

Free Study Material from All Lab Experiments



**Electromagnetic Theory
for NET/Gate Physical Sciences
>Conversion Laws in Electrodynamics<**

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Conservation laws in Electrodynamics :-

1. Conservation of charge:- Cons. of charge follows from continuity eqⁿ

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

2. Cons. of Energy (Poynting theorem):-

$$\frac{dw}{dt} = -\frac{d}{dt} \int_V \frac{1}{2} \left(\frac{B^2}{\mu_0} + \epsilon_0 E^2 \right) d\tau - \frac{1}{\mu_0} \oint (\vec{E} \times \vec{B}) \cdot d\vec{s}$$

This is the integral form of Poynting theorem.

This is the Work-Energy theorem of electrodynamics.

Statement - Work done on the charges by the electromagnetic forces is equal to decrease in the energy stored in the fields, less the energy flow out through the surface.

If we have a close surface filled charge inside. Vol. of close surface $\rightarrow V$ then energy

If EM fall on surface, then charges displaced. EM wave contain energy

This energy can be used in 2 ways

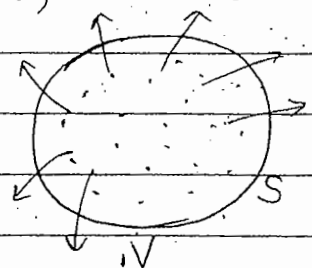
- (i) Working on the charges
- (ii) Energy flowing out through the surface's.

$$\boxed{\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}}$$

$\Rightarrow$ Energy flux density

Poynting vector tells

$\rightarrow$ Per unit area, per unit time, how much energy is flowing out through the surface S



Generally $\vec{S} = \vec{E} \times \vec{H}$

$$\frac{dw}{dt} = \frac{d}{dt} \int_V u_{\text{mech}} \cdot d\tau$$

$u_{\text{mech}} \rightarrow$ Mechanical Energy density

By incident EM wave, there will be mechanical motion.

By multiplying $u_{\text{mech}} \times d\tau \Rightarrow$ Energy

$$\frac{d}{dt} \int_V \frac{1}{2} \left(\frac{B^2}{\mu_0} + \epsilon_0 E^2 \right) d\tau = \frac{d}{dt} \int_V u_{\text{em}} d\tau$$

We can write

$$\oint_S \left(\frac{\vec{E} \times \vec{B}}{\mu_0} \right) \cdot d\vec{a} = \int_S \vec{S} \cdot d\vec{a} = \int_V (\vec{\nabla} \cdot \vec{S}) d\tau$$

$d\tau =$ Arbitrary so

$$\frac{d}{dt} (u_{\text{mech}} + u_{\text{em}}) = -\vec{\nabla} \cdot \vec{S}$$

$$u_{\text{mech}} + u_{\text{em}} = u_{\text{Total}}$$

This S is equivalent to energy density $\vec{J}$ & energy density is equivalent to charge

$$\text{density } \frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{J}$$

Cons. of Linear Mom. :- Total linear mom. is always conserved in electrodynamics & linear momentum density of electromagnetic field is

$$\boxed{\vec{p}_{\text{em}} = \mu_0 \epsilon_0 \vec{S}}$$

Angular Mom. in Electromag. field :-

(Cons. of Ang. mom) :- Total ang. mom. is conserved in electromag. field. Angular

mom. density of EM field is given by

$$\vec{L}_{em} = (\vec{r} \times \vec{p}_{em})$$

$$\vec{p}_{em} = \mu_0 \epsilon_0 \vec{S} = \frac{\mu_0 \epsilon_0}{\mu_0} \left(\frac{\vec{E} \times \vec{B}}{\mu_0} \right)$$

$$\vec{L}_{em} = (\vec{r} \times \vec{p}_{em}) = \epsilon_0 [\vec{r} \times (\vec{E} \times \vec{B})]$$

exercise - 1

Q.1 If $\vec{E} = \hat{i} A \sin(12y) \sin(\alpha z) \cos \omega t$, describes an electric field in free space. What should be the numerical value of α for which electric field satisfies Maxwell's eqn. Given that, $\omega = 15 \times 10^8$ rad/sec.

Q.2 In a region of empty space, the mag. field is described by $\vec{B} = B_0 e^{ax} \sin(ky - \omega t) \hat{z}$. Find Electric field.

Q.3 If 2 EM waves given by $\vec{E}_1(z,t) = (E_0 \hat{x} + E_0 e^{-i\frac{\pi}{2}} \hat{y}) e^{i(kz - \omega t)}$ & $\vec{E}_2(z,t) = (E_0 \hat{x} + E_0 e^{i\frac{\pi}{2}} \hat{y}) e^{i(kz - \omega t)}$ superimposed. Find polarisation of resultant wave.

Q.4 The elec. vector of a wave is given by $\vec{E} = E_0 \left(\frac{8\hat{x} + 6\hat{y} + 5\hat{z}}{\sqrt{125}} \right) e^{i(\omega t + 3x - 4y)}$ & its

freq. is 1 GHz. (2)

a) Investigate whether wave is plane wave or not?

- b) Find wave vector $\vec{K}$
- c) Phase velocity along y-dirⁿ
- d) dielectric const. & refractive index assuming medium is Non-magnetic.

Q. Calculate Plasma freq & maxi. penetration depth for a plasma containing 10^8 e⁻s per m^3 .

Q. A He-Ne laser beam of wavelength $\lambda = 632$ nm of intensity 1 W/ m^2 is travelling along x-axis in vacuum. (a) Find amplitudes of electric & magnetic fields associated with laser beam. (b) Express $\vec{E}(x,t)$ & $\vec{B}(x,t)$ if electric field is polarised along y-dirⁿ.

Potential Formulation :- We have 2 type of potⁿ,

- (i) Scalar Potⁿ $\vec{V}(\vec{r}, t)$
- (ii) Vector Potⁿ $\vec{A}(\vec{r}, t)$

Both potⁿ are freeⁿ of time in electrostatics & Magnetostatics, they are independent of time.

We have $\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \boxed{\vec{B} = \vec{\nabla} \times \vec{A}} \quad \text{--- (A)}$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} \times \vec{E} = -\vec{\nabla} \times \left(\frac{\partial \vec{A}}{\partial t} \right) \quad (\text{On putting } \vec{B} = \vec{\nabla} \times \vec{A})$$

$$\vec{\nabla} \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0 \quad (\text{in electrodynamics})$$

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\vec{\nabla} V$$

$$\boxed{\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}} \quad \text{--- (B)}$$

for ^(unique) same value of $\vec{E}$ & $\vec{B}$, we can have different value of $\vec{V}$ & $\vec{A}$. ($\vec{E}$ & $\vec{B}$ are dependent on each other).

If we choose a new mag. vector potⁿ $\vec{A}'$ s.t.

$$\boxed{\vec{A}' = \vec{A} + \vec{\nabla} \lambda}$$

$\vec{A}$ is corresponding to $\vec{B}$

$\vec{A}'$ " " " $\vec{B}'$

We have to proof for different $\vec{A}$ & $\vec{A}'$, we will get same $\vec{B}$.

Simultaneously, we have to subtract time derivative from scalar potⁿ.

$$\boxed{V' = V - \frac{\partial \lambda}{\partial t}}$$

$\lambda \rightarrow$ Gauge function

Corresponding to $\vec{A} \longrightarrow V, \vec{E}$

" $\vec{A}' \longrightarrow V', \vec{E}'$

$$\vec{B}' = (\vec{\nabla} \times \vec{A}')$$

$$= \vec{\nabla} \times \vec{A} + 0$$

$$\boxed{\vec{B}' = \vec{B}}$$

On changing $\vec{A}$ by $\vec{A}'$, $\vec{B}$ (mag. field) remains same.

$$\vec{E}' = -\vec{\nabla} V' - \frac{\partial \vec{A}'}{\partial t}$$

$$\vec{E}' = -\vec{\nabla} V + \vec{\nabla} \left(\frac{\partial \lambda}{\partial t} \right) - \frac{\partial \vec{A}}{\partial t} - \vec{\nabla} \left(\frac{\partial \lambda}{\partial t} \right)$$

$$\vec{E}' = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

* To solve Maxwell's eqⁿ in form of potⁿ, we use their restrictions (gauge) [C.G. & L.G.]

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$$\vec{E}' = \vec{E}$$

Hence, for same value of $\vec{E}$ & $\vec{B}$, we get different sets of $\vec{A}$ & V .

Coulomb Gauge :-

Condition of Coulomb gauge is

$$\vec{\nabla} \cdot \vec{A} = 0$$

We have

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Subs. the value of $\vec{E}$, from (B),

$$\Rightarrow \vec{\nabla} \cdot \left(-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \right) = \frac{\rho}{\epsilon_0} \quad (\vec{\nabla} \cdot \vec{A} = 0)$$

$$\Rightarrow \boxed{\nabla^2 V = -\frac{\rho}{\epsilon_0}} \Rightarrow \text{Poisson's Eqⁿ}$$

This is the wave eqⁿ of V under Coulomb gauge.

$$\text{We have } \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Subs. the value of $\vec{B}$ & $\vec{E}$ from (A) & (B)

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \right)$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \vec{\nabla} \left(\frac{\partial V}{\partial t} \right) - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} + \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = \mu_0 \vec{J} - \mu_0 \epsilon_0 \vec{\nabla} \left(\frac{\partial V}{\partial t} \right)$$

$$\Rightarrow \boxed{\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J} + \mu_0 \epsilon_0 \vec{\nabla} \left(\frac{\partial V}{\partial t} \right)}$$

This is the wave eqⁿ for vector potⁿ $\vec{A}$.

Under Coulomb Gauge, it is easy to find V but difficult to find $\vec{A}$.

Hence this gauge is of little use.

Lorentz Gauge :-

Condⁿ of Lorentz gauge is

$$\boxed{\vec{\nabla} \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t} = 0}$$

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

$$\Rightarrow \vec{\nabla} \cdot \left(-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \right) = \frac{\rho}{\epsilon_0}$$

$$\Rightarrow -\nabla^2 V - \frac{\partial}{\partial t} (\nabla \cdot \vec{A}) = \frac{\rho}{\epsilon_0}$$

$$\boxed{\nabla^2 V - \mu_0 \epsilon_0 \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}} \quad \text{--- (1)}$$

This is the wave eqⁿ for scalar potⁿ V .

$$\text{Now, } \vec{\nabla} \times \vec{E} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \right)$$

$$\Rightarrow \vec{\nabla} (\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J} - \mu_0 \epsilon_0 \vec{\nabla} \left(\frac{\partial V}{\partial t} \right) - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$\Rightarrow -\nabla^2 \vec{A} + \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} + \underbrace{\vec{\nabla} (\vec{\nabla} \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t})}_{=0} = \mu_0 \vec{J}$$

$$\Rightarrow \boxed{\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}} \quad \text{--- (2)}$$

This is the wave eqⁿ of vector potⁿ $\vec{A}$.

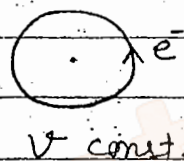
eqⁿ (1) & (2) are the wave eqⁿ under Lorentz Gauge.

Leinard-Weichard potential :- This is potⁿ of a moving point charge moving at velocity very close to speed of light (relativistic)

If charge q is moving in straight line with const. vel. $\vec{v}$, then it can $q \xrightarrow{\vec{v}_{\text{const.}}}$ not emit electromag. radiation

'Only accelerating charge can emit EM radiation.

If charge q is moving with constant vel. v in a circle then it will emit EM radiation bcoz v is const. but dirⁿ is changing. So it is accelerated motion



$$\vec{a} = \frac{d\vec{v}}{dt}$$

It depend on both dirⁿ & velocity

Retarded time (t_r) :- (means time lag)

L-W potⁿ is Retarded potⁿ,

Potⁿ (not freeⁿ of time)

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(r)}{r} d\tau$$

Potⁿ at time t is due to charge of retarded time,

$$V(r, t) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(r, t_r)}{r} d\tau$$

Sun-light reaches on earth in 8 min (ret. time) & light of stars are the concept of ret. time.

Retarded time, $t_r = t - \frac{r}{c}$

$t \rightarrow$ present time

$\frac{r}{c} \rightarrow$ time of travel

Illy. for $\vec{A}$,

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(\vec{r}', t_r)}{r} d\tau$$

$\rightarrow$ We can not find $\vec{E}$ & $\vec{B}$ directly through retarded time but we can find potⁿ

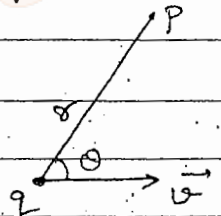
Bcoz $\vec{E}$ & $\vec{B}$ are not linear with $\frac{1}{r}$

$\rightarrow$ We can find $\vec{E}$ & $\vec{B}$ by $\vec{B} = \vec{\nabla} \times \vec{A}$

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

- A point charge q moving with const. vel^y velocity in a straight line then its potⁿ at r distance

$$V(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{r \sqrt{1 - \frac{v^2 \sin^2 \theta}{c^2}}}$$



If charge is stationary then

potⁿ will be same on the surface of sphere (if we sketch a sphere of radius r) at a inst^y time

But if charge is moving then potⁿ is not same. Potⁿ at P_1 is more than at P_2 .

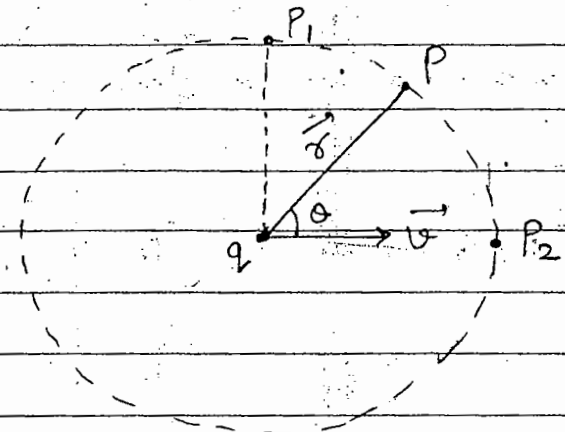
$$V_{P_1} > V_{P_2}$$

Here $v \approx c$, if

$v \ll c$ then

no effect of

potⁿ v .



$$V_{\text{static}} = \frac{q}{4\pi\epsilon_0 r} \quad \text{Static means } v=0$$

Vector potⁿ of a point charge

$$\vec{A} = \frac{\vec{v}}{c^2} V(\vec{r}, t)$$

if charge is static then $v=0 \Rightarrow \vec{A}=0$

Vector potⁿ $\vec{A}$ depends on both strength of charge q as well as velocity $\vec{v}$.

Electric field, of a pt. charge moving in straight line with const. vel^y v ,

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{(1 - \frac{v^2}{c^2})}{(1 - \frac{v^2}{c^2} \sin^2 \theta)^{3/2}} \frac{\hat{r}}{r^2}$$

If charge particle q is stationary then $\vec{E}$ will be same But if q is moving then

$$\vec{E}_{\text{static}} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$$

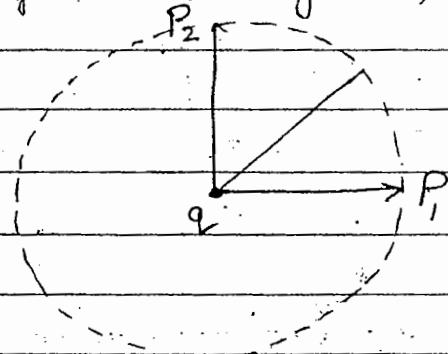
for P_1 , $\theta = 0$

$$\vec{E}_{P_1} = \frac{q}{4\pi\epsilon_0} \frac{(1 - \frac{v^2}{c^2})}{r^2} \hat{r}$$

At P_2 , $\theta = 90^\circ$

$$\vec{E}_{P_2} = \frac{q}{4\pi\epsilon_0} \frac{1}{\sqrt{(1 - \frac{v^2}{c^2})} r^2} \hat{r}$$

$$\vec{E}_{P_2} > \vec{E}_{P_1}$$

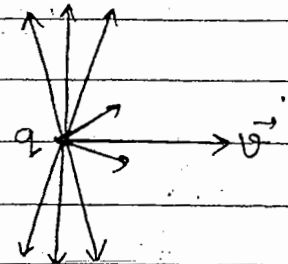


$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\vec{E}_{P_1} = \frac{\vec{E}_{\text{static}}}{\gamma^2}$$

$$\vec{E}_{P_2} = \gamma \vec{E}_{\text{static}}$$

Field lines :- will be symmetrical but their strength will be different in diff. dirⁿ. If charge particle q move with c then in the dirⁿ of motion of q there will be no field.



$$E_p \rightarrow 0$$

$$E_p \rightarrow \infty$$

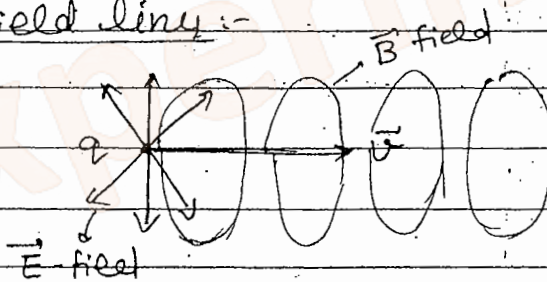
i.e. electric field will have no component, Mag. field,

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{qv(1 - \frac{v^2}{c^2}) \sin \theta}{[1 - \frac{v^2}{c^2} \sin^2 \theta]^{3/2}} \hat{\phi}$$

if $\theta = 0$ then $\vec{B} = 0$

Mag. field can never exist in dirⁿ of motion of q .

Field lines :-



Thumb will be in the dirⁿ of motion of particle. Mag. field will curl around dirⁿ of motion.

Radiation :- Radiation is propagation of energy in the form of EM waves.

Radiation means EM wave or photon. For a stationary charge, energy/intensity/power/poynting vector is same thing.

	E	B	$\vec{S}$	Power
Stationary charge	$\propto \frac{1}{r^2}$	0	0	0
Charge moving with const. vel. $\rightarrow \vec{v}$	$\propto \frac{1}{r^2}$	$\propto \frac{1}{r^2}$	$\propto \frac{1}{r^4}$	$\propto \frac{1}{r^2}$
	($\vec{E}$ & $\vec{B}$ are independent)			

We know,

Intensity, $I = \langle \vec{S} \rangle = \text{energy per unit area/unit time}$

Power, $\boxed{P = \int_S \langle \vec{S} \rangle d\vec{a}}$

For charge particle moving with const. v . Power is dec. as $\frac{1}{r^2}$. After some time $P=0$.

Accelerated Motion, $\vec{E}$ & $\vec{B}$ of an accelerated charge are $\propto \frac{1}{r}$.

	E	B	S	P
	$\propto \frac{1}{r}$	$\propto \frac{1}{r}$	$\propto \frac{1}{r^2}$	constant

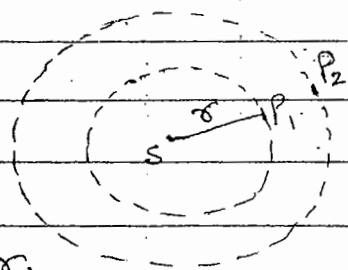
Quantized / Accelerated Radiation :- Mag. of $\vec{E}$ & $\vec{B} \propto \frac{1}{r}$

EM wave coming from Sun, $E \propto \frac{1}{r}$ & $B \propto \frac{1}{r}$.

Both spheres have same power at surface

But energy/unit area at P_1 is more than " " " " P_2

But power is const. independent on r .



If Power dec. by some value of r & area is $\uparrow$ by the same value of r .

i.e. Power is constant over a spherical surface.
(if power decay then we can't receive sun rays)

For a, point charge, dipole, Quadrupole,
Accelerated part of electric & mag. field
defined as $E_a \propto \frac{1}{r}$ & $B_a \propto \frac{1}{r}$

$S \propto \frac{1}{r^2}$, $P \rightarrow$ const. over a given surface.

Electric dipole Radiation:-

Consider a electric dipole, along z -axis.

dipole will be oscillating

if q is "free" of time t .

& if q is not a

"free" of t then dipole
will be static & radiation

will not emit in

that case.

i.e. charge

If dipole is oscillating with freq. ω
then

$$q(t) = q_0 \cos \omega t$$

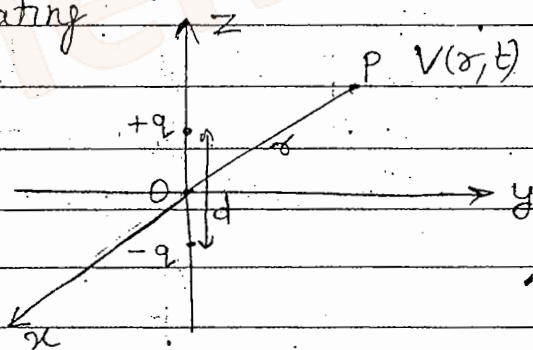
Its dipole moment $\vec{p} = qd$

$$\vec{p} = q_0 d \cos \omega t \hat{z}$$

$$\vec{p} = p_0 \cos \omega t \hat{z}$$

& freq. of oscillation is ω .

Now find at point P , what is the



scalar & vector potⁿ, Ele. & mag. field, Poynting vector S & power radiated by the dipole.

Use the concept of Retarded time.

$$t_r = t - \frac{r}{c}$$

The changes in dipole at time t will happen in retarded time t_r . We have to take approximation.

Approximation :- P is at a far distance so Power at P will be due to Accelerated potⁿ.

$$E = E_s + E_v + E_a$$

static velocity acceleration

Static & velocity part is very very less than accelerated part at far distance

$$\left. \begin{array}{l} \text{(i)} \quad d \ll r \\ \text{(ii)} \quad d \ll \frac{c}{\omega} \\ \text{(iii)} \quad r \gg \frac{c}{\omega} \end{array} \right\} \Rightarrow \text{Radiation Zone} \\ \text{or far-field zone}$$

Combiningly, $d \ll \frac{c}{\omega} \ll r$

$$\boxed{d \ll \lambda \ll r}$$

• Scalar Potⁿ,

$$V(r, \theta, t) = \frac{-p_0 \omega}{4\pi\epsilon_0 c} \left(\frac{\cos\theta}{r} \right) \sin\left(\omega t - \frac{\omega r}{c}\right)$$

• Vector Potⁿ,

$$\vec{A}(r, \theta, t) = -\frac{\mu_0 p_0 \omega}{4\pi r} \sin\left(\omega t - \frac{\omega r}{c}\right) \hat{z}$$

$V, \vec{A}, E, B \propto \frac{1}{r}$ for accelerated charge or radiation.

- Electric field

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{E} = -\frac{\mu_0 p_0 \omega^2}{4\pi} \left(\frac{\sin \theta}{r} \right) \cos \left[\omega t - \frac{\omega r}{c} \right] \hat{\theta}$$

by $\vec{\nabla} V \rightarrow$ radial part $\rightarrow$ drop the θ part

- Magnetic field

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{B} = -\frac{\mu_0 p_0 \omega^2}{4\pi c} \left(\frac{\sin \theta}{r} \right) \cos \left[\omega t - \frac{\omega r}{c} \right] \hat{\phi}$$

- Poynting Vector

$\vec{E}$ is along $\hat{\theta}$, $\vec{B}$ is along $\hat{\phi}$ so $\vec{S}$ is along $\Rightarrow \hat{\theta} \times \hat{\phi} = \hat{r}$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\vec{S} = \frac{\mu_0 p_0^2 \omega^4}{16\pi^2 c} \left(\frac{\sin^2 \theta}{r^2} \right) \cos^2 \left[\omega t - \frac{\omega r}{c} \right] \hat{r}$$

$\hat{r}$ means power (energy) is going out.

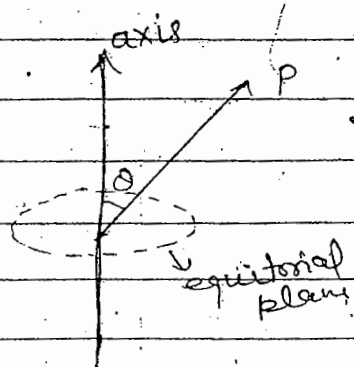
- Intensity

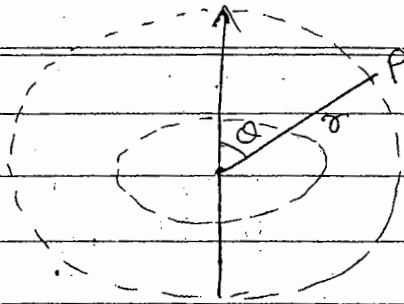
$$I = \langle \vec{S} \rangle = \frac{\mu_0 p_0^2 \omega^4 \sin^2 \theta}{32\pi^2 c r^2}$$

- * If P is along axis then $\theta = 0$
i.e. Power = 0

- ✓ No power is radiated along axis of the dipole.

- ✓ Maxi. power radiated along equatorial plane





if we sketch a sphere then

Power radiated over the surface of sphere

$$P = \int_S \langle \vec{S} \rangle d\vec{a}$$

$$d\vec{a} = r^2 \sin\theta d\theta d\phi$$

$$P = \frac{\mu_0 k^2 \omega^4}{32 \pi^2 c} \int_0^{2\pi} d\phi \int_0^\pi \sin^3\theta d\theta$$

$$P = \frac{\mu_0 k^2 \omega^4}{12 \pi c} = \text{const.}$$

So 'Power radiated of a dipole is proportional to ω^4 .

$$P \propto \omega^4$$

Magnetic Dipole Radiation:- Suppose we

have a magnetic dipole in the form of circular ring of radius

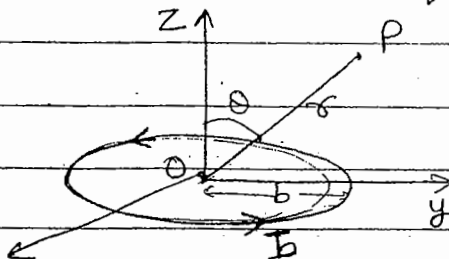
b & current flowing

$I(t)$ which is time

dependent (if I is

not time dependent then

change it will not radiate.



This circular ring is uncharged.

$$I(t) = I_0 \cos \omega t$$

Mag. mom. $\vec{m} = I \cdot A \vec{e}_z$

$$\vec{m} = I \cdot \pi b^2 \hat{z}$$

$$\vec{m} = I_0 \pi b^2 \cos \omega t \hat{z}$$

$$\vec{m} = m_0 \cos \omega t \hat{z}$$

m_0 is the amp. of mag. mom.

- Because loop is uncharged so scalar potⁿ $V=0$ but $A \neq 0$ bcoz V arise from charge & A arise from current.
Approximation

$$b \ll \frac{c}{\omega} \ll r$$

$$b \ll \lambda \ll r$$

} far-field zone/
radiation zone

- Vector potⁿ:- $\vec{A}$ is || or anti|| to current.

$$\vec{A}(r, \theta, t) = -\frac{\mu_0 m_0 \omega}{4\pi c} \left(\frac{\sin \theta}{r} \right) \sin \left(\omega t - \omega \frac{r}{c} \right) \hat{\phi}$$

- Electric field:-

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

Since $V=0$ so $\vec{E} = -\frac{\partial \vec{A}}{\partial t}$

$$\vec{E} = \frac{\mu_0 m_0 \omega^2}{4\pi c} \left(\frac{\sin \theta}{r} \right) \cos \left[\omega t - \omega \frac{r}{c} \right] \hat{\phi}$$

- Magnetic field:-

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{B} = -\frac{\mu_0 m_0 \omega^2}{4\pi c^2} \left(\frac{\sin \theta}{r} \right) \cos \left[\omega t - \omega \frac{r}{c} \right] \hat{\theta}$$

$\vec{B}$ at centre will be in $\hat{z}$ dirⁿ.

- Poynting Vector:-

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\vec{S} = \frac{\mu_0 m_0^2 \omega^4}{16 \pi^2 c^3} \left(\frac{\sin^2 \theta}{r^2} \right) \cos^2 \left[\omega t - \frac{\omega r}{c} \right] \hat{r}$$

Again, No energy radiated along z-dir.

- Intensity :- average of $\vec{S}$ over one cycle of time.

$$I = \langle \vec{S} \rangle = \frac{\mu_0 m_0^2 \omega^4}{32 \pi^2 c^3} \left(\frac{\sin^2 \theta}{r^2} \right) \hat{r}$$

- Power Radiated :-

$$P = \int \langle \vec{S} \rangle \cdot d\vec{a}$$

$$P = \frac{\mu_0 m_0^2 \omega^4}{12 \pi c^3}$$

Comparison of Electric dipole & Magnetic dipole :-

$$\left(\frac{P_{ele}}{P_{mag}} \right) = \left(\frac{p_0 c}{m_0} \right)^2$$

This is the energy comparison.

- # Energy around a dipole whether electric or magnetic forms a Donut shape.

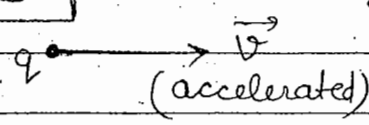
⇒ shape in which energy is maxi. along eq. equatorial plane & no energy along the axis.

Power radiated due to Quadrupole :-

$$P \propto \omega^6$$

Power Radiated by an Accelerated point charge,

$$P = \frac{\mu_0 q^2 \gamma^6}{6\pi c} \left[a^2 - \left| \frac{\vec{v} \times \vec{a}}{c} \right|^2 \right] \Rightarrow \text{General formula}$$

where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$  (accelerated)

$a \rightarrow$ acceleration

$v \rightarrow$ velocity

This formula is called Leinard Formula.

There may be 3 cases,

(i) $v \ll c$, (ii) v & a are $\perp$ (iii) v & a are collinear.

(i) $v \ll c$

charge particle is moving with non-relativistic speed. i.e. $\gamma \rightarrow 1$, power

$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

This is called Larmor formula.

Here power radiated $P \propto q^2 \& a^2$.

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \times \frac{\epsilon_0}{\epsilon_0}$$

$$\mu_0 \epsilon_0 = \frac{1}{c^2}$$

$$P = \frac{q^2 a^2}{6\pi \epsilon_0 c^3}$$

This is another form of Larmor formula.

$\Rightarrow$ If e^- & α particle are accelerated with same acceleration then α particle will be accelerated more. α -particle will emit 4 times power than e^- bcoz it depends of magnitude of charge.

{ e^- & p will emit same power if they acc. with same acc. }

(ii) Synchrotron Radiation

v & a are $\perp$ to each other.

If charge particle moves in a circle then v & a are $\perp$ to each other.

$$P = \frac{\mu_0 q^2 \gamma^6 a^2 (1 - v^2/c^2)}{6\pi c (1 - v^2/c^2)}$$

$$P = \frac{\mu_0 q^2 \gamma^6 a^2}{6\pi c \gamma^2}$$

$$P = \frac{\mu_0 q^2 \gamma^4 a^2}{6\pi c}$$

(iii) v & a are colinear (Bremsstrahlung Radiation)
then angle b/w them = 0
this is possible if charge particle is moving in a straight line.

$$P = \frac{\mu_0 q^2 \gamma^6 a^2}{6\pi c}$$

This is also called Breaking Radiation.

This is found, when high speed e^- s decelerated.

⇒ When high speed e^- fall on metal surface then e^- will be decelerated & energy will radiates, it will be Continuous x-ray, & this radiation is called Bremsstrahlung Radiation.

Exercise-2

Q.1:- Find the fields, charge & current distributions corresponding to $V(\vec{r}, t) = 0$

$$\vec{A}(\vec{r}, t) = -\frac{1}{4\pi\epsilon_0} \frac{qz}{r^2} \hat{z}$$

Q.2:- Suppose $V=0$ & $\vec{A} = A_0 \sin(kx - \omega t) \hat{y}$ where A_0 , ω & k are constants. Find $\vec{E}$ & $\vec{B}$ & check whether they satisfy Maxwell's eqⁿ in vacuum or not & what condition is to be imposed on ω & k .

Q.3:- Use Gauge funcⁿ $\lambda = -\frac{1}{4\pi\epsilon_0} \frac{qz}{r}$ to transform the potⁿs given by $V(\vec{r}, t) = 0$ & $\vec{A}(\vec{r}, t) = -\frac{1}{4\pi\epsilon_0} \frac{qz}{r^2} \hat{z}$ i.e. V' & $\vec{A}' = ?$

Q.4:- Found which is Coulomb Gauge, Lorentz Gauge or both in the following potentials:-

(i) $V(\vec{r}, t) = 0$

$$\vec{A}(\vec{r}, t) = -\frac{1}{4\pi\epsilon_0} \frac{qz}{r^2} \hat{z}$$

(ii) $V=0$, $\vec{A} = A_0 \sin(kx - \omega t) \hat{y}$

Exercise - 1

Q.1 :- $\vec{E} = \hat{i} A \sin(12y) \sin(\alpha z) \cos \omega t$

($\alpha = 13$)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A \sin(12y) \sin(\alpha z) \cos \omega t & 0 & 0 \end{vmatrix}$$

$$\vec{\nabla} \times \vec{E} = \hat{j} \left[\frac{\partial}{\partial z} \{ A \sin(12y) \sin(\alpha z) \cos(\omega t) \} \right] - \hat{k} \left[\frac{\partial}{\partial y} \{ A \sin(12y) \sin(\alpha z) \cos(\omega t) \} \right]$$

$$= \hat{j} A \alpha \sin(12y) \cos(\alpha z) \cos(\omega t) - \hat{k} A (12) \cos(12y) \sin(\alpha z) \cos(\omega t)$$

$$-\frac{\partial \vec{B}}{\partial t} = A \cos \omega t \left[\hat{j} \alpha \sin(12y) \cos(\alpha z) - \hat{k} 12 \cos(12y) \sin(\alpha z) \right]$$

$$\vec{B} = -\int dt \left[A \cos \omega t \left\{ \hat{j} \alpha \sin(12y) \cos(\alpha z) - \hat{k} 12 \cos(12y) \sin(\alpha z) \right\} \right]$$

$$\vec{B} = -\frac{A \sin \omega t}{\omega} \left[\hat{j} \alpha \sin(12y) \cos(\alpha z) - \hat{k} 12 \cos(12y) \sin(\alpha z) \right]$$

Now $\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ ————— ①

$$\Rightarrow \vec{\nabla} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & -\alpha \sin(12y) \cos \alpha z & 12 \cos(12y) \sin \alpha z \end{vmatrix}$$

$$= \hat{i} \left[-144 \sin(12y) \sin(\alpha z) - \alpha^2 \sin(12y) \sin(\alpha z) \right]$$

① $\Rightarrow \frac{A \sin \omega t}{\omega} \hat{i} \left[-144 - \alpha^2 \right] \sin(12y) \sin(\alpha z)$

$$= +\mu_0 \epsilon_0 \omega \hat{i} A \sin(12y) \sin(\alpha z) \sin \omega t$$

$$\Rightarrow \frac{1}{\omega} [-144 - \alpha^2] = \omega \mu_0 \epsilon_0$$

$$-144 - \alpha^2 = \frac{\omega^2}{c^2}$$

$$\Rightarrow \alpha^2 = -144 - \left(\frac{15 \times 10^8}{3 \times 10^8} \right)^2 = -144 - 25$$

$$\Rightarrow \alpha^2 = -169$$

$$\Rightarrow \boxed{\alpha = 13i}$$

Q.2:- $\vec{B} = B_0 e^{\alpha x} \sin(ky - \omega t) \hat{z}$ in empty

space. If for a plane wave $\vec{k} \cdot \vec{B} = 0$

$$\text{then } \vec{E} = -\frac{c^2}{\omega} (\vec{k} \times \vec{B})$$

from $\vec{B}$, we get $\vec{k} = k \hat{y}$

& $\vec{k} \cdot \vec{B} = 0$ Here, so

$$\vec{E} = -\frac{c^2}{\omega} [k B_0 e^{\alpha x} \sin(ky - \omega t)] \hat{x}$$

{ $\hat{y} \times \hat{z} = \hat{x}$ }

$$\boxed{\vec{E} = \frac{c^2}{\omega} k B_0 e^{\alpha x} \sin(ky - \omega t) (-\hat{x})}$$

Another Method,

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \vec{E} = \int \frac{1}{\mu_0 \epsilon_0} (\vec{\nabla} \times \vec{B}) dt$$

$$\vec{\nabla} \times \vec{B} = \hat{i} \left[\frac{\partial}{\partial y} B_z \right] - \hat{j} \left[\frac{\partial}{\partial x} B_z \right] + \hat{k} (0)$$

$$= \hat{i} [B_0 e^{\alpha x} k \cos(ky - \omega t)] - \hat{j} [B_0 \alpha e^{\alpha x} \sin(ky - \omega t)]$$

$$\text{So } \vec{E} = c^2 \int [\hat{i} B_0 e^{\alpha x} k \cos(ky - \omega t) - \hat{j} B_0 \alpha e^{\alpha x} \sin(ky - \omega t)] dt$$

$$\vec{E} = c^2 \left[\hat{i} B_0 e^{\alpha x} k \frac{\sin(ky - \omega t)}{(-\omega)} + \hat{j} B_0 \alpha e^{\alpha x} \frac{\cos(ky - \omega t)}{-\omega} \right]$$

$$\vec{E} = \frac{c^2}{\omega} \left[(-\hat{i}) B_0 e^{\alpha x} k \sin(ky - \omega t) + (\hat{j}) B_0 \alpha e^{\alpha x} \cos(ky - \omega t) \right]$$

Q.3:- $\vec{E}_1(z,t) = (E_0 \hat{x} + E_0 e^{-i\frac{\pi}{2}} \hat{y}) e^{i(kz - \omega t)}$

$\vec{E}_2(z,t) = (E_0 \hat{x} + E_0 e^{i\frac{\pi}{2}} \hat{y}) e^{i(kz - \omega t)}$

Put $z=0$, in $\vec{E}_1$ then

$\vec{E}_1 = [E_0 \hat{x} + E_0 (-i) \hat{y}] e^{-i\omega t}$

→ At $\omega t = 0$ then

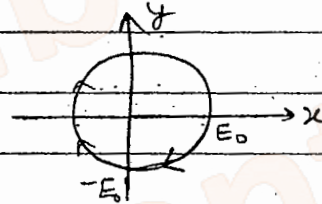
$\vec{E}_1 = E_0 \hat{x} - E_0 i \hat{y}$

$\vec{E}_1 = E_0 \hat{x}$

→ At $\omega t = \pi/2$, $\vec{E}_1 = (E_0 \hat{x} - i E_0 \hat{y}) (-i)$

$\vec{E}_1 = -i E_0 \hat{x} - E_0 \hat{y}$

$\vec{E}_1 = -E_0 \hat{y}$



So acc. to Ang. mom. → LCP

So Acc. to Optics RCP.

clockwise

Now, Put $z=0$ in $\vec{E}_2$

$\vec{E}_2 = (E_0 \hat{x} + i E_0 \hat{y}) e^{-i\omega t}$

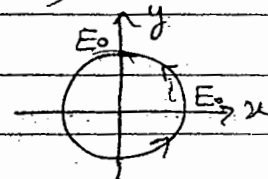
At $\omega t = 0$, $\vec{E}_2 = E_0 \hat{x} + i E_0 \hat{y}$

$\vec{E}_2 = E_0 \hat{x}$

At $\omega t = \frac{\pi}{2}$, $\vec{E}_2 = (E_0 \hat{x} + i E_0 \hat{y}) (i - i)$

$\vec{E}_2 = -i E_0 \hat{x} + E_0 \hat{y}$

$\vec{E}_2 = E_0 \hat{y}$



So acc. to optics LCP

Now $\vec{E}_1(z,t) = (E_0 \hat{x} - i E_0 \hat{y}) e^{i(kz - \omega t)}$

$\vec{E}_2(z,t) = (E_0 \hat{x} + i E_0 \hat{y}) e^{i(kz - \omega t)}$

$\vec{E} = \vec{E}_1 + \vec{E}_2$

$\vec{E} = 2 E_0 \hat{x} e^{i(kz - \omega t)}$

Resultant wave polarised along x-dirⁿ.

Q.4 :- $\vec{E} = \frac{E_0 (8\hat{x} + 6\hat{y} + 5\hat{z})}{\sqrt{125}} e^{i(\omega t + 3x - 4y)}$

freq. $\nu = 1 \text{ GHz}$

(a) $\vec{E} = \frac{E_0 (8\hat{x} + 6\hat{y} + 5\hat{z})}{\sqrt{125}} e^{i\{\omega t - [(-3\hat{x} + 4\hat{y})(x\hat{x} + y\hat{y})]\}}$

$\vec{K} = -3\hat{x} + 4\hat{y}$

$\vec{K} \cdot \vec{E} = -24 + 24 = 0 \Rightarrow \vec{K} \cdot \vec{E} = 0$

So it is a plane wave.

(b) $\vec{K} = -3\hat{x} + 4\hat{y}$

(c) $v_g = \frac{\omega}{K_y} = \frac{2\pi\nu}{K_y}$

$v_g = \frac{2 \times 3.14 \times 1 \times 10^9}{4} = \frac{6.28}{4} \times 10^9$

$(v_p)_y = 1.57 \times 10^9$

This is more than c . v_p may be $> c$ but v_g is $< c$ s.t. $v_p v_g = c^2$

(d) $\epsilon_r, n = ?$

$v = \frac{\omega}{K} = \frac{2 \times 3.14 \times 1 \times 10^9}{\sqrt{9+16}} = \frac{6.28 \times 10^9}{5}$

$v = 1.256 \times 10^9$

$v = \frac{c}{n} \Rightarrow n = \frac{c}{v} = \frac{3 \times 10^8}{1.256 \times 10^9}$

$n = 2.38854 \times 10^{-1} \Rightarrow n = 0.23885$

$\epsilon_r = n^2 = (0.23885)^2 = 0.05705$

$\epsilon_r = 0.05705$

Q.5 - Plasma freq. = ? & S_{max} = ?

$$n = 10^{18} \text{ e}^{-}/\text{m}^3$$

$$f_c = \frac{1}{2\pi} \sqrt{\frac{n e^2}{m}}$$

$$\approx \frac{1}{2\pi} \sqrt{10^{18}} = 9 \times 10^9 \text{ Hz}$$

$$f_c = \frac{\omega_p}{2\pi} \Rightarrow \omega_p = 2\pi f_c$$

$$\omega_p = 2 \times 3.14 \times 9 \times 10^9$$

$$\omega_p = 5.66 \times 10^{10}$$

$$\omega_p = 5.66 \times 10^{10} \text{ Hz}$$

Now max. penetration depth

$$S_{max} = \frac{c}{\omega_p}$$

$$= \frac{3 \times 10^8}{5.66 \times 10^{10}}$$

$$= 0.53004 \times 10^{-2}$$

$$S_{max} = 0.0053$$

$$\text{or } S_{max} = 5.3 \times 10^{-3}$$

Note :- $\omega_p = \left(\frac{n e^2}{\epsilon_0 m} \right)^{1/2}$

Q.6 - He-Ne laser

$$\lambda = 632 \text{ nm}, I = 1 \text{ W/cm}^2$$

$$\vec{E} \text{ \& \; } \vec{B} = ? \quad = 10000 \text{ W/m}^2$$

$$I = \langle \vec{S} \rangle = \frac{1}{2} c \epsilon_0 E_0^2$$

$$\Rightarrow E_0^2 = \frac{2 \langle \vec{S} \rangle}{c \epsilon_0}$$

$$\begin{aligned}
 E_0 &= \sqrt{\frac{2 \langle S \rangle}{c \epsilon_0}} \\
 &= \sqrt{\frac{2 \times 10^9}{3 \times 10^8 \times 8.85 \times 10^{-12}}} \text{ V/m} \\
 &= 0.2744 \times 10^4 \\
 &= 2744 \text{ V/m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Now } B_0 &= \frac{E_0}{c} \\
 &= \frac{0.2744 \times 10^4}{3 \times 10^8} = 0.09147 \times 10^{-4} \\
 B_0 &= 9.146 \times 10^{-6}
 \end{aligned}$$

(b) Express $\vec{E}(x,t)$ & $\vec{B}(x,t)$ in polarised along y-dirⁿ.

$$\vec{E}(x,t) = E_0 e^{i(kx - \omega t)} \hat{y}$$

$$K = \frac{2\pi}{\lambda} \quad \& \quad \frac{\omega}{K} = c$$

$$\begin{aligned}
 K = \frac{2\pi}{\lambda} &= \frac{2 \times 3.14 \times 10^9}{632} = 0.00994 \times 10^9 \\
 &= 9.9 \times 10^6
 \end{aligned}$$

$$\omega = cK = 3 \times 10^8 \times 9.9 \times 10^6$$

$$\omega = 29.7 \times 10^{14}$$

$$\vec{E}(x,t) = 2744 e^{i(9.9 \times 10^6 x - 2.97 \times 10^{14} t)} \hat{y}$$

$$\& \vec{B}(x,t) = B_0 e^{i(kx - \omega t)} \hat{z}$$

$$\vec{B} = \frac{K \times \vec{E}}{\omega}$$

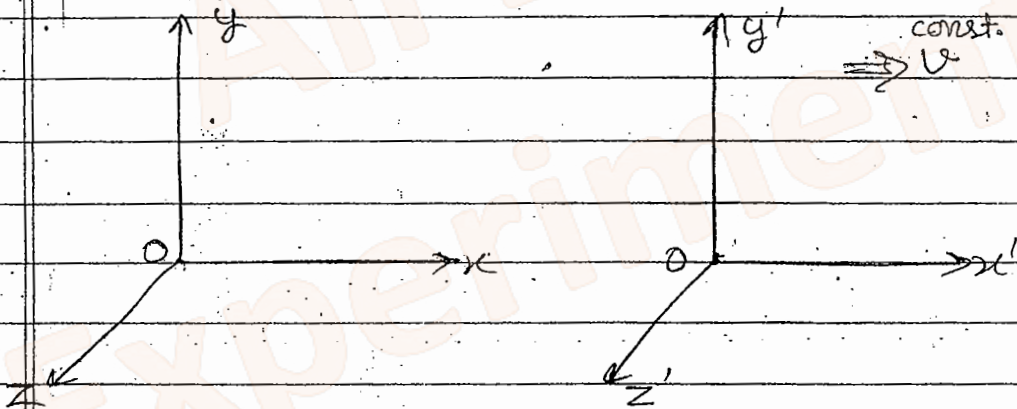
Special Theory of Relativity & Electrodynamics:-

It is apply only on inertial frame of references. Inertial frame of ref. are non accelerated & non rotated.

Postulates:-

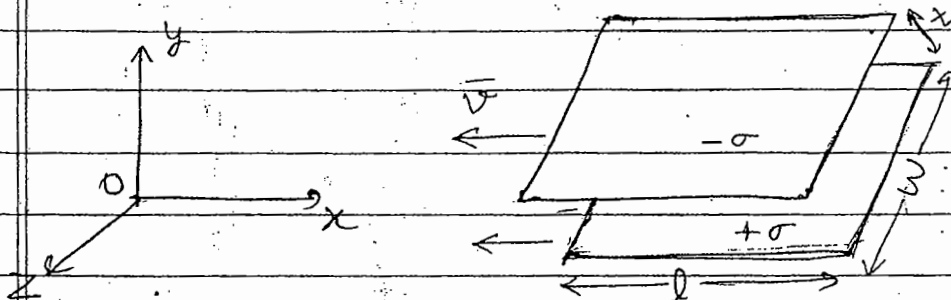
- 1) Speed of light is same for all inertial frame.
- 2) Law of physics are " " " "

Inertial frame - Newton's 1st law is valid.
How the fields are modified from one frame of reference to another.



We know ele. & mag. field in rest frames & have to find in moving frame S' .

Let us consider a parallel plate capacitor,



charge q is relativistically (Lorentz) invariant,

In Rest frame, Electric field

$$\vec{E}_0 = \frac{\sigma}{\epsilon_0} \hat{y}$$

When this capacitor moves then elec. field will be increased γ times as shown below

$$\sigma_0 = \frac{q}{A} = \frac{q}{lw}$$

When capacitor is moving then $\sigma = \frac{q}{\frac{l}{\gamma} w}$

$$\sigma = \frac{\gamma q}{lw} = \gamma \sigma_0 \Rightarrow \boxed{\sigma = \gamma \sigma_0}$$

$$\vec{E} = \frac{\gamma \sigma_0}{\epsilon_0} \hat{y}$$

$$\boxed{\vec{E} = \gamma \vec{E}_0}$$

$\Rightarrow$ When Eloc. field is $\perp$ to the motion then it $\uparrow \gamma$ times & when Eloc. field is $\parallel$ ^{or anti} to the motion then $\vec{E}$ will be No change in electric field. (length contraction in l & w but Not in t)

Hence,

$$\boxed{E_{\text{motion}}^{\parallel} = E_{\text{rest}}^{\parallel}}$$

&

$$\boxed{E_s^{\perp} = \gamma E_s^{\perp}}$$

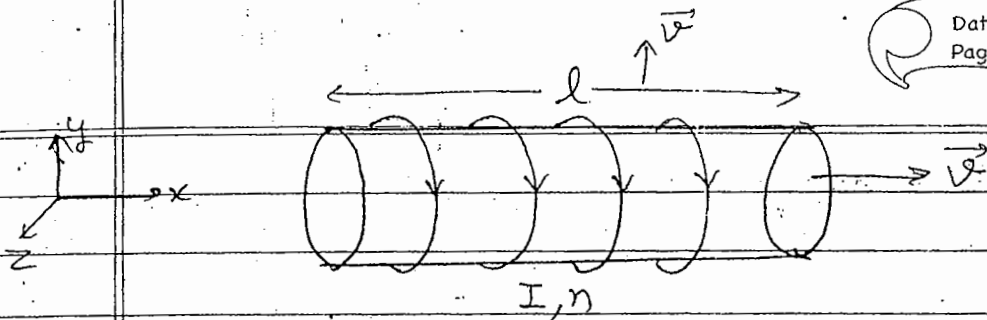
This is for independent E. field.

For independent Mag. field

Consider a solenoid & current flowing in its wires I & n be the no. of wires per unit length.

When it is at rest then its mag. field will be

$$\boxed{\vec{B} = \mu_0 n I \hat{x}}$$



When solenoid is in motion then current in it (charge is invariant)

$$I_{s'} = \frac{q}{t}$$

$$(q = It)$$

$$I_{s'} = \frac{I}{\gamma}$$

& No. of turns per unit length while it is moving, $\uparrow$ by γ times i.e.

$$n_{s'} = \gamma n$$

$$\text{So } \vec{B}_{s'} = \mu_0 n I \hat{x}$$

$$\boxed{\vec{B}_{s'} = \vec{B}_s} \quad \therefore$$

Conclusion:- When solenoid is moving in x-dir.

If Any comp. of $\vec{B}$ is in x-dir then it will be unaffected. i.e.

$$\boxed{B_{s'}^{\parallel} = B_s^{\parallel}}$$

(only 2 vectors $\neq$ will contract)

Now If solenoid is moving in upward dir then $\perp$ comp of mag. field is also $\uparrow$ by γ times

$$\boxed{B_{s'}^{\perp} = \gamma B_s^{\perp}}$$

bcz in this case simultaneously length of 2 vectors $\downarrow$ (contract)

(s, t)

$$n' = \gamma^2 n$$

$$B_{s'} = \mu_0 \gamma^2 n \frac{I}{\gamma} \hat{x} \Rightarrow \vec{B}_{s'} = \gamma \vec{B}_s$$

$$\text{i.e. } \boxed{B_{s'}^{\perp} = \gamma B_s^{\perp}}$$

Hence, No change in $\parallel$ comp of $\vec{E}$ & $\vec{B}$ & γ times increase the $\perp$ " " " " " "

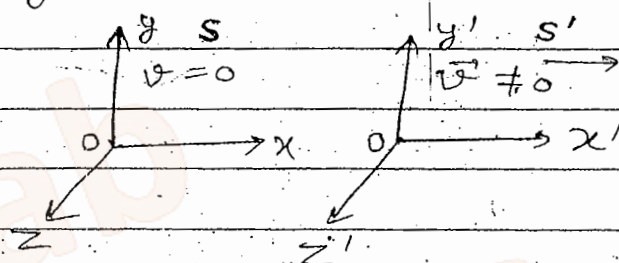
Combiningly for $\vec{E}$ & $\vec{B}$:-

Transformation eqⁿ :-

If motion is along x -dirⁿ (i.e. comp of $\vec{E}$ -field which is in the dirⁿ of motion - No change) i.e.

$$\boxed{E'_x = E_x}$$

$$\& \boxed{B'_x = B_x}$$



If v is the velocity of motion then

$$\boxed{E'_y = \gamma(E_y - vB_z)}$$

$$\boxed{E'_z = \gamma(E_z + vB_y)}$$

$$\boxed{B'_y = \gamma(B_y + \frac{vE_z}{c^2})}$$

$$\boxed{B'_z = \gamma(B_z - \frac{vE_y}{c^2})}$$

These are the modifications in Elec. & mag. field when motion is in x -dirⁿ.

All these eqⁿs are known as Transformation eqⁿs.

If Motion is in y -dirⁿ then

$$E'_x = \gamma(E_x + vB_z)$$

$$E'_y = E_y$$

Special Cases:-

1.) If Mag. field is 0 in rest frame

$$\vec{B}_s = 0 \quad \text{but} \quad \vec{E}_s \neq 0$$

$$\text{i.e. } B_x = B_y = B_z = 0$$

then in motion frame

$$\boxed{\vec{B}_{s'} = -\frac{1}{c^2}(\vec{v} \times \vec{E}_s)} \quad \text{or} \quad \vec{B}_{s'} = -\frac{\gamma}{c^2}(\vec{v} \times \vec{E}_s)$$

It is not necessary if $\vec{B} = 0$ in rest place then it is 0 in motion frame bcoz mag. field & E. field are not Lorentz invariant itself.

2.) If $\vec{E} = 0$, $\vec{B} \neq 0$

$$\text{then} \quad \boxed{\vec{E}_{s'} = \vec{v} \times \vec{B}_{s'}}$$

Lorentz Invariant Quantities:-

(i) charge q

(ii) $\vec{E} \cdot \vec{B}$

(iii) $E^2 - c^2 B^2$

$$\text{or } E^2 - \frac{1}{\mu_0 \epsilon_0} B^2$$

$$\text{or } \left(\epsilon_0 E^2 - \frac{B^2}{\mu_0} \right)$$

or $E^2 - B^2$ (in Gaussian Unit i.e. CGS)

$$\text{or } |\vec{E}|^2 - |\vec{B}|^2$$

(iv) $x^2 + y^2 + z^2 - c^2 t^2$

$$(v) \quad \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$$

(vi) $E^2 - p^2 c^2$ (rel^y energy)

if product $\vec{E} \cdot \vec{B}$ is 0 in rest frame then it will be 0 in moving frame

Conversions :-

From M.K.S. to Gaussian, the replacements are

$$\epsilon_0 \rightarrow \frac{1}{4\pi}$$

$$\mu_0 \rightarrow \frac{4\pi}{c^2}$$

$$\vec{B} \rightarrow \frac{\vec{B}}{c}, \quad \vec{A} \rightarrow \frac{\vec{A}}{c}$$

$$\vec{E} \rightarrow \vec{E}$$

$$\left\{ \begin{array}{l} \vec{B} = \nabla \times \vec{A} \\ \frac{\vec{B}}{c} = \nabla \times \frac{\vec{A}}{c} \end{array} \right\}$$

S.I.	Gaussian
1. $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$	$\nabla \cdot \vec{E} = 4\pi\rho$
2. $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$
3. $\nabla \cdot \vec{B} = 0$	$\nabla \cdot \vec{B} = 0$
4. $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$	$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}$
5. Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$	$\vec{F} = q(\vec{E} + \vec{v} \times \frac{\vec{B}}{c})$
6. energy density $u = \frac{1}{2} (\epsilon_0 E^2 + \frac{B^2}{\mu_0})$	$u = \frac{1}{8\pi} (E^2 + B^2)$
7. Poynting vector $\vec{S} = \frac{(\vec{E} \times \vec{B})}{\mu_0}$ $\vec{S} = \frac{\epsilon_0 \vec{E} \times \vec{B}}{\mu_0 \epsilon_0} = \frac{\epsilon_0 c^2 \vec{E} \times \vec{B}}{c^2}$	$\vec{S} = \frac{c}{4\pi} (\vec{E} \times \vec{B})$
8. Lorentz Gauge $\nabla \cdot \vec{A} = 0$	$\nabla \cdot \vec{A} = 0$
9. Coulomb Gauge $\nabla \cdot \vec{A} + \frac{1}{c^2} \frac{\partial V}{\partial t} = 0$	$\nabla \cdot \vec{A} + \frac{1}{c} \frac{\partial V}{\partial t} = 0$
10. Power radiated by a point charge , $P = \frac{\mu_0 q^2 a^2}{6\pi c}$ or $P = \frac{q^2 a^2}{6\pi \epsilon_0 c^3}$ Larmor formula	$P = \frac{2}{3} \frac{q^2 a^2}{c^3}$
11. $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$	$\vec{D} = \vec{E} + 4\pi \vec{P}$

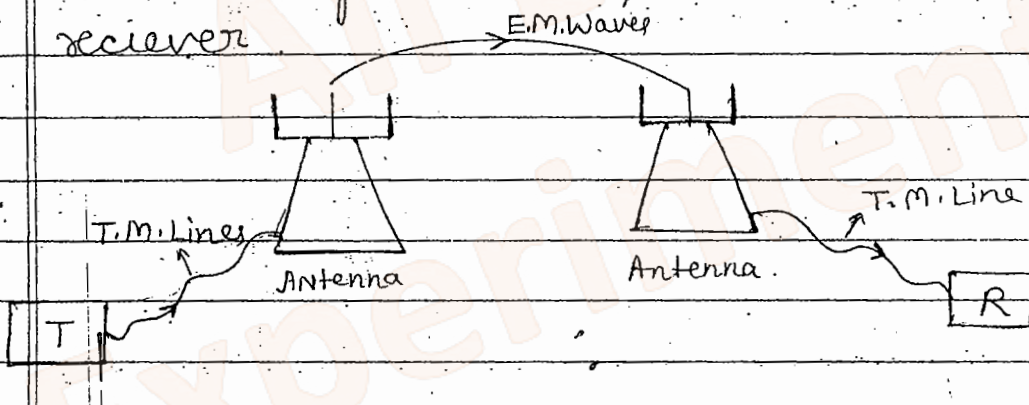
Exercise-3

Q.1:- Let $\vec{B} = B\hat{y}$ & $\vec{E} = 0$ in a rest frame S . Find all components of $\vec{E}'$ & $\vec{B}'$ in a frame S' which is moving with a constant velocity along x dirⁿ. $\vec{v} = v\hat{x}$.

Q.2:- Suppose in a rest frame $\vec{E} = \alpha\hat{x}$ & $\vec{B} = \beta\hat{y}$. Find electric & mag. fields in a frame moving with $\vec{v} = v\hat{y}$.

Transmission Lines:-

Transmission lines are the connector b/w i.e. connecting medium b/w transmitter & receiver.



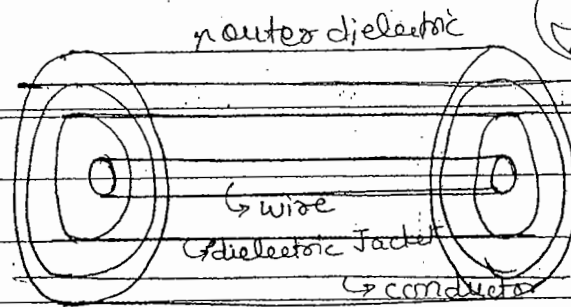
In T.M. lines, current & voltages are in the form of transverse electromagnetic i.e. TEM waves.

Transmission lines are of 2 types

- i) Two wire line
- ii) Co-axial cable

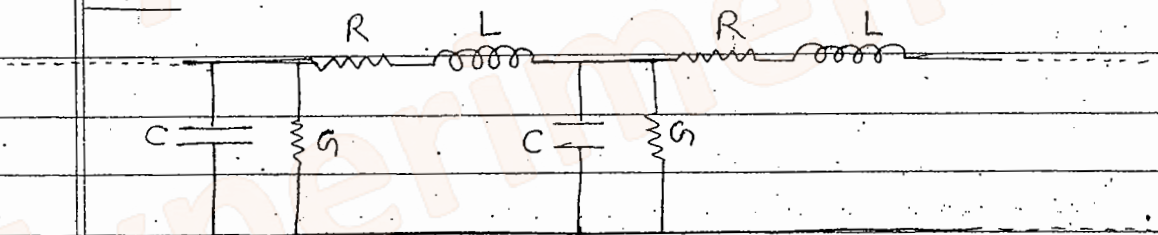
→ In Two wire line, 2 wires are always connected || to each other.

→ Now a days, Two wire lines are replaced by coaxial cable, in which there is a central wire, over which dielectric & then conductor.



- Transmission of information of transmission line always occur in the form of TEM wave. Transmission lines are commonly used in power distribution at low frequencies & in communication at high frequencies. They are of two type - Two wire line & coaxial cable.
balanced line Unbalance line

Equivalent Circuit-diagram of Transmission Line :-



These are distributed parameters. They are distributed over all the trans. line.

$G \rightarrow$ Conductance / unit length

$L \rightarrow$ Inductance / " "

$R \rightarrow$ Resistance / " "

$C \rightarrow$ capacitance / " "

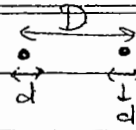
Expressions for L & C ;

(i) for two wire line,

$$L = \frac{\mu}{\pi} \ln\left(\frac{2D}{d}\right) \text{ H/m}$$

$$C = \frac{\pi \epsilon}{\ln\left(\frac{2D}{d}\right)} \text{ F/m}$$

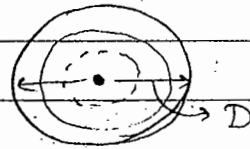
$d \rightarrow$ diameter of each wire
 $D \rightarrow$ distance b/w 2 wires



(ii) Coaxial Line) Cable :-

$$L = \frac{\mu}{2\pi} \ln\left(\frac{D}{d}\right) \text{ H/m}$$

$$C = \frac{2\pi\epsilon}{\ln\left(\frac{D}{d}\right)} \text{ F/m}$$



$d \rightarrow$ is the diameter of inner conductor (wire)
 $D \rightarrow$ " " " outer dielectric

Phase Velocity of T.M. Line:-

For any medium which have permeability μ & permittivity ϵ then phase vel.

$$v_p = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{\sqrt{\epsilon_r}} \quad \text{--- (1)}$$

Wavelength :- $\lambda = \frac{c}{f_0} = \frac{v_p}{f}$ (freq. is same in all medium)

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}} \quad \text{--- (2)}$$

If wave freq. vel. $\downarrow$ then W.L. will also $\downarrow$
s.t. their ratio must be same.

Product of L & C for both cases,

$$LC = \mu\epsilon$$

so
$$v_p = \frac{1}{\sqrt{LC}}$$

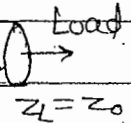
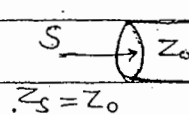
where $L \rightarrow$ inductance / unit length
 $C \rightarrow$ capacitance / " "

Characteristic Impedance (Z_0):

for max power transmission, impedance matching is required. i.e. impedance of T.M. Line of source & impedance of T.M. Line of receiver will match.

$$Z_L = Z_0$$

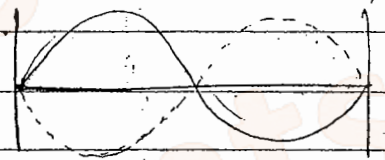
$$Z_S = Z_0$$



If impedance is not

matching & wave will be

reflected back but phase change - This is called Standing wave.



Expression for characteristic impedance,

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \Omega$$

(i) When T.M. Line is loss-less,

$R = 0$ & $G = 0$ then

$$Z_0 = \sqrt{\frac{L}{C}}$$

(ii) Distortionless transmission line

$$\frac{R}{G} = \frac{L}{C}$$

$$Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

(iii) At low frequencies,

$R \gg \omega L$ & $G \gg \omega C$

$$Z_0 = \sqrt{\frac{R}{G}}$$

(iv) At high frequencies,
 $R \ll \omega L$ & $G \ll \omega C$

$$Z_0 = \sqrt{\frac{L}{C}}$$

Propagation Coefficient (γ) :-

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

For Loss-less transmission line,

$$\gamma = j\omega \sqrt{LC}$$

For Distortion less T.M. line, ($\frac{R}{L} = \frac{G}{C}$)

$$\gamma = \sqrt{L \left(\frac{R}{L} + j\omega \right) C \left(\frac{G}{C} + j\omega \right)}$$

$$\gamma = \sqrt{LC \left(\frac{R}{L} + j\omega \right)^2}$$

$$\gamma = \left(\frac{R}{L} + j\omega \right) \sqrt{LC}$$

$$\gamma = \frac{1}{L} (R + j\omega L) \sqrt{LC}$$

$$\gamma = \sqrt{\frac{C}{L}} (R + j\omega L)$$

Reflection Coefficient :- Γ_L

i.e. How much part of incident voltage will be reflected back.

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Matched Load

(i) If $Z_L = Z_0$ then $\Gamma_L = 0$

i.e. No part will be reflected back.

(i) Short Circuit, $Z_L = 0$

then $\Gamma_L = -1$

(iii) Open Circuit (load), $Z_L = \infty$

then $\Gamma_L = 1$

So Range of Reflection coefficient is

$$-1 \leq \Gamma_L \leq 1$$

Voltage Standing Wave Ratio (VSWR) :-

$$VSWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

Case (i) $\rightarrow$ Matched Load

$Z_L = Z_0$ then $\Gamma_L = 0$

$$VSWR = 1$$

(ii) Short Circuit, $Z_L = 0$, $\Gamma_L = -1$

$$VSWR = \infty$$

(iii) Open circuit,

$Z_L = \infty$, $\Gamma_L = 1$

$$VSWR = \infty$$

Range of VSWR

$$1 \leq VSWR \leq \infty$$

Wave Guides :- Wave guides are hollow metallic pipes of uniform cross-section. They can transmit electromag. waves by successive reflection from inner walls of wave guide.

Optical fiber $\rightarrow$ dielectric wave guide.

Reason for using Wave Guide :- At high frequencies above 3 GHz, transmission of electromag. wave through transmission line becomes difficult due to the losses occurs in dielectric jacket as well as on the conductor.

If signal is of less than 3 GHz then for transmission of EM wave, transmission lines can be used. Working Range of TM lines ~~300~~ $0 \rightarrow 3$ GHz.

TM Line Waveguide support TEM wave.

$\rightarrow$ Working range of WG, 300 MHz - 300 GHz (Microwave region)

$\rightarrow$ Waveguide can not support TEM waves

But It supports TE & TM waves.

$\rightarrow$ Waveguide acts as a high pass filter. i.e. it can not support the EM waves whose freq. is less than the cut off freq. ω_c .
i.e. $\omega < \omega_c$.

It will support only those EM waves for which $\omega > \omega_c$

$\rightarrow$ Hence we can not transmit dc.

Boundary Conditions on the wave guide surface :-

At the surface

$$E'' = 0 \Rightarrow \boxed{E^t = 0}$$

Tangential comp. of E-field.

$$B_1^\perp = B_2^\perp$$

$$B_1^\perp = 0 \Rightarrow \boxed{B^n = 0}$$

i.e. Tangential comp. of E-field & normal comp. of mag. field must be zero.

(i) Transverse Electric (TE) wave :-

E-field is entirely transverse to the dirⁿ of wave propagation.

If wave propagation is only along z-dirⁿ

$$E_z = 0$$

$$B_z \neq 0$$

This is also called H-Wave.

(ii) Transverse Magnetic (TM) wave :-

$$B_z = 0, E_z \neq 0$$

This is also called E-Wave

(iii) TEM Wave :-

$$E_z = 0, B_z = 0$$

(iv) If $E_z \neq 0, B_z \neq 0$ then wave is called Mixed wave or Hybrid wave.

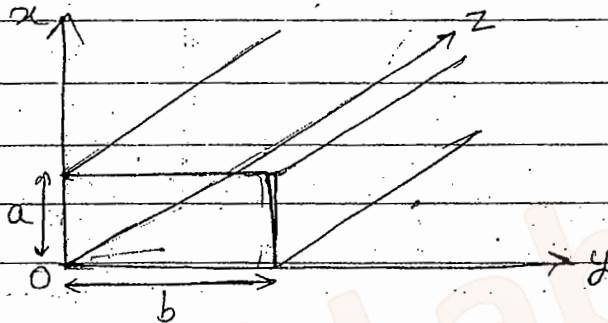
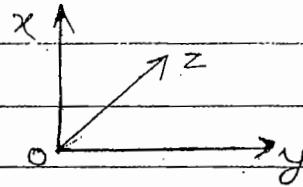
$$\text{TE :- } E_z = 0, H_z \neq 0$$

$$\text{TM :- } E_z \neq 0, H_z = 0$$

Transverse Electric Mode:-

$$E_z = 0 \quad \& \quad B_z \neq 0$$

Consider Rectangular
WG (most useful)



(a > b)

Comp. of mag. field along dirⁿ of wave propagation

$$B_z = B_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

By using B_z we can find other comp. of elec. & mag. field $\rightarrow E_x, E_y, B_x, B_y$

$z \rightarrow$ dirⁿ of wave propagation

$x \rightarrow$ dimensions along x-dirⁿ

$y \rightarrow$ " " " " y - "

mn , n & $m \rightarrow$ decides modes supported by the WG.

Dominant Mode:- TE_{mn}

$$TE_{10} \Rightarrow m=1, n=0$$

for TE_{10} ,

$$B_z = B_0 \cos\left(\frac{\pi x}{a}\right) e^{-\gamma z}$$

WG behaves as high pass filter.

Cut off freq.:- $f_c = \frac{wc}{2\pi}$

$$\checkmark \left[f_c = \frac{c}{2} \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 \right]^{1/2} \right]$$

Modes - how many waves can propagate at a time through a waveguide (WG).

classmate

Date

Page

depends upon modes & dimensions.

- Cut off Wavelength :-

$$v = \frac{c}{\lambda} \Rightarrow \lambda = \frac{c}{v}$$

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

- Guide Wavelength :- Wave travel inside the WG is called Guide wave. And WL of Guide wave is

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

Where, $\lambda_0 \rightarrow$ free space WL.

- Phase Velocity :- phase vel. of Guide wave is

$$V_p = \frac{c}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

- Group Velocity :-

$$V_g = c \sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

such that $V_p V_g = c^2$

V_p may be $> c$ but V_g always $< c$.

- Wave Guide Impedance :-

TE $\rightarrow$

$$Z_E = \frac{Z_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

TM $\rightarrow$

$$Z_M = Z_0 \sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

s.t.

$$Z_E Z_M = Z_0^2$$

Note: For different wave, cut off freq. will be different.

- If incident freq. $f = 10^9$ Hz

& cut off freq. $(f_c)_{mn}$

$$(f_c)_{10} = 10^8 \text{ Hz}$$

$$(f_c)_{11} = 10^9 \text{ Hz}$$

So 10 mode will propagate but 11 not.
 bcoz $f > (f_c)_{10}$ & $f < (f_c)_{11}$

- For TE Mode: Dominant Mode:-

$$B_z = B_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

1.) Lowest value of mode,

$$m=0, n=0$$

for these values all B_x, B_y, E_x, E_y are zero.

So TE_{00} mode is not possible.

2.) $m=0, n=1$

$$m=1, n=0$$

$$\text{for } m=0, n=1, (f_c)_{01} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$(f_c)_{01} = \frac{c}{2a}$$

$$\text{for } m=1, n=0, (f_c)_{01} = \frac{c}{2b}$$

As $a > b$, so $(f_c)_{10} < (f_c)_{01}$

lowest freq. mode or longest WL. mode is called dominant mode.

⇒ So dominant mode → TE_{10} if $a > b$
 for rectangular wa TE_{01} if $b > a$

for Square WG $\rightarrow$ both TE_{10} & TE_{01} will be dominant mode.

TM Wave :-

$$E_z \neq 0, H_z = 0$$

Comp. of elec. field along the dirⁿ of wave propagation.

$$E_z = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

By this expression, we can find out $E_x, E_y, B_x, B_y, B_z = 0$

Dominant Mode :- Mode 00, 01, 10 are not possible.

So Lowest possible mode for TM wave is TM_{11} Mode i.e.

Dominant Mode for TM wave $\rightarrow TM_{11}$

- If we compare TE & TM then,

Lowest possible freq. :- is for TE mode either TE_{01} or TE_{10} . If we combinely propagate both TE & TM wave in a WG then TE wave will start to propagate first.

{ Dominant Mode $\rightarrow$ for which mode, the wave propagate first. }

$$* \lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

This comes from $K_c^2 + K_g^2 = K_0^2$

$K_c \rightarrow$ Cut-off Wave vector

$K_g \rightarrow$ Guide "

$K_o \rightarrow$ free space "

Now, $K_c^2 + K_g^2 = K_o^2$

$$\Rightarrow \left(\frac{2\pi}{\lambda_c}\right)^2 + \left(\frac{2\pi}{\lambda_g}\right)^2 = \left(\frac{2\pi}{\lambda_o}\right)^2$$

$$\Rightarrow \boxed{\frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2} = \frac{1}{\lambda_o^2}}$$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi \nu}{c} = \frac{\omega}{c}$$

$$K \propto \omega$$

So $K_c^2 + K_g^2 = K_o^2 \Rightarrow \omega_c^2 + \omega_g^2 = \omega_o^2$

$\omega_o \rightarrow$ free space freq.

$$\omega_o = c K_o$$

$$(K = K_o)$$

So $\omega_c^2 + \omega_g^2 = c^2 K_o^2$

$$\boxed{\omega_g^2 = c^2 K^2 - \omega_c^2}$$

Q.21- What modes are possible at an operating freq. of 3 GHz in a hollow rectangular WR of inner dimensions $3.44 \times 7.22 \text{ cm}^2$ and corresponding values of phase velocity, group velocity, cut off WL, cut off freq. & phase constant.

$$f = 3 \text{ GHz}$$

TE₀₁ is the dominant mode as $(3.44 < 7.22)$

$$f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \quad \& \quad \lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$(f_c)_{10} = \frac{c}{2a}$$

$$(\lambda_c)_{10} = 2a = 2 \times 3.44 = 6.88 \text{ cm}$$

$$(f_c)_{01} = \frac{c}{2b}$$

$$(\lambda_c)_{01} = 2b = 2 \times 7.22 = 14.44 \text{ cm}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m}$$

$$\text{or } \lambda = 10 \text{ cm}$$

for Propagation, $\lambda < \lambda_c$

$$\lambda < (\lambda_c)_{01}$$

The only mode that can propagate is TE₀₁
Cut off freq.

$$(f_c)_{01} = \frac{c}{2b} = \frac{3 \times 10^8}{2 \times 7.22} =$$

$$V_p = ?$$

$$V_p = \frac{c}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - \left(\frac{10}{14.44}\right)^2}} \text{ m/sec.}$$

$$V_p = \frac{3 \times 10^8}{0.72139793} = 4.1586 \times 10^8 \text{ m/sec}$$

$$V_g = ?$$

$$V_p V_g = c^2 \Rightarrow V_g = \frac{c^2}{V_p}$$

$$V_g = \frac{(3 \times 10^8)^2}{4.1586 \times 10^8} = 2.1642 \times 10^8 \text{ m/sec}$$

$$\text{Phase const. } \beta = \frac{2\pi}{\lambda}$$

$$\beta = \frac{2 \times 3.14}{10} = 0.628$$

- Q.2:- A hollow rectangular WG has $a=6\text{cm}$, $b=4\text{cm}$. Freq. of the incident signal is 3GHz . Calculate (i) λ_c of dominating mode.
(ii) λ_g of dominant mode
(iii) Intrinsic wave impedance
(iv) V_p (v) V_g

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m} = 10 \text{ cm}$$

(i) $(\lambda_c)_{10} = 2a = 12 \text{ cm}$

$(\lambda_c)_{01} = 2b = 8 \text{ cm}$

Only Mode which can propagate i.e.

Dominant Mode $\rightarrow \text{TE}_{10}$

(ii) $\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{10}{\sqrt{1 - \left(\frac{10}{12}\right)^2}} \text{ cm}$

$$\lambda_g = \frac{10}{0.55277} = 18.0907 \text{ cm}$$

(iii) Intrinsic Impedance $Z_E = \frac{Z_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$

$$Z_E =$$

Q.2:- At 15 GHz an air field $5 \times 2 \text{ cm}^2$ WG has. $E_z = 20 \sin 40\pi x \sin 50\pi y e^{-j\beta z} \text{ V/m}$. Find the mode of propagation and cut off W.L. of this mode.

$$\left\{ \begin{array}{l} f = 15 \text{ GHz} \\ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{15 \times 10^9} = \frac{1}{50} = 0.02 \text{ m} \\ \lambda = 2 \text{ cm} \end{array} \right. \quad (\text{No Need})$$

$\therefore E_z \neq 0$ So TM wave can propagate
dimension $\rightarrow 5 \times 2 \text{ cm}^2 = 0.05 \times 0.02 \text{ m}^2$

$$E_z = 20 \sin(40\pi x) \sin(50\pi y) e^{-j\beta z}$$

compare with $E_z = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$

$$\Rightarrow \frac{m\pi x}{a} = 40\pi x$$

$$\Rightarrow m = 40 \times a = 40 \times 0.05$$

$$\boxed{m = 2}$$

$$\& \frac{n\pi y}{b} = 50\pi y \Rightarrow n = 50 \times 0.02$$

$$\boxed{n = 1}$$

So Dominant Mode $\rightarrow \text{TM}_{21}$

Cutoff

$$\text{W.L. } (\lambda_c)_{21} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$(\lambda_c)_{21} = \frac{2}{\sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{1}{2}\right)^2}} = \frac{2}{\sqrt{\frac{4}{25} + \frac{1}{4}}} = \frac{20}{\sqrt{16 + 25}}$$

$$(\lambda_c)_{21} = \frac{20}{6.4031} \text{ cm}$$

$$(\lambda_c)_{21} = 3.12348 \text{ cm}$$

Q.4:- In a rectangular W.G. for which $a = 1.5 \text{ cm}$
 $b = 0.8 \text{ cm}$, $\mu = \mu_0$, $\epsilon = \epsilon_0$,
 $H_x = 2 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) \sin(\pi \times 10^{11} t - \beta z) \text{ A/m}$

Determine Mode of operation, cut off freq. & cut off wavelength for this mode.

Q.5:- A standard air filled rectangular W.G. with dimensions $a = 8.6 \text{ cm}$ & $b = 4.3 \text{ cm}$ fed by 4 GHz carrier from a coaxial cable. Determine whether TE_{10} mode will be propagated or not. If yes, determine its group & phase velocity.

Q.6:- A loss-less transmission line has a characteristic impedance of 70 ohms & phase velocity as $2 \times 10^7 \text{ m/sec}$. Calculate inductance per meter & capacitance per meter of the T.M. Line.

$$Z_0 = 70 \Omega$$

$$v_p = 2 \times 10^7 \text{ m/s}$$

for loss-less T.M. Line,

$$Z_0 = \sqrt{\frac{L}{C}} \quad \text{--- (1)}$$

$$\& \quad v_p = \frac{1}{\sqrt{LC}} \quad \text{--- (2)}$$

$$\text{eqn (1) } \times \text{ (2)} \Rightarrow Z_0 v_p = \sqrt{\frac{L}{C}} \frac{1}{\sqrt{LC}} = \frac{1}{C}$$

$$\Rightarrow C = \frac{1}{Z_0 v_p}$$

$$C = \frac{1}{70 \times 2 \times 10^7} = \frac{1}{140 \times 10^7} \text{ F/m} = \frac{100 \times 10^{-10}}{14}$$

$$C = 7.14 \times 10^{-10} \text{ F/m}$$

$$\& (2) \Rightarrow v_p^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{C v_p^2}$$

$$L = \frac{(2 \times 10^7) \times 70}{(2 \times 10^7)^2}$$

$$L = 3.5 \times 10^{-6} \text{ H/m}$$

(Henri/m)

Q.7:- A loss-less T.M. Line of length 50 cm with $L = 10 \mu\text{H/m}$ & $C = 40 \text{ pF/m}$ is operated at 30 MHz. If T.M. Line is assumed to be non-magnetic. Find the dielectric constant of the T.M. Line.

$$L = 10 \mu\text{H/m}, \quad C = 40 \text{ pF/m}$$

$$= 10 \times 10^{-6} \text{ H/m}, \quad = 40 \times 10^{-12} \text{ F/m}$$

$$f = 30 \text{ MHz} = 30 \times 10^6 \text{ Hz}$$

$$\text{We know, } v_p = \frac{1}{\sqrt{LC}} \quad \& \quad \text{Also } v_p = \frac{c}{\sqrt{\epsilon_r}}$$

$$\text{i.e. } \frac{c}{\sqrt{\epsilon_r}} = \frac{1}{\sqrt{LC}}$$

$$\frac{c^2}{\epsilon_r} = \frac{1}{LC} \Rightarrow \epsilon_r = L C^3$$

$$\epsilon_r = (3 \times 10^8)^2 \times L \times C$$

$$= 9 \times 10^{16} \times 10^{-5} \times 40 \times 10^{-12}$$

$$= 360 \times 10^{-1}$$

$$\epsilon_r = 36$$

Q.4 :- $H_x = 2 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) \sin(\pi \times 10^{10} t - \beta z) \text{ A/m}$

$a = 1.5 \text{ cm}$ & $b = 0.8 \text{ cm}$

$\frac{m\pi x}{a} = \frac{\pi x}{a} \Rightarrow m = 1$

$\frac{n\pi y}{b} = \frac{3\pi y}{b} \Rightarrow n = 3$

Dominant mode will be TE_{13} or TM_{13}

Cut off freq. $f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$(f_c)_{13} = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{1}{1.5}\right)^2 + \left(\frac{3}{0.8}\right)^2}$
 $= \frac{3 \times 10^8}{2} (3.80879)$
 $= 5.7132 \times 10^8$

Cut off W.L. $\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$

$(\lambda_c)_{13} = \frac{2}{3.80879}$

$(\lambda_c)_{13} = 0.52510115$

Q.5 :- $a = 8.6 \text{ cm}$, $b = 4.3 \text{ cm}$
 $f = 4 \text{ GHz}$

$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{4 \times 10^9} = \frac{0.75}{10} = 0.075 \text{ m}$
 $= 7.5 \text{ cm}$

$(\lambda_c)_{10} = 2a = 2 \times 8.6 = 17.2 \text{ cm}$

Here $(\lambda_c)_{10} \neq \lambda_c$

TE_{10} mode will not propagate.

Phase & group vel. = ?

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - \left(\frac{7.5}{17.2}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - 0.19014}}$$
$$v_p = \frac{3 \times 10^8}{0.89992} = 3.33363 \times 10^8 \text{ cm/s}$$

Now, $v_p v_g = c^2$

$$\Rightarrow v_g = \frac{c^2}{v_p} = \frac{(3 \times 10^8)^2}{(3.33363 \times 10^8)} = 2.69976 \times 10^8 \text{ cm/s}$$

चौधरी PHOTOSTAT

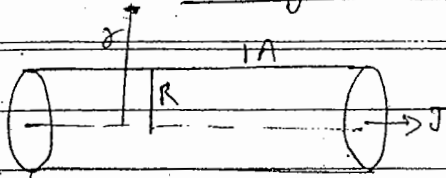
39, Jia Sarai, Near IIT, Hauz Khās

New Delhi-16

Mobile No. : 9818909565, 9211212600

Assignment

(36)



$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{B} = \frac{\mu_0}{2\pi r} \hat{\phi} \quad \text{at distance } r.$$

$\vec{J}$ is uniformly distributed.

$$\nabla \times \vec{B} = \frac{1}{r} \begin{vmatrix} \hat{r} & r\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \frac{\mu_0}{2\pi r} & 0 \end{vmatrix}$$

$$\nabla \times \vec{B} = \frac{1}{r} (0) = 0$$

(38)

force that compensate the ele. field

$$\vec{F} = q\vec{E}$$

(c) ✓

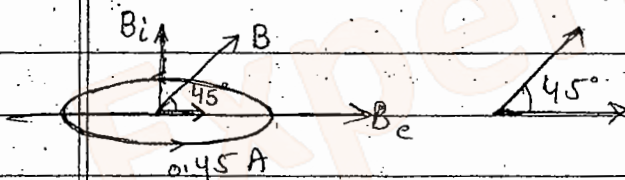
(65)

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

In Gaussian units $\vec{D} = \vec{E} + 4\pi \vec{P}$

(88)

Circular coil is || to the earth's mag field



$$\tan 45^\circ = \frac{y}{x} = \frac{B_i}{B_e}$$

$$1 = \frac{B_i}{B_e}$$

$$B_e = B_i = \frac{\mu_0 I}{2R} \quad (\text{at centre mag. field})$$

If N no. of turns then

$$B_e = \frac{\mu_0 N I}{2R}$$

$$= \frac{10 \times 0.45 \times 4\pi \times 10^{-7}}{2 \times 0.12} \text{ T}$$

$$= 75 \times 3.14 \times 10^{-7} \text{ T}$$

$$= 23.5 \mu\text{T}$$

No

- Kirchoff's Current law $\Rightarrow \sum i = 0 \Rightarrow \nabla \cdot \vec{J} = 0$
voltage law $\Rightarrow \sum V = 0 \Rightarrow \nabla \times \vec{E} = 0$

(108) $\vec{E} = E_0$, $\vec{B} = B_0$

for a uniform mag. field, $\vec{A} = \frac{1}{2} (\vec{B}_0 \times \vec{r})$

$$\vec{A}' = \vec{A} + \nabla \lambda$$

$$V' = V - \frac{\partial \lambda}{\partial t}$$

for Lorentz T. $\vec{E}' = E$ & $\vec{B}' = B$

$$\vec{E} = -\nabla V \text{ or } -\nabla \phi$$

(i) If $\phi = 0$ then $\nabla \phi = 0$ i.e. $E = 0$ so ϕ must not be 0.

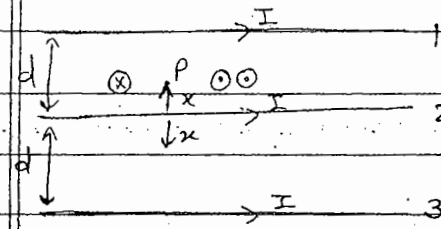
(ii) ∇

$$\nabla \phi = \nabla (\vec{E}_0 \cdot \vec{r}) = E_0$$

Note:- For a constant uniform Electric field $\vec{E}$,
 $\vec{A} = \frac{1}{2} (\vec{B}_0 \times \vec{r})$ & $\phi = -\vec{E}_0 \cdot \vec{r}$ mag. field $\vec{B}$

Level - II

(2)



$B_1 = \frac{\mu_0 I}{2\pi(d-x)}$ $\otimes$
 $B_2 = \frac{\mu_0 I}{2\pi x}$ $\otimes$
 $B_3 = \frac{\mu_0 I}{2\pi(d+x)}$ $\otimes$

$$\begin{aligned} \vec{B} &= \frac{\mu_0 I}{2\pi} \left[\frac{1}{d-x} - \frac{1}{x} - \frac{1}{d+x} \right] \\ &= \frac{\mu_0 I}{2\pi} \left[\frac{x(d+x) - (d+x)(d-x) - x(d-x)}{x(d-x)(d+x)} \right] \\ &= \frac{\mu_0 I}{2\pi} \left[\frac{dx + x^2 - d^2 + x^2 - dx + x^2}{x(d-x)(d+x)} \right] \end{aligned}$$

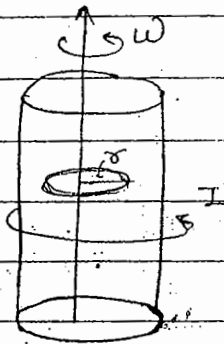
$$dx^2 + x^2 - d^2 + x^2 = dx + x^2 = 0$$

$$3x^2 = d^2$$

$$x = \pm \frac{d}{\sqrt{3}}$$

At this distance above & below mag field = 0

(10)



$$\vec{K} = \sigma v \hat{\phi}$$

$$= \sigma \omega R \hat{\phi} \quad (v = R\omega)$$

$$(11) \quad \vec{B}_{in} = \mu_0 K$$

$$= \mu_0 \sigma \omega R \hat{z}$$

It'll behave as a solenoid.

$$\oint \vec{A} \cdot d\vec{l} = \int \vec{B} \cdot d\vec{S}$$

$$= \int_S (\nabla \times \vec{A}) \cdot d\vec{S}$$

$$\phi = B_{in} A$$

Flux pass through the loop, $\Phi_m = \mu_0 \sigma \omega R \cdot \pi r^2$

$$\vec{A} = \frac{\mu_0 \sigma \omega R r}{2} \hat{\phi}$$

(12)

$$\phi = \frac{\mu_0 I a}{2\pi} \ln\left(\frac{b+a}{a}\right)$$

$$= \frac{\mu_0 I a}{2\pi} \ln(2)$$

for induced current, I find induced emf,

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt} \left(\frac{\mu_0 I a}{2\pi} \ln(2) \right) = -\frac{d}{dt} \frac{\mu_0 a I_0 \cos \omega t}{2\pi} \ln(2)$$

$$= \frac{\mu_0 I_0 \omega a}{2\pi R} \ln(2) = \frac{\mu_0 I_0 \omega a}{2\pi R} \ln(2) \sin(\omega t)$$

$$I = \frac{\mathcal{E}}{R} = -\frac{d\phi}{dt} \frac{1}{R}$$

$$I = \frac{\mu_0 I_0 \omega a}{2\pi R} \ln(2) \sin(\omega t)$$

(13)

$$E_{or} = \left(\frac{n_1 - n_2}{n_1 + n_2} \right) E_{oi}$$

$$E_{ot} = \left(\frac{2n_1}{n_1 + n_2} \right) E_{oi}$$

$$n_1 = 1$$

$$n_2 = 1.5$$

$$E_{oi} = 2$$

$$E_{or} = \frac{1 - 1.5}{1 + 1.5} \times 2 = \frac{-0.5}{2.5} \times 2 = \frac{-2}{5}$$

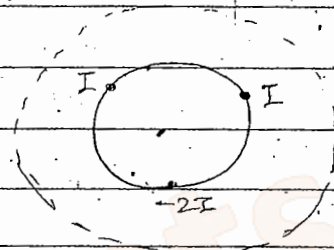
$$E_{ot} = \frac{2 \times 1}{1 + 1.5} \times 2 = \frac{2}{2.5} \times 2 = \frac{8}{5}$$

(17)

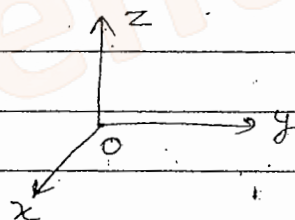
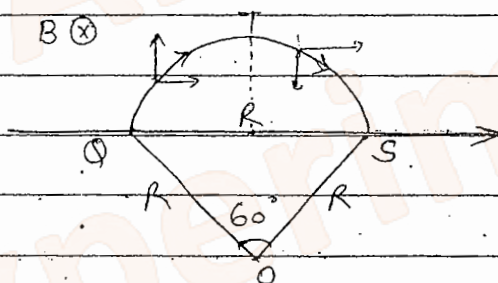
$$\oint B \cdot dl = \mu_0 I_{enc}$$

$$I_{enc} = 0$$

$$\therefore \oint B \cdot dl = 0$$



(21)



$$d\vec{l}_1 = dy \hat{y} + dz \hat{z}, \quad d\vec{l}_2 = dy \hat{y} - dz \hat{z}$$

$$\text{Total } d\vec{l} = d\vec{l}_1 + d\vec{l}_2 = 2 dy \hat{y}$$

$$\text{Force } \vec{F} = I \int d\vec{l} \times \vec{B}$$

$$= I \int_0^{R/2} dy \hat{y} \times \hat{z}$$

$$= I \int_0^{R/2} dy \hat{x}$$

$$= I B R \hat{x}$$

B → Into the page
($-\hat{z}$)

$$\hat{y} \times (-\hat{z}) = +\hat{x}$$

Note :- For half circle $\vec{F} = 2 I B R \hat{x}$

Exercise-2

Q.1:- $V(r, t) = 0$

$$A(r, t) = \frac{-1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{r}$$

Find $\vec{E}$, $\vec{B}$, q & $\rho = ?$

$$\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t} = 0 + \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$= \frac{1}{r \sin \theta} \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ -\frac{1}{4\pi\epsilon_0} \frac{qt}{r^2} & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{r \sin \theta} [\hat{r}(0) + \hat{\theta}(0) + \hat{\phi}(0)]$$

$$\boxed{\vec{B} = 0}$$

charge = q

charge distribution,

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\rho = -\epsilon_0 (\vec{\nabla} \cdot \vec{E})$$

$$\rho = \epsilon_0 \frac{q}{4\pi\epsilon_0} 4\pi \delta^3(r)$$

$$\boxed{\rho = q \delta^3(r)}$$

Current distribution,

' $\vec{J} = 0$ ' because there is no displacement current & no conduction current.

Q.2:- $V=0$, $\vec{A} = A_0 \sin(kx - \omega t) \hat{y}$

$\vec{E}$ & $\vec{B} = ?$

$$\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t} \quad \Rightarrow 0 + A_0 \omega \cos(kx - \omega t) \hat{y}$$

$$= 0 + A_0 \omega \cos(kx - \omega t) \hat{y}$$

$$\boxed{\vec{E} = A_0 \omega \cos(kx - \omega t) \hat{y}}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & A_0 \sin(kx - \omega t) & 0 \end{vmatrix}$$

$$\vec{B} = \hat{z} [A_0 \cos(kx - \omega t) \cdot k]$$

$$\boxed{\vec{B} = A_0 k \cos(kx - \omega t) \hat{z}}$$

Now, verify Maxwell's eqⁿ,

$$(1) \vec{\nabla} \cdot \vec{E} = \frac{\partial}{\partial y} [A_0 \omega \cos(kx - \omega t)] = 0$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$(2) \vec{\nabla} \cdot \vec{B} = 0$$

$$(3) \vec{\nabla} \times \vec{E} = -A_0 \omega k \sin(kx - \omega t) \hat{z}$$

$$-\frac{\partial B}{\partial t} = -A_0 \omega k \sin(kx - \omega t) \hat{z}$$

$$\therefore \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$(4) \vec{\nabla} \times \vec{B} = A_0 k^2 \sin(kx - \omega t) \hat{y}$$

$$\frac{\partial \vec{E}}{\partial t} = A_0 \omega^2 \sin(kx - \omega t) \hat{y}$$

$$\therefore \vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow A_0 k^2 \sin(kx - \omega t) = \frac{1}{c^2} A_0 \omega^2 \sin(kx - \omega t)$$

$$\Rightarrow \frac{\omega^2}{k^2} = c^2 \Rightarrow \frac{\omega}{k} = c \Rightarrow \boxed{\omega = ck}$$

This is the condⁿ to be imposed on ω & k .

Q.3:- $\lambda = \frac{-1}{4\pi\epsilon_0} \frac{qt}{r}, V(r,t) = 0$

$$\vec{A}(r,t) = \frac{-1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{r}$$

$$V' \text{ \& } A' = ?$$

$$V' = V - \frac{\partial \lambda}{\partial t}$$

$$V' = 0 - \frac{-1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$V' = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$\vec{A}' = \vec{A} + \vec{\nabla} \lambda$$

$$\vec{A}' = \frac{-1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{r} + \frac{1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{r}$$

$$\vec{A}' = 0$$

Q.4 (i) $V(r,t) = 0$

$$\vec{A}(r,t) = \frac{-1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{r}$$

Coulomb Gauge $\vec{\nabla} \cdot \vec{A} = 0$

Lorentz Gauge $\vec{\nabla} \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t} = 0$

$$\begin{aligned} \text{Here, } \vec{\nabla} \cdot \vec{A} &= \frac{-qt}{4\pi\epsilon_0} 4\pi \delta^3(r) \\ &= \frac{-qt}{\epsilon_0} \delta^3(r) \end{aligned}$$

$$\& \vec{\nabla} \cdot \vec{A}' + \mu_0 \epsilon_0 \frac{\partial V'}{\partial t} = \frac{-qt}{\epsilon_0} \delta^3(r) + 0 \neq 0$$

So Neither C.G. nor L.G. is followed.

(ii) $V = 0, \vec{A} = A_0 \sin(Kx - \omega t) \hat{y}$

Here $\vec{\nabla} \cdot \vec{A} = 0$

$$\vec{\nabla} \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t} = 0$$

} both C.G. & L.G. followed.

Exercise - 3

Q.1 :- $\vec{B}' = B \hat{y}$, $\vec{E}' = 0$ in rest S frame
find all components of $\vec{E}$ & $\vec{B}$ in S' frame
 $\vec{v} = v \hat{x}$

$$\boxed{\begin{aligned} \vec{B}_x' &= B_x = 0 \\ \vec{E}_x' &= E_x = 0 \end{aligned}}$$

$$\vec{E}_y' = \gamma(E_y - vB_z), \quad E_y = 0, B_z = 0$$

$$\boxed{\vec{E}_y' = 0}$$

$$\vec{B}_y' = \gamma(B_y + \frac{vE_z}{c^2}), \quad \vec{B}_y = B_y, E_z = 0$$

$$\boxed{\vec{B}_y' = \gamma B_y = \frac{B_y}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$\vec{E}_z' = \gamma(E_z + vB_y)$$

$$E_z = 0, \vec{B}_y = B_y$$

$$\boxed{\vec{E}_z' = \gamma v B_y = \frac{v B_y}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$\vec{B}_z' = \gamma(B_z - \frac{vE_y}{c^2})$$

$$B_z = 0, E_y = 0$$

$$\boxed{\vec{B}_z' = 0}$$

So, $\boxed{\vec{B}' = \frac{B}{\sqrt{1 - \frac{v^2}{c^2}}} \hat{y}}$

$$(\vec{B}' = \vec{B}_x' + \vec{B}_y' + \vec{B}_z')$$

$$\text{f. } \vec{E}' = \vec{E}_x' + \vec{E}_y' + \vec{E}_z'$$

$$\boxed{\vec{E}' = \frac{vB}{\sqrt{1 - \frac{v^2}{c^2}}} \hat{z}}$$

$$\vec{E} \cdot \vec{B} = 0 \text{ (in rest)}$$

$$\text{Also } \vec{E}' \cdot \vec{B}' = 0$$

Q.2 :- $\vec{E}' = \alpha \hat{x}$ & $\vec{B}' = \beta \hat{y}$
 $\vec{E}$ & $\vec{B} = ?$

$$\vec{v} = v \hat{y}$$

$$\boxed{B_y' = B_y = \beta \hat{y}}$$

$$\boxed{E_y' = E_y = 0}$$

$$\vec{B}_x' = \gamma \left(B_x + \frac{v}{c^2} E_z \right)$$

$$\vec{B}_x' = 0$$

$$E_x' = \gamma (E_x + v B_z) = \gamma E_x$$

$$E_x' = \frac{\alpha}{\sqrt{1 - v^2/c^2}}$$

$$\begin{aligned} \vec{B}_z' &= \gamma \left(B_z + \frac{v}{c^2} E_x \right) = \gamma E_x \frac{v}{c^2} \\ &= \frac{v \alpha}{c^2 \sqrt{1 - v^2/c^2}} \end{aligned}$$

$$\vec{E}_z' = \gamma (E_z - v B_x) = 0$$

$$\text{So } \vec{B}_x' + \vec{B}_y' + \vec{B}_z' = \vec{B}'$$

$$\Rightarrow \vec{B}' = \beta \hat{y} + \frac{v \alpha}{c^2 \sqrt{1 - v^2/c^2}} \hat{z}$$

$$\& \vec{E}' = \vec{E}_x' + \vec{E}_y' + \vec{E}_z'$$

$$\vec{E}' = \frac{\alpha}{\sqrt{1 - v^2/c^2}} \hat{x}$$

$$\& \boxed{\vec{E} \cdot \vec{B} = 0 = \vec{E}' \cdot \vec{B}'}$$

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