Free Study Material from

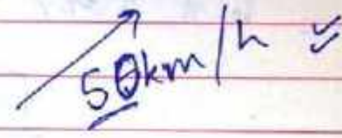
All Lab Experiments

Mathematical Physics for JAM/NET/Gate Physics

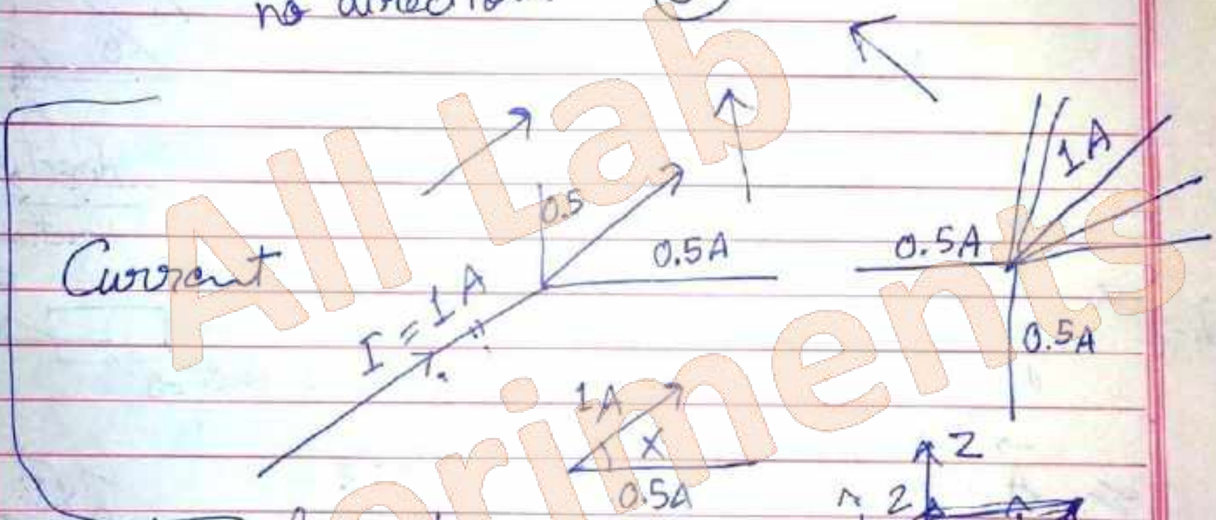
> Vector Algebra & Calculus <

Vectors → Algebra
Vectors → Calculus

Vector



Scalar (number)
no direction 5

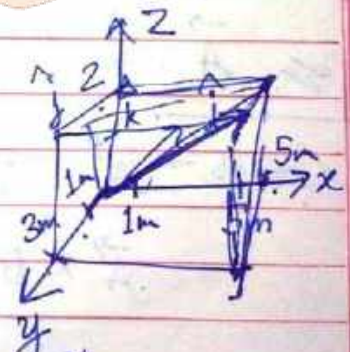


Magnitude of a vector →

$$\vec{A} = 5\hat{i} + 3\hat{j} + 2\hat{k}$$

a_1 a_2 a_3

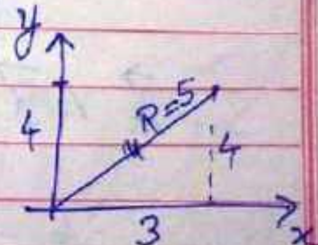
=



$$|\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

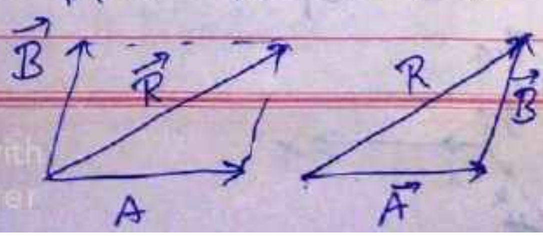
Resultant Vector

$$R = \sqrt{|A|^2 + |B|^2 + 2|A||B|\cos\theta}$$



$3^2 + 4^2 = R^2$

$R = 5$



Vector Product → Dot, Cross

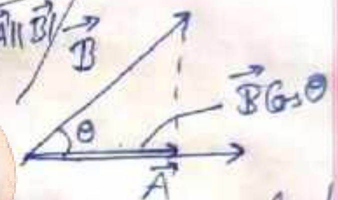
$\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Dot

$\vec{A}\cdot\vec{B} = |A||B|\cos\theta \rightarrow \cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$

$|A| = \sqrt{a_1^2 + a_2^2 + a_3}$



$W = \int \vec{F}\cdot d\vec{s}$

direction of applicatin

direction of actin

$|\hat{i}| = 1$

$\hat{i}\cdot\hat{i} = 1.1\cos 0^\circ = 1$

$\hat{i}\cdot\hat{j} = 0$

$\hat{j}\cdot\hat{j} = 1 \qquad \hat{k}\cdot\hat{k} = 1$

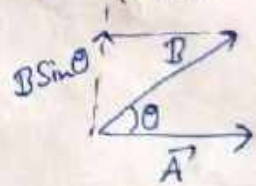
$\hat{i}\cdot\hat{j} = 1.1\cos 90^\circ = 0$

$\vec{A}\cdot\vec{B} = a_1b_1 + a_2b_2 + a_3b_3$

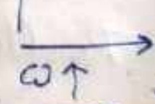
Cross

Cross → $\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta \rightarrow \sin\theta = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}||\vec{B}|}$

$\vec{A}\times\vec{B} = 0$ (|| vectors)



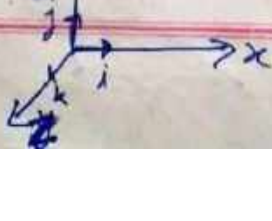
$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \underline{\underline{\text{det}}}$$

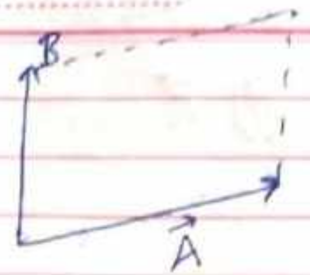


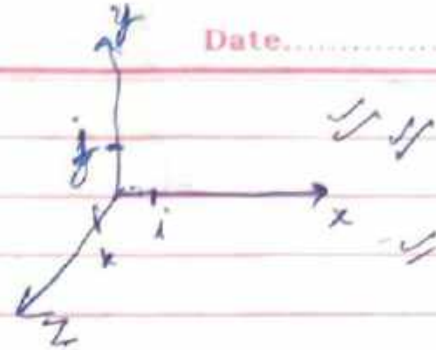
$\hat{i}\times\hat{j} = \hat{k}$

$\hat{j}\times\hat{k} = \hat{i}$

$\hat{k}\times\hat{i} = \hat{j}$

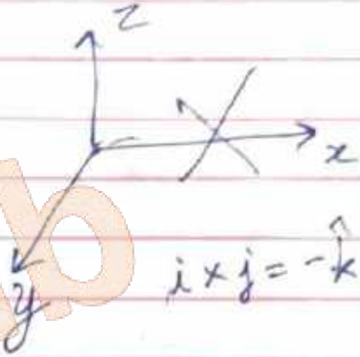






∥gram

$\vec{A} \times \vec{B}$ = Area of this ∥gram



$i \times j = -\hat{k}$

$\vec{A} \times \vec{B} = \vec{C}$

$\vec{B} \times \vec{A} = -\vec{C}$

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

Triple Product → Scalar triple Product
→ Vector triple Pro. $\vec{A} \times (\vec{B} \times$

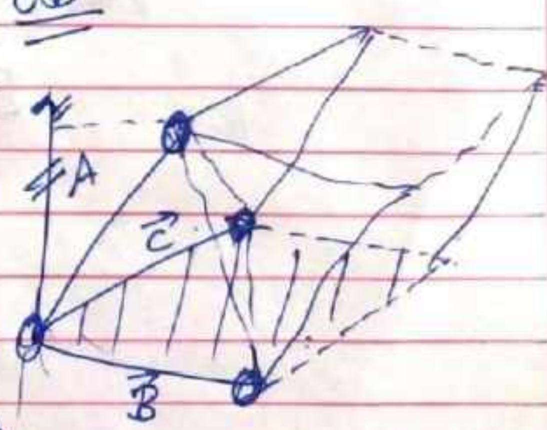
$\underset{a_1 a_2 a_3}{\vec{A}}$ $\overset{b_1 b_2 b_3}{\vec{B}}$ $\vec{C}\ c_1 c_2 c_3$

$\vec{A} \cdot (\vec{B} \times \vec{C})$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \underline{\text{det}}$$

|||

~~Area~~ Volume of ∥piped.



$\frac{1}{6}$ V. of ∥piped ≡ V. of tetrahedron

$$A.(\vec{B}\times\vec{C}) = C.(A\times B) = \vec{B}.(\vec{C}\times\vec{A})$$

$$\vec{A}.(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B}).\vec{C}$$

$$\vec{B}\times\vec{C} = -(\vec{C}\times\vec{B})$$

$$A.(\vec{B}\times\vec{C}) = -\vec{A}.(\vec{C}\times\vec{B})$$

Vector triple product

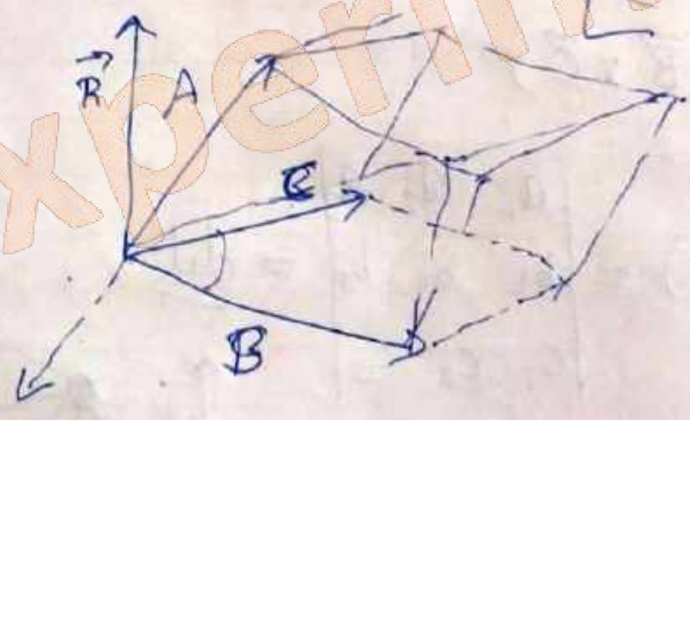
$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$$

$$\vec{A}\times(\vec{B}\times\vec{C}) + \vec{B}\times(\vec{A}\times\vec{B}) + \vec{B}(\vec{C}\times\vec{A}) = 0$$

Jacobi's identity

$$\vec{A}\times(\vec{B}\times\vec{C}) \neq (\vec{A}\times\vec{B})\times\vec{C} = -\vec{C}\times(\vec{A}\times\vec{B})$$

$$= -[(\vec{C}.\vec{B})\vec{A} - (\vec{C}.\vec{A})\vec{B}]$$



4 Vectors

Scalar $(\vec{A}\times\vec{B})\cdot\underbrace{(\vec{C}\times\vec{D})}_{\vec{R}} = (\vec{A}\times\vec{B})\cdot\vec{R} = \vec{A}\cdot(\vec{B}\times\vec{R})$

$= \vec{A}\cdot(\vec{B}\times(\vec{C}\times\vec{D})) = \vec{A}\cdot((\vec{B}\cdot\vec{D})\vec{C} - (\vec{B}\cdot\vec{C})\vec{D})$

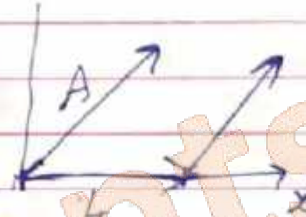
$= (\vec{B}\cdot\vec{D})(\vec{A}\cdot\vec{C}) - (\vec{B}\cdot\vec{C})(\vec{A}\cdot\vec{D})$

Vector $\underbrace{(\vec{A}\times\vec{B})}_{\vec{R}}\times(\vec{C}\times\vec{D})$

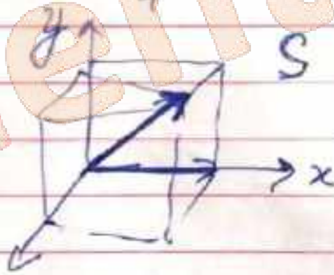
$= [(\vec{A}\times\vec{B})\cdot\vec{D}]\vec{C} - [(\vec{A}\times\vec{B})\cdot\vec{C}]\vec{D}$

$A = a_1 i + a_2 j + a_3 k$

$= (a_1+4)i + a_2 j + a_3 k$



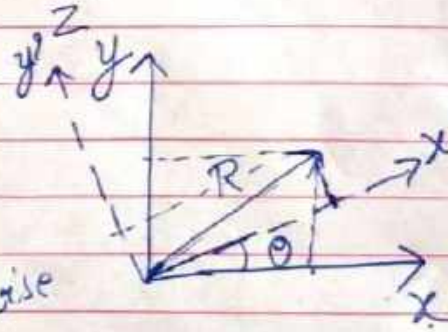
Rotation of a vector →



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$



XY frame Rotates at an angle θ anticlockwise

|||

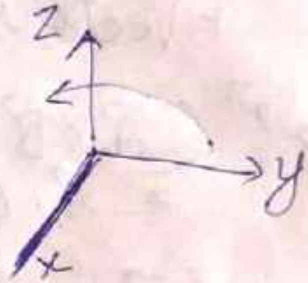
Vector R Rotates an angle θ clockwise

Date..................

in 3D $\Rightarrow$

along X $\rightarrow$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



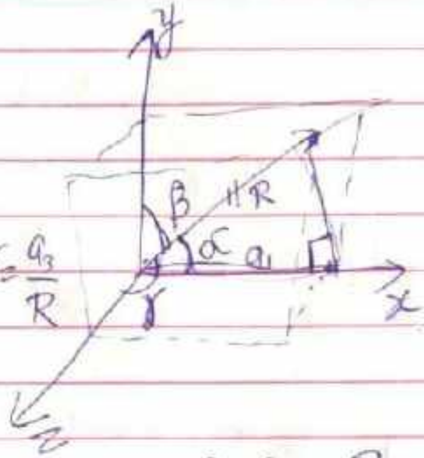
along y axis

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Directional Cos →

$\vec{A} = a_1 \hat{i} + a_2 \hat{j} + a_3 k$

$R = \sqrt{a_1^2 + a_2^2 + a_3^2}$



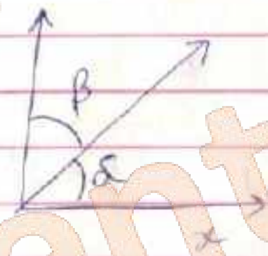
$\cos\alpha = \dfrac{a_1}{R}$ $\quad \cos\beta = \dfrac{a_2}{R}$ $\quad \cos\gamma = \dfrac{a_3}{R}$

$R^2 = a_1^2 + a_2^2 + a_3^2$

$1 = \dfrac{a_1^2}{R^2} + \dfrac{a_2^2}{R^2} + \dfrac{a_3^2}{R^2}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos\theta = \dfrac{B}{H}$

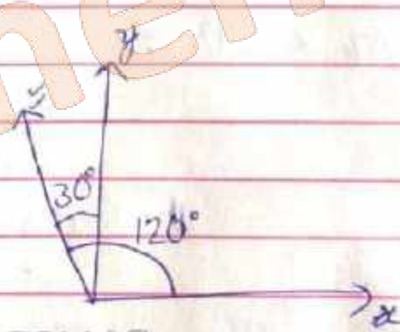


$n = \cos 120° \hat{i} + \cos 30° \hat{j}$

$= \cos(90+30)\, \hat{i} + \sqrt{3}/2\, \hat{j}$

$= -\dfrac{\sqrt{3}}{2} \hat{i} + \sqrt{3}/2\, \hat{j}$

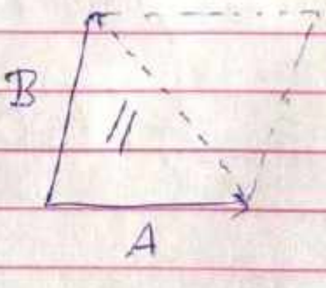
$A = 10 \hat{i} + 7 \hat{j}$



$\vec{A} =$

$\vec{B} =$

$\Delta = \dfrac{1}{2} [A \times B]$



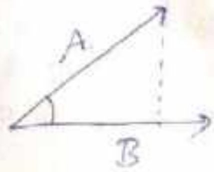
Tetrahedron $= \dfrac{1}{6} A \cdot [B \times C]$

Projection of vec. A on B is $\frac{19}{9}$

$\vec{A} = i + mj + k \qquad \vec{B} = 4i - 4j + 7k$ Date.................

$\frac{19}{9}$

$$\hat{B} = \frac{4i - 4j + 7k}{\sqrt{4^2 + 4^2 + 7^2}} = \frac{\boxed{}}{\sqrt{16+16+49}}$$



$$= \frac{4i - 4j + 7k}{\sqrt{81}} = \frac{\boxed{}}{9}$$

$$\vec{A} \cdot \hat{B} = [i + mj + k]\left[\frac{4i - 4j + 7k}{9}\right] = \frac{19}{9}$$

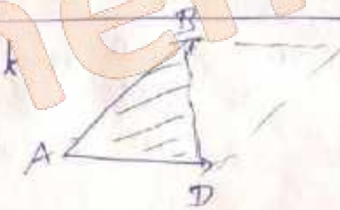
$$= 4 - 4m + 7 = 19$$

$$11 - 19 = 4m$$

$$-8 = 4m$$

$$m = -2$$

$\vec{AB} = i - 2j + 3k \qquad \vec{AD} = 2i + j + 4k$

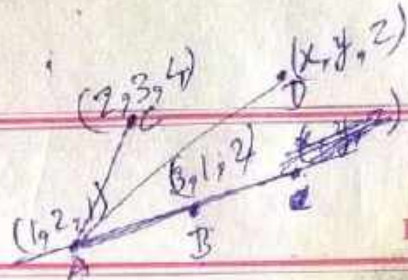


$$\vec{AB} \times \vec{AD} = \begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ 2 & 1 & 4 \end{vmatrix} =$$

$$= 5i + 10j + 5k$$

Area of ||gm $= \sqrt{5^2 + 10^2 + 5^2} = \sqrt{25+25+100} = \sqrt{150}$

Area of $\Delta = \frac{1}{2}\sqrt{150}$



$(x_1 y_1 z_1)$ $(x_2 y_2 z_2)$

$(x_2-x_1)i + (y_2-y_1)j + (z_2-z_1)k$

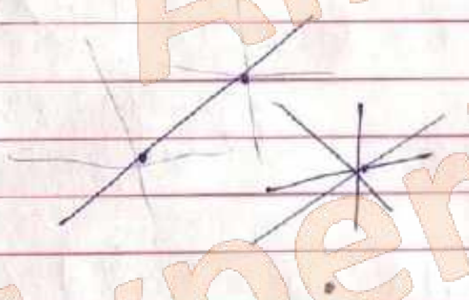
$\vec{AB} = (3-1)i + (1-2)j + (2-1)k = 2i - 1j + 1k$

$\vec{AD} = (x-1)i + (y-2)j + (z-1)k = (x-1)i + (y-2)j + (z-1)k$

$\vec{AC} = (2-1)i + (3-2)j + (4-1)k = 1i + 1j + 3k$

$\vec{AB} \cdot (\vec{AD} \times \vec{AC}) = 0$

$$\begin{vmatrix} 2 & -1 & 1 \\ x-1 & y-2 & z-1 \\ 1 & 1 & 3 \end{vmatrix} = 0$$

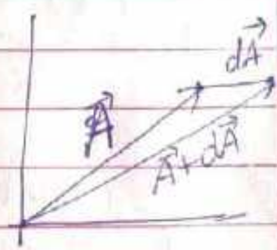


$+2x + 3y + 5z = 0$

$[\qquad] = 0$

$\vec{A}$ m $\vec{B} =$ ____ $\vec{C} =$ ____

E&M Vector calculus



$$\frac{d\vec{A}}{dt} = \frac{(\vec{A}+d\vec{A}) - \vec{A}}{dt} = v$$

$$\frac{d^2A}{dt^2} = a$$

A particle moves along the curve

$x = 2t^2 \quad y = t^2 - 4t \quad z = 3t - 5$ m

at $t = 1$ sec. position vel. acc.

$\vec{R} = x\,i + y\,j + z\,k = (2t^2)i + (t^2 - 4t)j + (3t-5)k$
$= 2i + (-3)j + (-2)k$ m

$\frac{d\vec{R}}{dt} = 4t\,i + (2t-4)j + 3k = 4i + (-2)j + 3k$ m/s

$\frac{d^2\vec{R}}{dt} = 4i + 2j + 0k = 4i + 2j$ m/s²

$\frac{d}{dt}(\vec{A}\cdot\vec{B}) = \vec{A}\cdot\frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt}\cdot\vec{B}$

$\frac{d}{dt}(\vec{A}\pm\vec{B}) = \frac{dA}{dt} \pm$ ...

$\frac{d}{dt}(\vec{A}\times\vec{B}) = \vec{A}\times\frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt}\times\vec{B}$

$\frac{d}{dt}(c\vec{A}) = c\,\frac{d\vec{A}}{dt}$

$f = \underline{2t+3}$

$\frac{d}{dt}(f\vec{A}) = f\,\frac{d\vec{A}}{dt} + \frac{df}{dt}\vec{A}$

f → scalar function

Teacher's Sign.............

partial diff. $\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$ $\frac{\partial}{\partial z}$

$$\frac{\partial x^2 y}{\partial x} = y \frac{\partial x^2}{\partial x} = y(2x)$$

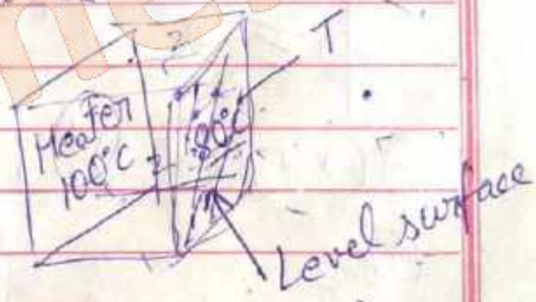
$$\frac{d}{dx} x^2 y = x^2 \frac{dy}{dx} + y \frac{dx^2}{dx} =$$

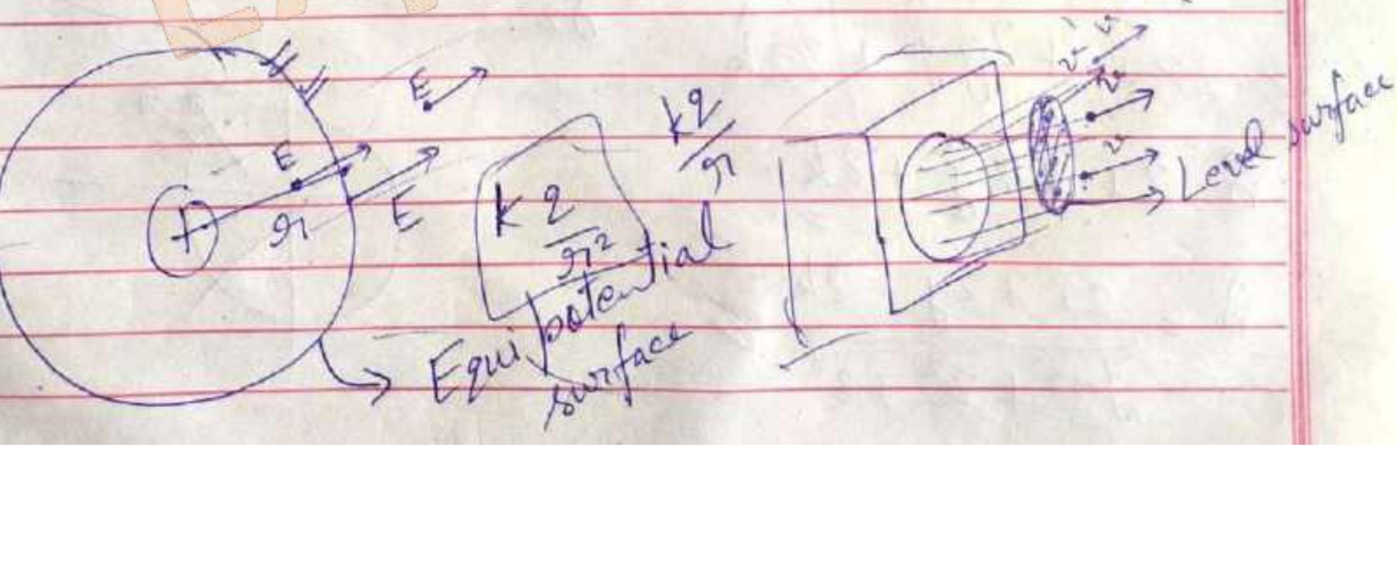
Del operator

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$$

Gradiant Divergence Curl

field — Scalar field, Vector field

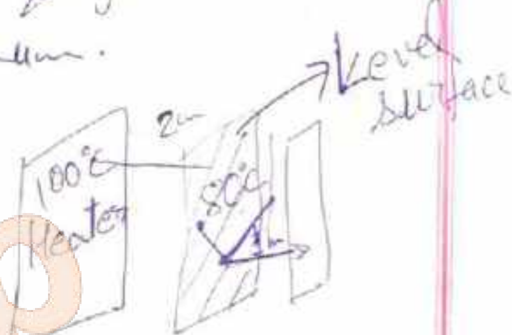




Rate = variation wrt time

Gradient ↳ variation with r.t. space $\quad v + \frac{\partial v}{\partial x} \quad \frac{d}{dx}$

Grad = The direction in which change in field quantity is maximum.



(1) How to apply

$$\nabla \equiv \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) \phi$$

$$\underline{\nabla \phi} \equiv \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

Scalar function

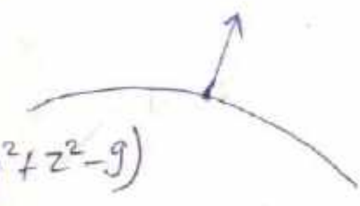
$$\phi \equiv x^2 + y^2 + z^2 = 3^2$$

$(1,1,1)$

$$\boxed{E = -\nabla V}$$

$$\phi \equiv x^2 + y^2 + z^2 - 9$$

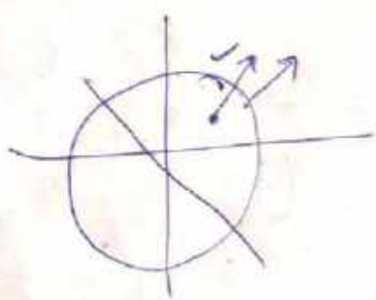
$$\frac{\partial}{\partial x}(x^2+y^2+z^2-9)\hat{i} + \frac{\partial}{\partial y}(x^2+y^2+z^2-9)\hat{j} + \frac{\partial}{\partial z}(x^2+y^2+z^2-9)$$



$$2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

$$\vec{n} \quad 2i + 2j + 2k$$

$$\hat{n} = \frac{2i + 2j + 2k}{\sqrt{2^2 + 2^2 + 2^2}}$$



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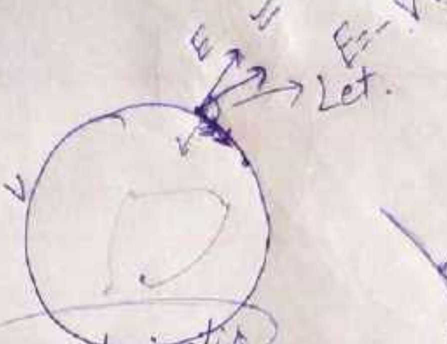
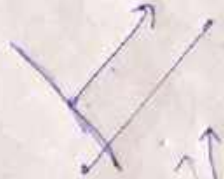




direc. derivation

$\phi = x^2 y z^3$ along $\underbrace{2i + 2j + 1k}_{\vec{A}}$

Step 1 $\nabla\phi = \frac{\partial \phi}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$

$= 2xyz^3 \hat{i} + x^2 z^3 \hat{j} + x^2 y (3z^2) \hat{k}$

$(1, 1, 1)$

Max. change in field. $= 2\hat{i} + j + 3\hat{k}$

directional derivative

Step 2. $\nabla\phi \cdot \hat{A} = (2i + j + 3k) \cdot \left(\frac{2i + 2j + 1k}{\sqrt{2^2 + 2^2 + 1^2}} \right) =$

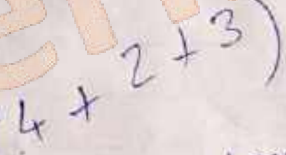
$= \frac{1}{3}(4 + 2 + 3) = \frac{9}{3} = 3$

dot product

$\vec{r} = xi + yj + zk$

$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

projection $\rightarrow$ dot



$\cos\theta = \frac{B}{H}$

Base = Hypo. $\cos\theta$

$= \vec{B} \cos\theta$

$\vec{A} \cdot \vec{B} =$

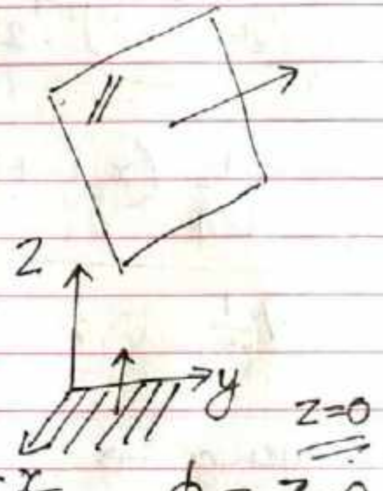
Teacher's Sign..................

$\nabla\vec{r} = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) \left(\sqrt{x^2 + y^2 + z^2} \right)$

Del. →

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$$

Grad. →



$$\nabla \phi = \underbrace{\frac{\partial z}{\partial x}}_{=0} \hat{i} + \underbrace{\frac{\partial z}{\partial y}}_{=0} \hat{j} + \frac{\partial z}{\partial z} \hat{k}$$

$$= \hat{k}$$

$Z=0$

$\phi \equiv Z-0$

find the unit normal vector to

$$\phi \equiv x^2+y^2-Z=1 \quad (1,1,1)$$

$$\nabla\phi = 2x\hat{i} + 2y\hat{j} - 1\hat{k}$$

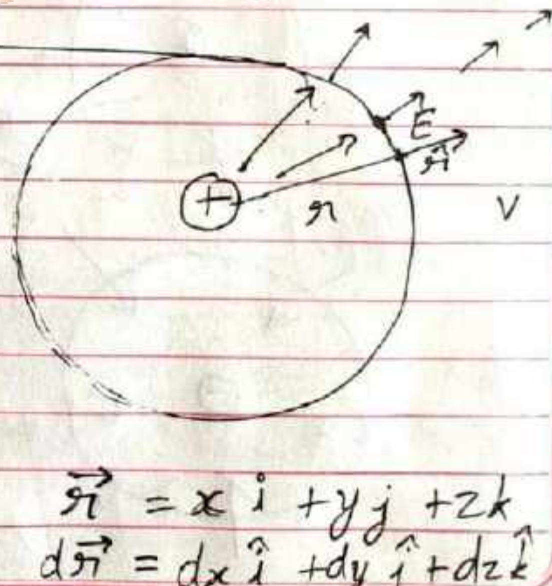
$$\nabla\phi_{(1,1,1)} = 2\hat{i} + 2\hat{j} - 1\hat{k}$$

$$\frac{\nabla\phi}{|\nabla\phi|} = \frac{2i+2j-1k}{\sqrt{2^2+2^2+1^2}} = \frac{2i+2j-1k}{3}$$

$$E = -\nabla V$$

$$E = -\nabla\, k\frac{q}{r}$$

$$= kq\, -\nabla \frac{1}{r}$$



$$k\frac{q}{r^2}\hat{r} = kq\left(-\nabla \frac{1}{r}\right)$$

$$-\frac{1}{r^2}\hat{r} = \nabla \frac{1}{r}$$

$$\vec{r} = x\,i + y\,j + z\,k$$

$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

∇r

$$|\vec{r}| = \sqrt{x^2+y^2+z^2}$$

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Parul Date..................

$|\vec{r}| = \sqrt{x^2+y^2+z^2}$

$\vec{r}$ = bold r
$|\vec{r}|$ = normal r

$\nabla|\vec{r}|$

$\frac{\partial}{\partial x}(\quad)i + \frac{\partial}{\partial y}(\quad)j + \frac{\partial}{\partial z}(\quad)k$

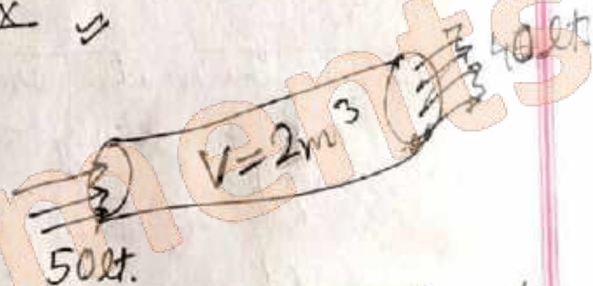
$\frac{1}{2}(x^2+y^2+z^2)^{\frac{1}{2}-1}\,2x\,i + \frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}}\,2y\,j + \frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}}\,2z\,k$

$\frac{1}{|\vec{r}|}(x\,i + y\,j + z\,k) = \frac{\vec{r}}{|\vec{r}|} = \hat{r}$

$\nabla\frac{1}{|\vec{r}|} = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}\hat{k}\right)(x^2+y^2+z^2)^{-\frac{1}{2}} = -\frac{\vec{r}}{r^3} = -\frac{\hat{r}}{r^2}$

Divergence $\rightarrow \dfrac{\text{Change in flux}}{\text{Volume}}$

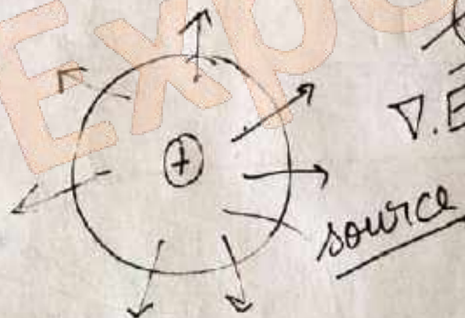
$\phi = \int E.ds$



$\nabla.\vec{A}$

Change in Q. of water

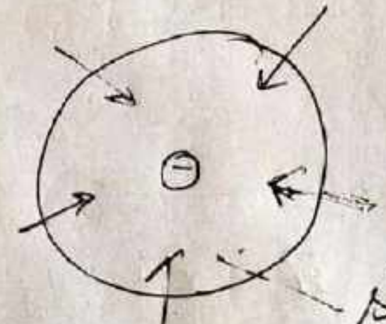
$\nabla.\vec{E}$ = +tive



source

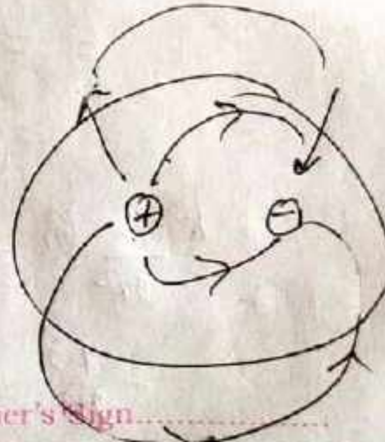
$40 - 50 = -10\,lt$

$= \dfrac{-10\,lt}{2m^3} = -5\,lt/m^3$



$\nabla.\vec{E}$ = −tive

sink



$\nabla.E = 0$

Teacher's Sign..................

Div $A = \vec{\nabla}\cdot\vec{A} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(A_1\hat{i} + A_2\hat{j} + A_3\hat{k}\right)$

$= \frac{\partial}{\partial x}A_1 + \frac{\partial}{\partial y}A_2 + \frac{\partial}{\partial z}A_3$

$\vec{V} = (x+3y)\hat{i} + (y-3z)\hat{j} + (x-2z)\hat{k}$

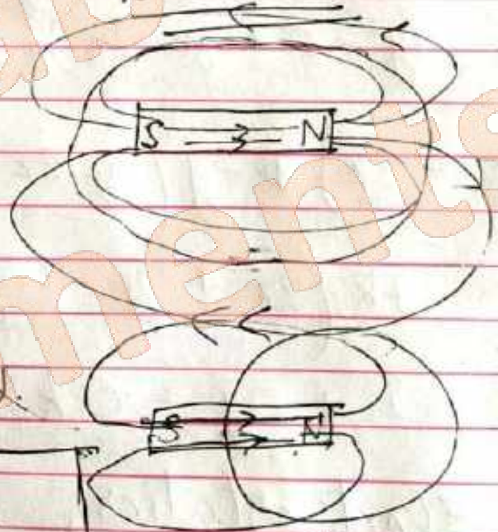
$\vec{\nabla}\cdot\vec{V} = \frac{\partial(x+3y)}{\partial x} + \frac{\partial(y-3z)}{\partial y} + \frac{\partial(x-2z)}{\partial z}$

$= 1 + 1 - 2 = 0$

→ This field is solenoidal

$\nabla\cdot B = 0$

monopoles can't exist



$\vec{B} = B_0(x\,i + y\,j)$

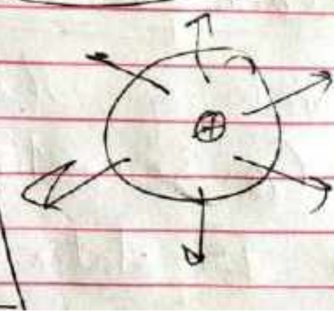
$\nabla\cdot\vec{B} = 2B_0$ This can't be mag. field

$\vec{A} = (xyz)i + (3x^2y)j + (xz^2 - y^2z)k$

$(2,-1,1)$

$\vec{\nabla}\cdot\vec{A} = yz + 3x^2 + x(2z) - y^2$

$= -1 + 12 + 4 = \underline{\underline{15}}$

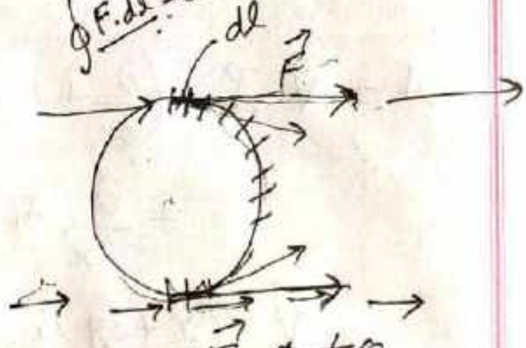


$\nabla\cdot E =$

Date

$\oint F.dl = 0$

Curl $\rightarrow$ $\dfrac{\text{line integral}}{\text{Area}}$



$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} \quad \vec{F}$$

$\nabla \times \vec{F} \neq 0$

$$i\left(\frac{\partial}{\partial y}F_3 - \frac{\partial}{\partial z}F_2\right) - j\left(\frac{\partial}{\partial x}F_3 - \frac{\partial}{\partial z}F_1\right) + k\left(\frac{\partial}{\partial x}F_2 - \frac{\partial}{\partial y}F_1\right)$$

$\boxed{\nabla \times \vec{F} = 0}$

$\Downarrow$

irrotational

$$\vec{A} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$$

$$i(3x - 3x) - j(3y - 2z - (-2z + 3y)) + k(3z + 2y - (2y + 3z))$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$0i + 0j + 0k$$

Electrostatics $\quad \nabla \times \vec{E} = 0$

which of this field represents electrostatic field.

$\rightarrow$ Conservative

$\rightarrow$ Irrotational

$E = E_0(xy\,i + zj)$

$\nabla \times E = \frac{\partial B}{\partial t}$

$$E_0 \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & z & 0 \end{vmatrix} = -1j - j(0) + xk \neq 0$$

$$\vec{\nabla}\cdot(\phi\vec{A}) = (\vec{\nabla}\phi)\cdot\vec{A} + \phi(\vec{\nabla}\cdot\vec{A})$$
$$= (grad\,\phi)\cdot\vec{A} + \phi(div\,A)$$

$$\vec{\nabla}\times(\phi\vec{A}) = \phi(\vec{\nabla}\times\vec{A}) + (\vec{\nabla}\phi)\times\vec{A}$$
$$= \phi\,(Curl\,A) + (grad\,\phi)\times\vec{A}$$

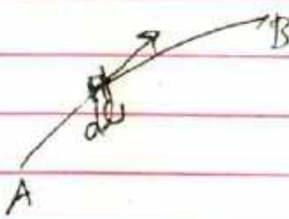
$$\boxed{xy^2z^4}\ \boxed{(3xy\,i + 2j + 3x\,\hat{k})}$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})$$

Div = $\dfrac{\text{Change in flux}}{\text{Volume}}$ Curl = $\dfrac{\text{line integral}}{\text{area.}}$

Line integral, Surface Int. & Volume Int.

$$W_{AB} = \int_A^B \vec{F}\cdot\underline{d\vec{l}} \longrightarrow dr$$



$$\vec{F} = F_1 i + F_2 j + F_3 k$$
$$l = r = x\,i + y\,j + z\,k$$
$$dl = dr = dx\,i + dy\,j + dz\,k$$

line Integral $= \displaystyle\int_A^B F_1\,dx + F_2\,dy + F_3\,dz$

$$Z = \pm\sqrt{\tfrac{3}{2}x^2 + \tfrac{3}{2}y^2} \qquad A\left(\sqrt{\tfrac{2}{3}}, 0, 1\right)$$

You need unit normal vector

$$z^2 = \tfrac{3}{2}x^2 + \tfrac{3}{2}y^2$$

$$2z^2 = 3x^2 + 3y^2 \Rightarrow \phi \equiv 2z^2 - 3x^2 - 3y^2$$
$$= 3x^2 + 3y^2 - 2z^2 =$$

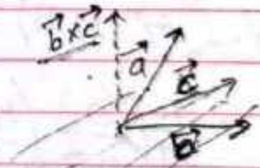
$$\nabla\phi = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k = -6xi - 6yj + 4zk$$
$$= 6x\, i + 6y\, j - 4z k$$

$$= 6\sqrt{\tfrac{2}{3}}\, i - 4k = \frac{6\sqrt{\frac{2}{3}}i - 4k}{\sqrt{\left(6\sqrt{\frac{2}{3}}\right)^2 + 4^2}} = \frac{\sqrt{3}}{\sqrt{5}} i - \frac{2}{\sqrt{10}}\hat{k}$$

$$\vec{\nabla}(\vec{F}\cdot\vec{r}) \qquad \vec{F} = \text{Constant } (F_1 i + F_2 j + F_3 k)$$
$$\vec{r} = \text{position vector } (x i + y j + z k)$$

$$\left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)\left(F_1 x + F_2 y + F_3 z\right) = F_1 i + F_2 j + F_3 k = \vec{F}$$

$$\vec{a} = j + k \qquad \vec{b} = 2i + 3j - 5k \qquad \vec{c} = \hat{j} - \hat{k}$$
$$\vec{a}\times(\vec{b}\times\vec{c})$$



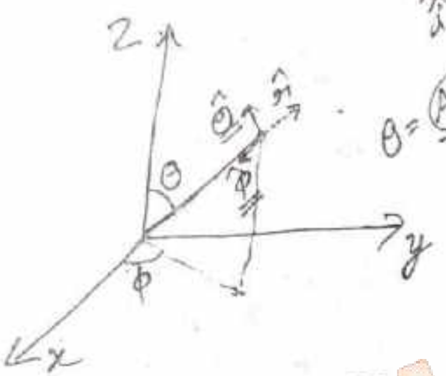
$$\vec{b}\times\vec{c} = \begin{vmatrix} i & j & k \\ 2 & 3 & -5 \\ 0 & 1 & -1 \end{vmatrix} = i(-3+5) - j(-2+0) + k(2) = 2i + 2j + 2k$$

$$\vec{a}\times(\vec{b}\times\vec{c}) = \begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ 2 & 2 & 2 \end{vmatrix} = i(0) - j(-2) + k(-2) = 2j - 2k =$$
$$= 2(\hat{j} - \hat{k})$$
$$= 2\vec{c} =$$

$dx\, dy\, dz = r^2 \sin\theta\, dr\, d\theta\, d\phi$

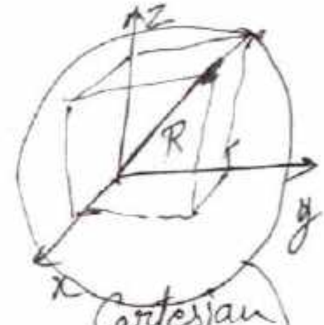
Spherical →

Cylindrical



$\theta = \frac{Arc}{R}$

$Arc = r\theta$



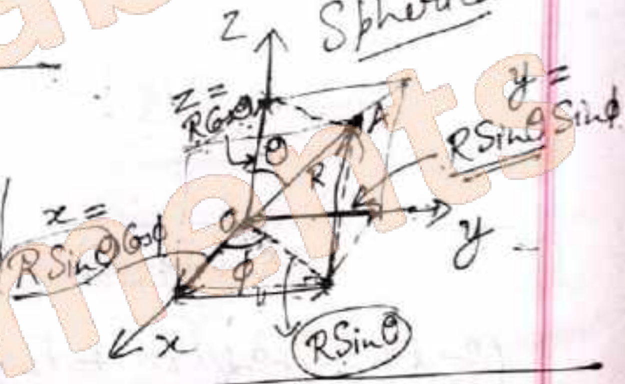
Cartesian → Spherical

$\vec{R} = \sqrt{x^2+y^2+z^2}\ \hat{r}$

$$\nabla = \hat{r}\frac{\partial}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial}{\partial \theta} + \frac{\hat{\phi}}{r\sin\theta}\frac{\partial}{\partial \phi}$$

$$\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$$

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$



$R = \sqrt{x^2+y^2+z^2}$

Q. $\nabla\cdot(r^n\vec{r}) = \nabla\cdot((x^2+y^2+z^2)^{n/2}|r|\hat{r})$

$\nabla\cdot(r^n\, r\,\hat{r}) = \nabla\cdot(r^{n+1}\hat{r})$

$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cdot r^{n+1}\right)$

$= \frac{1}{r^2}\left(\frac{\partial}{\partial r} r^{n+3}\right) = \cancel{\frac{1}{r^2}(n+3)}$

$= \frac{1}{r^2}(n+3)\, r^{n+2}$

$= (n+3)\, r^n$

$\tan(\theta) = \frac{P}{B} = \frac{R\sin\theta}{z} = \frac{\sqrt{x^2+y^2}}{z}$

$\theta = \tan^{-1}\left(\frac{x^2+y^2}{z}\right)$

$\tan(\phi) = \frac{P}{B} = \frac{y}{x}$

$\phi = \tan^{-1}\frac{y}{x}$

Teacher's Sign.........

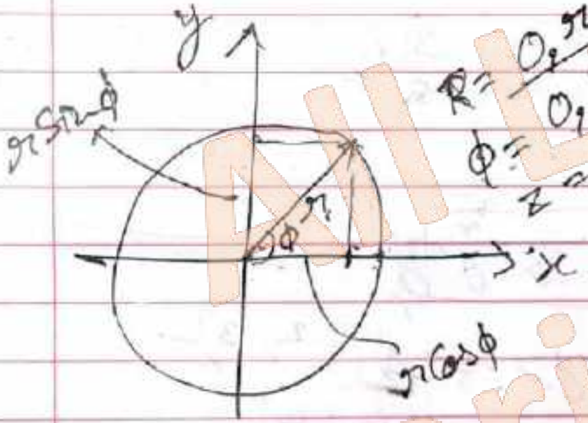
Cylindrical → Carti.

Cylinderical

$z = z$

$x = r \cos\phi$

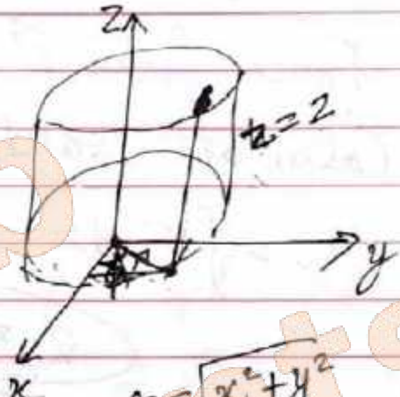
$y = r \sin\phi$



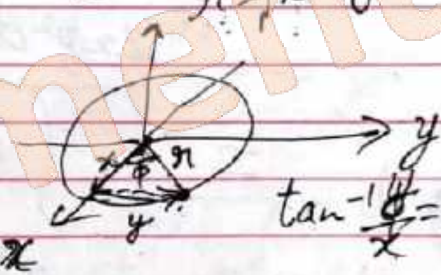
$R = 0, r$

$\phi = 0, 2\pi$

$z = 0, z$



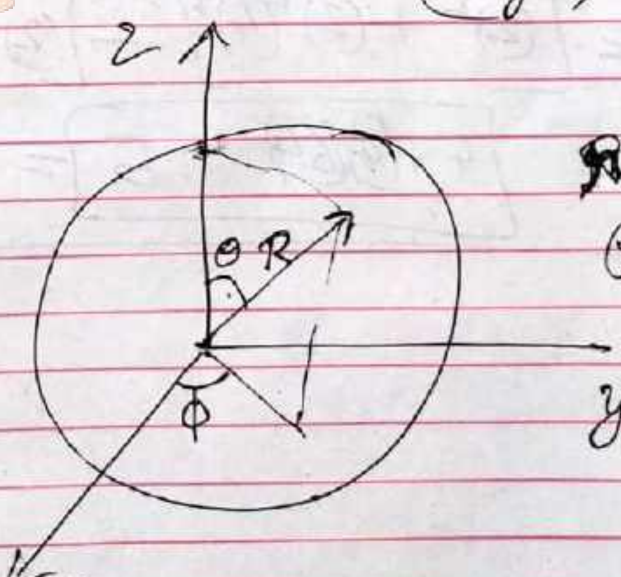
$r = \sqrt{x^2+y^2}$



$\tan^{-1}\frac{y}{x} = \phi$

$x^2 + y^2 = r^2$

$(x\ y\ z) \equiv (r\ \phi\ z)$



$R = 0, R$

$\theta = 0, \pi$

$\phi = 0, 2\pi$

Line Integral →

Parul

$$= \int \vec{F} \cdot dr$$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$= \int (F_1 \hat{i} + F_2 \hat{j} + F_3 \hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

Force field $\vec{F} = 2x(y^2 + z^3)\hat{i} + 2x^2y\hat{j} + 3x^2z^2\hat{k}$

Calculat workdone $(1, 2, 1) \to (2, 1, 4)$

$$W = \int F \cdot dr = \int (2xy^2 + 2xz^3)dx + 2x^2y\,dy + 3x^2z^2dz$$

$$= \frac{2x^2}{2}y^2 + \frac{2x^2y^2}{2} + \frac{2x^2}{2}z^3 + \frac{3x^2z^3}{3}$$

$$= 2xy^2dx + 2x^2y\,dy$$

$$= \int d(x^2y^2) + \int d(x^2z^3)$$

$d(x^2y^2)$

$d(x^2z^3)$

$$= x^2y^2 + x^2z^3 \Big|_{(1,2,1)}^{(2,1,4)}$$

$$= \left[(2)^2 + (2)^2(4)^3\right] - \left[(2)^2 + 1\right]$$

$$= \boxed{4 + (4)(64) - 5} = \quad =$$

force field → $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$

along $y = x^3$ $(1,1)-(2,8)$

$$W = \int F \cdot dr = \int (5xy - 6x^2)dx + (2y - 4x)dy$$

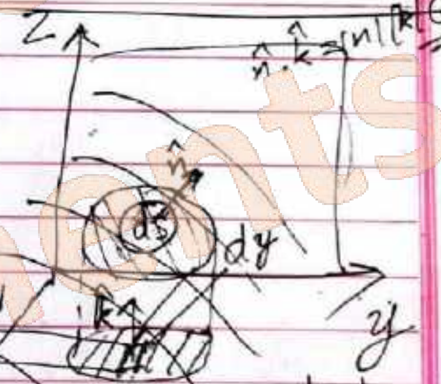
$y = x^3$ $\quad dy = 3x^2\,dx$

$$= \int_1^2 (5x(x^3) - 6x^2)\,dx + (2x^3 - 4x)(3x^2\,dx)$$

$=$

Surface Integral →

$$\iint \vec{E} \cdot d\vec{s} = \vec{E} \cdot \hat{n}\,ds \Rightarrow d\vec{s} = \hat{n}\,ds$$

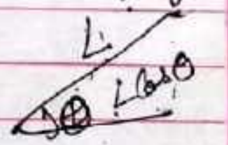


$$dx\,dy = ds\cos\theta = ds(\hat{n}\cdot\hat{k})$$

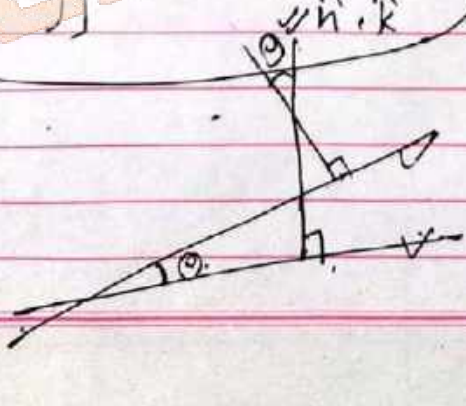
$$ds = \frac{dx\,dy}{\hat{n}\cdot\hat{k}}$$

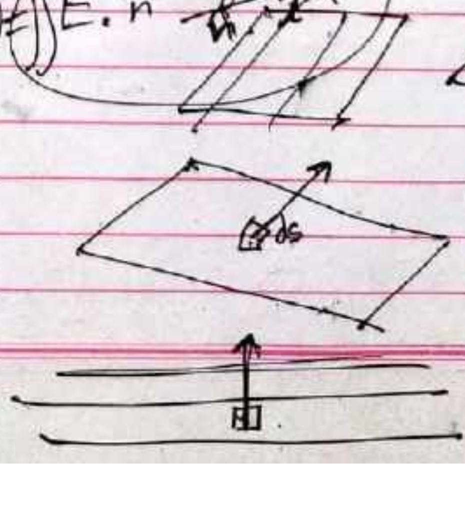
$$= \iint E \cdot \hat{n}\,\frac{dx\,dy}{\hat{n}\cdot\hat{k}} = \iint E \cdot \hat{n}\,\frac{dy\,dz}{\hat{n}\cdot\hat{i}}$$

$$\frac{dx\,dz}{\hat{n}\cdot\hat{j}}$$



ds cos θ





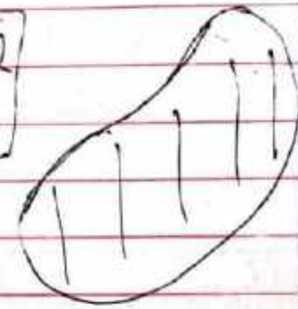
JAM 2019 The grad. of a scalar field $S(x,y,z)$ has the following characters.

~~a)~~ Line integ. is path indep. ~~b)~~ Closed line intg. is 0.

c) grad of S gives max. change in field ~~d)~~ Grad of S is a scalar Quantity

$\vec{\nabla} S$ = vector.

$$\vec{\nabla} \times \underline{\underline{\vec{\nabla} S}} = 0 = \left[\frac{\oint \text{line integral}}{\text{Area}}\right]$$



$$\vec{\nabla} S = \frac{\partial S}{\partial x} i + \frac{\partial S}{\partial y} j + \frac{\partial S}{\partial z} k$$

$$\vec{\nabla} \times \vec{\nabla} S = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial S}{\partial x} & \frac{\partial S}{\partial y} & \frac{\partial S}{\partial z} \end{vmatrix} = i\left(\frac{\partial}{\partial y}\frac{\partial S}{\partial z} - \frac{\partial}{\partial z}\frac{\partial S}{\partial y}\right) \pm (0) j + (0) k$$

$$\underset{0}{\parallel}$$

$\phi(x,y,z)$ satisfies Laplace Eqn.. $\nabla^2 \phi = 0 \Rightarrow$

$\vec{\nabla} \cdot \vec{\nabla} \phi = 0 \Rightarrow$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\phi = 0$$

$$\boxed{\nabla \cdot E = \frac{\rho}{\varepsilon_0} \quad \text{electrodynamics} \quad E = -\nabla V}$$

$\nabla \cdot E = 0$ electrostatic

$\nabla \phi$

$\Downarrow$

irrotational

solenoidal

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \quad \text{(Poission Eqn.)}$$

$$\nabla^2 V = 0 \quad \text{(Laplace Eqn.)}$$

$\Downarrow$

$$\nabla \cdot E = 0 \quad — (1)$$

(2) $\vec{\nabla} \times \vec{E} = 0$ $E = -\nabla V$

Parul

Date..................

$f(x,y) = x^3 - 3y^3 \rightarrow \nabla^2 f = 0 \rightarrow \vec{\nabla}\cdot\nabla f$

This fun. follows Laplace Eq^n.

along which curve this scalar fun. follows Laplace Eq.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) f = 0$$

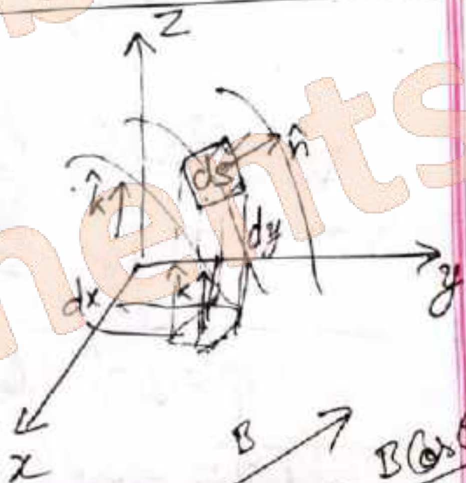
$$6x + (-18y) = 0$$

$$6x = 18y$$

$$x = 3y$$

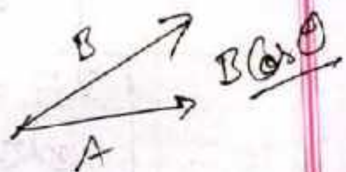
Surface Integral →

$$\iint \vec{E}\cdot d\vec{S} = \vec{E}\cdot\hat{n}\, ds$$

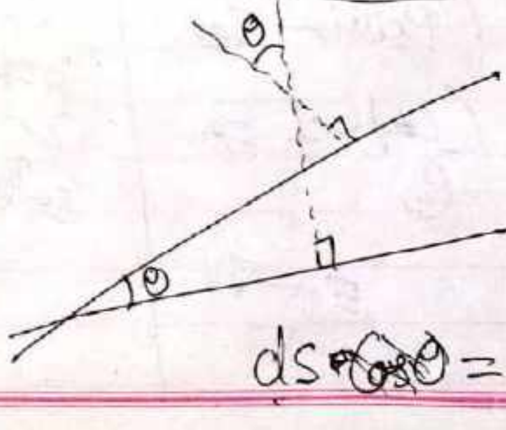


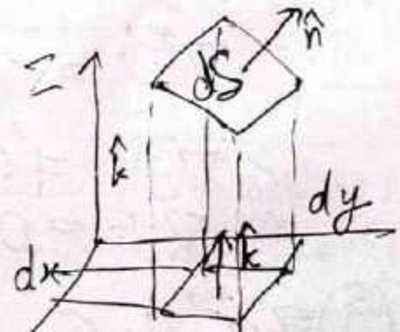
$$ds \underbrace{\cos\theta}_{\hat{n}\cdot\hat{k}} = dx\, dy$$

$$ds = \frac{dx\, dy}{\hat{n}\cdot\hat{k}}$$



$$\iint \vec{E}\cdot d\vec{S} = \iint \vec{E}\cdot\hat{n}\, \frac{dx\, dy}{\hat{n}\cdot\hat{k}}$$





$$ds \cos\theta = \frac{dx\, dy}{\hat{n}\cdot\hat{k}} \quad \Bigg| \quad \frac{dy\, dz}{\hat{n}\cdot\hat{i}}$$

Teacher's Sign..................

$$ds = \frac{dx\,dy}{\hat{n}\cdot\hat{k}} = \frac{dy\,dz}{\hat{n}\cdot\hat{i}} = \frac{dz\,dx}{\hat{n}\cdot\hat{j}}$$

$\vec{A} = 18z\,\hat{i} - 12\hat{j} + 3y\,\hat{k}$

Calculate its surface integral when eq. of surface is $2x + 3y + 6z = 12$ (first octant.)

$$\iint_S \vec{A}\cdot d\vec{s} = \iint \vec{A}\cdot\hat{n}\,ds = \iint \vec{A}\cdot\hat{n}\,\frac{dx\,dy}{\hat{n}\cdot\hat{k}}$$

Step 1 find $\hat{n}$

$\phi \equiv 2x + 3y + 6z - 12$

$\vec{n} = \nabla\phi = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$$\hat{n} = \frac{2i + 3j + 6k}{\sqrt{2^2 + 3^2 + 6^2}} = \frac{2i + 3j + 6k}{7}$$

Step 2. $\hat{n}\cdot\hat{k} = 6/7$

$$\vec{A}\cdot\hat{n} = \left((18z)\hat{i} - 12\hat{j} + 3y\,\hat{k}\right)\frac{(2\hat{i} + 3j + 6k)}{7}$$

$$= \frac{36z - 36 + 18y}{7}$$

Step 3 $\rightarrow$

$$\iint \frac{36z - 36 + 18y}{7}\,\frac{dx\,dy}{6/7} = \iint 6z - 6 + 3y\,dx\,dy$$

$$z = \frac{1}{6}(12 - 2x - 3y)$$

$$\int_0^6 \int_0^{\frac{1}{3}(12-2x)} (6 - 2x \;\;)\,dx\,dy$$

$2x = 12$

$x = 6$

$2x + 3y + 6z = 12$

$$\int_0^6 (6-2x)\left[\frac{1}{3}(12-2x)-0\right] dx$$

$$= \frac{1}{3}\int_0^6 72-24x-12x+4x^2\, dx$$

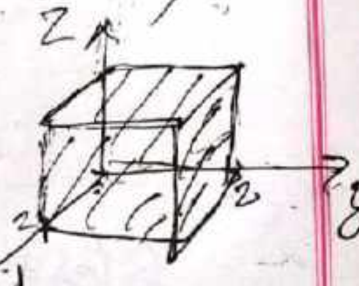
$$= \frac{1}{3}\int_0^6 (4x^2-36x+72)\, dx$$

$$= \frac{1}{3}\left[\frac{4x^3}{3} - 36\frac{x^2}{2} + 72x\right]_0^6 = \underline{\underline{24}}$$

Volume Integral $\rightarrow \iiint \vec{F}.dV = \iiint F.\,dx\,dy\,dz$

$\vec{F} = 2z\hat{i} - x\hat{j} + y\hat{k}$

(Volume integ. for a Box.
$x=0 \quad x=2$
$y=0 \quad y=2$
$z=0 \quad z=2$)



$$\int_0^2\int_0^2\int_0^2 (2z\hat{i} - x\hat{j} + y\hat{k})\,dx\,dy\,dz =$$

$$= \int_0^2\int_0^2 2zx\hat{i} - \frac{x^2}{2}\hat{j} + yx\hat{k}\Big|_0^2 \,dy\,dz$$

$$= \int_0^2\int_0^2 (4z\hat{i} - 2\hat{j} + 2y\hat{k})\,dy\,dz$$

$$= \int_0^2 4zy\hat{i} - 2y\hat{j} + \frac{2y^2}{2}\hat{k}\Big|_0^2 dz$$

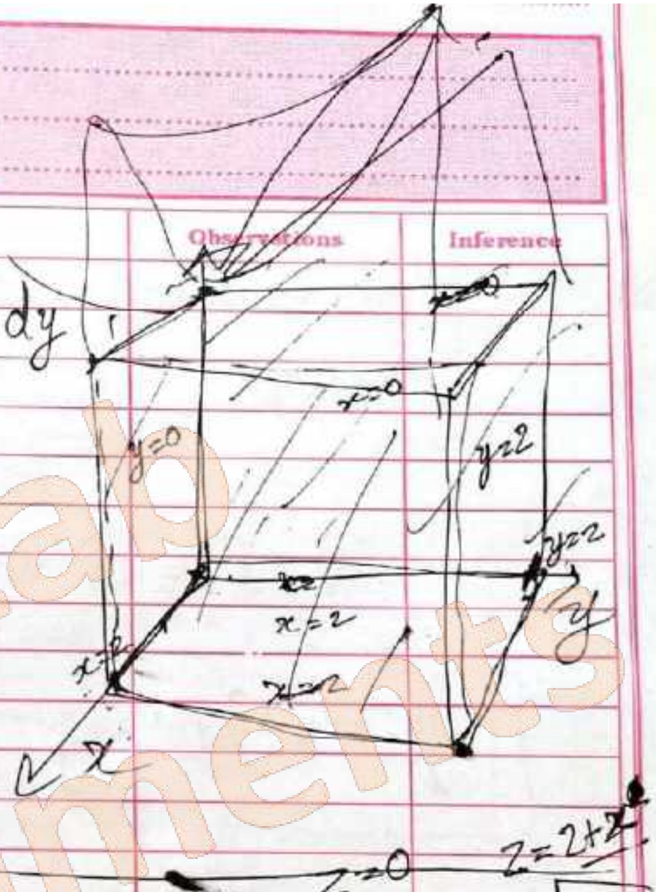
$$= \int_0^2 (8z\hat{i} - 4\hat{j} + 4\hat{k})\,dz$$

$$= 8z^2\hat{i} - 4z\hat{j} + 4z\hat{k}\Big|_0^2 = 16\hat{i} - 8\hat{j} + 8\hat{k}$$

Aim:

S. No.	Experiments	Observations	Inference

$$\int_0^2 \int_0^2 \int_0^{2+x} \boxed{}\, dz\, dx\, dy$$



Gauss Div. Thm.

$$\vec{\nabla}\cdot\vec{F} = \frac{\text{Change in flux}}{\text{Volume}}$$

$$\vec{\nabla}\cdot\vec{F} = \frac{\oiint \vec{F}\cdot d\vec{S}}{\text{Volume}}$$

a surface can make volume only when it is closed.

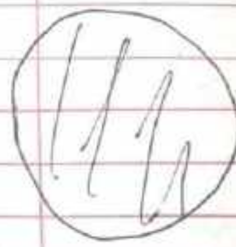
$$\boxed{\iiint \vec{\nabla}\cdot\vec{F}\, dV = \oiint \vec{F}\cdot d\vec{S}}$$

Gauss Div Theorem.

S. No. | Experiments | Observations | Inference

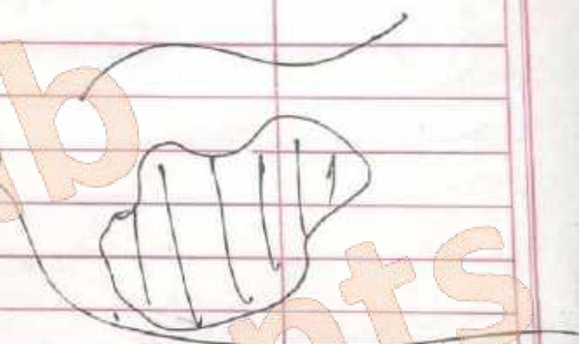
Stokes's Theorem →

$$\vec{\nabla} \times \vec{F} = \frac{\text{line Integral}}{\text{Area}}$$



$$\vec{\nabla} \times \vec{F} = \frac{\oint F \cdot dl}{\text{Area}}$$

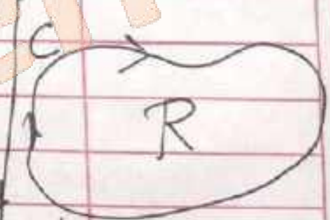
$$\boxed{\iint (\vec{\nabla} \times \vec{F}) \cdot d\vec{s} = \oint F \cdot dl}$$



Green's Theorem →

Closed curve C defines a region R

This is on XY plane.



$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y} \right) dx\,dy$$

$\vec{F} \cdot d\vec{r} \rightarrow$ $F_1 i + F_2 j + F_3 k$

$d\vec{r} = dx\,i + dy\,j + dz\,k$

Closed Curve.

$$\oint_C \boxed{F_1} dx + F_2\, dy + \underset{\substack{\| \\ 0}}{F_3} dz.$$

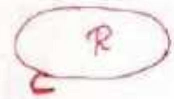
① Gauss Div. Thm →

$$\vec{\nabla}\cdot\vec{F} = \frac{\text{Change in flux}}{\text{Volume}} \qquad \boxed{\iiint \vec{\nabla}\cdot\vec{F}\,dV = \oiint \vec{F}\cdot\hat{n}\,ds}$$

② Stoke's Thm →

$$\vec{\nabla}\times\vec{F} = \frac{\text{line integral}}{\text{Area}} \qquad \boxed{\iint \vec{\nabla}\times\vec{F}\,d\vec{s} = \oint \vec{F}\cdot d\vec{l}}$$

③ Green's Thm →

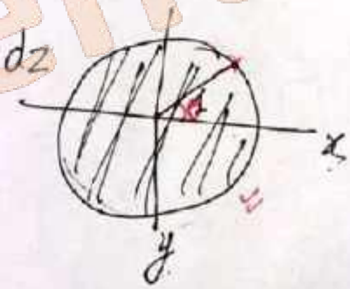


$$\oint_C M\,dx + N\,dy = \iint_R \frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}\,dx\,dy$$

Stoke's Theorem Qn. → $\vec{F} = (2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$

$\oint \vec{F}\cdot d\vec{r}$ ⟶ Closed path $x^2+y^2=1$

$$\oint F\cdot dr = \oint (2x-y)dx - yz^2dy - y^2z\,dz$$



$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (2x-y) & -yz^2 & -y^2z \end{vmatrix}$$

$$= \hat{i}(-2yz + 2yz) - \hat{j}(0-0) + \hat{k}(0+1) = +\hat{k}$$

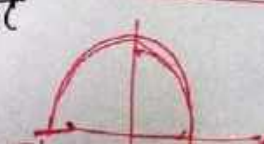
$$\iint \vec{\nabla}\times\vec{F}\,\hat{n}\,ds$$

$\hat{n} = \hat{k}$

$$= \iint \hat{k}\cdot\hat{n}\,\frac{dx\,dy}{\hat{n}\cdot\hat{k}}$$

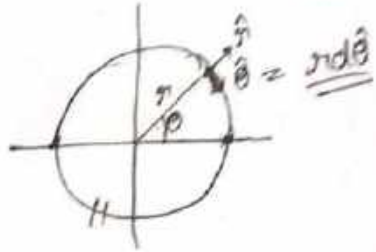
$$= \boxed{\iint dx\,dy} = \pi r^2 = \pi$$

$$\iint dx\,dy = \int_0^{2\pi}\int_0^1 r\,dr\,d\theta = \int_0^{2\pi} \frac{r^2}{2}\,d\theta = \frac{r^2}{2}\cdot 2\pi = \pi r^2$$



Cartesian → polar.

$dx\ dy$ $\qquad$ $dr\ rd\theta$



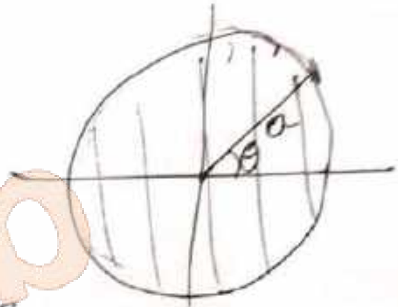
⇒ line integral r = Constant

$$\int F.dl = \int_0^{2\pi} F.\ rd\theta$$

(complete circle)

⇒ surface integral

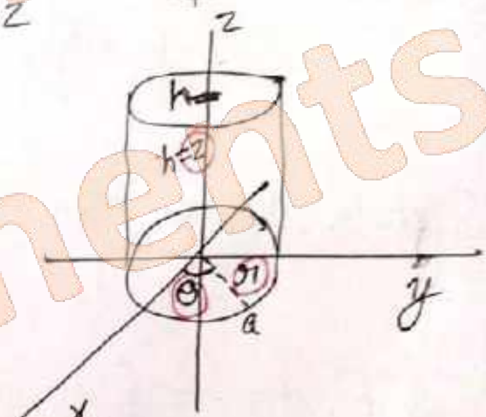
$$\iint F\, dx\, dy = \int_0^{2\pi}\int_0^{a} F\ dr\ rd\theta$$



Cartesian → cylindrical

$$\iiint F\ dx\,dy\,dz = \int_0^{h}\int_0^{2\pi}\int_0^{a} F\ rd\theta.dr.dz$$

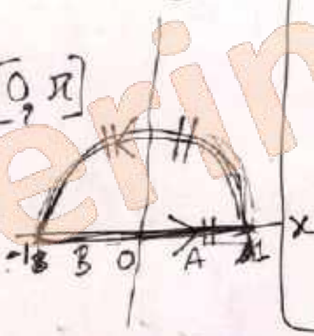
$\boxed{\text{Solve this}}$



Line Integral

r = Constant $\theta = [0, \pi]$

$$\oint F.dl = \int_0^{\pi} F\, rd\theta + \int_{-1}^{+1} F dx$$



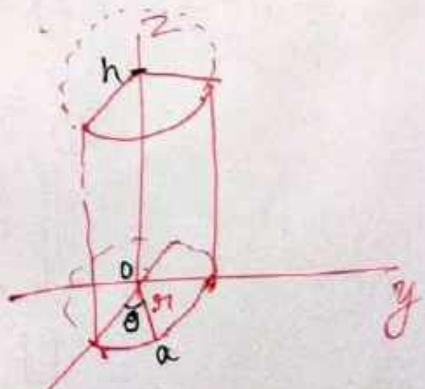
~~y=0~~

$$dl = dx\hat{i} \xleftarrow{y=0} dl = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

Page 1 Q.3

Volume

$$\int_0^{h}\int_0^{\pi/2}\int_0^{a} rd\theta\ dr\ dz$$



Cartesian → Spherical

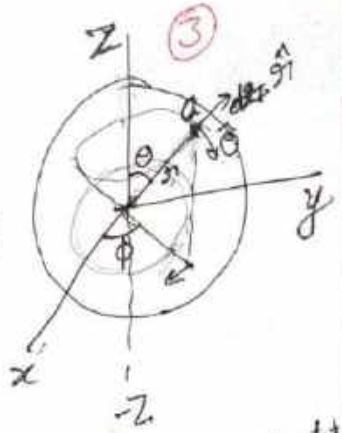
Volume Integral →

$$\iiint \vec{F}\, dx\, dy\, dz = \quad d\hat{r}\ \ \underline{r\, d\hat{\theta}}\ \ \underline{r \sin\theta\, d\hat{\phi}}$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{a} F\, r^2 \sin\theta\, d\hat{r}\, d\hat{\theta}\, d\hat{\phi}$$

$\parallel 1$

Solve this



Surface Integral →

$r =$ Costant

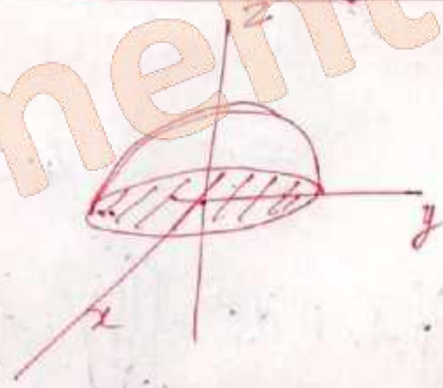
$$\iiint F\, ds = \int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta\, d\hat{\theta}\, d\hat{\phi}$$

Solve this

$\theta =$ Constant

$dr\ r\sin\theta\, d\phi$

Volume $\displaystyle\int_0^{2\pi}\int_0^{\pi/2}\int_0^{a} r^2 \sin\theta\, d\hat{r}\, d\hat{\theta}\, d\hat{\phi}$



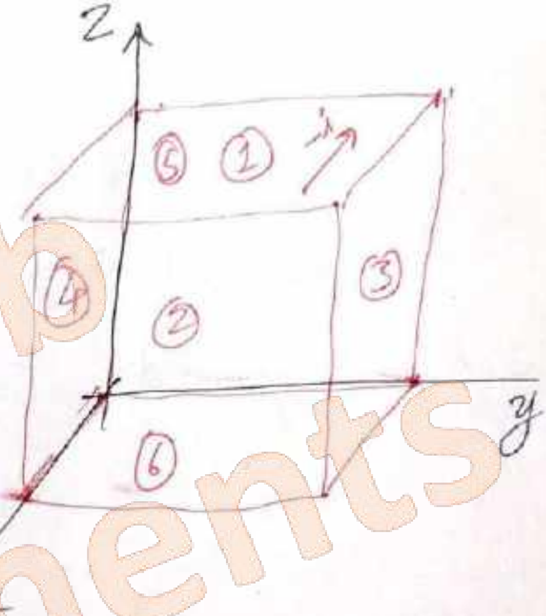
Surface $\displaystyle\int_0^{2\pi}\int_0^{\pi/2} r^2 \sin\theta\, d\hat{\theta}\, d\hat{\phi}$

Example Gauss Div. Theorem.

$$\vec{A} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$$

Calculat $\iint \vec{A}\cdot d\vec{s}$ along a surface of cube $x=0$ $y=0$ $z=0$, $x=1$ $y=1$ $z=1$

(1) $x=0$ $\quad dx=0$



$$\iint \vec{A}\cdot d\vec{s} = \vec{A}\cdot\hat{n}\,\frac{dy\,dz}{\hat{n}\cdot\hat{i}}$$

$$= \vec{A}\cdot(-\hat{i})\,\frac{dy\,dz}{-\hat{i}\cdot\hat{i}}$$

$$= \int_0^1\int_0^1 0\; dy\,dz$$

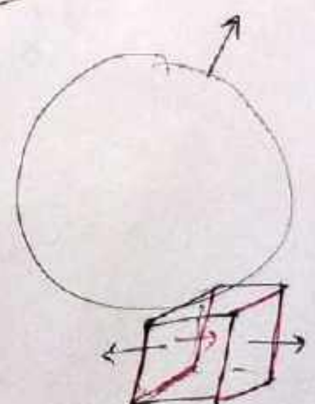
$$= 0$$

$$\vec{\nabla}\cdot\vec{A} = 4z \cancel{+} -2y + y = \underline{\underline{4z-y}}$$

$$\oiint \vec{A}\cdot\hat{n}\,d\vec{s} = \iiint \vec{\nabla}\cdot\vec{A}\,dV$$

$$= \int_0^1\int_0^1\int_0^1 (4z-y)\,dx\,dy\,dz$$

$\boxed{\text{Home work}}$



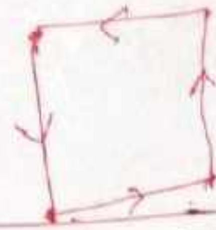
Evaluate $\oint F \cdot dl$ vertices of that closed path are (5)

$(0,0)\ (1,0),\ (1,2)\ (0,2)$

$F = (3x-2y)\hat{i} + x^2 z\,\hat{j} + y^2(z+1)\,\hat{k}$

Home work

(4)



Consider a cylinder of radius R & height h

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

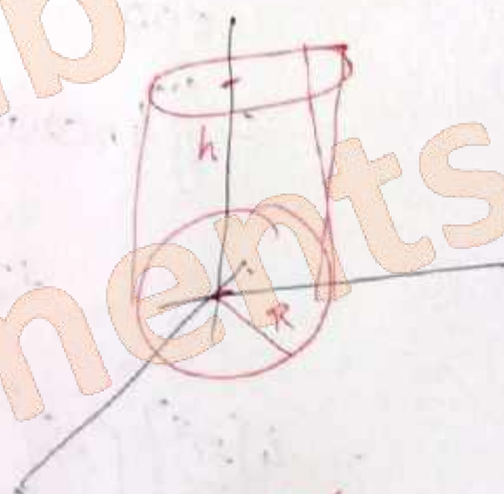
Calculate $\iint \vec{r} \cdot \hat{n}\, d\vec{s}$

$\vec{\nabla} \cdot \vec{r} = 1 + 1 + 1 = 3$

$\iiint \vec{\nabla} \cdot \vec{r}\, dV$

$3 \int_0^h \int_0^{2\pi} \int_0^R r\,dr\,d\theta\,dz$

$3 \left[\pi R^2 h\right]$



$z = 0$

$z = h$

$\phi = x^2 + y^2 = R^2$

$\nabla\phi = \boxed{2x\,i + 2y\,j}$

$\hat{n}$

⑥

Calculate the flux of a vector field

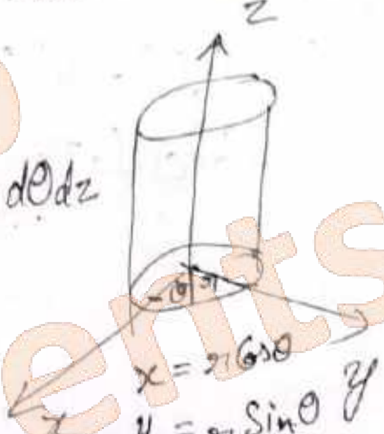
$$\vec{F} = 4x\,\hat{i} - 2y^2\,\hat{j} + z^2\,\hat{k}$$

$x^2+y^2=4$
$z=0 \quad z=3$
cylinder

$$\vec{F}\cdot d\vec{s} = \vec{\nabla}\cdot\vec{F}\,dV$$

$$= \iiint 4 - 4y + 2z \;\, dx\,dy\,dz$$

$$= \int_0^3\int_0^{2\pi}\int_0^2 \boxed{4 - 4r\sin\theta + 2z}\;\, dr\; r\,d\theta\,dz$$



$$= \iiint 4r\,dr\,d\theta\,dz - 4\iiint \sin\theta\, r^2\,dr\,d\theta\,dz$$

$$+ 2\iiint z\,r\,dr\,d\theta\,dz$$

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

$$= 4\left[\frac{r^2}{2}\right]_0^2 \left[\theta\right]_0^{2\pi} \left[z\right]_0^3$$

$$- 4\iiint \left[\frac{r^3}{3}\right]_0^2 \left[-\cos\theta\right]_0^{2\pi} \left[z\right]_0^3$$

$$+ 2\iiint \left[\frac{r^2}{2}\right]_0^2 \left[\theta\right]_0^{2\pi} \left[\frac{z^2}{2}\right]_0^3$$

Green's Thm → $\oint M\,dx + N\,dy = \iint \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\,dx\,dy$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \\ M & N & 0 \end{vmatrix}$$

$$\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\hat{k}$$

$\vec{F} = x^2y\, i + x^2 j$

Calculate $\vec{F}\cdot d\vec{l}$ Counterclockwise of a triangle $(0,0)$ $(1,0)$ $(1,1)$

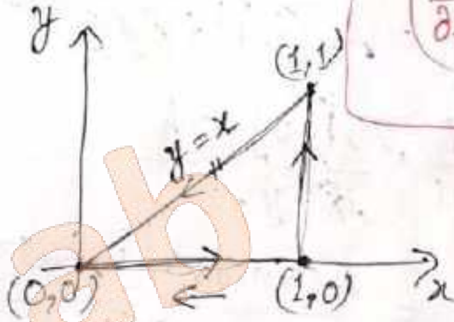
$$\oint \vec{F}\cdot d\vec{l} = \oint x^2y\,dx + x^2\,dy$$

$$= \int_0^1\int_0^x (2x - x^2)\hat{n}\,\frac{dx\,dy}{\hat{n}\cdot\hat{k}}$$

$$= \int_0^1 (2x - x^2)\, y\Big|_0^x\, dx$$

$$= \int_0^1 (2x - x^2)x\, dx$$

$$= \frac{2x^3}{3} - \frac{x^4}{4}\Big|_0^1 = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$



$y = mx + c$

$y = x$

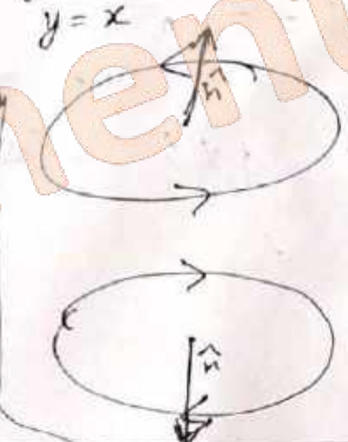
E. $\vec{F} = \sin y\,\hat{i} + x(1+\cos y)\hat{j}$

$\vec{F} = \sin y\,\hat{i} + x(1+\cos y)\hat{j}$

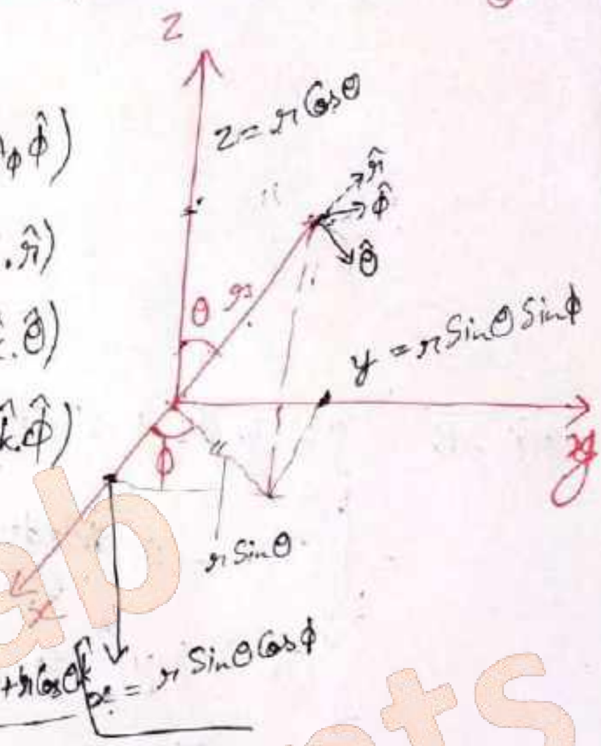
$\oint F\cdot dr$ along $x^2 + y^2 = a^2$

Home Work

πa^2



Transformation of vectors →



Cartesian → Spherical

$\vec{A}(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \rightarrow (A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi})$

$\vec{A}_r = (\vec{A}\cdot\hat{r}) = A_x(i\cdot\hat{r}) + A_y(j\cdot\hat{r}) + A_z(\hat{k}\cdot\hat{r})$

$\vec{A}_\theta = (\vec{A}\cdot\hat{\theta}) = A_x(i\cdot\hat{\theta}) + A_y(j\cdot\hat{\theta}) + A_z(\hat{k}\cdot\hat{\theta})$

$\vec{A}_\phi = (\vec{A}\cdot\hat{\phi}) = A_x(i\cdot\hat{\phi}) + A_y(j\cdot\hat{\phi}) + A_z(\hat{k}\cdot\hat{\phi})$

$$\hat{r} = \frac{\vec{r}}{|r|} = \frac{xi + yj + zk}{\sqrt{x^2+y^2+z^2}}$$

$$= \frac{r\sin\theta\cos\phi\,\hat{i} + r\sin\theta\sin\phi\,\hat{j} + r\cos\theta\,\hat{k}}{r}$$

$$\hat{\phi} = \frac{\hat{z}\times\hat{r}}{\sin\theta} \Rightarrow |z||r|\sin\theta\,\hat{\phi} = \hat{z}\times\hat{r}$$

$$= -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$$

$$\hat{\theta} = \hat{\phi}\times\hat{r} = \cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j} - \sin\theta\,\hat{k}$$

	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{i}$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\hat{j}$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\hat{k}$	$\cos\theta$	$-\sin\theta$	0

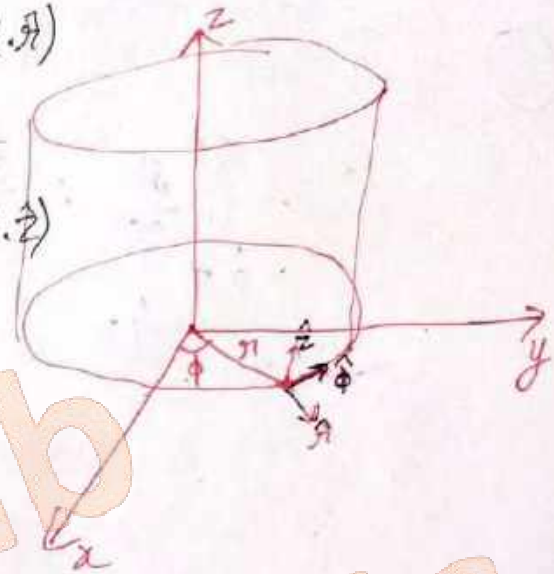
Cartesian → Cylinderical ③

$\vec{A} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \to (A_r \hat{r} + A_\phi \hat{\phi} + A_z \hat{z})$

$A_r = \vec{A}\cdot\hat{r} = A_x(\hat{i}\cdot\hat{r}) + A_y(\hat{j}\cdot\hat{r}) + A_z(\hat{k}\cdot\hat{r})$

$A_\phi = \vec{A}\cdot\hat{\phi} = \ - \quad - \quad -$

$A_z = \vec{A}\cdot\hat{z} = 0 + 0 + A_z(\hat{k}\cdot\hat{z})$



Dot	$\hat{r}$	$\hat{\phi}$	$\hat{z}$
$\hat{i}$	$\cos\phi$	$-\sin\phi$	0
$\hat{j}$	$\sin\phi$	$\cos\phi$	0
$\hat{k}$	0	0	1

Jest 2013 → $\vec{A} = xz\hat{i} + y\hat{j} \longrightarrow$ Cylinderical

$A_r = \vec{A}\cdot\hat{r} = xz(\hat{i}\cdot\hat{r}) + y(\hat{j}\cdot\hat{r}) + 0$

$x = r\cos\phi$

$y = r\sin\phi$

$A_\phi = \vec{A}\cdot\hat{\phi} = xz(\hat{i}\cdot\hat{\phi}) + y(\hat{j}\cdot\hat{\phi}) + 0$

$A_z = \vec{A}\cdot\hat{z} = xz\underbrace{(\hat{i}\cdot\hat{z})}_{0} + y\underbrace{(\hat{j}\cdot\hat{z})}_{0} + 0$

$A_r = r\cos^2\phi\, z + r\sin^2\phi$

$A_\phi = r\cos\phi\sin\phi\,(1-z)$

$\vec{A} = A_r\hat{r} + A_\phi\hat{\phi} + 0$

(1) $\nabla \cdot \vec{A} = \vec{\nabla} \cdot \frac{\hat{r}}{r^2} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \cdot \frac{1}{r^2} = \frac{1}{r^2}\frac{\partial 1}{\partial r} = \underline{\underline{0}}$ (4)

(2) $\iiint \nabla \cdot \vec{A}\, dV = \iint \vec{A} \cdot d\vec{s}$

$E = -\nabla V$

$V = k\frac{q}{r}$

$E = k\frac{q}{r^2}\hat{r}$

$0 = \int_0^{2\pi}\int_0^{\pi} \frac{\hat{r}}{r^2} r^2 \sin\theta\, d\theta\, d\phi$

$= 4\pi$

$\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\, \delta(r)$

Dirac Delta fun.

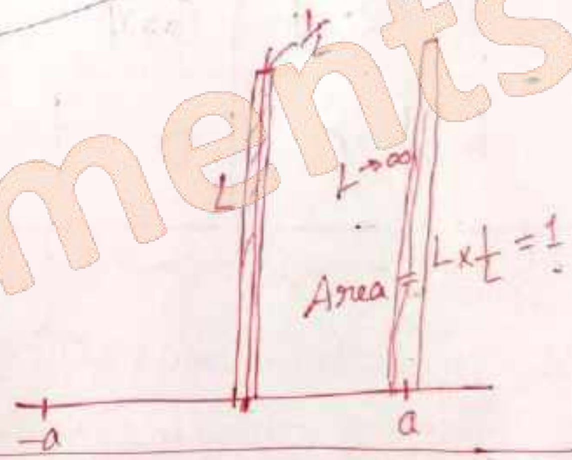
$\int_{-\infty}^{\infty} \delta^3(r-0)\, dV = 1$

$\int_{-\infty}^{\infty} \delta(x-0)\, dx = 1$

$\delta(r-0)$

$\delta(r-a)$

$f(x) \longrightarrow f(x \pm a)$



$\iiint_{r=0} 4\pi\, \underline{\delta(r)}\, dV = 4\pi$

NU 2015 → which of the following field represents Electrostatics →

a) $(2xz-y^2)\hat{i} + (2xy-z^2)\hat{j} + (2yz-x^2)\hat{k}$

b) $y^2\hat{i} + (2xy+z^2)\hat{j} + (2yz+x^2)\hat{k}$

c) $yz\hat{i} - xz\hat{j} + xy\hat{k}$

d) $y^2\hat{i} + (2xy+z^2)\hat{j} + (2yz+x^2)\hat{k}$

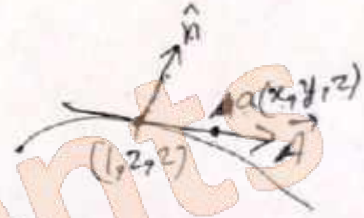
$\vec{\nabla}\times\vec{E} = 0$ (5)

Home work

Jest 2017 find the eqn. of plane which is tangent to the surface $xyz=4$ at $(1,2,2)$

$\cancel{\phi = xyz-4}$

$\vec{n} = yz\,i + xz\,j + xy\hat{k} = 4\hat{i}+2\hat{j}+2\hat{k}$



$\vec{A} = (x-1)i + (y-2)j + (z-2)\hat{k}$

$\vec{n}\cdot\vec{A} = 4(x-1) + 2(y-2) + 2(z-2) = 0$

$4x-4+2y-4+2z-4 = 0$

$4x+2y+2z = 12$

$\boxed{2x+y+z=6}$

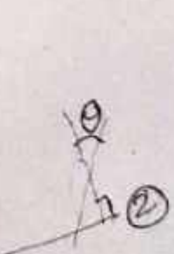
Jest 2019 find the angle b/w $y^2+z^2=2$ (1) $y^2-x^2=0$ (2) at point $(1,-1,1)$

grad of 1 → $\vec{A} =$

grad of 2 → $\vec{B} =$

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$



Gate2013 → If A & B are two constant vectors & $\vec{r}$ is the position vector. Then find $\vec{\nabla}(\vec{A}\cdot(\vec{B}\times\vec{r}))$. 6

$$\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k} \qquad \vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$$

$$\vec{r} = x\,i + y\,j + z\hat{k}$$

Q. If the surface integral of a vector field $\vec{A} = 2kx\,i + ly\,\hat{j} - 3mz\,\hat{k}$ over an sphere of radius 2 cm. is zero. Then what is the relation b/w k, l, & m ?

Ans. $\vec{\nabla}\cdot\vec{A}$ must be zero [Gauss Div. Theo.]

$$2k + l - 3m = 0$$

BHU 2010 To define a unique vector field what is ur required

If you define div. & curl of a vector field. it means that vector field is uniquely defined.

$\vec{\nabla}\cdot\vec{E} =$ $\qquad$ $\vec{\nabla}\cdot\vec{B} =$

$\vec{\nabla}\times\vec{E} =$ $\qquad$ $\vec{\nabla}\times\vec{B} =$

when $\vec{\nabla}\times\vec{E} = 0$
Then that vector field can be defined as

$$\vec{E} = \vec{\nabla}V$$

gradiant of a scalar field potential

$$\vec{\nabla}\times\vec{\nabla}V = 0$$

electrostatic

when $\vec{\nabla}\cdot\vec{B} = 0$
Then that vector field can be defined as

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

The curl of a vector potential

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$$

BHU2010 → There are two arbitrary vectors $\vec{A}$ $\vec{B}$ $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$ ⇒ what is the angle b/w vectors $\vec{A}$ & $\vec{B}$.

$$\sqrt{|A|^2+|B|^2+2|A||B|\cos\theta} = \sqrt{|A|^2+|B|^2-2|A||B|\cos\theta}$$

$$4|A||B|\cos\theta = 0$$

~~|A|~~ $\vec{A}$ & $\vec{B}$ are not null vectors.

$$\theta = \pi/2$$

BHU 2011 for an incompressible fluid $\vec{\nabla}.\rho\vec{v}$ is =
where ρ → density v → velocity of fluid.

$$\frac{\partial}{\partial t} = \vec{\nabla}.\rho\vec{v} = 0$$

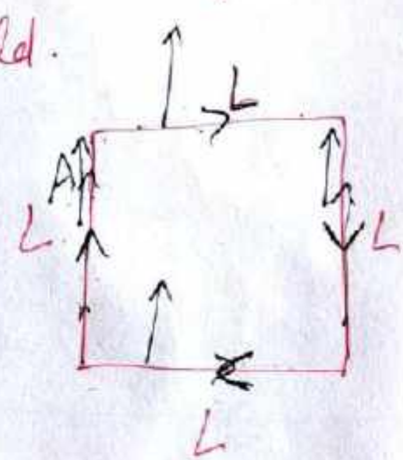
because change in flux is zero.

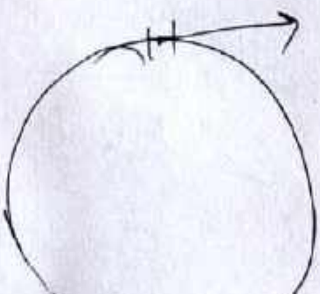
BHU 2011 ~~A~~ Let $\vec{A}$ is a non-conservative vector field. then
~~which definatio~~

$\oint A.dl = 0$ $\quad \vec{\nabla}\times\vec{A}=0$ $\quad \vec{A} = \vec{\nabla}f$ $\quad \boxed{\vec{\nabla}.\vec{A}=0}$

Q. find $\oint\vec{A}.d\vec{l}$ along a square loop of side L when $\vec{A}$ is a uniform field.

a) 2AL b) 4AL c) AL d) 0





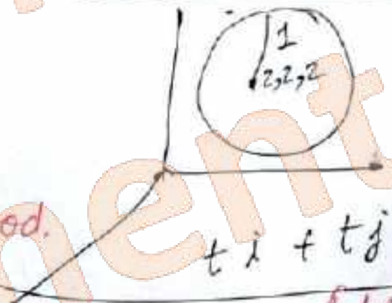
Q. What is $\oint \frac{\vec{r}\cdot d\vec{s}}{r^3}$ where S is a closed surface enclosing the origin.

$$\oint \frac{|r|\,\hat{r}\cdot d\vec{s}}{r^3} = \oint \frac{\hat{r}}{r^2}\,ds$$

$$\int \nabla\cdot\frac{\hat{r}}{r^2}\,dV = \int 4\pi\,\delta(r)\,dV = 4\pi$$

If exclude origin then This is zero.

$$\boxed{(x-2)^2+(y-2)^2+(z-2)^2 = 1}$$



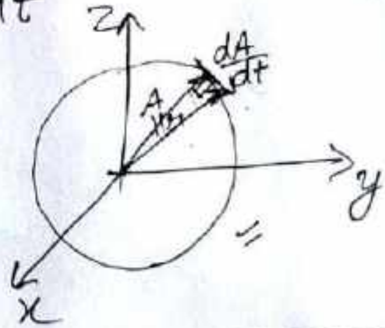
$t\hat{i} + t\hat{j} + t\hat{k}$

Q. If $\vec{A}$ has a constant magnitude. then which relation holds good.

a) $\frac{d\vec{A}}{dt}\times\vec{A}=0$ b) $\frac{d\vec{A}}{dt}=0$ c) $\frac{dA}{dt}\cdot A=0$ $\cancel{\frac{dA}{dt}\times A}$ x

$$\vec{A}\cdot\vec{A} = A^2 \Rightarrow 2A\cdot\frac{dA}{dt} = 0 \Rightarrow A\cdot\frac{dA}{dt} = 0$$

$A = t\hat{i} + \ldots$



Jest 2019 Let $\vec{r}$ be the position vector on a closed contour C Then find $\oint \vec{r}\cdot d\vec{r}$?

$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

Stokes theorem. →

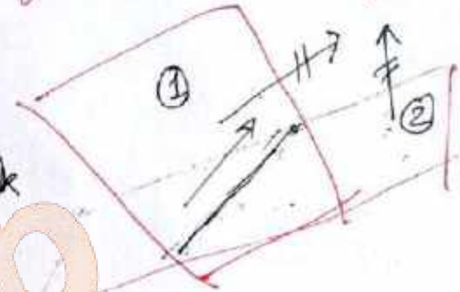
$$\vec{\nabla}\times\vec{r} = 0 \qquad \oint \vec{r}\cdot d\vec{r} = \iint \underbrace{\vec{\nabla}\times\vec{r}}_{=0}\cdot d\vec{s}$$

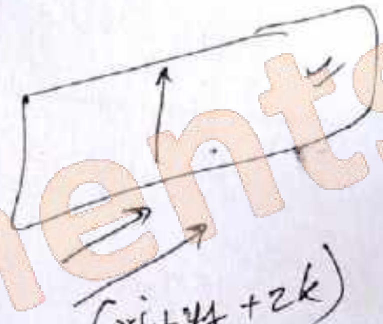
Jest 2019 which one of the following vectors lies along the line of interaction of two planes

$x+3y-z=5$ (1) $2x-2y+4z=3$ (2)

a) $10i-2j+5k$ b) $10i-6j-8k$ c) $10i+2j+5k$ d) $10i-2j-5k$

Home work





$a(xi+yj+zk)$

Gate2012 → $\vec{a}=3i+2j$ $\vec{b}=i+2j$

a) Linearly dependent b) Linearly independent

c) orthogonal d) parallel

$$\vec{A}=a_1\vec{B}+a_2\vec{C}+a_3\vec{D}+ \_\_\_\_$$

$$\vec{a}=m\vec{b}$$

$$3i+2j=mi+2mj$$

$m=3$

$2m=2 \Rightarrow m=1$