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All Lab Experiments

Nuclear & Particle Physics Notes for NET/GATE Physical Sciences

Nuclear Physics - 1

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25.9.2015

Nuclear Physics

→ Basic nuclear properties.
→ nuclear models
- → Liquid drop model (LDM)
- → Shell model
- → Collective model.

→ nuclear decay
- → α decay, Gammow theory
- → β decay, fermi theory, selection rule
- → γ decay, multipole radiation, selection Rules.

→ nuclear Reactions
- compound nuclear reaction, cross-section,
- direct reaction.

→ nuclear forces
- Deutron problems and information about two nuclear force.
- nucleon-nucleon scattering
  - Low energy,
  - High energy

→ nuclear detectors.
- different types of detectors
- limitations.

→ tools for study of nuclear physics:-

1. physical tools
   - accelerators
   - spectrographs
   - detectors. etc.

2. Mathematical tools
   - Quantum nuclear,
   - mechanical laws nuclear.
   - statistical concept.
   - probability etc.

22·2·2016
23·2·2016

nuclear spin = total angular momentum of neutron/nucleus.

⇒ Basic nuclear ~~and~~ properties:—

Nucleus:— nucleus is a quantum system which has discreat energy levels which are are charactarized by same spin and parity.

I - ~~pa~~ spin.
P - parity.

$J_4 P_4$
$J_3 P_3$ } Excited states.
$J_2 P_2$
$J_1 P_1$ — ground state.

notation:— $^{A}_{Z}X$ A - mass no. Z - atomic no.

Types of nuclei:—

Isotopic nuclei:— • same Z but different A.
Ex:— $^{12}_{6}C$, $^{13}_{6}C$, $^{14}_{6}C$.

• they have similar chemical properties.

Isobaric nuclei:— • same A but different Z.
Ex:— $^{16}_{8}O$, $^{16}_{7}N$

chemical properties are different.

Isotones:— • same no. of neutrons.
Ex $^{14}_{6}C$, $^{15}_{7}N$, $^{16}_{8}O$ ⇒ n=8 in all nuclei.

Isomeric nuclei:— • same Z, same A but different internal nuclear structures (life times of different nuclear state may be different).
→ explanation of nuclear isomerism can be given by shell model.

magnetic moment. $\mu_p = \frac{e\hbar}{\pi 4 m_p}$. $m_p \simeq 1000 m_e$.

$\mu_e = \frac{e\hbar}{\pi 4 m_e}$

5. <u>Mirror nuclei</u>:– nuclei have p and n no. interchanged.

<u>Ex</u>:– $^{7}_{3}Li$ and $^{7}_{4}Be$

$p = 3$ $p = 4$

$n = 4$ $n = 3$

- no. of protons = no. of ~~atomic no.~~ neutron.
  no. of neutrons = mass no. – atomic no.

$z_1 - z_2 = 1$ if $z_1 > z_2$.

$z_2 - z_1 = 1$ if $z_2 > z_1$

- ~~different~~ difference in atomic no.'s of mirror nuclei is of unity. also the mass no. A is same for both nuclei

- $^{A_1}_{z_1}X$ and $^{A_2}_{z_2}Y$

if x and y are mirror nuclei then.

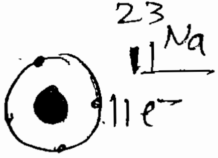
$A_1 = A_2$.

$z_1 - z_2 = \pm 1$

- These nuclei have same radii and the difference in the Colombian energies of mirror nuclei can be used to measure the nuclear charge radius.

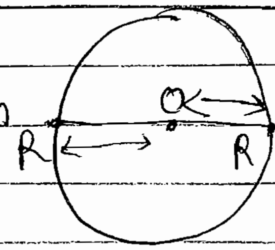
⇒ <u>Non-existence of e⁻ inside the nucleus</u>:– before the discovery of neutron it was assumed that e⁻ exist inside the nucleus. (theory was known as p–e theory) but there were many objections for the existence of e⁻ inside nucleus.

* ⇒ p-e theory assumptions.—



Assumptions:— no. of protons = A inside nucleus.
no. of e^-s inside the nucleus = (A−Z).
circulating e^-s = Z.

1. uncertainty principle:—



maximum uncertainty in the position of e^- $\Delta x \simeq 2R$.

so $\Delta p \Delta x \sim \hbar$

$\Rightarrow \Delta p \sim \frac{\hbar}{\Delta x}$.

$$\Delta p \sim 10^{-20}\ kg\ m/sec.$$

this implies that momentum must be atleast of above order. i.e. $p \sim 10^{-20}$ kg m/sec.

so energy of e^- inside the nucleus.

$$E^2 = p^2c^2 + m^2c^4.$$

$$E \simeq 20\,MeV.$$

but e^-s come from the nucleus in β decay have ~~any~~ only 4-5 MeV energy.

∴ e^- do not exist inside the nucleus.

2. Magnetic moment consideration:—

$$\mu_e = \frac{e\hbar}{4\pi m_e} \qquad m_p \simeq 1000 m_e.$$

~~$m_e \simeq 1000 \mu_p$~~.

$$\mu_p = \frac{e\hbar}{4\pi m_p}$$

$$\mu_e \simeq 1000 \mu_p.$$

so the dominating part of magnetic moment of nucleus should be due to e^-s.

but the observed magnetic moment is found to be of order of proton magnetic moment.

⇓

e^-s do not exist ~~on~~ inside the nucleus.

$m_p = 1836\, m_e$

I miss my friend ☹

3. Angular momentum consideration:– the predicted value of angular momentum of the nucleus is not in ~~resultant~~ resemblance with the observed angular momentum so this also supports the non existence of e^-s inside the nucleus.

4. De-Broglie wavelength:– $\lambda = \frac{h}{p}$

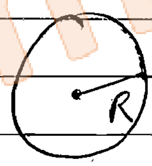
for proton, $\lambda_p = \frac{h}{p_p}$

for electron, $\lambda_e = \frac{h}{p_e}$

It is observed that $\lambda_p < 2R$ – exists inside the nucleus.
$\lambda_e > 2R$ – does not exist inside the nucleus.

Properties:– 1. nuclear size (Radius):– treating nucleus to be spherical.

Volume $\propto A$



$\frac{4}{3}\pi R^3 \propto A$

$\Rightarrow R^3 \propto A$

$\Rightarrow R \propto A^{1/3}$

$\Rightarrow \boxed{R = R_0 A^{1/3}}$

R_0 is called nuclear radius parameter
$R_0 \simeq 1.2 - 1.3\,fm$ $(1fm = 10^{-15} m)$

20.10.2015

⇒ Measurement of nuclear radius

nuclear charge Radius

→ methods are based on the assumption how the charge e^- is distributed inside the nucleus.

↓

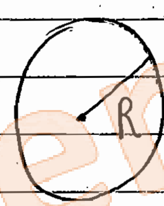
ex – e^- scattering, μ^- scattering ($R_0 \simeq 1.2\,fm$)

nuclear potential radius (nuclear mass radius).

→ methods are based on the distribution of mass of nucleons inside the nucleus

ex – neutron scattering, scattering by light nuclei ($R_0 \simeq 1.3 - 1.4\,fm$).

2. Nuclear mass density :-



${}^{A}_{Z}X$ → given nucleus.

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{Zm_p + (A-Z)m_n}{4/3\,\pi R^3}$$

$m_p \simeq m_n \simeq m_N$ – mass of nucleon.

$$\rho \simeq \frac{A m_N}{4/3\,\pi R_0^3 A}$$

$$\rho = \frac{m_N}{4/3\,\pi R_0^3}$$

$m_N \simeq 1.67 \times 10^{-27}\,Kg$.

$R_0 \simeq 1.2\,fm$.

$$\boxed{\rho \simeq 10^{17}\,Kg/m^3}$$

when a charge particle move in circular path, current will produced, then $\mu_N = I \times \vec{a}$ (due to angular momentum).
↓ area.

---

thus the nuclear density is independent of its volume and is constant for all nuclei.

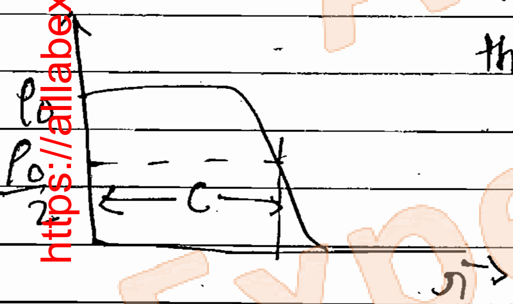
3. Nucleon concentration:- (no. of nucleons/m^3)

$$\text{no. of nucleons}/m^3 = \frac{\text{nuclear density}}{\text{mass of a nucleus}}$$

$$\simeq \frac{10^{17}}{1.67\times10^{-27}} /m^3$$

$$\simeq 10^{44} \text{ nucleons}/m^3$$

4. Nuclear charge distribution:- the charge distribution inside the nucleus is not uniform. It remains constant upto certain region and after that it falls off.



the fermi model for nuclear charge distribution gives good fit to the experimental values.

$$\boxed{\rho = \frac{\rho_0}{1+e^{k(r-c)}}}$$

ρ_0 - central charge density
C - distance from the centre at which density falls to 50% of central value.
K - constant
C - varies with A as $C = 1.07 A^{1/3}$ fm

5. nuclear magnetic moment:- $\vec{\mu} = \vec{\mu_l} + \vec{\mu_s}$
$\vec{\mu_l}$ - orbital magnetic moment.
$\vec{\mu_s}$ - intrinsic magnetic moment.

for proton — we have both orbital and intrinsic magnetic moment

$$\mu_l \neq 0, \mu_s \neq 0.$$

for neutron :—

$$\mu_l = 0.$$
$$\mu_s \neq 0.$$

for proton :— $\mu_p = 2.79\,\mu_N$ — intrinsic magnetic moment.

for ~~prot~~ neutron :— $\mu_n = -1.91\,\mu_N$ — intrinsic magnetic moment.

μ_N – nuclear magneton. $= \frac{eh}{4\pi m_p}$

- $\vec{\mu_l} = g_l \vec{l}$
- $\vec{\mu_s} = g_s \vec{s}$

for proton → $\boxed{g_l = 1,\ g_s = 5.58}$

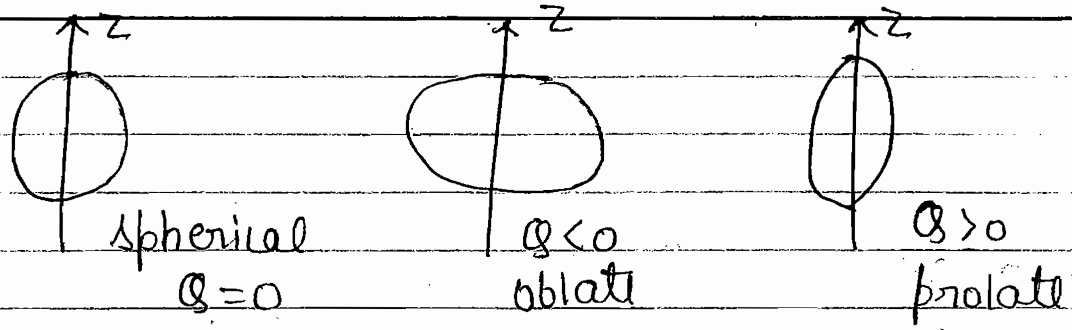
for neutron → $\boxed{g_l = 0,\ g_s = -3.82}$.

the magnetic moment of a nucleus is vector sum of magnetic moments of all protons and neutrons.

why neutron have negative magnetic moment →

- asymmetric charge distribution of neutron results -ve magnetic moment.

7. Electric Quadrupole moment (Q) :— it measures the departure from the spherical charge distributions



Even-Even nuclei have quadrupole moment Q=0 and therefore they are spherical.

• Other nuclei have been found to have finite value of +ve and -ve magnetic moment which indicates that they are ellipsoidal in shape.

Quantum mechanically: —

$$\boxed{Q = \frac{1}{e}\int (3z^2 - r^2)\rho \, d\tau}$$

ρ - charge density.
$d\tau$ - small volume element.
Unit → barn (b).
$1b = 10^{-28} \, m^2$.

8. Binding energy of nuclei: —

m_p - mass of proton.
m_n - mass of neutron.
${}^{A}_{Z}X$ - nucleus.

Theoretical mass (calculated mass) $= Zm_p + (A-Z)m_n$.

but the observed mass is found to be less than the theoretical mass.

$$M_{obs} < Zm_p + (A-Z)m_n$$

AB

the difference

$\Delta m = [\text{theoretical mass} - \text{observed mass}]$

(mass defect) $\Delta m = [Zm_p + (A-Z)m_n - M^{obs}_{nucl}]$

the significance of the mass defect is related with the B.E. of nucleus (STR).

$$\boxed{B.E. = \Delta m c^2}$$

$$B.E. = [Zm_p + (A-Z)m_n - {}^{A}_{Z}M]c^2$$

$$B.E. = [Zm_p + (A-Z)m_n - {}^{A}_{Z}M^{\text{nuclear}}]$$

if we are giving Zm_p, we will consider ${}^{A}_{Z}M^{n}$ is consider, nuclear mass.

$$B.E. = [Zm_p + (A-Z)m_n + Zm_e - ({}^{A}_{Z}M + Zm_e)]c^2$$

$$= [Zm_H + (A-Z)m_n - {}^{A}_{Z}M^{\text{atomic}}]$$

∴ if m_H is given ${}^{A}_{Z}M$ is considered as atomic mass.

- B.E. can be defined as the energy liberated when a nucleus is formed by its individual nucleus.

(or)

- it is energy required to break a nucleus into its individual nucleons.

Binding energy per nucleon $\left(\frac{B}{A}\right)$:— it is used to measure the relative stability of different nuclei.

more is the $\frac{B}{A}$ more will be the stability.

Ex:—

Deutron $({}^2_1H)$:— $B.E. = 2.25\,MeV$, no. of neucleons $= 2$

$$\frac{B}{A} = 1.125\,MeV.$$

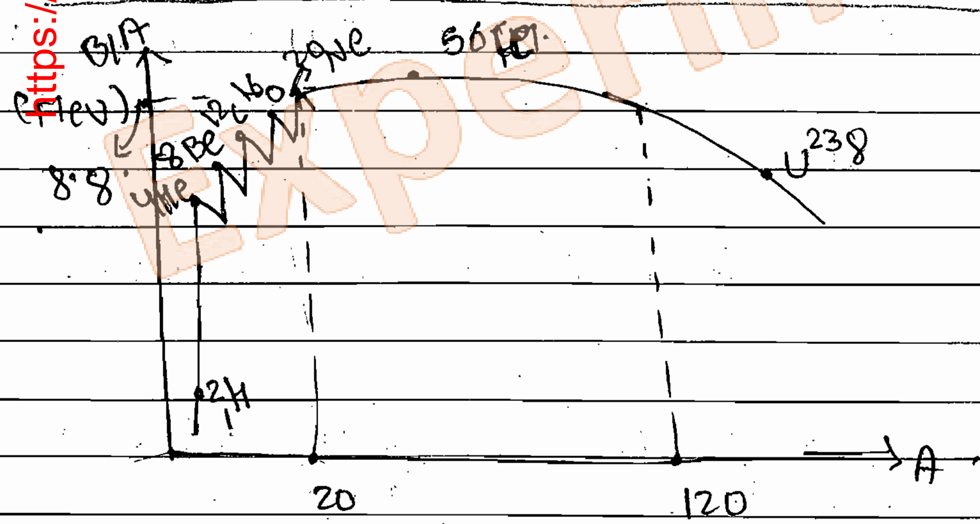
${}^4_2He$:— $B.E. = 28.4\,MeV.$

no. of nucleons $= 4.$

$$\therefore \frac{B}{A} = 7.1\,MeV.$$

$\Rightarrow$ ${}^4_2He$ is more stable as compared to ${}^2_1H$.

Binding energy curve:— the binding energy per nucleon $\left(\frac{B}{A}\right)$ vs mass no. A curve is called B.E. curve.



features of the curve:—

1. curve shows peaks corresponding to $A = 4, 8, 12, 16, 20$ for nuclei $A \leq 20$. and this means they have more stability corresponding to their neighbouring

nuclei ⇒ this is due to the formation of (2p+2n)
? (α particle combination) of nucleus in these nuclei.
⇒ also these are even nuclei so more stable.
it is also called, $A = 4n$ rule :-

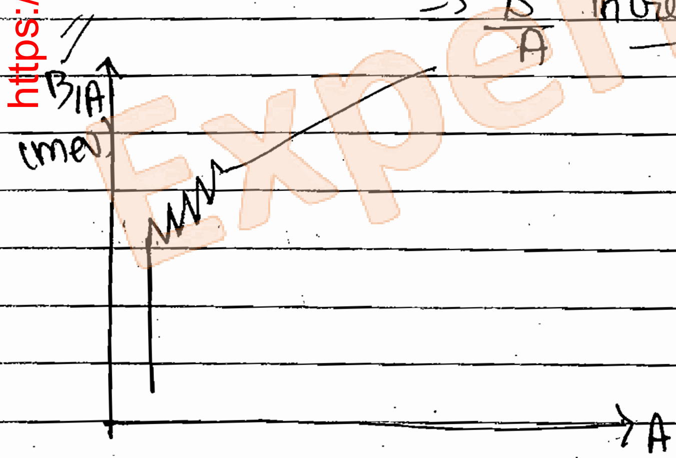
2. for nuclei in the range $A > 20$ but < 120 the $\frac{B}{A}$ is roughly constant (8.5 MeV) which shows the saturation prop. of nuclear forces.

Saturation property :- a particular nucleon interacts with limited no. of its neighbouring nucleons. (not with all nucleus). of nuclear forces would have the nature like EM interaction then.

$$B.E. \propto \frac{A(A-1)}{2}$$

$$\frac{B.E.}{A} \propto \frac{(A-1)}{2}$$

$\Rightarrow \frac{B}{A}$ increases linearly with mass no. A.



3. the maximum $\frac{B}{A}$ is corresponding to ^{56}Fe which is 8.8 MeV
^{56}Fe → magic no. (more stability).

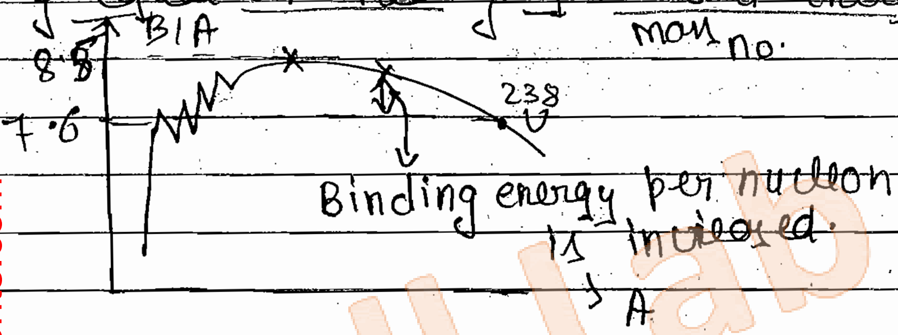
4. for heavy nuclei the B.E. falls of due to the decrease in their stability.

* $\frac{B}{A}$ increases in the nuclear fission as well as fusion process.

for $^{238}U \rightarrow \frac{B}{A} \approx 7.6$ MeV.

⇒ Explanation of energy liberation in nuclear fusion and fission:-

fission — A heavy nucleus breaks up into two lighter nuclei of equal or nearly equal and energy is liberated. mass no.



for $U^{238} \longrightarrow \frac{B}{A} \approx 7.6$ MeV.

The fission fragments have $\frac{B}{A} \approx 8.5$ MeV.

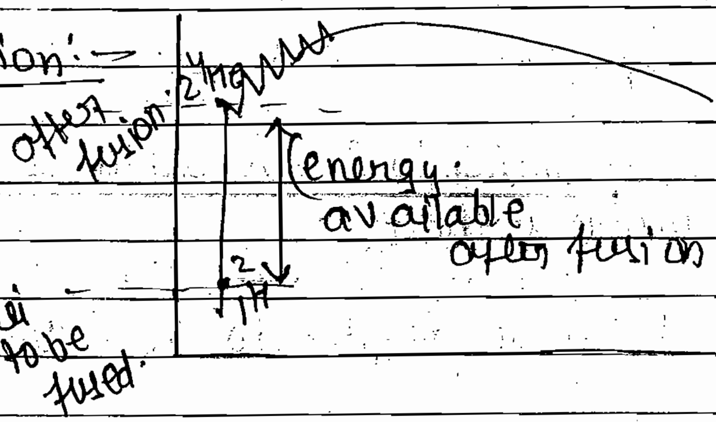
⇒ $\frac{B}{A}$ increases in the nuclear fission process increased.

$\frac{B}{A} = 8.5 - 7.6 = .9$ MeV.

so total energy leberated $= .9 \times 238 \approx 214.2$ MeV.

Hence roughly 200 MeV energy is liberated in nuclear fission.

⇒ fusion:-



(energy available after fusion $\frac{B}{A}$ increases).

238 / 119

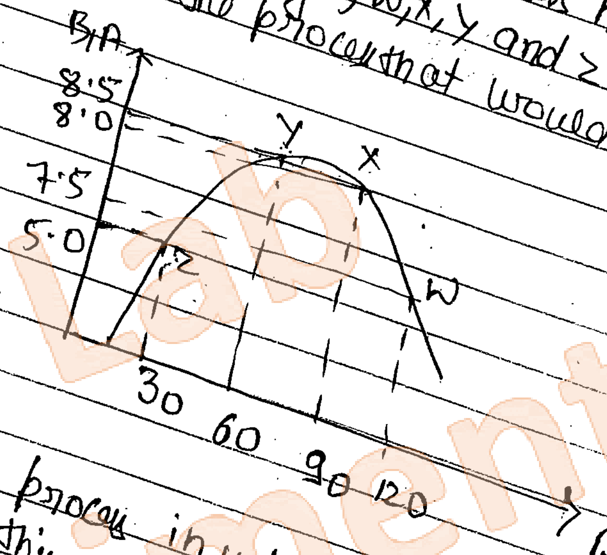
... two lighter nuclei fuse together to give ... heavier nucleus.

- In this process the $\frac{B}{A}$ is also increased. So energy is liberated in nuclear fusion.

Ex: Binding energy per nucleon verses mass no. curve for nuclei is given in the fig., W, X, Y and Z are for nuclei indicated on the curve. The process that would release energy

$\frac{B}{A}\uparrow$ High E

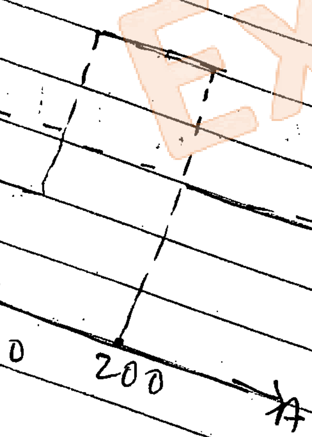
(a) $Y \to 2Z$ low E
(b) $W \to X + Z$.
(c) $W \to 2Y$ ✓
(d) $X \to Y + Z$.



diff. should be +ve.

$W \to 2Y (Y + Y)$ is process in which $\frac{B}{A}$ is increased. Hence energy will be liberated in this reaction.

Ex: Assume the nuclear binding energy per nucleon $(\frac{B}{A})$ versus mass no. A as shown in the fig. use this plot to the choose correct choice.



(a) fusion of two nuclei with mass no lying in the range $1 < A < 50$ will release energy. — will not release energy.
(b) ✓ fusion of two nuclei with mass no. lying in the range $51 < A < 100$ will release energy.

(d) → energy increase.

(c) fission of a nucleus lying in the ~~entire~~ range 100 < A < 20
will release energy when broken into two equal parts.

(d) fission of a nucleus lying in the range 200 < A < 26
will release energy when broken into two equal parts
which is correct option.

(i) a and c (ii) a and d (iii) b and d (iv) a

⇒ Internal nuclear structure and stability: —

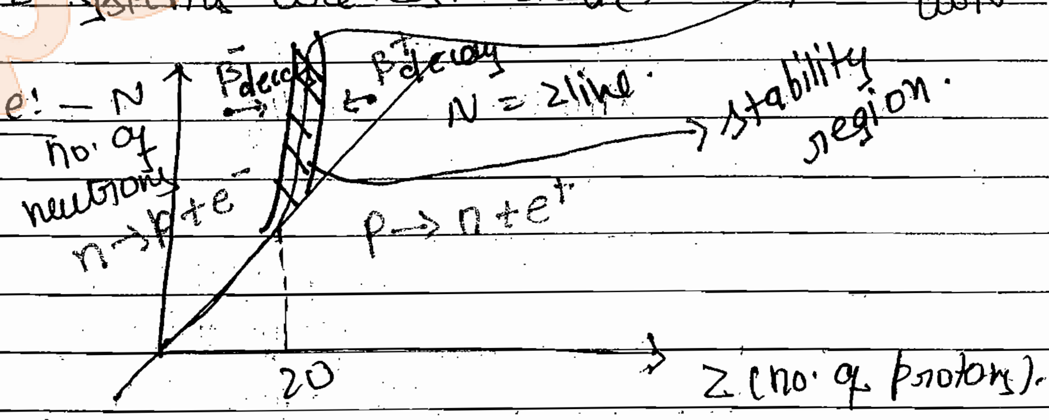
natural observed data.

Protons	neutrons	no. of nuclei
Even	even	160
Even	odd	56
odd	even	52
odd	odd	4.

2+2

- even-2 nuclei are most abundant.
- nature prefers even-2 systems.
- even-2 systems are most stable, even-odd and odd even systems have moderate stability.
- odd-2 systems are least stable.

stability curve! — N no. of neutrons



β^- → lie above the stability region.
and β^+ → lie below the stability region.

- for nuclei having their $Z \leq 20$ the stability curve coincides with $N = Z$ line.
- But for $Z > 20$, the curve bents towards the neutron axis as the the no. of neutrons increases more rapidly.
- the region around the stability curve in which nuclei are found to be stable is called stability region.
- the nuclei lying above the stability region have tendency to β^- decay & while the nuclei lying below the stability region have tendency to β^+ decay.

$(n \to p + e^-)$ $\quad$ $p \to n + e^+$

* $\frac{N}{P}$ ratio for stable nuclei is $\simeq 1.7$.

* β decay is a process which modifies $\frac{n}{p}$ ratio.

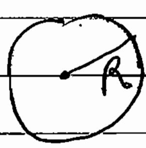
$\Rightarrow$ Semi-Emperical mass formula (SEMF):-

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{A^{3/4}}$$

0 for even-odd, odd-even

+ for even-even $\quad$ − for odd-odd

$\Rightarrow$ Volume



$V \propto A$.

$E_V \propto V$

$\propto \frac{4}{3}\pi R^3$

- $a_v \simeq 14$ MeV.
- $a_s \simeq 13$ MeV.
- $a_c \simeq .6$ MeV.
- $a_a \simeq 34$ MeV
- $a_p \simeq 19$ MeV.

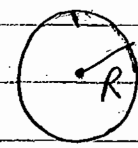
Bethe and Weizacker SEMF for B.E. of a nucleus ${}^A_Z X$ is given by

:

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_P}{A^{3/4}}$$

- $a_v A$ → vol. energy term
- $a_s A^{2/3}$ → surface energy term
- $a_c \frac{Z(Z-1)}{A^{1/3}}$ → coloumbian energy term.
- $a_a \frac{(A-2Z)^2}{A}$ → asymetric energy term.
- $\frac{a_P}{A^{3/4}}$ → Pairing energy term.

1. Volume energy term :- treating nucleus to be spherical as in Liquid drop model



Volume $\propto A$.

$E_V \propto V$.

$E_V \propto \frac{4}{3}\pi R^3$

$E_V \propto \frac{4}{3}\pi R_0^3 A$

a_v - volume energy constant.

$\boxed{E_V = a_v A}$

2. Surface energy term :- all nucleons inside the nucleus do not contribute equally to the B.E. Surface nucleons feel less force as compared to the inner nucleus.

$E_S \propto$ surface area.

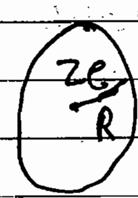
$E_S \propto 4\pi R^2$

$E_S \propto 4\pi R_0^2 A^{2/3}$

a_s - surface energy constant.

$\boxed{E_S = a_s A^{2/3}}$

3. Coulomb energy term :- protons inside the nucleus have coulombian repulsive force also. therefore the coulombian repulsive energy will decrease the B.E. of nucleus.



Learn

$$E_C = \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{Z(Z-1)e^2}{R}.$$

coulombian energy

$$E_c = \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{Z(Z-1)e^2}{R_0 A^{1/3}}$$

$$\boxed{E_c = a_c \frac{Z(Z-1)}{A^{1/3}}} \qquad a_c = \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{e^2}{R_0}.$$

a_c - coulomb energy constant.

4. Asymmetry energy term:- nuclei for which N=Z - more stable

→ the departure from N=Z line results in the decrease in stability so in B.E. this effect should be taken into account.

(N-Z) - measures the asymmetry.

the observed data fit in the expression

$$E_a \propto \frac{(N-Z)^2}{A}$$

$$E_a = a_a \frac{(N-Z)^2}{A} = a_a \frac{(A-2Z)^2}{A}$$

$$\boxed{E_a = a_a \frac{(A-2Z)^2}{A}}$$

a_a - asymmetric energy constant.

5. Pairing energy term:- even-even nuclei - most stable.
odd-odd nuclei - least stable.

even-odd }
odd-even } medium stability.

this effect should ~~be~~ also be incorporated in the expression of B.E.

$$E_p = \pm \frac{a_p}{A^{3/4}} \rightarrow \text{even}-2.$$

$$= -\frac{a_p}{A^{3/4}} \rightarrow \text{odd}-2.$$

$$= 0 - \text{even-odd}$$
odd - even.

Hence, the B.E. of ${}^{A}_{Z}X$ is :-

$$\boxed{B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{A^{3/4}}.}$$

where.

$a_v \simeq 14$ MeV.
$a_s \simeq 13$ MeV.
$a_c \simeq 0.6$ MeV.
$a_a \simeq 19$ MeV.
$a_p \simeq 34$ MeV.

⇒ Applications of SEMF :-

1. Determination of B.E. of nuclei
2. Determination of atomic masses of nuclei.
3. Study of energetics of α and β-decay
4. Q value of fission processes.
5. charge radii of mirror nuclei.
6. determination of stable isobars.

Atomic masses in term of SEMF :-

$$B.E. = Zm_p + (A-Z)m_n - {}^{A}_{Z}M^{\text{nuclear}}$$

⋮

$${}^{A}_{Z}M = M(Z, A).$$

⇒ nuclear mass –

$${}^{A}_{Z}M = Zm_p + (A-Z)m_n - B.E.$$

$$\boxed{{}^{A}_{Z}M = Zm_p + (A-Z)m_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \mp \frac{a_p}{A^{3/4}}}$$

Atomic mass –

$${}^{A}_{Z}M^{atomic} = ZM_H + (A-Z)m_n - B.E.$$

$$\boxed{{}^{A}_{Z}M^{atomic} = ZM_H + (A-Z)m_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \mp \frac{a_p}{A^{3/4}}}$$

3. $${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + Q.$$

$$M(Z,A) = M(Z-2, A-4) + M(2,4).$$

$$\boxed{Q = M(Z,A) - M(Z-2, A-4) - M(2,4)}$$

if $Q = +ve$, then energy liberate → α decay.

if $Q = -ve$, then energy donot liberate → no α-decay.

this can be solved with SEMF.

4. $${}^{A}_{Z}X \rightarrow {}^{A_1}_{Z_1}Y_1 + {}^{A_2}_{Z_2}Y_2 + kn + Q$$ } discuss in liquid drop

for mirror nuclei, $A = 2Z \pm 1$

* the mass diff. b/w mirror nuclei can be calculated by SEMF.

5. Charge Radius of mirror nuclei :— a pair of nuclei is called a mirror nuclei if their proton and neutron no.s are interchanged.

ex:— $^{7}_{3}Li$ $\quad$ $^{7}_{4}Be$

$p = 3 \qquad p = 4$
$n = 4 \qquad n = 3$

Other ex:— $^{13}_{6}C$, $^{13}_{7}N$

$^{23}_{12}Mg$, $^{23}_{11}Na$

Properties:— 1. the atomic no. difference b/w mirror nuclei is of unity.

i.e. $Z_1 - Z_2 = 1$ if $Z_1 > Z_2$.
Or $Z_2 - Z_1 = 1$ if $Z_2 > Z_1$.

2. the mass no. A of both nuclei is same.

3. $^{A_1}_{Z_1}X \qquad ^{A_2}_{Z_2}X$

$p = Z_1 \qquad p = Z_2$
$N = A_1 - Z_1 \qquad N = A_2 - Z_2$

$Z_1 = A_2 - Z_2$ and $Z_2 = A_1 - Z_1$.

$\boxed{Z_1 + Z_2 = A}$ $\qquad A_1 = A_2 = A$.

Let $Z_1 = Z$ if $Z_1 > Z_2$.
$Z_2 = Z-1$

$Z_1 = Z$
$Z_2 = Z+1$ if $Z_2 > Z_1$

So, $\underbrace{Z_1 + Z_2}_{A} = 2Z-1$. Hence for mirror nuclei.
$\boxed{A = 2Z-1}$ $\boxed{A = 2Z \pm 1.}$

$\underbrace{Z_1 + Z_2}_{A} = 2Z+1$ $A =$

$\boxed{A = 2Z+1}$

- the mass defect difference b/w mirror nuclei can be calculated by SEMF.

$$M(Z,A) = Zm_p + (A-Z)m_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \mp \frac{a_p}{A^{3/4}}.$$

$A =$ constant,
$Z_1 - Z_2 = 1$, if $Z_1 > Z_2$.

$$M(Z_1,A) = Z_1 m_p + (A-Z)m_n - a_v A + a_s A^{2/3} + a_c \frac{Z_1(Z_1-1)}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \mp \frac{a_p}{A^{3/4}} \quad \text{---} (1)$$

$$M(Z_2,A) = Z_2 m_p + (A-Z)m_n - a_v A + a_s A^{2/3} + a_c \frac{Z_2(Z_2-1)}{A^{1/3}} + a_a \frac{(A-Z_2)^2}{A} \mp \frac{a_p}{A^{3/4}} \quad \text{---} (2)$$

so mass difference, $\Delta M = M(Z_1, A) - M(Z_2, A)$

$Z_1 - Z_2 = 1$

$Z_2 - Z_1 = -1.$

$$\Delta M = (Z_1 - Z_2)m_p + (Z_2 - Z_1)m_n + \frac{a_c}{A^{1/3}}\left[Z_1(Z_1 - 1) - Z_2(Z_2 - 1)\right]$$

$$\Delta M = (m_p - m_n) + \frac{a_c}{A^{1/3}}\left[Z_1(Z_1 - 1) - (Z_1 - 1)(Z_1 - 2)\right].$$

if $Z_1 > Z_2$

the mass diff. b/w mirror nuclei

$$\Rightarrow \boxed{\Delta M = (m_p - m_n) + \frac{2a_c}{A^{1/3}}(Z_1 - 1).}$$

from above expression we can calculate 'a_c'

$$a_c = \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{e^2}{R_0}.$$

$\Rightarrow$ R_0 can be estimated.

$\Rightarrow$ $R = R_0 A^{1/3}$ gives the charge radius.

note:- 1. Let $Z_1 = Z$ and neglecting mass difference b/w proton and neutron.

$$\boxed{\Delta M = \frac{2a_c}{A^{1/3}}(Z - 1)}$$

2. if Coulombian term is taken to be $\frac{a_c Z^2}{A^{1/3}}$ instead of $\frac{a_c Z(Z-1)}{A^{1/3}}$ i.e. $Z(Z-1) \approx Z^2$.

then $\Delta M = (m_p - m_n) + \frac{a_c}{A^{1/3}}\left[Z_1^2 - Z_2^2\right]$ $\left\{\begin{array}{l} Z_1 - Z_2 = 1 \\ Z_1 + Z_2 = A \end{array}\right.$

$$= (m_p - m_n) + \frac{a_c}{A^{1/3}} (Z_1 + Z_2)(Z_1 - Z_2).$$

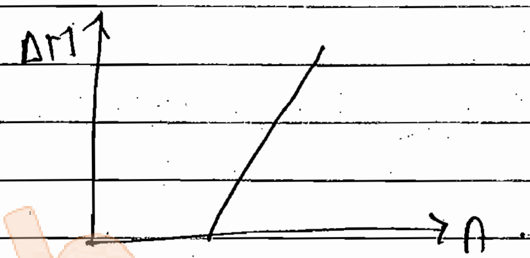
$$\Rightarrow \Delta M = (m_p - m_n) + \frac{a_c}{A^{1/3}} A.$$

$$\Rightarrow \boxed{\Delta M = (m_p - m_n) + a_c A^{2/3}}$$

if $(m_p - m_n)$ is neglected.

$$\boxed{\Delta M = a_c A^{2/3}}$$

$$\Rightarrow \boxed{\Delta M \propto A^{2/3}}$$



3. Coulombian energy difference b/w mirror nuclei.

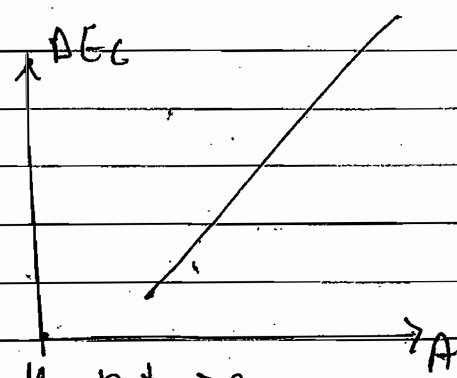
$$E_c = \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{Z^2 e^2}{R_0 A^{1/3}} = \frac{a_c Z^2}{A^{1/3}}.$$

$$\Delta E_c = \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{e^2}{R} [Z_1^2 - Z_2^2]$$

$$= \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{e^2}{R} [\underbrace{(Z_1 + Z_2)}_{A} \underbrace{(Z_1 - Z_2)}_{1}]$$

$$= \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{e^2}{R_0 A^{1/3}} \cdot A^1.$$

$$\boxed{\Delta E_c = \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{e^2}{R_0} A^{2/3}}$$



exactly not zero
some value.

$$M(Z,A) = Zm_p + (A-Z)m_n - B.E.$$

Mass Parabola and stable isobars: —

nuclear mass

$$M(Z,A) = Zm_p + (A-Z)m_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \mp \frac{a_p}{A^{3/4}}.$$

atomic mass

$$M(Z,A) = Zm_H + (A-Z)m_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \mp \frac{a_p}{A^{3/4}}$$

Mass-parabola eqn.

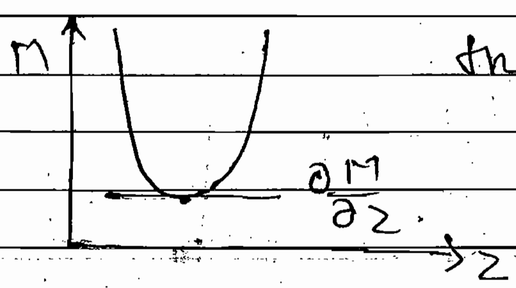
$$\boxed{M(Z,A) = \alpha A + \beta Z + \gamma Z^2 \mp \delta.} \quad --- (*) \qquad \delta = \frac{a_p}{A^{3/4}}$$

where,
$$\alpha = m_n - a_v + \frac{a_s}{A^{1/3}} + a_a.$$

$$\beta = m_H - m_n - \frac{a_c}{A^{1/3}} - 4a_a.$$

$$\gamma = \frac{a_c}{A^{1/3}} + \frac{4a_a}{A}$$

eqn. (*) represents a parabola



the vertex of which corresponds to $\left(\frac{\partial M}{\partial Z}\right)_A = 0$.
(shows the stability).

$$\left(\frac{\partial M}{\partial z}\right)_A = \beta + 2\gamma z = 0$$

$$\Rightarrow \boxed{z_0 = -\frac{\beta}{2\gamma}}$$ this is condition for most stable isobar.

<u>Case I</u>:— when A is odd.

Odd A.

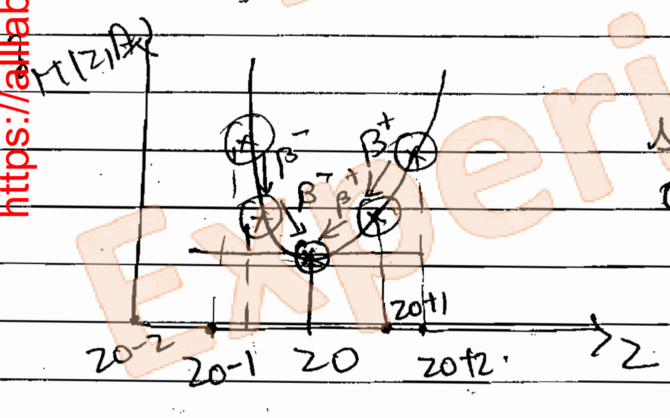
even z — odd N — $a_p = 0$, $\delta = 0$

odd z — even N — $a_p = 0$, $\delta = 0$

Hence,

$$\boxed{M(z,A) = \alpha A + \beta z + \gamma z^2}$$

Hence, we get 1 parabola for odd A nuclei.



stable isobars will at point z_0. others isobars approches to stable point.

The most stable isobars lies at the bottom or in the neighbourhood of it. others isobars lie on the left and right arm of parabola.

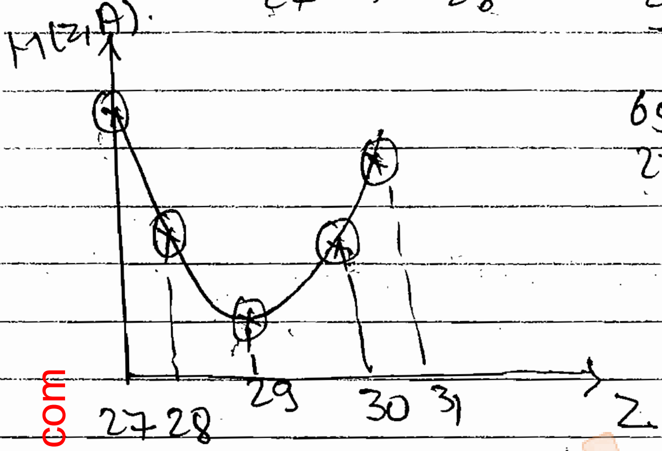
left arm → β^- emittery.

right arm → β^+ emittery.

ex:— $A = 65$

different isobars are.

$^{65}_{27}Co$, $^{65}_{28}Ni$, $^{65}_{29}Cu$, $^{65}_{30}Zn$, $^{65}_{31}Ga$.



$$^{65}_{27}Co \xrightarrow{\beta^-} {}^{65}_{28}Ni$$

$$\downarrow \beta^-$$

$$^{65}_{29}Cu.$$

$$^{65}_{31}Ga \xrightarrow{\beta^+} {}^{65}_{30}Zn \xrightarrow{\beta^+} {}^{65}_{29}Cu.$$

Case II:— Even A

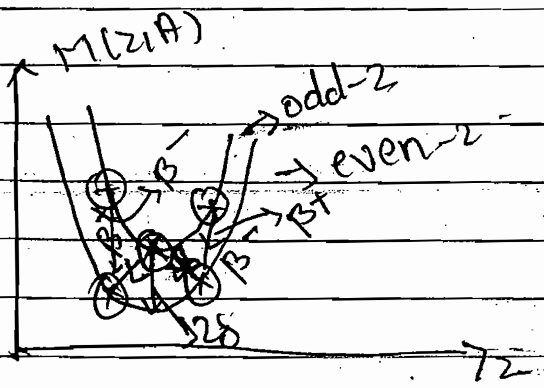
even Z, even N: $\delta = +ve$

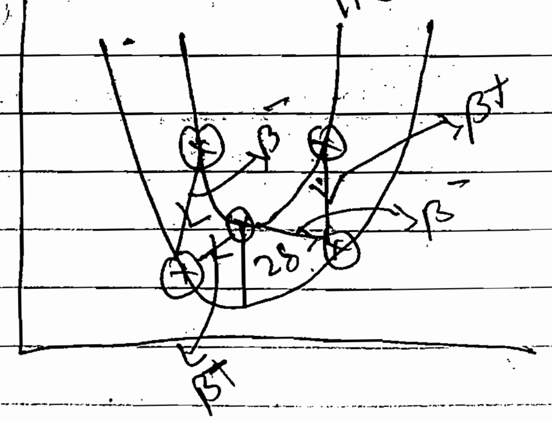
odd Z, odd N: $\delta = -ve$.

Thus we get two parabola.

$$\boxed{M(Z,A) = aA + \beta Z + \gamma Z^2 - \delta}$$ → even-Z — most stable, lie downward

$$\boxed{M(Z,A) = aA + \beta Z + \gamma Z^2 + \delta}$$ → odd-odd — less stable, lie upward.



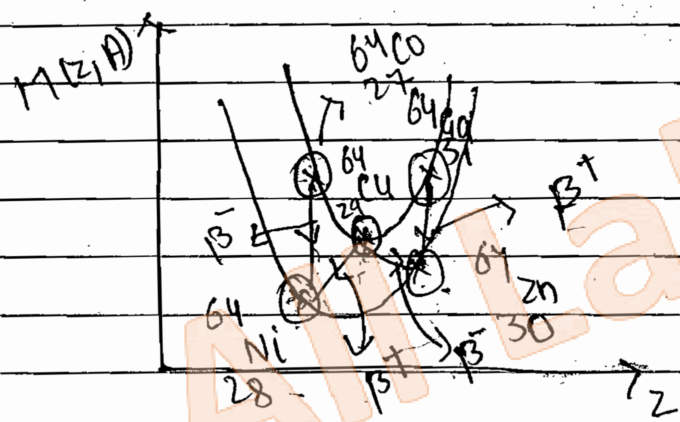


Here the vertical mass difference b/w two parabola is 28.

→ in this case more than one stable isobars are obtained

for ex:— $A = 64$

$^{64}_{27}Co$, $^{64}_{28}Ni$, $^{64}_{29}Cu$, $^{64}_{30}Zn$, $^{64}_{31}Ga$.



⇒ Condition of β^- and β^+ decay:—

β^- decay (increase in 1 proton).

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}_{-1}e^0 + Q$$

nuclear mass $^{A}_{Z}M$ ⟶ $^{A}_{Z+1}M$ ⟶ m_e

$$\boxed{Q = \left({}^{A}_{Z}M - {}^{A}_{Z+1}M - m_e\right)}$$

for β^- decay to occur $Q > 0$.

$$\Rightarrow {}^{A}_{Z}M > {}^{A}_{Z+1}M + m_e.$$

$$\therefore \Rightarrow \boxed{\left({}^{A}_{Z}M - {}^{A}_{Z+1}M\right) > m_e}$$ (nuclear mass).

In terms of atomic masses:—

$${}^{A}_{Z}M + Zm_e > {}^{A}_{Z+1}M + \underbrace{Zm_e}_{(Z+1)m_e} + m_e$$

atomic mass:→ ${}^{A}_{Z}M > {}^{A}_{Z+1}M$

$$\Rightarrow \boxed{{}^{A}_{Z}M - {}^{A}_{Z+1}M > 0}$$ atomic masses.

β^{+1} decay:—

decrease in proton

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + {}_{+1}e^{0} + Q.$$

nuclear mass ↓ ${}^{A}_{Z}M$ ↓ ${}^{A}_{Z-1}M$ ↓ m_e

$$Q = ({}^{A}_{Z}M - {}^{A}_{Z-1}M - m_e).$$

for β^{+} decay to occur $Q > 0$.

$$\Rightarrow {}^{A}_{Z}M > {}^{A}_{Z-1}M + m_e.$$

$$\Rightarrow \boxed{({}^{A}_{Z}M - {}^{A}_{Z-1}M) > m_e}$$ nuclear masses.

In terms of atomic masses:—

$${}^{A}_{Z}M > {}^{A}_{Z-1}M + m_e.$$

atomic masses :— ${}^{A}_{Z}M + Zme > {}^{A}_{Z-1}M + Zme + me$

$${}^{A}_{Z}M > {}^{A}_{Z-1}M + 2me$$

$$\boxed{{}^{A}_{Z}M - {}^{A}_{Z-1}M > 2me}$$ atomic masses.

Q:— Check whether β transition occur b/w ${}^{7}_{3}Li$ and ${}^{7}_{4}Be$ isotropic atomic masses are.

$${}^{7}_{3}Li = 7.01600 4 \text{ amu}.$$

$${}^{7}_{4}Be = 7.016926 \text{ amu}.$$

How do we know, to implies atomic mass condition. b/c we are given atomic mass (amu).

Sol— Possibilities are:—

$${}^{7}_{3}Li \rightarrow {}^{7}_{4}Be + {}_{-1}e^{0} \quad \text{— I}$$ β^- (addition in 1 proton)

$${}^{7}_{4}Be \rightarrow {}^{7}_{3}Li + {}_{+1}e^{0} \quad \text{— II}$$ β^+. (subtraction of 1 proton)

for I to occur :—

$$M({}^{7}_{3}Li) - M({}^{7}_{4}Be) > 0.$$

$$\downarrow -0.000922$$

but this is not the case, so I will not occur

for IInd to occur:—

$$M({}^{7}_{4}Be) - M({}^{7}_{3}Li) > 2me.$$

$$\begin{array}{r} 7.016929 \\ -7.016004 \\ \hline .000925 \text{ amu}. \end{array}$$

$= 0.000925 \times 931 \text{ meV}$

$= 0.8613 \text{ meV}$

$2m_e = 2 \times 0.511$ MeV.
$= 1.022$ MeV.
It will also not occur.

21.10.2015

⇒ Separation Energy of last proton and Neutron:–

The energy required to remove last proton or neutron from a nucleus is called separation energy of last proton or neutron.

Notation –

S_p – separation energy of proton.
S_n – separation energy of neutron.

S_n – the separation energy of last neutron can also be defined as the energy released when a nucleus absorbs a neutron to form a nucleus with higher mass no.

$${}^{A-1}_{Z}X + {}^{1}_{0}n \rightarrow {}^{A}_{Z}Y \qquad {}^{1}_{1}H \rightarrow \text{proton.}$$

$$S_n = [{}^{A-1}_{Z}M + m_n - {}^{A}_{Z}M]c^2.$$

$$S_n = B(Z, A) - B(Z, A-1).$$

S_p :– $${}^{A-1}_{Z-1}X + {}^{1}_{1}H \longrightarrow {}^{A}_{Z}Y$$
(proton).

$$S_p = [{}^{A-1}_{Z-1}M + m_p - {}^{A}_{Z}M]c^2$$

Or.

$$S_p = B(Z, A) - B(Z-1, A-1)$$

Illustrative Examples:-

Q:- Find the binding energy of last added neutron in the isotopes of ${}^{206}_{82}Pb$ and ${}^{208}_{82}Pb$. Given that atomic masses.

$${}^{205}Pb = 204.9744 \text{ amu}. \qquad {}^{208}Pb = 207.9767 \text{amu}.$$
$${}^{206}Pb = 205.9745 \text{ amu}. \qquad m_n = 1.0086 \text{ amu}.$$
$${}^{207}Pb = 206.9759 \text{ amu}.$$

Sol:- (i) In ${}^{206}_{82}Pb$

$${}^{205}_{82}Pb + {}^{1}_{0}n \longrightarrow {}^{206}_{82}Pb$$

$$S_n = [M({}^{205}Pb) + m_n - M({}^{206}Pb)].$$
$$= [204.9744 + 1.0086 - 205.9745] \text{ amu}.$$
$$= 0.0085 \text{ amu}.$$
$$= 0.0085 \times 931 \text{ mev}.$$
$$= 7.91 \text{ mev}.$$

$\boxed{1 \text{ amu} = 931 \text{ meV}.}$

(ii) ${}^{208}Pb$

$${}^{207}Pb + {}^{1}_{0}n \longrightarrow {}^{208}Pb.$$

$$S_n = [M({}^{207}Pb) + m_n - M({}^{208}Pb)].$$
$$S_n = 206.9759 + 1.0086 - 207.9767.$$
$$S_n = .0078 \text{ amu}.$$
$$= .0078 \times 931. \text{ Mev}.$$
$$\boxed{S_n = 7.26 \text{ mev}}$$

Q:- Calculate the B.E. of last added.

(a) n in ${}^{4}_{2}He$

(b) p in ${}^{16}_{8}O$

Compare these values with B.E. per nucleon.

Given that
$${}^{3}_{2}M = 3.0160$$
$${}^{4}_{2}M = 4.0026U$$
$${}^{15}_{7}M = 15.0001U.$$
$${}^{16}_{8}M = 15.9949U$$

01:— (a) $^{3}_{2}He + ^{1}_{0}n \rightarrow ^{4}_{2}He$

$S_n = [^{3}_{2}M + m_n - ^{4}_{2}M]$

$= [3.016 + 1.0086 - 4.0026]$ amu.

$= 4.0246 - 4.0026$

$= .022$ amu.

$= .022 \times 931$ MeV.

$= 20.48$ mev.

Binding energy in terms of SEMF $= Zm_p + (A-Z)m_n - ^{A}_{Z}M$

(a) total binding energy $= [2m_p + 2m_n - 4.0026]$ amu.

$m_p = 1.0078$ amu.

$m_n = 1.0086$ amu.

$B.E. = [2 \times 1.0078 + 2 \times 1.0086 - 4.0026]$.

$\simeq 28.4$ Mev.

B.E per nucleon $= \frac{28.4}{4} = 7.1$ Mev.

(b) for proton — for last proton added = B.E. (p).

$^{15}_{7}N + ^{1}_{1}H \rightarrow ^{16}_{8}O$

$B.E.(p) = [M(^{15}N) + m_p - M(^{16}O)]$.

$= 15.0001 + 1.0078 - 15.9949$

$= 0.130$ amu.

$= 0.130 \times 931$ mev

$= 12.103$ mev.

Gate Problems :-

Q:- If the nuclear radius of ^{27}Al is 3.6 fm, then approx. nuclear radius of ^{64}Cu in fm is.

(a) 4.8 (b) 3.6 (c) 2.4 (d) 1.2.

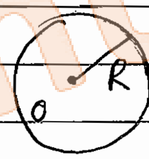
Sol:-
$$R = R_0 A^{1/3}$$
$$R_1 = 3.6\,fm, \quad A_1 = 27$$
$$R_2 = ?, \quad A_2 = 64.$$
$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{64}\right)^{1/3} = \frac{3}{4}.$$
$$R_2 = \frac{4}{3} \times 3.6 = 4.8\,fm.$$

Gate

Q:- the mean momentum of a nucleon in a nucleus with mass no. A varies as.

(a) A (b) A^2 (c) $A^{-2/3}$ (d) $A^{-1/3}$

Sol:- the max. uncertainity in the position of a nucleus



$$\Delta x \approx 2R$$
$$\Delta p . \Delta x \sim \hbar$$
$$\Delta p \sim \frac{\hbar}{2R}.$$
$$\Delta p \sim \frac{\hbar}{2R_0 A^{1/3}}$$

$\Delta p \propto A^{-1/3}$

$\therefore$ the mean momentum of a nucleon.

$$\boxed{p \propto A^{-1/3}}$$

Energy of nucleon $E = \frac{p^2}{2m}$

$$E \propto p^2 \Rightarrow \boxed{E \propto A^{-2/3}}$$

Gate-2011

Q:- the semiemperical formula for B.E. of nucleus contains a surface correction term. this term depends upon the mass no. A of the nucleus as.

(a) $A^{-1/3}$ (b) $A^{1/3}$ (c) $A^{2/3}$ (d) A.

Q:- An ^{16}O nucleus in spherical has a charge radius R and volume $V = \frac{4}{3}\pi R^3$. according to the emperical observations of charge radius the volume of $^{128}_{54}Xe$ nucleus will be

Sol:- $\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{16}{128}\right)^{1/3} = \frac{1}{2}$.

$$R_2 = 2R_1 = 2R.$$

$$V_2 = \frac{4}{3}\pi R_2^3 = \frac{4}{3}\pi 8R^3$$

$$= 8V.$$

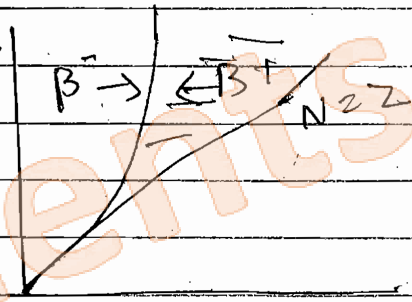
Q:- nuclei which are β^- emitters lie.

(a) Below β stability line

(b) On the β stability line.

(c) above β stability line.

(d) below $N = Z$ line.



Q:- A nucleus with A = 225 splits up into two nuclei whose radii are in the ratio 2:1 then their mass no.s will be

(a) 200, 25

(b) 175, 50

(c) 125, 100

(d) 25, 200.

Sol:- $A \longrightarrow A_1 + A_2$

$$A_1 + A_2 = 225 \quad \text{--- (1)}$$

$$\frac{R_1}{R_2} = 2 \Rightarrow R_1 = 2R_2.$$

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$\frac{A_1}{A_2} = \frac{R_1^3}{R_2^3} = (2)^3 = 8.$$

$A_1 = 8A_2$.

$9A_2 = 225$

$\boxed{A_2 = 25}$

$\boxed{A_1 = 200}$

Q:- the mass ~~defects~~ difference between two mirror nuclei of mass no. A varies as

(a) A (b) $A^{1/3}$ (c) $A^{-2/3}$ (d) $A^{2/3}$.

Sol:- $\Delta M = m_p - m_n + a_c A^{2/3}$

$\Delta M \propto A^{2/3}$.

Gate-2008

Q: The mass difference b/w ~~two~~ a pair of mirror nuclei $^{11}_{6}C$ and $^{11}_{5}B$ is given to be Δ MeV/c^2. According to the SEMF the mass ~~defect~~ difference between the pair of mirror nuclei $^{17}_{9}F$ and $^{17}_{8}O$ will approx be (rest mass of proton $m_p = 938.27$ MeV/c^2 and rest mass of neutron $m_n = 939.57$ MeV/c^2).

(a) 1.39Δ MeV/c^2 (b) $(1.39\Delta + 0.5)$ MeV/c^2.

(c) 1.86Δ MeV/c^2 (d) $(1.6\Delta + 0.78)$ MeV/c^2.

Sol:- for $^{11}_{6}C$ and $^{11}_{5}B$

mass diff. of mirror nuclei $\Delta = (m_p - m_n) + \frac{2a_c}{A^{1/3}}(Z_1 - 1)$ $(Z_1 > Z_2)$.

$Z_1 = 6,\ Z_2 = 5$.

$\Delta = -1.3 + \frac{2a_c}{A^{1/3}} \times 5$ --- (1)

for $^{17}_{9}F$ and $^{17}_{8}O$

$$\Delta' = -1.3 + \frac{2a_c}{A^{1/3}} \times 8 \quad \text{---} (2)$$

from (1), $\frac{(\Delta + 1.3)}{5} = \frac{2a_c}{A^{1/3}}$

putting in (2) $\quad \Delta' = -1.3 + \frac{(\Delta+1.3)}{5} \times \cancel{8}\ 1.6$

$$\Delta' = -1.3 + 1.6\Delta + 2.08$$

$$\boxed{\Delta' = 1.6\Delta + 0.78}$$

gate-2008

Q: Consider the following expressions for the mass of a nucleus with Z protons and A nucleons.

$$M(A, Z) = \frac{1}{c^2}\left[f(A) + \gamma Z + \lambda Z^2\right].$$

Here f(A) is function of A

$$\gamma = -4a_A$$

$$\lambda = a_c A^{-1/3} + 4a_A A^{-1}$$

a_A and a_c are constants of suitable dimensions. For a fixed A, the expression of Z for the most stable nucleus of A is:

(a) $Z = \frac{A/2}{1 + \left(\frac{a_c}{a_A}\right) A^{2/3}}$

(b) $Z = \frac{A/2}{1 - \left(\frac{a_c}{4a_A}\right) A^{2/3}}$

(c) ✓ $Z = \frac{A/2}{1 + \left(\frac{a_c}{4a_A}\right) A^{2/3}}$

(d) $Z = \frac{A}{1 + A^{2/3}}$

- nuclear force is independent of charge.
- even - even nuclei have spin = 0.

$^{20}_{10}Ne \to$ even -2 nuclei.

Sol:- $\left(\frac{\partial M}{\partial z}\right)_A = 0 \Rightarrow \frac{1}{c^2}[Y + 2\gamma z] = 0$

$\Rightarrow \cancel{\frac{1}{c^2}} [\ z = \frac{-Y}{2\gamma}$.

$z = \frac{+4a_A}{2[a_c A^{-1/3} + 4a_A A^{-1}]}$.

$= \frac{4a_A}{2 \times 4a_A A^{-1}\left[1 + \left(\frac{a_c}{4a_A}\right) A^{2/3}\right]}$

$z = \frac{A/2}{1 + \left(\frac{a_c}{4a_A}\right) A^{2/3}}$

Gate-2008

Q:- The following gives a list of pair containing
(a) a nucleus
(b) One of its properties
find the pair which is inappropriate
(a) (i) $^{20}_{10}Ne$ Nucleus (b) stable nucleus. (b/c even-even)
(b) (i) A spheroidal nucleus (b) an electric quadrupole moment
(c) (i) $^{16}_{8}O$ nucleus (ii) nuclear spin, $I = 1/2$.
(d) (i) ^{238}U nucleus (ii) B.E. = 1785 MeV (approx.).

NET-2013

Q:- The B.E. of a light nucleus (Z, A) in MeV is given by approx. formula.

$B(Z,A) \simeq 16A - 20A^{2/3} - \frac{3}{4} Z^2 A^{-1/3} + 30\frac{(N-Z)^2}{2}$ $(A-2Z)$

where $N = A - Z$ is the neutron no. The value of Z of the most stable isobar for a given A is.

(i) $\frac{A}{2}\left(1 - \frac{A^{2/3}}{160}\right)^{-1}$ (ii) $\frac{A}{2}$

(iii) $\frac{A}{2}\left(1-\frac{A^{2/3}}{120}\right)^{-1}$ (iv) $\frac{A}{2}\left(1+\frac{A^{4/3}}{04}\right)$.

Sol:- $\frac{\partial B}{\partial Z} = 0$.

$$-\frac{3}{4}\, 2Z\, A^{-1/3} + \frac{30}{A}\, 2(A-2Z)(-2) = 0.$$

$$-\frac{Z}{4A^{1/3}} - \frac{20}{A}(A-2Z) = 0.$$

$$-\frac{Z}{4A^{1/3}} - 20 + \frac{40Z}{A} = 0$$

$$\frac{Z}{A}\left[-\frac{A^{+2/3}}{4} + 40\right] = 20.$$

$$Z = \frac{20A}{\frac{40}{2}\left[1-\frac{1}{160}A^{2/3}\right]} = \frac{A}{2}\left[1-\frac{1}{160}A^{-2/3}\right]^{-1}.$$

NET

Q:- $^{27}_{14}Si$ and $^{27}_{13}Al$ are mirror nuclei having identical ground state except for charge. if their mass difference is 6 MeV, estimate their radius, neglecting n-p mass difference.

Sol:-

$$\Delta M = \underbrace{(m_p - m_n)}_{\text{neglected.}} + \frac{2a_c}{A^{1/3}}(Z_1-1)$$

$$6\,\text{MeV} = \frac{2a_c}{(27)^{1/3}}(13)$$

$$6 = \frac{2a_c}{3}\times 13$$

$$\boxed{a_c = \frac{9}{13}\,\text{MeV}}$$

$e = 1.6 \times 10^{-19} C$

$e^2 - 1 MeV - fm$

$F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$

$Fr^2 = \frac{1}{4\pi\epsilon_0} q^2$.

$a_c = \frac{3}{5} \times \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{R}$

$R_0 = \frac{r}{0}$

$N t m^2$

$\underbrace{m t m + m}$

$MeV - fm$

$F = \frac{dU}{dr}$.

$dU = F dr$

Q: NET-2014 The difference in the coulomb energy between the mirror nuclei $^{49}_{24}Cr$ and $^{49}_{25}Mn$ is 6.0 MeV. Assuming that the nuclei have a spherically symmetric charge distribution and that e^2 is approx 1.0 MeV-fm, the radius of $^{49}_{25}Mn$ nucleus is

(a) $4.9 \times 10^{-13} m$ (b) $4.9 \times 10^{-15} m$.

(c) $5.1 \times 10^{-13} m$ (d) $5.1 \times 10^{-15} m$.

Sol:- $E_c = \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{Z^2 e^2}{R}$.

$\Delta E_c = \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{e^2}{R} [Z_1^2 - Z_2^2]$.

$6 = \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{e^2}{R} [\underbrace{(Z_1 + Z_2)}_{A} \underbrace{(Z_1 - Z_2)}_{1}]$

$6 = \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{e^2}{R} \times A$.

$\cancel{6}^{2} MeV = \frac{\cancel{3}}{5} \times 1 \times \frac{49}{R}$.

$R = \frac{49}{10} fm = 4.9 fm = 4.9 \times 10^{-15} m$.

magic no. – 2,8,20,28,50,82,126.

21.10.2015

Nuclear models: –

1. liquid drop model (LDM)
2. Shell model.
3. Collective model.

1. Liquid drop model (LDM): –

Magic no. – 2, 8, 20, 28, 50, 82, 126.

Magic nuclei –

nuclei in which

$$\left.\begin{array}{l} p = 2,\ 8,\ 20, 28, 50,\ 82 \\ n = 2,\ 8,\ 20,\ 28,\ 50,\ 82,\ 126 \end{array}\right\}$$ they have extra stability as compared to their neighbouring nuclei.

Doubly magic nuclei: – nuclei in which both p and n are magic no.'s

Ex: – $^{4}_{2}He$, $^{16}_{8}O$, $^{40}_{20}Ca$ etc.

p = 2, n = 2   p = 8, n = 8   p = 20, n = 20

semi magic no.: – 40.

Liquid Drop model: – • this model was proposed by Bohr

• model is based on the similarities of a nucleus with a liquid drop.

• Here the properties of a nucleus are due to the collective behaviour of all nucleons.

Similarities of Nucleus with liquid drop:—

1. the density of liquid drop is independent of its volume. similarly the density of nucleus is also independent of its volume [$\rho \approx 10^{17}\ kg/m^3$)

2. the cohesive forces among the molecules of a liquid drop are short range. similarity the nuclear forces among nucleons are short range.

3. the evaporations of molecules from the liquid drop surface is analogues to the emission of particles from the nucleus (α, β decay)

4. Latent heat of vaporization is similar to the constant B.E. per nucleon for most of the nuclei.

5. A liquid drop on ~~going~~ gaining energy oscillates and may break up into two parts. similarly is the process of nuclear ~~per~~ fission.

SEMF

Based on the liquid drop model:—

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{A^{3/4}}.$$

⇒ Bohr wheeler Theory of Nuclear fission:—

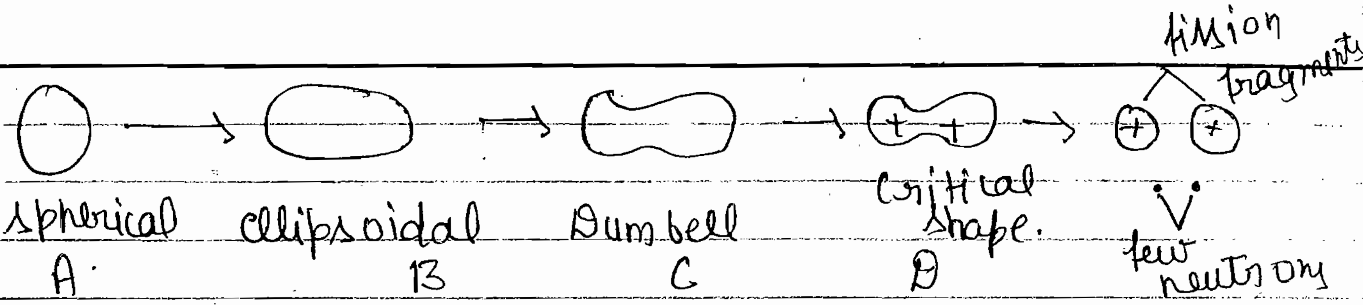
- the fission of a nucleus is similar to the fission of a liquid drop.
- the nuclear fission can be induced by bombardment of some particles like ~~proton~~ neutron.

ex:— $^{1}_{0}n + {}^{235}U \longrightarrow {}^{236}U^{*} \longrightarrow$ excited state nucleus. compound nucleus.

- $^{1}_{0}n + {}^{A}_{Z}X \longrightarrow C^{*}$ $\quad (C^{*} = {}^{A+1}_{Z}Y)$.

$\downarrow$
C^{N}

- the compound nucleus is highly energetic



- this extra energy sets up a series of oscillations and nucleus shape is deformed into ellipsoidal shape (B)
- Here two forces come to play Regaining forces (nuclear forces) have tendency to return back in stage A while determining forces tend towards more deformation.
- If oscillations are large, then dumbell shape is formed (stage C).
- again if oscillations are vident then nucleus goes to critical stage (D) where two positive charge centres are creted, repelling each other.
- finally nucleus breaks-up into two smaller fragments with emission of few neutrons.

Merits and Demerits :-

1. Merits → 1. model is able to explain B.E. and stability of nuclei.
   2. LDM is able to explain the nuclear fission.

2. Demerits :- 1. Unable to explain the magic nos.
   2. spin and parity of nuclear states could not be explained.
   3. magnetic moments and electric quadrupole moments of various nuclei could not be explained.

27.10.2015

Fission :–

Types of fission — – spontaneous
– induced.

Spontaneous – no need of bombarding particles (rare phenomenon).

induced – Bombarding particles needed.

by thermal neutrons (<1 MeV) for odd-A nuclei
ex – Th^{233}, U^{235}
(less stable)

by fast neutrons (>1 MeV).
for even-A nuclei.
ex – Th^{232}, U^{236}.
(more stable).

Symmetric and Asymmetric fission –

• Symmetric – • Heavy nucleus breaks up in two equal parts.

$$(Z, A) \rightarrow \left(\frac{Z}{2}, \frac{A}{2}\right) + \left(\frac{Z}{2}, \frac{A}{2}\right)$$

• the prob. of symmetric fission is very small at low energy of bombarding particles.
• but the prob. of symmetric fission increases with the increase in the energy of bombarding particle.

• Asymmetric – Heavy nucleus breaks up into two unequal parts.

$$(Z, A) \not\rightarrow \left(\frac{Z}{2}, \frac{A}{2}\right) + \left(\frac{Z}{2}, \frac{A}{2}\right)$$

i.e.

$$(Z, A) \longrightarrow (Z_1, A_1) + (Z_2, A_2)$$

where $Z_1 \neq Z_2$
$A_1 \neq A_2$.

Typical fission curve → Let us take the example of ^{235}U

$$ {}^{1}_{0}n + {}^{235}U \rightarrow {}^{236}U^{*} \longrightarrow X + Y + \nu n + Q $$

$^{236}U^{*}$ → compound nucleus (C.N.)

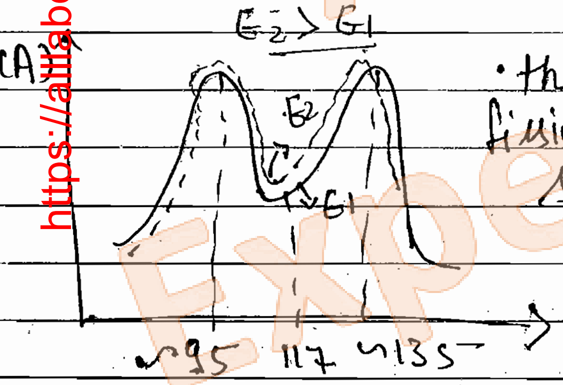
$X + Y$ → fission fragments

νn → no. of neutrons

- decay products are not unique.
  other modes of decay.
  $X_1 + Y_1$
  $X_2 + Y_2$ etc.

- the % yield = $\dfrac{\text{no. of nuclei of mass no. A produced } \{N(A)\}}{\text{total no. of fission} \quad N_0} \times 100$

$$ \% \, Y(A) = \frac{N_A}{N_0} \times 100 $$



- these peaks represents asymmetric fission and the minimum curve represents symmetric fission.

- peaks corresponds to asymmetric fission. valley corresponds to symmetric fission.

- the probability of symmetric fission is very small as evident from the fission curve.
- as energy increases the prob. of symmetric fission increases.

Estimation of liberated energy in fission: —

by masses of fission fragments | by using binding energy curve.

$${}_0^1n + {}^{235}U \longrightarrow X + Y + un + Q$$

$$Q = [M(U^{235}) + m(n) - M(X) - M(Y) - m(un)]$$

Ex: — if $X = {}^{98}Mo$,

$Y = {}^{136}Xe$.

$M(X) = 97.936$ amu.

$M(Y) = 135.951$ amu

$m(n) = 1.008$.

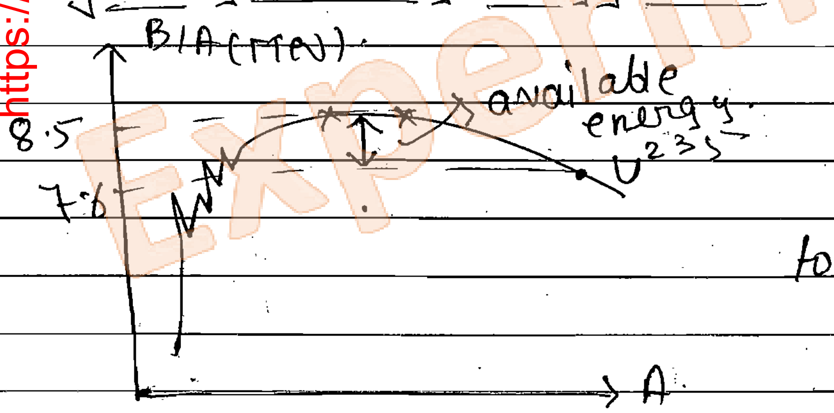
$M(U^{235}) = 235.1290$

$= [235.1290 + 1.0086 - 97.936 - 135.951 - 2 \times 1.0086]$ amu

$= 0.228 \times 931$ MeV.

$= 212$ MeV.

By using Binding Energy curve: —



Available energy

$= 8.5 - 7.6$

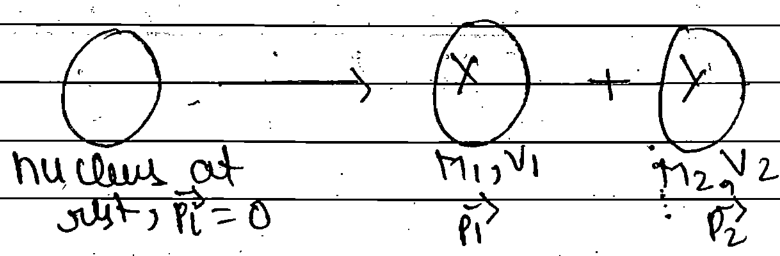
$= .9$ MeV/nucleon.

total energy liberated.

$= .9 \times 235$

$\simeq 214$ MeV.

⇒ Energy and momentum conservation in fission: —



neglecting neutron mass.

$$\vec{p_f} = \vec{p_1} + \vec{p_2}$$

$\vec{P_i} = \vec{P_f}$

$\boxed{P_1 = P_2}$ numerically.

$$M_1 V_1 = M_2 V_2$$

$$\boxed{V_2 = \frac{M_1}{M_2} V_1}$$

⇒ Ratio of energies of ~~fission~~ fragments:

$$\frac{E_1}{E_2} = \frac{\frac{1}{2} M_1 V_1^2}{\frac{1}{2} M_2 V_2^2} = \frac{M_1}{M_2} \times \left(\frac{M_2}{M_1}\right)^2$$

$$\boxed{\frac{E_1}{E_2} = \frac{M_2}{M_1}}$$

Condition of spontaneous symmetric fission: —

• $\frac{Z^2}{A}$ is a parameter which measures the stability of a nucleus against fission.

In symmetric fission: —

$$^{A}_{Z}X \rightarrow {}^{A_1}_{Z_1}Y_1 + {}^{A_2}_{Z_2}Y_2 + Q.$$

where, $Z_1 = Z_2 = \frac{Z}{2}$

$$A_1 = A_2 = \frac{A}{2}.$$

$$M(Z, A) \longrightarrow M\left(\frac{Z}{2}, \frac{A}{2}\right) + M\left(\frac{Z}{2}, \frac{A}{2}\right).$$

~~SEMA~~ →

$$Q = \left[M(Z, A) - 2M\left(\frac{Z}{2}, \frac{A}{2}\right)\right] \quad \text{---} (1)$$

SEMF →

$$M(Z,A) = Zm_H + (A-Z)m_n - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}}$$

$$+ \underbrace{a_a \frac{(A-2Z)^2}{A} + \frac{a_p}{A^{3/4}}}_{\text{neglected.}}$$

$$M\left(\frac{Z}{2}, \frac{A}{2}\right) = \frac{Z}{2} m_H + \frac{1}{2}(A-Z)m_n - a_v \frac{A}{2}$$

$$+ a_s \left(\frac{A}{2}\right)^{2/3} + a_c \frac{(Z/2)^2}{(A/2)^{1/3}}.$$

$$Q = a_s A^{2/3}\left[1 - 2\times\frac{1}{(2)^{1/3}}\right] + a_c \frac{Z^2}{A^{1/3}}\left[1 - 2\times\frac{(1/2)^2}{(1/2)^{1/3}}\right] 6.$$

$$Q = a_s A^{2/3}\left[1 - 2^{1/3}\right] + a_c \frac{Z^2}{A^{1/3}}\left[1 - \frac{1}{2^{2/3}}\right]$$

$$\boxed{Q = \underset{-ve}{-0.26 a_s A^{2/3}} + \underset{+ve.}{0.37 a_c \frac{Z^2}{A^{1/3}}}} \quad \text{---} (2).$$

- if coulombian term dominates over the surface term then $Q > 0 \rightarrow$ fission will occur.

if surface term dominate over the coloumbian term $Q < 0 \rightarrow$ fission will not occur.

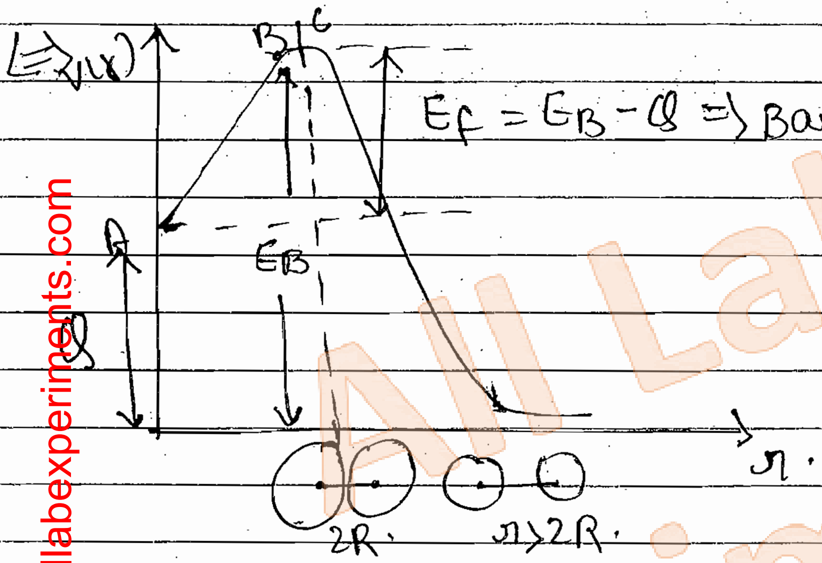
for $Q \geq 0$

$$0.37 a_c \frac{Z^2}{A^{1/3}} \geq 0.26 a_s A^{2/3}$$

$$\Rightarrow \frac{Z^2}{A} \geq \frac{.26}{.37} \frac{a_s}{a_c} \qquad a_s \simeq 13\,\text{MeV}$$
$$a_c \simeq 0.5\,\text{MeV}.$$

then $\boxed{\frac{Z^2}{A} \geq 15}$

this condition is for the spontaneous symmetric fission the observed results do not match with this condition. ∵ Bohr and wheeler used the concept of potential barrier at the time of fission.



- the shape of the potential barrier b/w two fragments is shown in the fig.
- for $r > 2R$

only coulombian force b/w two fragments so,

$$V = \frac{1}{4\pi\epsilon_0} \frac{(Z/2\,e)(Z/2\,e)}{r} = \frac{1}{4\pi\epsilon_0} \frac{Z^2 e^2}{4r}.$$

- for $r = 2R$ - when they are just to separate out (just touch).

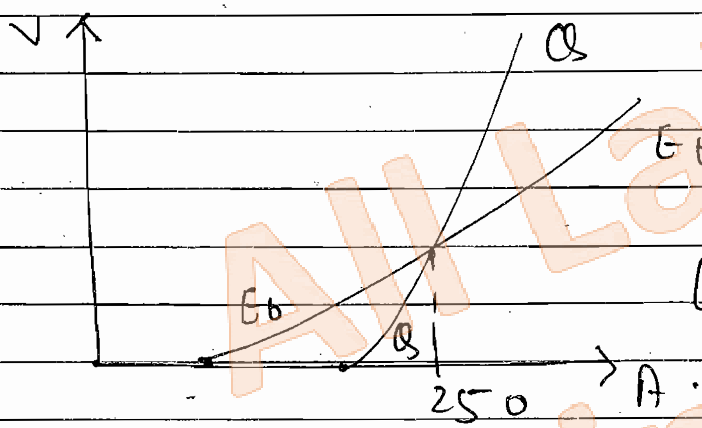
nuclear force also comes to play along with the coulombian force so the peak of barrier is B not C.

- for $r < 2R$ — the nuclear forces dominate over the coulombian force and the curve ~~for~~ falls to A.

→ for fission to occur the two bells must cross the barrier height.

$$E_B = \frac{1}{4\pi\epsilon_0} \frac{Z/2e \times Z/2e}{2R} = \frac{Z^2e^2}{32\pi\epsilon_0 R}.$$

i.e. $Q > E_b$ for fission.



but for nuclei upto $A = 250$, $Q < E_b$ - so fission will not occur. (However there is chance of fission by tunneling — very low probability).

- for $A > 250$, $Q > E_b$ so spontaneous fission will occur

$Q > E_b$.

$$\Rightarrow \left[-0.26 a_s A^{2/3} + 0.37 a_c \frac{Z^2}{A^{1/3}}\right] > \frac{Z^2e^2}{32\pi\epsilon_0 R}.$$

On simplification we get.

$$\boxed{\frac{Z^2}{A} > 52}$$

- if Quantum mechanical tunneling is also taken into account then condition becomes.

$$\boxed{\frac{Z^2}{A} \geq 44}$$

Problems

Q:- The typical energies released in fission and fusion are
(a) 50 MeV and 1000 MeV.
(b) 200 MeV and 1000 MeV.
(c) 1000 MeV and 50 MeV.
(d) 200 MeV and 10 MeV.

Q:- Consider the following statement about the asymmetric fission of a heavy nucleus.
1. The magnitude of momenta of light and heavy fragments are equal
2. The K.E. of light and heavy fragments are equal.
3. The momenta as well as K.E. of two fragments are equal.
Which of the above statements are correct.

Q:- $E_B(Z,N) = 16A - 17A^{2/3} - 0.6\frac{Z^2}{A^{1/3}} - 25\frac{(N-Z)^2}{A}$

in units of MeV.
in case of symmetric fission $A = Z = N$, show that Q-value of symmetric fission is +ve only for large A, i.e. heavy nuclei.

Sol:- $M(Z,A) = Zm_H + (A-Z)m_n - 16A + 17A^{2/3} + 0.6\frac{Z^2}{A^{1/3}} + 25\frac{(N-Z)^2}{A}$ neglect

$M\left(\frac{Z}{2},\frac{A}{2}\right) = \frac{Z}{2}m_H +$

$$\frac{b^2}{a^2} = (1-\epsilon^2)$$

this condition occurs for nuclei having large A i.e. heavy nuclei.

NET Q:- Given SEMF for B.E. of a nucleus $^A_Z X$ as:

$$B(Z,A) = \alpha A - \beta A^{2/3}\left(1+\frac{2}{5}\epsilon^2\right) + \gamma Z^2 A^{-1/3}\left(1-\frac{1}{5}\epsilon^2\right)$$

where $\alpha = 14$ MeV.

$\beta = 13$ MeV.

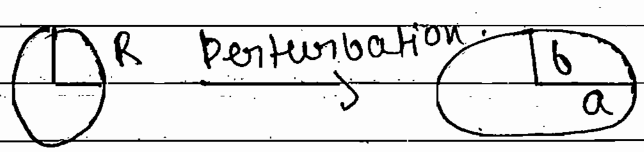
$\gamma = 0.6$ MeV.

ϵ = eccentricity

find the limiting conditions involving Z and A so that nucleus can undergo spontaneous fission.

Sol:- for undeformed nucleus $\epsilon = 0$

$$B(Z,A) = \alpha A - \beta A^{2/3} - \gamma Z^2 A^{-1/3}$$



$$\frac{b^2}{a^2} = 1-\epsilon^2.$$

undeformed. deformed.

no change in volume energy.
E_s and E_c both change.

Deformed nucleus:—

$$B(Z,A) = \alpha A - \beta A^{2/3}\left(1+\frac{2}{5}\epsilon^2\right) + \gamma Z^2 A^{-1/3}\left(1-\frac{1}{5}\epsilon^2\right).$$

net change in the combined surface and columbian energies $\Delta E = \Delta E_s + \Delta E_c$.

$$\Delta E = (E_s - E_{s_0}) + (E_c - E_{c0}) \text{ underformed}$$

deformed   undeformed   deformed

$$\Delta E = \left[\beta A^{2/3}\frac{2}{5}\epsilon^2 - \gamma Z^2 A^{-1/3}\frac{1}{5}\epsilon^2\right]$$

if ΔE = +ve bound.
ΔE = -ve unbound.

for fusion to occur

$$\gamma Z^2 A^{-1/3} \times \frac{1}{5}\cancel{\epsilon^2} > \beta A^{2/3} \times \frac{2}{5}\cancel{\epsilon^2}.$$

$$\boxed{\frac{Z^2}{A} > \frac{2\beta}{\gamma}}$$

$$\frac{Z^2}{A} > \frac{2\times 13}{0.6}$$

$$\frac{Z^2}{A} > \frac{260}{6} = 43.3$$

$$\therefore \boxed{\frac{Z^2}{A} > 43}$$

Q:- The energy produced in the fission of 1gm of ^{235}U will be about.

(i) 1.5×10^{23} mev.

(ii) 2.5×10^{23} mev.

(iii) 4×10^{23} mev.

(iv) 5×10^{23} mev.

(given that each ^{235}U gives 200 mev in fission).

Sol:- energy released in each fission = 200mev.

1 mole of ^{235}U = 235 gm.

no. of nuclei in 1mole = 6.023×10^{23}.

235gm = 6.023×10^{23} nuclei.

$\Rightarrow$ 1gm = $\frac{6.023\times10^{23}}{235}$ nuclei.

total energy = $\frac{6.023\times10^{23}}{235}\times200$ mev.

= $\frac{1204.6\times10^{23}}{235}$ mev.

= 5.12×10^{23} mev.