all magic no's (only limited 2, 8, 20 could be produced).
- therefore meyer, Jensen and Huxel (1949) added ~~and~~ a spin-orbit potential term to the central potential.

$$V(r) = \frac{1}{2} m\omega^2 r^2 \quad \text{--- (2)}$$

central Harmonic oscillator potential.

$V_{LS} = -\phi(r)(\vec{l}\cdot\vec{s})$. --- (3).

spin orbit potential.

Net potential:—

$V_T = V(r) + V_{LS}$.

$V_T = \frac{1}{2} m\omega^2 r^2 - \phi(r)(\vec{l}\cdot\vec{s})$ --- (4).

Spin-orbit potential.

$V_{LS} = -\phi(r)(\vec{l}\cdot\vec{s})$.

$\phi(r) = \frac{b}{r}\frac{\partial f}{\partial r}$. f(r) — spherically symmetric ~~potent~~ function.

- the strength of this term is nearly 20 times larger than atomic system.
- the sign of this term is opposite to that of atomic system.

putting eqn. (4) in eqn. (1) and solving we get.

$$\boxed{E = (2n + l - 1/2)\hbar\omega} \quad --- (5)$$

if we take $n = 1, 2, 3, ---$

$N = 2n + l - 2$, $l = 0, 1, 2 ---$

then,

$$\boxed{E = \left(N + \frac{3}{2}\right)\hbar\omega} \quad --- (6)$$

for 3-D harmonic oscillator.

Effect of spin-orbit term:— a level corresponding to l value splits up into two sub-levels.

$j = l + s = l + 1/2$ — lies lower
$j = l - s = l - 1/2$ — lies upper

} opposite to atomic case.

The splitting b/w two levels:

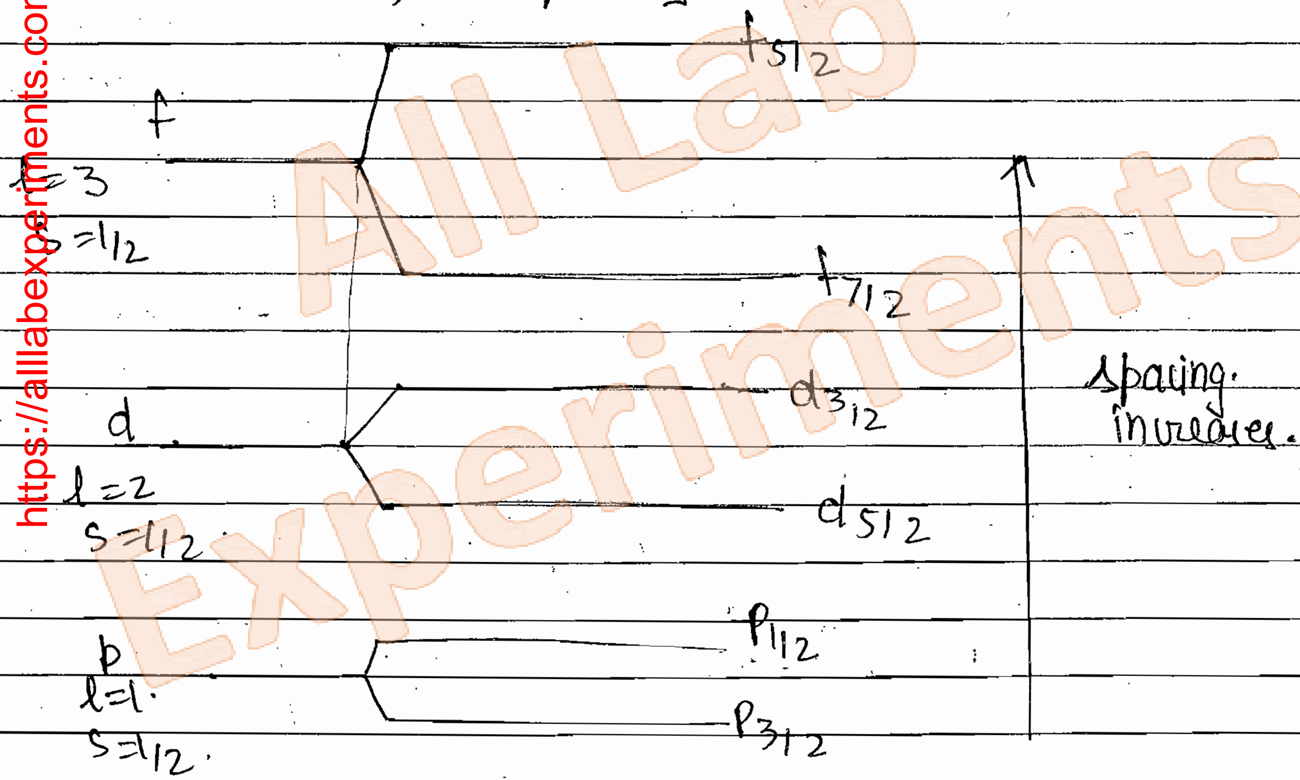
$$\Delta E = G(l+1/2) - G(l+1/2).$$

$$\boxed{\Delta E = \left(\frac{2l+1}{2}\right)\langle \phi(r)\rangle} \quad \text{---} (7)$$

$$\langle \phi(r)\rangle = \int \psi^* \phi(r)\psi \, d\tau.$$

ψ - nuclear wave function.

as l increases, the splitting b/w levels also increases.



2) Explanation of magic no.s :—

$$E = \left(N + \frac{3}{2}\right)\hbar\omega.$$

$$N = 2n + l - 2.$$

$n = 1, 2, 3, \ldots$

$l = 0, 1, 2, 3, \ldots$

$\cancel{2S+1=1}$
$\cancel{2\cdot\frac{1}{2}+1=1}$

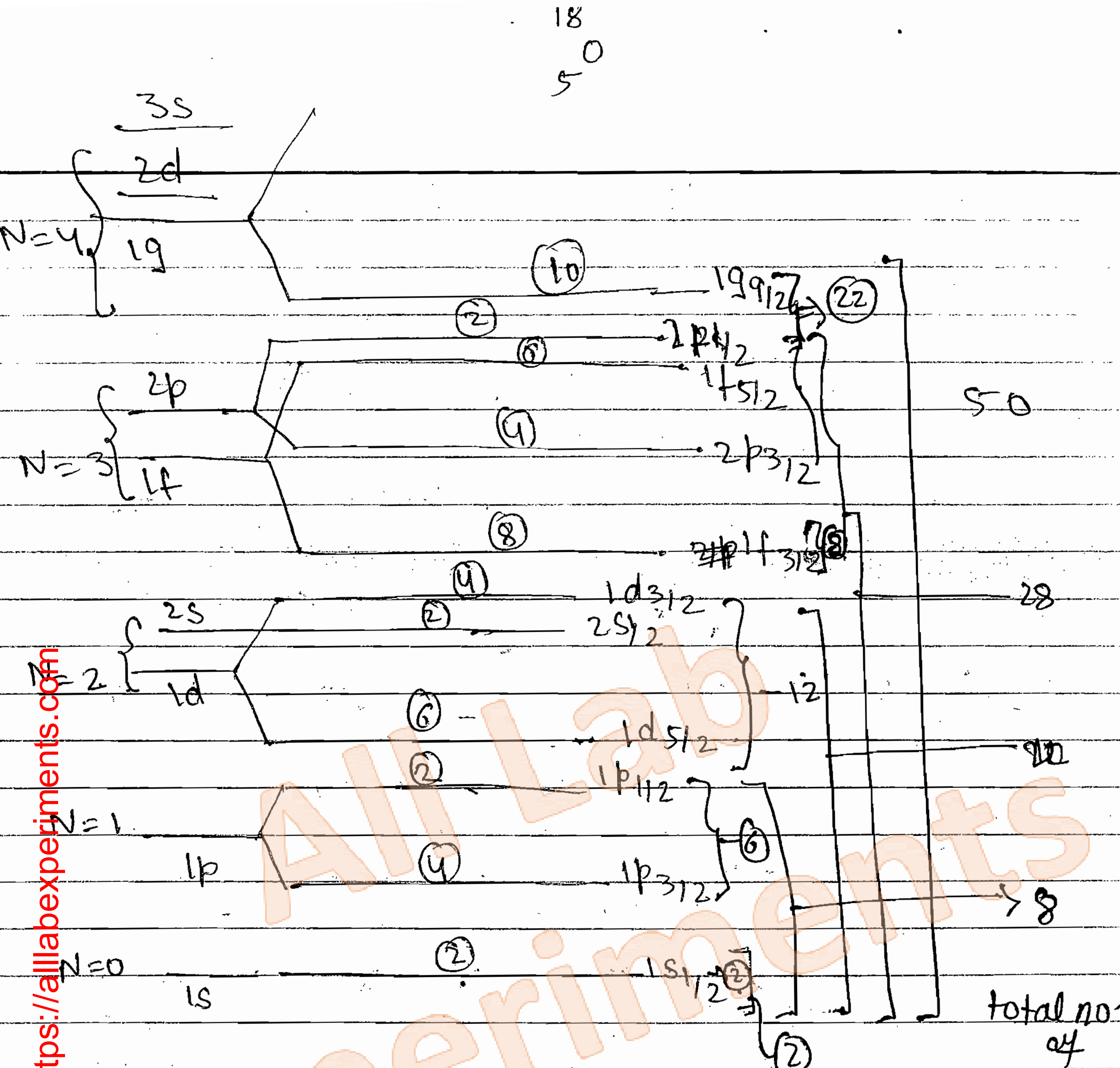
n - it is not the principal q. no. it just counts the no. of levels with that of l value.

1d - means the Ist d-level.

2d - means - IInd d level.

$N = 2n + l - 2$.

N	n	l, s	j	
• $N=0$	$n=1$	$l=0$, $s=1/2$	$j=1/2$	$1s_{1/2}$
• $N=1$	$n=1$	$l=1$, $s=1/2$	$j=\frac{1}{2}, \frac{3}{2}$	$1p_{1/2}, 1p_{3/2}$.
• $N=2$	$n=2$	$l=0$, $s=1/2$	$j=\frac{1}{2}$	$2s_{\frac{1}{2}}$.
	$n=1$	$l=2$, $s=\frac{1}{2}$	$j=\frac{3}{2}, \frac{5}{2}$	$1d_{3/2}, 1d_{5/2}$.
• $N=3$	$n=2$	$l=1$, $s=1/2$	$j=\frac{1}{2}, \frac{3}{2}$	$2p_{1/2}, 2p_{3/2}$,
	$n=1$	$l=3$, $s=1/2$	$j=\frac{5}{2}, \frac{7}{2}$	$1f_{5/2}, 1f_{7/2}$.
• $N=4$	$n=3$	$l=0$, $s=1/2$	$j=\frac{1}{2}$	$3s_{1/2}$.
	$n=2$	$l=2$, $s=1/2$	$j=3/2, 5/2$	$2d_{3/2}, 2d_{5/2}$.
	$n=1$	$l=4$, $s=1/2$	$j=\frac{7}{2}, \frac{9}{2}$.	$1g_{7/2}, 1g_{9/2}$.



no. of nucleus :— $(2j+1)$

the order of shell ~~no~~ filling is.

$1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$, $2p_{3/2}$, $2p_{1/2}$, $1g_{9/2}$ and so on.

* nuclear spin → total angular momentum of the nucleus.

Predictions of the single particle shell model (SPSM):—

(A) ~~Even~~ – Spin and parity →

(i) Even-even nuclei – in even-z nuclei both p and n are paired so resultant angular momentum.

$$I = 0,$$

$$\text{parity} \quad P = +1.$$

$$\therefore \boxed{I^P = 0^+}$$ no exception to this rule.

Ex:— $^{12}_{6}C$, $^{16}_{8}O$

for both $I^P = 0^+$

(B) odd-Even or Even-odd nuclei:— the spin-parity will be decided by last unpaired nucleon.

parity, $P = (-1)^{l_p}$ or $(-1)^{l_n}$
proton. neutron.

Ex:— $^{11}_{5}B$, $^{13}_{6}C$.

for $^{11}_{5}B$:— $p = 5 \Rightarrow (1s_{1/2})^2 (1p_{3/2})^3$ → last unpaired proton goes to $p_{3/2}$.

$\Rightarrow j = 3/2,\ l = 1$.

$n = 6 \Rightarrow (1s_{1/2})^2 (1p_{3/2})^4$ → no role.

So parity $P \Rightarrow (-1)^1 = -1$.

$$\boxed{J^P = 3/2^-}$$

$^{13}_{6}C$ → $p = 6 \quad (1s_{1/2})^2 (1p_{3/2})^4$ – no role.

$n = 7 \quad (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^1$ → last neutron goes to $p_{1/2}$ level

⇓

$J = 1/2,\ l = 1$.

parity $P = (-1)^1 = -1$.

$$\boxed{I^P = \frac{1}{2}^-}$$

* there are some exceptions to this rule.
for ex:— $^{23}_{11}Na$

$^{23}_{11}Na$:—
$p = 11 \rightarrow (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^3$
$n = 12 \rightarrow$ no role.
so $I = 5/2$, $l = 2$

$P = (-1)^2 \Rightarrow p = +1$.

$\boxed{I^P = \frac{5}{2}^+}$ but observed value $I = \frac{3}{2}^+$.

3. odd-odd nuclei— Here both p and n are unpaired so resultant angular momentum will be vector sum of $\vec{I_p}$ and $\vec{I_n}$.

$$\vec{I} = \vec{I_p} + \vec{I_n}$$

parity $= (-1)^{l_p} \times (-1)^{l_n} = (-1)^{l_p + l_n}$

Nordheim Rule— according to this rule if
$I_p + I_n + l_p + l_n =$ odd no.
then $\boxed{I = I_p + I_n}$

if $I_p + I_n + l_p + l_n =$ even no.

then, $\boxed{I = |I_p - I_n|}$

Ex:— $^{10}_{5}B$

$p = 5 \rightarrow (1s_{1/2})^2 (1p_{3/2})^3 \rightarrow I_p = \frac{3}{2}, \; l_p = 1.$
$n = 5 \rightarrow (1s_{1/2})^2 (1p_{3/2})^3 \rightarrow I_n = 3/2, \; l_n = 1.$

now $I_p + I_n + l_p + l_n = 5$ odd no.
so by Nordheim Rule.
$I = I_p + I_n = 3$ $\quad \therefore \boxed{I^P = 3^+}$

parity $= (-1)^{l_p} \times (-1)^{l_n} = (-1)^{l_p + l_n} = (-1)^{1+1} = +1.$

some exceptions exist to this rule.

⇒ Magnetic moment —

$\vec{\mu_l} = g_l \vec{l}$ — orbital magnetic moment.
$\vec{\mu_s} = g_s \vec{s}$ — intrinsic magnetic moment

$p - 2.79\,\mu_N$ $\quad \vec{\mu} = g_l \vec{l} + g_s \vec{s}.$ $\quad g_l = 1$ for p
$n - 1.91\,\mu_N.$ $\quad g_l = 0$ for n.

for $p \rightarrow$ intrinsic magnetic moment $= 2.79\,\mu_N \Rightarrow g_s = 5.58$ for

for n:— intrinsic magnetic moment $= -1.91\,\mu_N \Rightarrow g_s = -3.82$ for n.

(a) Even - Even nuclei — both p and n are paired
so mag. moment are cancelled out.
$\boxed{\text{Net } \mu = 0}$

(b) odd - even or even - odd nuclei —

$$\Rightarrow J = \sqrt{J(J+1)}$$

μ = sum of components of $\mu_{\vec{l}}$ and $\mu_{\vec{s}}$ along $\vec{s}$.

$$= \mu_l \cos(\vec{l}, \vec{J}) + \mu_s \cos(\vec{s}, \vec{J}).$$

$$= g_l \cancel{\sqrt{l(l+1)}} \frac{[J^2 + l^2 - s^2]}{2\sqrt{j(j+1)}\cancel{\sqrt{l(l+1)}}} + g_s \cancel{\sqrt{s(s+1)}} \frac{[J^2 + s^2 - l^2]}{2\sqrt{j(j+1)}\cancel{\sqrt{s(s+1)}}}$$

$$\Rightarrow \mu = g_l \left[\frac{j(j+1) + l(l+1) - s(s+1)}{2\sqrt{j(j+1)}} \right] + g_s \left[\frac{j(j+1) + s(s+1) - l(l+1)}{2\sqrt{j(j+1)}} \right] \quad \text{--- (1)}$$

two cases arise: —

case I: — $j = l + s = l + 1/2$.

case II: — $j = l - s = l - 1/2$.

using these values in eqn. (1), we get.

$$\left.\begin{aligned} \mu &= \left(J - \tfrac{1}{2}\right) g_l + \tfrac{1}{2} g_s \quad \text{for } j = l + \tfrac{1}{2}. \\ \mu &= \frac{J}{J+1}\left[\left(J + \tfrac{3}{2}\right) g_l - \tfrac{1}{2} g_s\right] \quad \text{for } j = l - \tfrac{1}{2} \end{aligned}\right\} \text{--- (A)}$$

Schmidth eqn.

- μ vs J curves are called schmidth curves.

on odd-even nuclei —

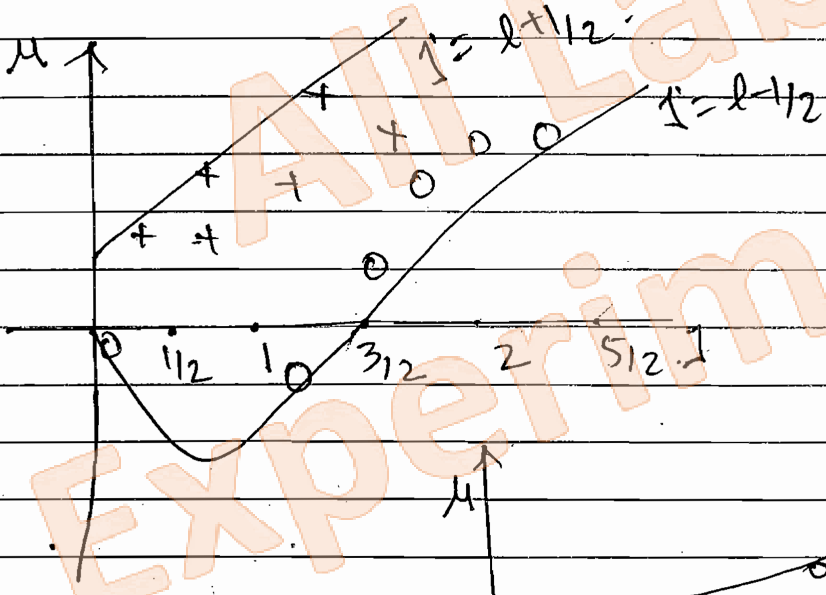
$$g_l = 1, \; g_s = 5.58 \text{ for } p.$$

$g_l = 0, \; g_s = -3.82$ for n.

$\checkmark \; \mu = I + 2.29$ for $I = l + 1/2$
$\checkmark \; \mu = I - 2.29 \frac{I}{I+1}$ for $j = l - 1/2$ $\Big\}$ --- (B).

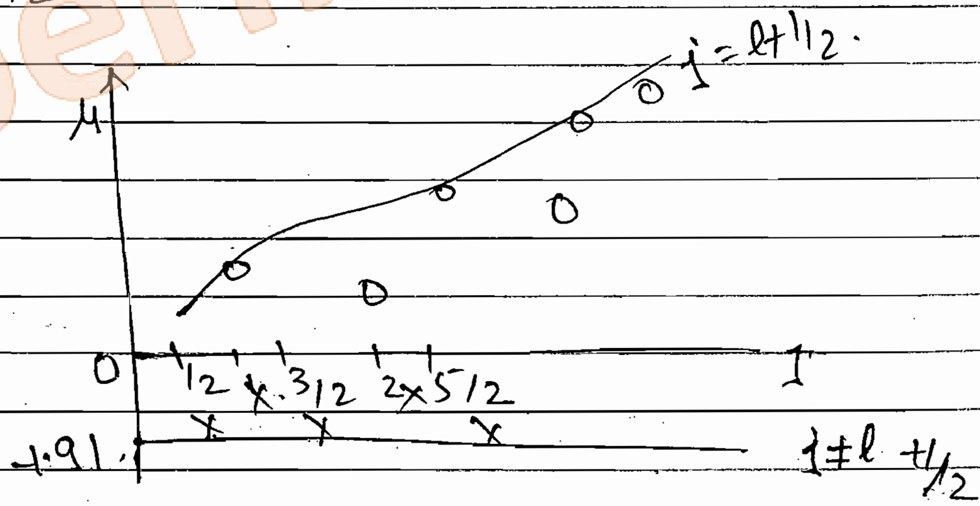
$\Rightarrow$ for Even-Z odd-N nuclei –

$\checkmark \; \mu = -1.91$ for $j = l + 1/2$.
$\mu = 1.91 \frac{I}{I+1}$ for $j = l - 1/2$ $\Big\}$ --- (C).



for odd-Z even-N.
$+ \rightarrow j = l + 1/2$
$o \rightarrow j = l - 1/2$

for even-Z odd N



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(Electric) Quadrupole moment :- measures the deviation from spherical system.

* for even-Z nuclei

$$Q = 0$$

* for odd-even or even-odd nuclei :-

$$Q = -\frac{(2I-1)}{(2I+2)} \langle r^2 \rangle$$

I - angular momentum or nuclear spin.

$\langle r^2 \rangle$ - mean square radius.

where $\langle r^2 \rangle = \frac{3}{5} R_c^2$

R_c - critical radius

$R_c = 1.2 A^{1/3}$ (fermi).

$$\Rightarrow \boxed{Q = \frac{-3(2I-1)}{5\,(2I+2)} R_c^2}$$

Ex:- ${}^{15}_{7}N$.

$$(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^1$$

p = 7

n = 8

$$I = \frac{1}{2}$$

$$Q = 0$$

the observed values in many cases are found to be larger than the predicted values (theoretical values) and are not negative always. In this way, shell model ~~are~~ is partially succesful in explaning Quadrupole moment of nuclei.

Ex:- find magnetic moment and electric Quadrupole moment of ${}^{17}_{8}O$ and ${}^{33}_{16}S$.

$x/n = 0$ * $\boxed{1\,fm = 10^{-15} m}$ * $1b = 10^{-28} m^2$

Sol:- $^{17}_{8}O$

$p = 8 \rightarrow$ no contribution.

$n = 9 \rightarrow (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^1$

$I = \frac{5}{2}$ Here $I = l + s = 2 + \frac{1}{2} = \frac{5}{2}$

$l = 2.$

$s = 1/2$

for neutron :- $\mu = -1.91$ for $I = l + \frac{1}{2}$

$\mu = +1.91 \frac{I}{I+1}$ for $I = l - \frac{1}{2}$.

$\boxed{\mu = -1.91 \mu_N}$ μ_N - nucleon magnetron.

$$Q = -\frac{3}{5} \frac{\left(2 \times \frac{5}{2} - 1\right)}{\left(2 \times \frac{5}{2} + 2\right)} R_c^2$$

$R_c = 1.2 \times (17)^{1/3}$

$R_c^2 = (1.2)^2 (17)^{2/3}$

$=$

$$= -\frac{3}{5} \frac{(4)}{7} R_c^2$$

$$= -\frac{12}{35} \times 1.2 \times 1.2 \times 6.48 \ fm^2$$

$$= -3.19 \times 10^{-30} \ m^2$$

$$= -0.32 \times 10^{-28} \ m^2$$

$$= -0.32b$$

Sol:- $^{33}_{16}S$ $p = 16 \rightarrow$ no contribution.

$n = 17$

$\frac{33}{16} \over 17$ $(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^6 (2s_{1/2})^2 (1d_{3/2}$

$I = \frac{3}{2}$

$l = 2$

$s = \frac{1}{2}$

$I = l - s = \cancel{\frac{3}{2} - \frac{1}{2}} \quad 2 - \frac{1}{2} = \frac{3}{2}$

Therefore,

$$\mu = 1.91 \cdot \frac{I}{I+1} = 1.91 \times \frac{3/2}{\frac{3}{2}+1}$$

$$= 1.91 \times \frac{\frac{3}{2}}{\frac{5}{2}}$$

$$\boxed{\mu_N = 1.146}$$

$$Q = -\frac{3}{5}\frac{\left(2 \times \frac{3}{2} - 1\right)}{\left(2 \times \frac{5}{2} + 1\right)} R^2_C$$

$R_C = 1.2 \times (33)^{1/3}$

$R^2_C = (1.2)^2 \times (33)^{2/3}$

$$= -\frac{3}{5}\left(\frac{2}{6}\right) \times$$

Q:- write down the configuration of proton and neutron of the following nuclei. find out the spin parity of these in their ground state.

(i) $^{41}_{21}Sc$ (ii) $^{57}_{28}Ni$ (iii) $^{129}_{53}I$

Calculate the magnetic moment and quadrupole moment of any two.

Sol:- (i) $^{41}_{21}Sc$

$p = 21$

$n = 20$: no contribution

$1s_{1/2}^2 \; 1p_{3/2}^4 \; 1p_{1/2}^2 \; 1d_{5/2}^6 \; 2s_{1/2}^2 \; 1d_{3/2}^4 \; 1f_{7/2}^1$

$I = \frac{7}{2}$ parity $= (-1)^3 = -1$

$3 + \frac{1}{2} = \frac{7}{2}$

$J = \frac{7}{2}^-$

$\mu = I + 2.29 = \frac{7}{2} + 2.29 = \frac{7 + 5.58}{2}$

$2.29 \times 2 = 5.58$

$Q = -\frac{3}{5} \frac{(2I-1)}{(2I+2)} R^2 c = -\frac{3}{5} \left(\frac{2 \times \frac{7}{2} - 1}{2 \times \frac{7}{2} + 2} \right) R^2 c.$

$= -\frac{3}{5} \frac{(7-1)}{(7+2)} 1.2 \times 1.2 \times (41)^{2/3}$

$57 - 28 = 29$

(ii) $^{57}_{28}Ni$ $p = 28 \rightarrow$ no contribution.

$n = 29$

$1s_{1/2}^2 \; 1p_{3/2}^4 \; 1p_{1/2}^2 \; 1d_{5/2}^6 \; 2s_{1/2}^2 \; 1d_{3/2}^4 \; 1f_{7/2}^8 \; 2p_{3/2}^1$

$I = \frac{3}{2}$ parity $= (-1)^1 = -1$

$1 + \frac{1}{2}$

$I = \frac{3}{2}^-$ $\mu = -1.91$

$$\begin{array}{r} 129 \\ -53 \\ \hline 76 \end{array}$$

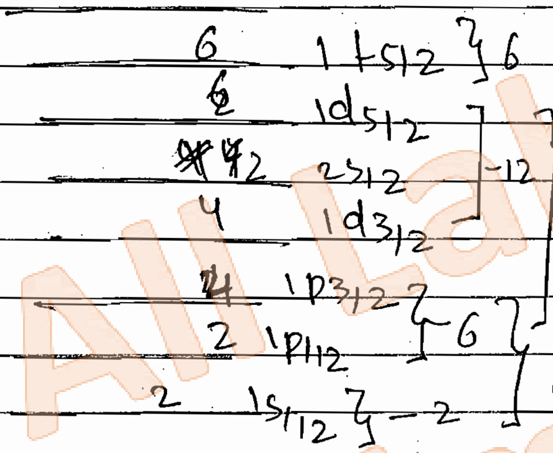
i) $^{129}_{53}I$:- $Z = 53$

$n = 76 \rightarrow$ no contribution.

NET

Q:- The sign of spin-orbit term is reversed from $-K\vec{l}\cdot\vec{s}$ to $+K\vec{l}\cdot\vec{s}$ what would be first five magic no.'s in new ~~spin~~ scheme? Compare the spin and parity of ^{17}O in new scheme with that of old scheme?

If spin orbit term changes from $l+s$ to $l-s$, then system behaves as atomic system, and lower j values lies lower and higher j values lies higher.



Q:- the ground state of radio isotope ~~has~~ $^{17}_{9}F$ has spin parity $I^P = \frac{5}{2}^+$ and first excited state has $I^P = \frac{1}{2}^-$

suggest the configuration for the excited state.

Sol:- $^{17}_{9}F$ $p = 9 \rightarrow (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^1$

$n = 8 \rightarrow$ no contribution

last proton goes to $d_{5/2}$ shell.

so $I = \frac{5}{2}$, $l = 2 \Rightarrow P = (-1)^2 = +1$.

$I^P = \frac{5}{2}^+ \rightarrow$ Ground state.

Excited state - $I^P = \frac{1}{2}^-$

this can be obtained by promoting a proton from $1p_{1/2}$ level to $1d_{5/2}$ level.

$(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^1 (1d_{5/2})^2$

odd proton.

$\Rightarrow I = \frac{1}{2}$

$l = 1$

parity $P = (-1)^1 = -1$.

$I^P = \frac{1}{2}^-$ Ans

Q:- the spin parity and energy about the ground state ^{15}O in some excited state is given by.

Ground state $\Rightarrow \frac{1}{2}^-$

excited state $\Rightarrow \frac{1}{2}^+$ − 5.18 MeV.

$\rightarrow \frac{5}{2}^+$ − 5.24 MeV.

$\rightarrow \frac{3}{2}^-$ − 6.18 MeV.

The order of shell filling is $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$ etc.

Use the shell model to find the reasonable configuration for excited state.

Sol:— ${}^{15}_{8}O$ $\quad p = 8 \rightarrow$ no contribution.

$n = 7 \rightarrow (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^1$

Ground state $\rightarrow I^P = \frac{1}{2}^-$

for excited state $\quad I^P = \frac{1}{2}^+$

$$(1s_{1/2})^1 (1p_{3/2})^4 (1p_{1/2})^2.$$

· Promotion of one n from $s_{1/2}$ level to $p_{1/2}$ level.

(ii) $I_P = \frac{3}{2}^-$ can be obtained by promoting a neutron from $p_{3/2}$ level to $p_{1/2}$ level.

$$(1s_{1/2})^2 (1p_{3/2})^3 (1p_{1/2})^2$$

(iii) $I^P = \left(\frac{5}{2}\right)^+$ can be obtained by promoting a neutron from $p_{1/2}$ level to next $1d_{5/2}$ level.

$$(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^0 (1d_{5/2})^1$$

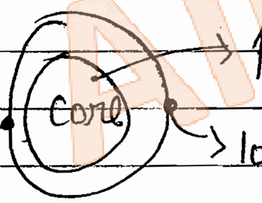
Collective Model: — shell model has certain limitations such as the magnetic moment and Quadrupole moment values predicted by this model, many time show large disagreement with the experimental values.

the collective model is an effort to explain / short out these limitations. it is the combination of some features of liquid drop model and some features of shell model.

LDM - Accounts for the collective behaviour of nucleons. ~~Or shell~~

shell model - accounts for the property which are due to individual nucleons.

nucleus = filled shells curve + extra loose nucleons
↓
Paired nucleons.



• due to the motion of loose nucleons the core can be deformed from spherical to spheroidal (ellipsoidal).

spherical $\xrightarrow[\text{of outer nucleons}]{\text{due to motion}}$ ellipsoidal.

the ~~ext~~ intensity of deformation of the core increases with increase in the no. of extra loose nucleons. this can explain the deviation ~~from~~ of magnetic moment and Quadrupole moment.

total energy of the system :—

$$E = \underbrace{E_{rot} + E_{vib.}}_{\text{due to collective behaviour of}} + \underbrace{E_{nucl}}_{\text{due to extra loose } e^-}$$

LDM Property. ⇒ due to collective behaviour of

shell property

* the rotation of whole nucleus does not take place, only the rotation of deformed portion of nuclear surface takes place.
* the coupling of extra nucleons with core may be weak, may be medium or strong.
  - weak coupling - ~~rotational model~~ vibrational model
  - strong coupling - rotational model.

Rotational Model:— $E = \frac{\hbar^2}{2I} J(J+1)$.

the rotational energy of the deformed nucleus is given by

$$E_{rot.} = \frac{\hbar^2}{2I} J(J+1) \rightarrow \text{Quantised}$$

I - moment of inertia.
J - nuclear spin.

due to the deformation of in case of ellipsoidal nucleus the deformation is symmetric at with respect to the reflection in nuclear centre. so J is restricted to only even values with even parity ($0^+, 2^+, 4^+, 6^+ - - -$)

$$E_{0^+} = 0$$
$$E_{2^+} = \frac{\hbar^2}{2I} 6$$
$$E_{4^+} = \frac{\hbar^2}{2I} \times 20$$
$$E_{6^+} = \frac{\hbar^2}{2I} \times 42$$

$\frac{6\hbar^2}{2I}$, $\frac{14\hbar^2}{2I}$, $\frac{22\hbar^2}{2I}$

spacing is increasing with J value.

fig:-

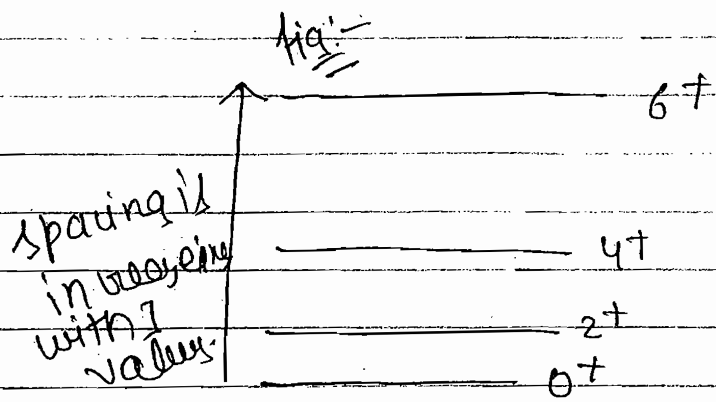
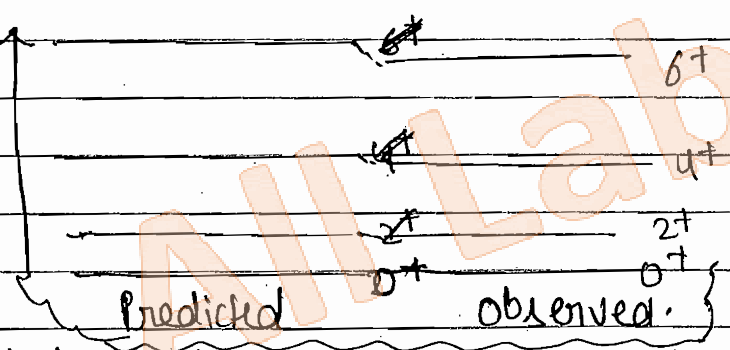


fig:- typical energy diagram of a rotational nucleus.



- the nature of observed rotational level is like predicted value but the gap b/w predicted and observed values increases with J value.

- Actually I - moment of inertia depends upon J values. To take this into account the modified expression is

$$E_{rot.} = \frac{\hbar^2}{2I} J(J+1) - \underset{\text{centrifugal constant}}{B} \underbrace{J^2(J+1)^2}_{\text{centrifugal term.}}$$

- now the agreement b/w predicted and experimental values is better.

NET

Q:- The rotational levels of some nuclei are given as below

6^+ ———— ?

4^+ ———— ?

2^+ ———— 93.3 MeV

0^+ ———— 0 MeV

The unknown energy levels have values.

(a) 305, 630 MeV.

(b) 311, 653 MeV.

(c) 311, 670 MeV.

(d) 300, 600 MeV.

Sol:-

$$E = \frac{\hbar^2}{2I} J(J+1).$$

$$E_{2^+} = \frac{\hbar^2}{2I} \times 2 \times 3$$

$$93.3 = \frac{\hbar^2}{2I} \times 6$$

$$\boxed{\frac{93.3}{6} = \frac{\hbar^2}{2I}.}$$

$$E_{4^+} = \frac{\hbar^2}{2I} \, 4 \times 5$$

$$= \frac{93.3}{6} \times 4 \times 5$$

$$= \frac{933}{3} = 311 \text{ MeV.}$$

$$E_{6^+} = \frac{\hbar^2}{2I} \, 6 \times 7 = \frac{93.3}{\cancel{6}} \times \cancel{6} \times 7$$

$$= 653.1 \text{ MeV.}$$

Q:- The rotational spectrum of a nucleus is characterised by which of the following properties.

(i) same parity of levels
(ii) same spin of levels
(iii) increase in angular momentum by $2\hbar$.
(iv) spacing b/w adjacent levels increases with spin.

Q:- The energy and the spin of the lower excited state of Ta^{182}

Energy (MeV)	0.100	0.329	0.680
Spin	2	4	6

Q:- What is the M.I. of the nucleus about the axis of rotation?

Q:- Calculate the energies of 12^+ and 16^+

Sol:-
$$E = \frac{\hbar^2}{2I} J(J+1).$$

$$E_{2^+} = \frac{\hbar^2}{2I} 2(2+1)$$

$$0.100 = \frac{\hbar^2}{\cancel{2}I} \cancel{2} \times 3$$

$$I = \frac{\hbar^2}{0.100} \times 3$$

$$I = \frac{(6.634 \times 10^{-34})^2}{0.100} \times 3$$

$$E_{12^+} = \frac{\hbar^2}{2I} 12 \times 13$$

$$= \frac{.100}{\cancel{6}\,1} \times \cancel{12}^{2} \times 13$$

$$= 2.6 \text{ mev.}$$

$$E_{16^+} = \frac{\hbar^2}{2I} \times 16 \times 17$$

$$= \frac{.100}{6} \times 16 \times 17$$

Q:- The energies of state 2^+ in the rotational band of a spheroidal nucleus is 0.1 mev

(a) evaluate m.I. of given nucleus.

(b) calculate the energies of 14^+ and 16^+ states of the same band and compare these values with experimentally observed values of 2.56 mev and 3.15 mev for 14^+ and 16^+. explain the reason behind the diff. b/w your answer and observed values.

(c) calculate the parameter that is empiracally used to explain these difference (B).

Sol:- (a) $E = \frac{\hbar^2}{2I} J(J+1)$.

$$0.1 = \frac{\hbar^2}{2I} 2 \times 3$$

$$\frac{0.1}{\cancel{6}\,3} \times I = \hbar^2 \Rightarrow I = \frac{\hbar^2}{0.1} \times 3$$

$$I =$$

(b) $E_{14^+} = \frac{1}{6} \times \frac{14 \times 15}{6} = 3.5\ \text{MeV}$

$E_{16^+} = \frac{1}{6} \times 16 \times 17 = 4.53\ \text{meV}$

} predicted values.

$E_{14^+} = 2.56\ \text{mev}$

$E_{16^+} = 3.15\ \text{mev}$

Since we have assumed, moment of interia fixed for different spin values but actually it is not fixed, it is very with spin that's why there is a diff. in energy.

(c) $E = \frac{\hbar^2}{2I} I(I+1) - B I^2 (I+1)^2$

$E_{14^+} = \frac{\hbar^2}{2I} 14 \times 15 - B(14)^2 (15)^2$

Q:- NET
The low lying energy levels of two hypothetical even-Z nuclei X and Y are shown below X is known to be a vibrational nucleus and Y is known to be a rotational nucleus. One level lying in b/w is missing in each case. Predict its energy, spin and parity.

1·0

1·05

0·0