Free Study Material from

All Lab Experiments

Electromagnetic Theory for NET/Gate Physical Sciences

> Electrodynamics Part-2 <

Distinction b/w Good Conductor & Bad Conductor :-

Conductor is good or bad - It depends upon its
→ Conductivity (σ)
→ Relaxation time (τ)

if we put a free static charge inside the conductor then how quickely that charge come out to the surface - that time will be the Relaxation time.

If conductor is bad, Its relaxation time will be more. it take more time to come out to the surface.

Relaxation time is given by permitivity to conductivity ratio.

$$\tau = \frac{\varepsilon}{\sigma}$$

Now If we fall an EM wave over a conductor of conductivity σ & per.. ε, we know $\vec{E}$ inside the conductor is 0. so amplitude of EM wave inside the conductor must be zero. This E. field is not electrostatic, it is dynamic (changing with time) Here ω is the freq. of oscillation of Elec. & mag. field.

τ will

So Conductor is good or bad → it is Not only depand of σ & τ but also depand on freq. ω.

→ If a conductor is good for a perticular freq. ω, It may be bad for some another freq.

If $\tau << \frac{1}{\omega}$

$\Rightarrow$ Good Conductor

If $\tau >> \frac{1}{\omega}$ $\Rightarrow$ Bad Conductor

(i) $\Rightarrow \frac{\varepsilon}{\sigma} << \frac{1}{\omega} \Rightarrow \frac{\sigma}{\varepsilon\omega} >> 1 \Rightarrow$ good conductor

(ii) $\Rightarrow \frac{\sigma}{\varepsilon\omega} << 1 \Rightarrow$ bad conductor

Inside the conductor, free volume charge density.

$$\rho_f = 0$$

At $t=0$, we put ρ_f inside conductor & after some time ρ_f becoms 0. So have to find that time in which ρ_f becom 0 from ρ_f.

Use <u>Continuity eqⁿ</u>,

$$\vec{\nabla}\cdot\vec{J_f} = -\frac{\partial \rho_f}{\partial t}$$

We have, $J_f = \sigma E$

So $\sigma(\vec{\nabla}\cdot\vec{E}) = -\frac{\partial \rho_f}{\partial t}$

$$\nabla\cdot E = \frac{\rho_f}{\varepsilon}$$

$$\Rightarrow \frac{\sigma}{\varepsilon}\rho_f = -\frac{\partial \rho_f}{\partial t}$$

$\nabla\cdot E = 0$ only when all the charge come out of surface.

$$\Rightarrow \frac{\partial \rho_f}{\rho_f} = -\frac{\sigma}{\varepsilon}dt$$

$$\Rightarrow \ln \rho_f(t) = -\frac{\sigma}{\varepsilon}t + C$$

At time $t=0$, $C = \ln \rho_f(0)$

$$\Rightarrow \ln \rho_f(t) = -\frac{\sigma}{\varepsilon}t + \ln \rho_f(0)$$

$$\Rightarrow \ln \frac{\rho_f(t)}{\rho_f(0)} = -\frac{\sigma}{\varepsilon}t$$

$$\Rightarrow \rho_f(t)/\rho_f(0) = e^{-\frac{\sigma}{\varepsilon}t}$$

$$\Rightarrow \boxed{\rho_f(t) = \rho_f(0)e^{-\frac{\sigma}{\varepsilon}t}}$$

It defines - How the free charge decays with time exponentially inside the conductor.

If We have a <u>Perfect Conductor</u>,

$$\sigma = \infty$$

So free charge inside $= 0$

Relaxation time $\tau = \frac{\varepsilon}{\sigma}$ (if $\sigma = \infty$)

$$\boxed{\tau = 0}$$

i.e. No time lag in putting the charge & come out to the surface.

for Insulator $\sigma = 0$

$$\boxed{\tau = \frac{\varepsilon}{\sigma} = \infty}$$

i.e. charge take ∞ time to come out to the surface.
i.e. It can never come out to the surface.

Note :- $\vec{\nabla}\cdot\vec{D} = \rho_f$ is valid for every medium.
$\vec{\nabla}\cdot(\varepsilon\vec{E}) = \rho_f$ " " only for isotropic medium.

for semiconductor
Conductivity is small but finite.
σ = very small
so τ = large
i.e. It will take more time to come out.
• $\vec{\nabla}\cdot\vec{E} = 0$ is valid only for perfect conductor.

Wave velocity inside good & bad conductor :-

$$v_{good} = \frac{\omega}{\alpha}$$

$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]^{1/2}$$

for good, $\frac{\varepsilon}{\sigma} << \frac{1}{\omega} \Rightarrow \frac{\sigma}{\varepsilon\omega} >> 1$ so neglect 1 as compare to $\frac{\sigma}{\varepsilon\omega}$

$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]^{1/2}$$

$$\alpha_{good} = \beta_{good} = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left(\frac{\sigma}{\omega\varepsilon}\right)^{1/2}$$

$$= \sqrt{\omega}\sqrt{\frac{\varepsilon\mu}{2}}\left(\frac{\sigma}{\varepsilon\omega}\right)^{1/2} = \sqrt{\frac{\sigma\omega\mu}{2}}$$

$$\boxed{v_{good} = \frac{\omega}{\alpha} = \sqrt{\frac{2\omega}{\sigma\mu}}}$$

for bad cond. $\frac{\sigma}{\varepsilon\omega} << 1$

$$V_{bad} = \frac{\omega}{\ }$$

$\alpha_{bad} = \omega\sqrt{\epsilon\mu}$

$v_{bad} = \frac{\omega}{\omega\sqrt{\epsilon\mu}}$

$$\boxed{v_{bad} = \frac{1}{\sqrt{\epsilon\mu}}}$$

This is similar to velocity inside the dielectric medium.

Skin Depth (δ) :-

If we incident EM wave on conductor. Inside the conductor, near the surface E field is not zero but it will travell some distance & then E-field become zero.

Skin depth is the distance at which amplitud of EM wave become 1/e value of the value at the surface.

If E-field at surface is E_0 then after travelling some distance i.e. skin depth, it become $\frac{E_0}{e}$.

More is β, less is the skin-depth.

$$\boxed{\delta = \frac{1}{\beta}}$$

Skin-depth of free space :- is ∞.

In free-space there is no decay in amp. of wave.

For Good Conductor,

$$\boxed{\delta_{good} = \frac{1}{\beta} = \sqrt{\frac{2}{\sigma\omega\mu}}}$$

for Bad Conductor,
(means it is Insulator)

$\boxed{\delta_{bad} = \infty}$ for perfect Insulator $\left(\begin{matrix}\sigma = 0 \\ \beta = 0\end{matrix}\right)$

for perfect dielectic (it is a " ")

If conductivity is finite but very small then there will be some skin depth. There will be decay so δ will be finite.

$$\beta = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\cancel{1}+\frac{1}{2}\left(\frac{\sigma}{\epsilon\omega}\right)^2-\cancel{1}\right]^{1/2}$$

$$= \omega\sqrt{\frac{\epsilon\mu}{2}}\;\frac{1}{\sqrt{2}}\left(\frac{\sigma}{\epsilon\omega}\right) = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$$

$$\boxed{\delta_{bad} = \frac{1}{\beta_{bad}} = \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}}$$

{for perfect Insulator, $\sigma = 0$ so $\delta = \infty$}

Phase :- If wave impedence is complex, it defines the phase difference b/w $\vec{E}$ & $\vec{B}$.

$$Z = \left|\frac{E}{H}\right| = \frac{\mu E}{B} = \frac{\mu\omega}{K^*} \qquad \left\{\frac{E}{B} = v = \frac{\omega}{K^*}\right.$$

$$= \frac{\mu\omega}{(\alpha+i\beta)} = \text{complex quantity}$$

Inside the conducting medium $\vec{E}$ & $\vec{B}$ are out of phase.

We know $K^* = \alpha + i\beta = Ke^{i\phi}$

where $K \rightarrow$ Amp. $\Rightarrow K = (\alpha^2+\beta^2)^{1/2}$

$\phi \rightarrow$ phase difference $\Rightarrow \tan\phi = \frac{\beta}{\alpha}$

We know that

$$\alpha = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{1/2}$$

$$\beta = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]^{1/2}$$

so $K = (\alpha^2+\beta^2)^{1/2} = [\omega$

$$\boxed{K = \omega\sqrt{\mu\epsilon}\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}$$

and $\tan\phi = \frac{\beta}{\alpha}$

$$\boxed{\phi = \tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)}$$

This is the phase diff. b/w $\vec{E}$ & $\vec{B}$. If $\sigma = 0$ then $\phi = 0$ i.e.

We have $\vec{B} = \frac{\vec{K^*} \times \vec{E}}{\omega}$

$\vec{K^*} = K^* \hat{z}$
$\vec{E} = E\hat{x}$
so $\vec{B}$ will be in $\hat{y}$

$$\vec{B} = \frac{K^* E}{\omega}\hat{y}$$

$$\vec{B} = \frac{K e^{i\phi}}{\omega} E_0 e^{-\beta(\hat{n}\cdot\hat{r})} e^{i(kz-\omega t)} \hat{y}$$

$K^* = K e^{i\phi}$
$E = E_0 e^{i(kz-\omega t)}$

On putting the values of K, we get

$$\boxed{\vec{B} = \sqrt{\mu\varepsilon}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4} E_0 e^{-\beta(\hat{n}\cdot\hat{r})} e^{i[kz-(\omega t-\phi)]}\hat{y}}$$

Phase of $\vec{E} = (kz-\omega t)$
$\vec{B} = [kz-(\omega t-\phi)]$

→ $\vec{B}$ is lagging behind $\vec{E}$ by phase diff ϕ i.e. $\vec{E}$ is leading by ϕ.

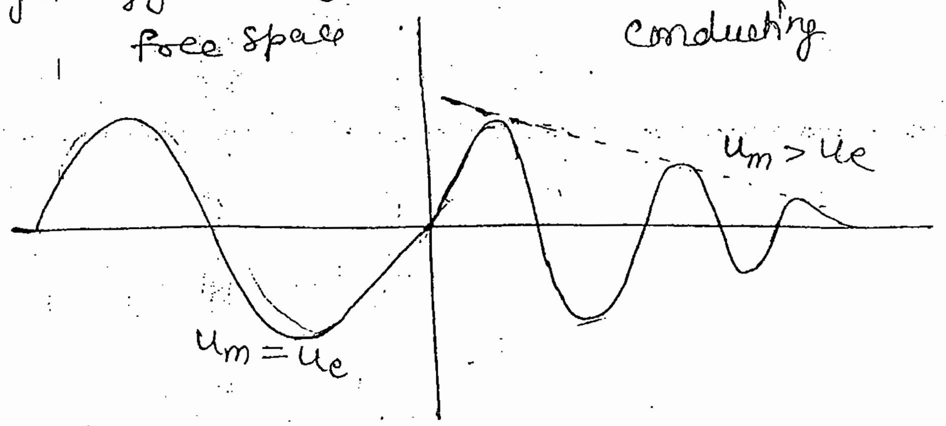
→ The amplitude of $\vec{B}$ is greater than $\vec{E}$ (as $\frac{\sigma}{\omega\varepsilon} >> 1$)
becoz, Amp of $\vec{B}$ contains amp of $\vec{E}$ & also a quantity multiplied by amp. of E (~~which~~ the quantity is greater than 1)

→ E decays faster that's why its amp. is small.

→ Energy density –
$u_m > u_e$

At any particular distance $\hat{z}$
$u_m \propto B^2$ & $u_e \propto E^2$

i.e. mag energy density is greater then electric energy density.



→ Poynting Vector :- $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu}\hat{z}$ $\quad$ ∵ $\vec{E} \to \hat{x}$, $\vec{B} \to \hat{y}$
∴ $S \to \hat{z}$

i.e. dirⁿ of energy flow is same as dirⁿ of wave propagation

We have $\vec{B} = \frac{K}{\omega} \vec{E}$.

$$\frac{\vec{B}}{E} = \sqrt{\mu\varepsilon}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}$$

so $$\vec{S} = \sqrt{\frac{\varepsilon}{\mu}}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4} E^2 \hat{z}$$

$$\vec{S} = \sqrt{\frac{\varepsilon}{\mu}}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4} E_0^2 e^{-2\beta z}\left[e^{2i(kz-\omega t)}\right] \hat{z}$$

Conclusions in conducting medium

① → Ele. & mag. field amplitude decays exponentially.

$E_0 e^{-\beta z}$

$= \boxed{E_0 e^{-\frac{z}{\delta}}}$ $\quad (\delta = \frac{1}{\beta})$

$\vec{K} = k.\hat{n}$

$\hat{n}$ = dirⁿ of prop

$(\hat{r}.\hat{n} = \hat{z})$

e.g.:- given that the skin depth for a certain material $\delta = 10$ nm. Calculate the amp. of $\vec{E}$ after 100 nm distance into the conductor.

E_0 → field at surface.

z → distance travelled into the medium.

$$E_0 e^{-\frac{z}{\delta}} = E_0 e^{-\frac{100\,nm}{10\,nm}} = E_0 e^{-10}$$

e^{-10} → very very small (negligible)

* If skin depth is given & find the amp. after a distance the use above formula.

② → $E \perp K$, $B \perp K$

i.e. $\boxed{E \perp B \perp K}$

EM waves are transvers.

③ → There is a phase diff. b/w $\vec{E}$ & $\vec{B}$ fields.

$\vec{E}$ leading in phase by angle $\phi = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$

④ → $U_m \neq U_e$ but $U_m > U_e$

so most of the energy lies in the mag. field

& decay of u is $\boxed{u \propto e^{-\frac{2z}{\delta}}}$

⑤ $\vec{S} \parallel \vec{K}$

i.e. dirn of Energy flow is along the dirn of wave propogation

i.e. $\vec{S} \propto e^{-2z/\delta}$

Relation b/w Conduction Current density & Displacement Current Density :-

for a medium having conductivity σ & permittivity ε then

$$\vec{J_c} = \sigma \vec{E}$$

$$\vec{J_d} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

If we incident a electromag. wave over such a medium $\vec{E} = E_0 e^{i(kz - \omega t)} \hat{x}$

then

$$\vec{J_c} = \sigma \vec{E}$$

$$\vec{J_d} = \varepsilon(-i\omega)\vec{E}$$

Then

$$\frac{J_c}{J_d} = \frac{\sigma}{\varepsilon(-i\omega)}$$

$$\left|\frac{\vec{J_c}}{\vec{J_d}}\right| = \left|\frac{\sigma}{\omega\varepsilon}\right|$$

for Good conductor, $\frac{\sigma}{\omega\varepsilon} >> 1$

so $\boxed{J_c >> J_d}$

for Poor Conductor, σ is less $\frac{\sigma}{\omega\varepsilon} << 1$

so $\boxed{J_c << J_d}$

{e.g. for a metal, $J_c = 10^6 A/m^2$ & $J_d = 10 A/m^2$ then it is Good Conductor.}

If $\sigma \uparrow$ then $J_c \uparrow$ (more)

If $\varepsilon \uparrow$ or $\omega \uparrow$ then $\vec{J_d} \uparrow$

i.e. $J_c \rightarrow$ depand on conductivity

$J_d \rightarrow$ " " permitivity & freq.

Propogation of EM Wave in Plasma :-

Plasma is the collection of charge particle, neutral particles & ions. It can consist of +ve & -ve ions. Its condⁿ is that one of the charge type must be mobi
Plasma contains equal concentration of +ve & -ve ion
So Plasma as a whole is Neutral.
(One of charge type must be mobile but if both type mobile → then no problem)

Plasma is found - in upper side of atmosphere
Ionosphere contains plasma.
Plasma can be made in ~~library~~ laboratory.

Use - Plasma is used in Communication. It can reflected only few freq EM waves.
~~It can ref~~ If we incident some EM wave on ionosphere, it will be reflected back if $\omega < \omega_p$
i.e. freq. of EM wave is less than a perticular freq called plasma freq. (ω_p)
ω_p → depends on free charge carrier concentration.
so Ionospher can reflect radio waves $(\omega < \omega_p)$ but can not reflect visible waves $(\omega > \omega_p)$

Conductivity of Plasma Medium :- Condⁿ are of 2 types -

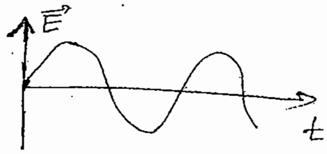
1) Static Conductivity
2) Dynamical "

Static Conductivity :- static means if we apply a const elec. field on a conductor then its condictivity will be

$$\boxed{\sigma_s = \frac{n e^2 \tau}{m}}$$

n → no. of free e^- per unit volume, e → e^- charge
τ → relaxation time, m → e^- mass

Dynamical Cond. :- When we apply a variable E. field (i.e. varying with time) ~~then~~ on a conductor then its conductivity will be variable with freq. (Not const.).
This is called Dynamical conductivity.



$$\boxed{\sigma_d = \frac{i n e^2}{m\omega}}$$

$\omega \rightarrow$ freq.

Total Conductivity,

$$\sigma_{Total} = \sigma_S + \sigma_d$$

$$= \frac{n e^2 \tau}{m} + \frac{i n e^2}{m\omega}$$

$$= \frac{n e^2}{m}\left(\tau + \frac{i}{\omega}\right)$$

$$\sigma_{Total} = \frac{n e^2}{m(\gamma - i\omega)} = \frac{n e^2}{m\left(\frac{1}{\tau} - i\omega\right)}$$

$\gamma \rightarrow$ damping factor.

$$\boxed{\sigma_{Total} = \frac{n e^2 \tau}{m(1 - i\omega\tau)}}$$

This is the total condn of any conducting material.

For Static Conductivity, $\omega = 0$, Condn reduces to
then $\sigma_S = \frac{n e^2 \tau}{m}$

Metal is also like plasma – In metal, there are +ve & –ve ions. One of charge type must be mobile.
But in actual plasma, charge types (e^-) moves ~~(e^-)~~ freely.
& in metal, charge move freely but during ~~b/w~~ moving they scattered with each other. & due to this relaxation time comes.

Ionospher contains dilute plasma so e^- are completely free. so there is No relaxation time i.e. No damping.

$\Rightarrow$ So Conductivity of a ionospher or plasma is

$$\sigma_d = \frac{i n e^2}{m\omega}$$

For ionosphere or plasma medium, we have to put the value of σ (this is the diff. from conducting medium)

Maxwell's eqn,

$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \vec{J}_f + \frac{\partial\vec{D}}{\partial t}$$

Inside the plasma → no free charge so $\rho_f = 0$

i.e. if we put some external free charge then it will go on surface of plasma.

For Dilute plasma i.e. for Ionosphere,

$$\varepsilon \simeq \varepsilon_0$$
$$\mu \simeq \mu_0$$

$$\Rightarrow \quad \vec{\nabla}\cdot\vec{E} = 0 \quad \text{——— (1)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{——— (2)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{——— (3)}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\sigma\vec{E} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \quad \text{——— (4)}$$

$\vec{J}_f = \sigma\vec{E}$

$D = \varepsilon_0\vec{E}$

Taking curl of eqn (3) & put the value of $\vec{\nabla}\times\vec{B}$ from (4)

we get
$$\nabla\times\nabla\times E = -\frac{\partial(\vec{\nabla}\times\vec{B})}{\partial t}$$

$$\nabla(\nabla\cdot E) - \nabla^2 E = -\frac{\partial}{\partial t}\left[\mu_0\sigma\vec{E} + \mu_0\varepsilon_0\frac{\partial E}{\partial t}\right]$$

$$\nabla^2\vec{E} - \mu_0\sigma\frac{\partial\vec{E}}{\partial t} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = 0 \quad \text{——— (5)}$$

Now take curl of (4), we get

$$\nabla^2\vec{B} - \mu\sigma\frac{\partial\vec{B}}{\partial t} - \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2} = 0 \quad \text{——— (6)}$$

The soln is $\vec{E} = E_0 e^{i(\vec{K}\cdot r - \omega t)}$ Put in (5),

$\vec{\nabla} \rightarrow i K \quad , \quad \frac{\partial}{\partial t} \rightarrow -i\omega$

$eq^n (5) \Rightarrow \; -K^2\vec{E} + i\mu_0\sigma\omega\vec{E} + \mu_0\varepsilon_0\omega^2\vec{E} = 0$

$\Rightarrow \; K^2 = i\mu_0\sigma\omega + \mu_0\varepsilon_0\omega^2$

Put $\sigma = \frac{i n e^2}{m\omega}$

$K^2 = -\frac{n e^2 \mu_0}{m} + \mu_0\varepsilon_0\omega^2$

$= \mu_0\varepsilon_0\omega^2\left[1 - \frac{n e^2}{m\varepsilon_0\omega^2}\right]$

where $\boxed{\omega_p = \left(\frac{n e^2}{\varepsilon_0 m}\right)^{1/2}}$ is called Plasma freq. —— (7)

$K^2 = \frac{\omega^2}{c^2}\left[1 - \frac{\omega_p^2}{\omega^2}\right]$

$$\boxed{K = \frac{\omega}{c}\left[1 - \frac{\omega_p^2}{\omega^2}\right]^{1/2}} \quad \text{—— (8)}$$

$K \rightarrow$ propogation coffi. or Amp. of wave vector

$\omega \rightarrow$ freq. of EM wave

(ii) $\omega_p \rightarrow$ plasma freq., it is the property of medium it depend on e^- concentration (power 1/2)

$$\boxed{\omega_p \propto n^{1/2}}$$

(i) If any EM wave have freq ω 4 $(\omega < \omega_p)$ then

$\omega < \omega_p$, $K \rightarrow$ imaginary $\rightarrow$ No propogation

if K is imaginary then ref. index will be imaginary.

& If ref. index is imaginary then wave propogation through that medium is Not possible & wave will be damped.

Wave can not propogate so it will reflect and go back into original medium.

All the EM wave having freq. $\omega < \omega_p$ can not travell through Plasma.

(ii) If $\omega > \omega_p$, $K \rightarrow$ Real , $n \rightarrow$ real

So wave will propogate. $\longrightarrow$ propogation

Refractive Index of plasma medium :-

$$n = \frac{c}{v}$$

and $v = \frac{\omega}{k} = c\left[1 - \frac{\omega_p^2}{\omega^2}\right]^{-1/2}$ (from 8)

So $n = \frac{c}{c}\left[1 - \frac{\omega_p^2}{\omega^2}\right]^{1/2}$

$$\boxed{n = \left[1 - \frac{\omega_p^2}{\omega^2}\right]^{1/2}}$$ (Real)

[if $\omega > \omega_p$ then $n \to$ real]

(for $\omega < \omega_p$ then $n \to$ imaginary)

Expression for Cut off frequency in Plasma medium :-

Plasma freq. is behaving like Cut off freq.

If $\omega < \omega_p \longrightarrow$ wave can not propogate

$\omega > \omega_p \longrightarrow$ wave can propogate.

So Plasma medium is High Pass filter.

$$\omega_p = \left(\frac{ne^2}{\varepsilon_0 m}\right)^{1/2}$$

Linear Plasma freq. $f_p = \frac{\omega_p}{2\pi} = \frac{1}{2\pi}\left(\frac{ne^2}{\varepsilon_0 m}\right)^{1/2}$

$n, e, \varepsilon_0, m \to$ all const. So $f_p \to$ const.

$f_c \to$ cut off freq. $\boxed{f_p = f_c \simeq 9\sqrt{n}}$

$n \to$ no. of e^- per meter^3 [$\not{e}/m^3$] (in M.K.S.)

Maximum Penetration (Skin) depth for plasma :-

$$\boxed{\delta_{max.} = \frac{c}{\omega_p}}$$

{ Word - skin depth is mostly used for metals }

Q. Find the Skin depth for a typical metal $\sigma = 10^7 \Omega^{-1} m^{-1}$ in visible range $\omega = 10^{15}/sec$. Assuming $\varepsilon \simeq \varepsilon_0$ & $\mu \simeq \mu_0$ & define why metals are opaque. Also find the phase diff. b/w Electric & magnetic field in this metal.

$\sigma = 10^7 \Omega^{-1} m^{-1}$

$\omega = 10^{15}/sec.$

It is a good conductor, & for good conductor, Skin depth is,

$$\delta_{good} = \sqrt{\frac{2}{\sigma \omega \mu}} = \sqrt{\frac{2}{10^7 \times 10^{15} \times 4\pi \times 10^{-7}}}$$

$$= \sqrt{\frac{10^{-16} \times 10}{2\pi}} = \sqrt{\frac{5}{\pi}} \times 10^{-8} \ m$$

$$= 1.3 \times 10^{-8} \ m$$

$$\delta_{good} = 13 \ nm$$

Now why Metal is opaque $\Rightarrow$

at every 13 nm, amp. decay by 1/e. Hence amplitude decay very fast. So electromag. wave can not pass through the ~~metals~~. Hence metals are opaque. But it has some limit.

$\omega > \omega_p$ then medium is transparent.

$\omega < \omega_p$, medium is opaque.

- Reflectivity of any metal remain ~~constant~~ upto UV region but beyond UV region it will ~~fall~~ drops.

Hence transmittance is also const. upto UV region.

$$R + T = 1$$

Reflectivity drops means transmitance increases.

- Hence, for X-rays & γ-rays, we can not make the mirrors. Because No metal exist which can reflect X-rays & γ-rays.

Visible $\longrightarrow$ UV $\longrightarrow$ X-ray $\longrightarrow$ γ-ray.

- X-rays can be reflected from an atomic plane but not by metal surface. $R \simeq 1$, $T \simeq 0$

$$\phi = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$$

$$= \frac{1}{2}\tan^{-1}\left[\frac{10^7\ \Omega^{-1}m^{-1}}{10^{15}/sec \times 8.85\times10^{-12}}\right]$$

$$= \frac{1}{2}\tan^{-1}(10^3) = \frac{1}{2}\times(89) \qquad \{89 \simeq 90\}$$

$$\simeq \frac{1}{2}\frac{\pi}{2} \simeq \frac{\pi}{4}$$

$$\boxed{\phi \simeq 45^\circ}$$

Phase diff. b/w $\vec{E}$ & $\vec{B}$ inside a good conductor is approximately $\underline{45^\circ}$.
i.e. $\vec{E}$ leads $\vec{B}$ by 45°.

Q. In free space an electromagnetic wave is given by
$\vec{E} = 20\cos(\omega t - 50x)\,\hat{y}$ V/m
Calculate
(i) $\vec{B}$ (ii) $\vec{J_d}$ (iii) ω

$$\vec{E} = 20\cos(\omega t - 50x)\,\hat{y}$$

$$\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega} \Rightarrow \frac{\omega}{K} = c \quad \text{then } \vec{B} = \frac{E}{c}$$

$$\vec{K} = 50\,\hat{x}$$

$$\frac{\omega}{|\vec{K}|} = c = 3\times10^8\ m/sec$$

$$\omega = 3\times10^8 \times 50 = \underline{1.5\times10^{10}/sec} \text{ Ans}$$

$$\vec{B} = \frac{20\cos(\omega t - 50x)\,\hat{z}}{3\times10^8}$$

$$\vec{B} = \frac{2}{3}\times10^{-7}\cos(\omega t - 50x)\,\hat{z} \quad wb/m^2 \text{ Ans}$$

$$\vec{J_d} = \varepsilon_0\frac{\partial\vec{E}}{\partial t} = -\varepsilon_0\, 20\sin(\omega t - 50x)\cdot\omega$$

$$\vec{J_d} = -20\,($$

$$\boxed{\vec{J_d} = \qquad A/m^2}$$

Q. An EM wave in free space is given by

$$\vec{E} = (10\hat{y} + 5\hat{z})\cos(\omega t + 2y - 4z) \ V/m$$

Calculate (i) $\vec{B}$ (ii) $\vec{k}$ (iii) ω (iv) $\vec{J_d}$

$$\vec{E} = (10\hat{y} + 5\hat{z})\cos(\omega t + 2y - 4z)$$

$$= (10\hat{y} + 5\hat{z})\cos[\omega t - \{(-2\hat{y} + 4\hat{z})\cdot(x\hat{x} + y\hat{y} + z\hat{z})\}]$$

$\therefore \ \vec{k} = -2\hat{y} + 4\hat{z}$ $\quad \{\cos(\omega t - \vec{k}\cdot\vec{r})$ & $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}\}$

This is lying in y-z plane.

(ii) $|\vec{K}| = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

In free space, the wave velocity is

$$v = \frac{\omega}{K} = c$$

(iii) $\Rightarrow \dfrac{\omega}{|K|} = c \quad \Rightarrow \omega = 3\times 10^8 \times 2\sqrt{5}$

$$\omega = 6\sqrt{5}\times 10^8 \text{ rad/sec.}$$

Now (i) $\vec{B} = \dfrac{\vec{K}\times\vec{E}}{\omega}$

$\because \ \vec{K}\times\vec{E} = (-10\hat{x} - 40\hat{x})\cdot\cos(6\sqrt{5}\times 10^8 t + 2y - 4z)$

$$\therefore \vec{B} = \frac{-50\hat{x}}{6\sqrt{5}\times 10^8}\cos(6\sqrt{5}\times 10^8 t + 2y - 4z)$$

$$\boxed{\vec{B} = \frac{5\sqrt{5}}{3\times 10^8}\cos(6\sqrt{5}\times 10^8 t + 2y - 4z)\ (-\hat{x})} \ Wb/m^2$$

(ii) $J_d = \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

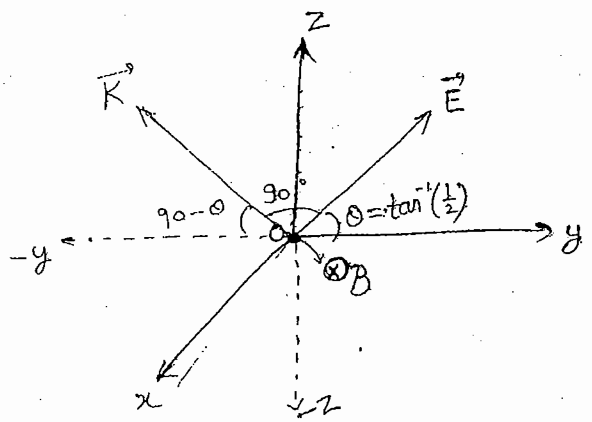
$$= \varepsilon_0 (10\hat{y} + 5\hat{z})[-\sin(\omega t + 2y - 4z)]\times\omega$$

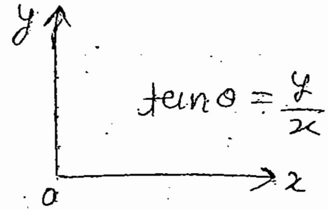
$$\boxed{\vec{J_d} = -\varepsilon_0(10\hat{y} + 5\hat{z})[6\sqrt{5}\times 10^8]\sin(\omega t + 2y - 4z)}$$

Poynting Vector →

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

dirⁿ of $\vec{S}$ is same as dirⁿ of $\vec{K}$.





$\tan\theta = \frac{y}{x}$

$\vec{E} = 10\hat{y} + 5\hat{z}$

$\tan\theta = \cancel{\frac{10}{5}}\ \frac{5}{10}$

$\theta = \tan^{-1}(\frac{1}{2})$

Mag. field is into the page. {Angle b/w $\vec{E}$ & $\vec{K}$ = $90°$ (in free space)}

Q. Mag. field of an EM wave is given by $\vec{B}(x,y,z,t) = B_0 \sin\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right]\hat{k}$ is given in free space. Find $\vec{E}$ and $\vec{S}$. Graphical rep^n = ?

$$\vec{B}(x,y,z,t) = B_0 \sin\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right]\hat{k}$$

$$= B_0 \sin\left[-\frac{(-x-y)K}{\sqrt{2}} + \omega t\right]\hat{k}$$

Standard from $\vec{B} = B_0 \sin(\omega t - \vec{k}\cdot\vec{r})$

$$= B_0 \sin\left[\omega t - \left\{\frac{(-\hat{x}-\hat{y})}{\sqrt{2}}K\cdot(x\hat{x}+y\hat{y}+z\hat{z})\right\}\right]$$

We get $\boxed{\vec{K} = \frac{K}{\sqrt{2}}(-\hat{x}-\hat{y})}$

We have
$$E = -\frac{c^2(\vec{K}\times\vec{B})}{\omega}$$

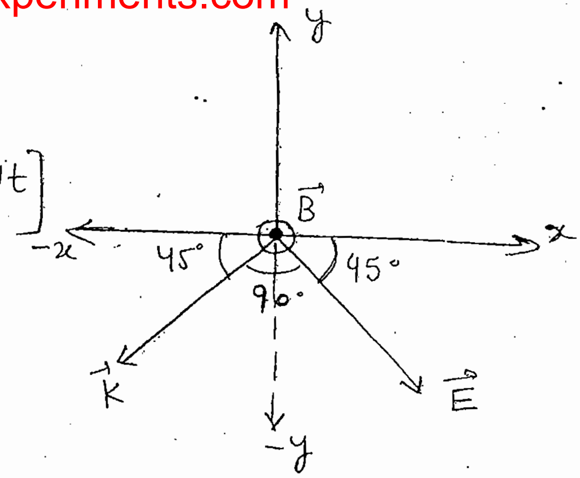
$$= -\frac{c^2}{\omega}\left[\frac{K}{\sqrt{2}}(-\hat{x}-\hat{y}) \times B_0 \sin\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right]\hat{z}\right]$$

$$= -\frac{c^2}{\omega}\frac{K}{\sqrt{2}}\left[-\hat{y}+\hat{x}\right] B_0 \sin\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right]$$

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

$$= \frac{1}{\mu_0}\left[-\frac{c^2}{\omega}\frac{K}{\sqrt{2}}\left[-\hat{y}+\hat{x}\right] B_0 \sin\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right] \times B_0 \sin\left[\frac{(x+y)K}{\sqrt{2}} + \omega t\right.\right.$$

$$\vec{S} = -\frac{c^2}{\mu_0 \epsilon_0}\frac{K}{\sqrt{2}}B_0^2\left[-\hat{x}-\hat{y}\right] \sin^2\left[\frac{(x+y)K}{\sqrt{2}}+\omega t\right]$$



Q. Plane EM Wave is propogating in lossless dielectric. Electric field of this wave is given by

$$\vec{E}(x,y,z,t) = E_0(\hat{x}+A\hat{z})\, e^{iK_0[-ct+(x+\sqrt{3}z)]}$$

Find (i) dielectric constant of the medium ε_r

(ii) Value of A

(iii) Poynting Vector S

$$\vec{E}(x,y,z,t) = E_0(\hat{x}+A\hat{z})\, e^{iK_0[(x+\sqrt{3}z)-ct]}$$

On Comparing with $\vec{E} = E_0\, e^{i(\vec{K}\cdot\vec{r}-\omega t)}$

We get $\vec{K} = K_0(\hat{x}+\sqrt{3}\hat{z})$ & $|\vec{K}| = 2K_0$

$\omega = K_0 c$

Wave velocity $v = \frac{\omega}{|\vec{K}|} = \frac{K_0 c}{2K_0}$

$$\left.\begin{array}{l} v = \frac{c}{2} \\ \text{Also } v = \frac{c}{n} \end{array}\right\} \text{On Comparing} \Rightarrow \boxed{n=2}$$

& $n = \sqrt{\varepsilon_r} \Rightarrow \varepsilon_r = n^2 \Rightarrow \boxed{\varepsilon_r = 4}$

for linear Isotropic dielectric (lossless)

$$\vec{K}\cdot\vec{E} = 0$$

$$K_0(\hat{x}+\sqrt{3}\hat{z})\cdot E_0(\hat{x}+A\hat{z})e^{iK_0(-ct+(x+\sqrt{3}z))} = 0$$

$$K_0E_0[1+\sqrt{3}A]e^{iK_0[-ct+(x+\sqrt{3}z)]} = 0$$

$$\Rightarrow \quad 1+\sqrt{3}A = 0$$

$$\Rightarrow \quad \boxed{A = -\frac{1}{\sqrt{3}}}$$

$$\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}, \quad \omega = K_0c$$

Now $\vec{E} = E_0\left[\hat{x}-\frac{1}{\sqrt{3}}\hat{z}\right]e^{iK_0[-ct+(x+\sqrt{3}z)]}$ & $\vec{K} = K_0(\hat{x}+\sqrt{3}\hat{z})$

$$\vec{K}\times\vec{E} = K_0E_0\left[\frac{\hat{y}}{\sqrt{3}}+\sqrt{3}\hat{y}\right]e^{iK_0[-ct+(x+\sqrt{3}z)]}$$

$$\therefore \vec{B} = \frac{K_0E_0}{K_0c}\left[\frac{4}{\sqrt{3}}\right]\hat{y}\,e^{iK_0[-ct+(x+\sqrt{3}z)]}$$

$$\boxed{\vec{B} = \frac{4E_0}{\sqrt{3}c}e^{iK_0[-ct+(x+\sqrt{3}z)]}\hat{y}}$$

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

dir^n of $\vec{S} \rightarrow$ dir^n of $\vec{K}$.

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{1}{\mu_0}\frac{4E_0^2}{\sqrt{3}c}\left[\hat{z}*+\frac{1}{\sqrt{3}}\hat{x}\right]e^{2iK_0[-ct+(x+\sqrt{3}z)]}$$

Q. A Plane wave E_q^n is given by

$$\vec{E} = \left(\frac{1}{\sqrt{2}}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)\cos(10^8 t - \sqrt{3}x + y)$$

in Isotropic linear Non-magnetic medium. Find

i) Wave vector k
ii) Refractive index n
iii) Dielectric constant ε_r
iv) Magnetic field $\vec{B}$
v) Wavelength of the wave λ
vi) Poynting vector $\vec{S}$
vii) Angle θ, wave vector $\vec{K}$ making from $+x$ axis.

$$\vec{E} = \left(\frac{1}{\sqrt{2}}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)\cos(10^8 t - \sqrt{3}x + y)$$

$$\vec{E} = \left(\frac{1}{\sqrt{2}}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right)\cos\left[10^8 t - (\sqrt{3}x - y)\right]$$

On comparing with $\vec{E} = E_0\cos(\omega t - k\cdot\vec{r})$, we get

$$\vec{K} = \sqrt{3}\hat{x} - \hat{y} \quad \& \quad \omega = +10^8$$

$$|\vec{K}| = \sqrt{3-1} = \sqrt{2} \Rightarrow \boxed{k = \sqrt{2}}$$

Wave velocity $v = \frac{\omega}{|\vec{K}|} = \frac{+10^8}{\sqrt{2}}$

$$n = \frac{c}{v} = \frac{3\times10^8}{v} \Rightarrow v = \frac{3\times10^8}{n},$$

$\frac{3}{n} = \frac{1}{\sqrt{2}}$

On Comparing, $n = +\sqrt{2}\times 3 \Rightarrow \boxed{n = +3\sqrt{2}}$

$$n = \sqrt{\varepsilon_r} \Rightarrow \varepsilon_r = n^2 = 9\times 2$$

$$\boxed{\varepsilon_r = 18}$$

$$\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}$$

$\vec{K}\times\vec{E} \neq \vec{K} = \sqrt{3}\hat{x} - \hat{y}$ & $\vec{E} = \left(\frac{1}{\sqrt{2}}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right)\cos(10^8 t - \sqrt{3}x + y)$

$$\vec{K}\times\vec{E} = \left(\frac{3}{2}\hat{z} + \frac{1}{\sqrt{2}}\hat{z}\right)\cos(10^8 t - \sqrt{3}x + y)$$

$$\omega = +10^8$$

So $$\boxed{\vec{B} = \frac{(3+\sqrt{2})\hat{z}}{10^8}\cos(10^8 t - \sqrt{3}x + y)}$$

Wave length $\lambda = ?$

$$K = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{K} \Rightarrow \lambda = \frac{2\pi}{\sqrt{2}}$$

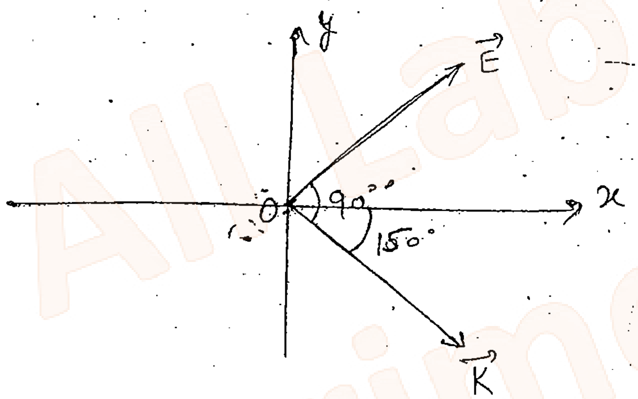
$$\boxed{\lambda = \sqrt{2}\,\pi} \quad \text{or} \quad \lambda = 1.414 \times 3.14 \Rightarrow \boxed{\lambda = 4.44062}$$

Poyinting Vector $\vec{S} = ?$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\vec{S} = \frac{1}{\mu_0}\left[\frac{3}{\sqrt{2}\times 10^8}\hat{y} - \frac{\sqrt{3}\cdot\sqrt{2}}{2\cancel{\times}10^8}\hat{x}\right]\cos^2(10^8 t - \sqrt{3}x + y)$$

$$\vec{S} = \frac{1}{\mu_0}\left[\frac{3}{\sqrt{2}}\hat{y} - \sqrt{\frac{3}{2}}\hat{x}\right]\frac{1}{10^8}\cos^2(10^8 t - \sqrt{3}x + y)$$



$$\vec{K} = \sqrt{3}\,\hat{x} - \hat{y}$$

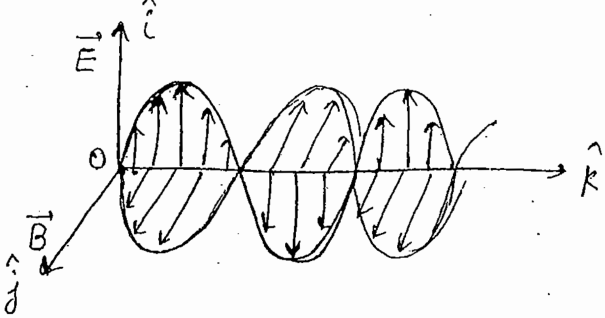
$$\tan\theta = \frac{-1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$$

$$\theta = 150°$$

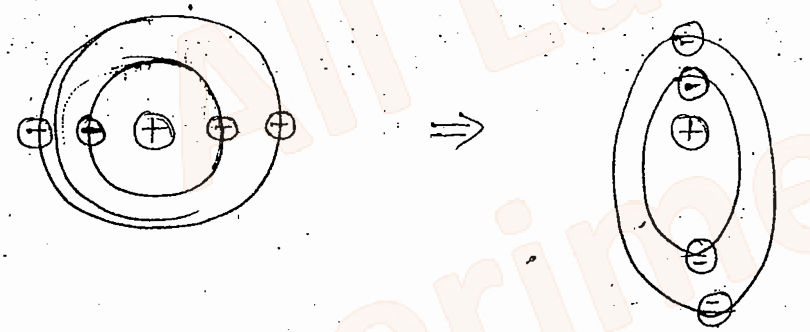
Polarisation in E.M. Waves :-

dirⁿ of polarisation vector is the dirⁿ of electric field $\vec{E}$.
suppose $\vec{E}$ is vibrating along $\hat{x}$-dirⁿ & wave is along $\hat{z}$.
so dirⁿ of polarisation is $\hat{x}$.



dirⁿ of mag. field $\vec{B}$ is $\perp^r$ to dirⁿ of $\vec{E}$. Hence $\perp^r$ to dirⁿ of polarisation.

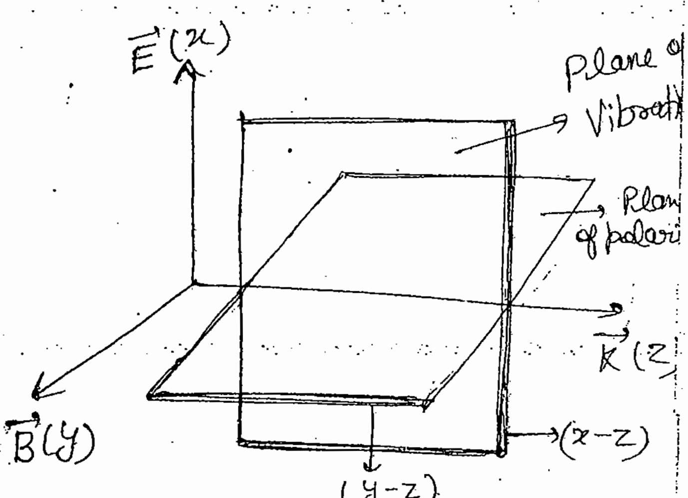
If we fall EM waves on any material (metal) which contains atom, then electric field will polarise the atom. for <u>I half cycle</u>, E is upward so dirⁿ of e^- is downward & dirⁿ of nucleus is upward so atom will oscillate.



<u>For lower half-cycle</u>
force of $e^- \rightarrow$ up.
nucleus $\rightarrow$ down
so atom will oscillate & shape will change.

Plane of Vibration :-

The plane that contains the dirⁿ of propogation & the dirⁿ of electric feild, is called <u>plane of Vibration</u>



Plane of Polarisation :-

The plane that contains the dirⁿ of propogation & No electric vibration, is called <u>plane of polarisation</u>

Plane of Vibration is $\perp^r$ to plane of polarisation.

Unpolarised Light :-

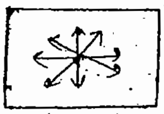
Light Waves → EM wave in visible region.

The concept of polarisation is valid only in Transvers wave. It is not found in longitudinal wave.

Sound wave (L.W.) can not be polarised but light waves ca

Plarisation is found in all electromag. wave (light sound ----). When light coming then $\vec{E}$ & $\vec{B}$ will confine in $\perp^r$ to th dir^n of propogation

→ If a light wave is coming & electric ~~field~~ vector is not confine, i.e. $\vec{E}$ vibrate randomly in particular dir^n in a plane. & dir^n of vibration is $\perp^r$ to. $\vec{E}$. ~~So~~ it is called Unpolarised l

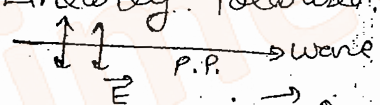
U.P.  $\vec{E}$

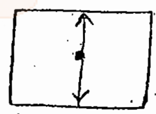
Polarisation may be of 3 types :-

Plane polarised, circularly pole., elliptically pol.

for plane polarisation, $\vec{E}$ is confine along (i.e. vibra along) a fix dir^n.

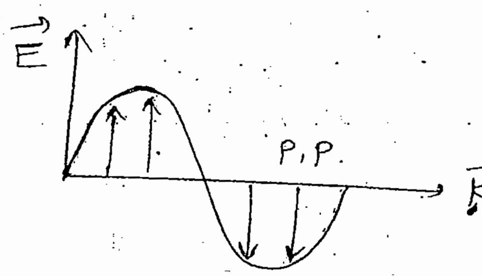
It is also called Linearly Polarised.



P.P. 

e.g. $E = E_0 e^{i(kz-\omega t)} \hat{x}$

This is Plane polarised wave.



If we superimpose, a another wave on it then

Dir^n of polarisation is depand upon the dir^n of polarisat of that other wave, their phase diff. & amplitude of both waves.

Note :- EM wave emit due to deexitaction of atoms. They will not produce only by single atom. When large no. of atom deexite from higher to lower level then EM wave ~~b/w~~ will produce & there will be No phase corelation b/w the electric vector.

Sun gives unpolarised light bcoz light is not coming from single nuclei, it is coming from different nuclei so their electric vector will vibrate in diff. dirn

If beam & wave → const. phase then → Cohrent Wave.

Plane Polarised → Electric vector always propogate along a fixed dirn. ↕

Elliptical & Circular Polarisation :-

If wave is coming towards us & electric vector rotate & traces a circle then it is Circularly polarised.
If it traces a ellipse then it is elliptically polarised. & dirn of propagation is ⊥r to the field.

$\vec{E_1} = E_0 e^{i(kz-\omega t)}\hat{x}$

$\vec{E_2} = E_0 e^{i(kz-\omega t)}\hat{x}$

$\vec{E_3} = E_0 e^{i(kz-\omega t)}\hat{y}$

If we superimpose $\vec{E_2}$ on $\vec{E_1}$ then wave will be plane polarised as these waves have same amp:, same phase (i.e. no phase diff. b/w them) & dirn of polarisation is same.

Amplitude of ~~Sump~~ Superimposed wave → $2E_0$

Q. Superimposition of which wave produce Interference.
(i) E_1 & E_2 (ii) E_2 & E_3 (iii) E_1 & E_3

Condn of Interfeece → The phase diff. b/w 2 waves should remain const. with time i.e. Cohrent.

Here all 3 waves are cohrent (i.e. have const. phase)

→ The superimposing ~~must~~ waves must have same state of polarisation.

So E_1 & E_2 will produce Interference as they have same state of polarisation.

$\vec{E_1} = E_0 e^{i(ky-\omega t)}\hat{x}$

$\vec{E_2} = E_0 e^{i(kz-\omega t)}\hat{x}$

Now these waves can not produce Interf-rence bcoz they can not superimpose bcoz both are propogating in different ~~state of polarisation~~ dir^n.

Cond^n of Interference

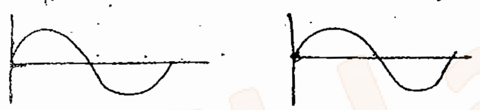
i) propogation in same dir^n

ii) same freq.

iii) phase diff. constant (cohrent)

iv) same state of polarisation.

If phase diff b/w 2 waves is 0 then $\rightarrow$ Constructive Inter.



$\rightarrow$ No phas diff.

If phase diff b/w 2 waves is $\pi/2$ then $\rightarrow$ distructive Inte.

P.D. $\rightarrow$ $0, 2\pi, 4\pi$ .... Constructive

$\pi, 3\pi, 5\pi$ .... Distructive

$\rightarrow$ By the superposition of E_1 & E_2 waves then their resultant wave will be plane wave & amplitued of resultant wave will be

$$\boxed{E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\phi}}$$

$\phi \rightarrow$ phase diff b/w 2 waves.

It is valid only when both waves have same state of polarisation

Here E_1 & $E_2 = E_0$ & ϕ = phase diff $= 0$

So $E = \sqrt{E_0^2 + E_0^2 + 2E_0^2} = \sqrt{4E_0^2}$

$E = 2E_0$

Superimposition of 2 wave which are different state of polarisation

We can never achieve circular or eliptical polarisation by the superposition of waves which are in same state of polarisation (only we get plane P.)

But ~~We can~~ By the superposition of waves which are in different state then we can achieve circularly or elliptically polarised wave under certain conditions & also can get plane P. wave.

First check the state of polarisation of 2 waves if same then → Plane wave.

Now if $\vec{E}_1$ & $\vec{E}_3$ superimpose then

$$\vec{E}_1 = E_0 e^{i(kz-\omega t)}\hat{x} \quad , \text{P.P. in } \hat{x} \text{ dirn}$$

$$\vec{E}_3 = E_0 e^{i(kz-\omega t)}\hat{y} \quad \text{P.P. '' } y \text{ ''}$$

Resultant Wave $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$\boxed{\vec{E} = E_0(\hat{x}+\hat{y})e^{i(kz-\omega t)}} \quad \text{——(A)}$$

(i) State of polarisation

If ∥ then plane polarised.

But If ⊥ then check

(ii) phase difference ϕ

(a) If $\phi = 0, \pi, 2\pi, 3\pi$ then wave will be plane polarised

(b) If $\phi = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ then check amp. of both wave

→ amplitude

1. if amp. same → $\boxed{E_1 = E_2}$ → Circularly Polarised
2. if amp. is not same $E_1 \neq E_2$ → Elliptically ''

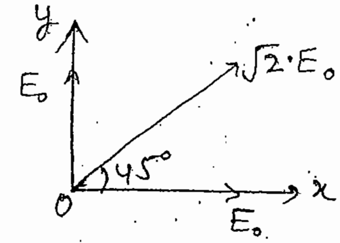
(c) If phase diff $\phi \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ then Surely it will Elliptically polarised.

Now for (A) $\Rightarrow$

$\vec{E} = E_0 (\hat{x} + \hat{y})\, e^{i(kz-\omega t)}$ } state of polarisation → is same, both plane wave.

This is plane polarised (No phase diff)

Superimposed wave is making angle 45° with x-axis & its magnitude is $\sqrt{2} E_0$.



- If we have

$$\vec{E} = \vec{E_1} + \vec{E_2} = E_0 (\hat{x} + e^{i\phi} \hat{y})\, e^{i(kz-\omega t)}$$

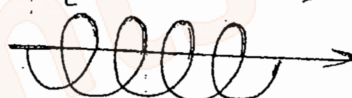
(i). $\phi = 0$ then Plane polarised making angle 45° with x-axis.

(ii) $\phi = \frac{\pi}{2}$ then

$$\vec{E} = E_0 (\hat{x} + i\hat{y})\, e^{i(kz-\omega t)}$$

$i \rightarrow$ phase diff of $\pi/2$ $\qquad \{ e^{i\frac{\pi}{2}} = i \}$

This is Circularly polarised wave.



amp. is const $\rightarrow E_0$

(iii) $\phi = \pi/4$

$$e^{i\pi/4} = \cos\frac{\pi}{4} + i \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}(i+1)$$

$$\vec{E} = E_0 \left[\hat{x} + \frac{(i+1)}{\sqrt{2}} \hat{y} \right] e^{i(kz-\omega t)}$$

This is Elliptically polarised

In Ellipse – distance from centre is not same but in circle → " " " is same so amplitude remain constant

* Most General type of Polarisation is Elliptical polarisation. Plane polarised & Circularly polarised waves are the special case of elliptically polarised wave.

e.g. $\vec{E} = E_0 (2\hat{x} + i\hat{y})\, e^{i(kz-\omega t)}$

phase diff $\rightarrow \pi/2$ & amp. is different

so elliptically polarised wave.

Circularly Polarised	Elliptically Polarised
→ Right Circular Polarised RCP	→ Right Elliptical Polarised REP
→ Left " " LCP	→ Left " " LEP

- There are 2 type of sep^n
  one along → angular mom. (30 % Probability)
  2nd along → optics (70 % Probability)

If electric vector is moving towards us doing clockwise motion i.e. wave is coming towards us & electric vector is rotating clockwise then
acc. to optics sep^n it is RCP (if amp. constant)
REP (if amp. Not constant)

acc. to Angular Mom. sep^n, it is

If electric vector is rotating anticlockwise then
acc. to optics sep^n it is LCP (const. amp.)
LEP (Not " ")

e.g. $\vec{E} = E_0 \cos(Kz - \omega t)\,\hat{x} + E_0 \sin(Kz - \omega t)\,\hat{y}$

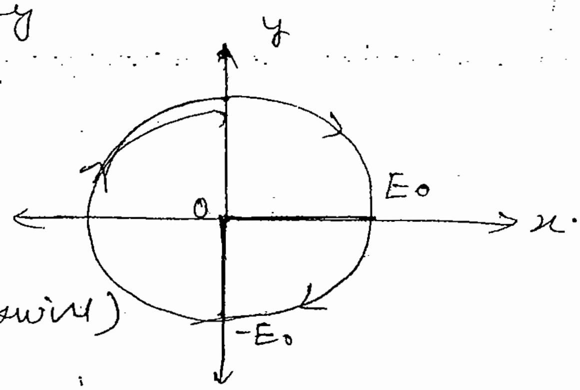
amp. same
Phase → $\pi/2$ } → Circularly Polarised

Take origin of wave $z = 0$ & dir of prop. $+\hat{z}$

$\vec{E} = E_0 \cos\omega t\,\hat{x} - E_0 \sin\omega t\,\hat{y}$

ωt → phase

At $\omega t = 0$, $\vec{E}$ along $\hat{x}$
$\omega t = \pi/2$, " $-\hat{y}$



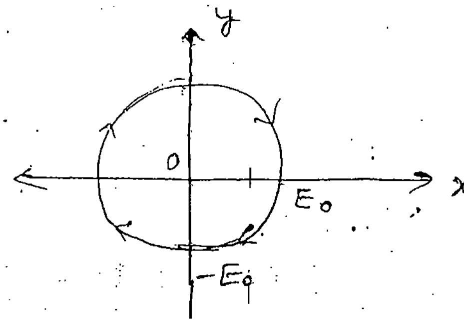
Acc. to ang. mom. LCP (clockwise)
so acc. to optics → RCP

RCP

Q. $\vec{E} = E_0 (\hat{x} + e^{i\frac{\pi}{2}} \hat{y}) \, e^{i(\omega t - kz)}$

Wave propogation along +z dirⁿ

elec. vector is vibrating in x-y plane.



$z = 0$,

$\vec{E} = E_0 (\hat{x} + i\hat{y}) \, e^{i\omega t}$

for $\omega t = 0$, $\vec{E} = E_0 (\hat{x} + i\hat{y})$

We need to take real amplitude only so

$\vec{E} = E_0 \hat{x}$

for $\omega t = \frac{\pi}{2}$, $\vec{E} = E_0 (\hat{x} + i\hat{y}) \, e^{i\pi/2} = E_0 (\hat{x} + i\hat{y}) \, i$

$= i E_0 \hat{x} - E_0 \hat{y}$

$\vec{E} = -E_0 \hat{y}$ (real)

Rotation is clockwise → so LCP acc. to ang mom. scp

so RCP acc. to optics.

So RCP

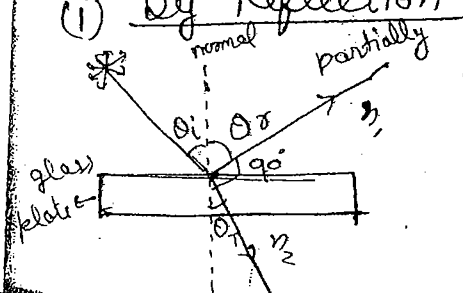
2/9/2012

Production of Plane Polarised Light :-

Plane polarised light can be produced by no. of methods

i) Reflection method
ii) Refraction " (Transmission)
iii) By scattering
iv) By phase retardation (doubly refracting crystals)
v) Beam splitter

(i) By Reflection :- If we have a transparent glass plate. If an unpolarised light is incident onto a crystal at an angle θ_i. Reflected light is partially plane polarised. θ_r is the reflection angle. And for a certain angle of incident, ~~refle~~ θ_i reflected



light is completely plane polarised with vibrations $\perp^r$ to plane of incidence. & that perticular angle of incidence is called Brewstor's angle θ_B. $\boxed{\theta_i = \theta_B}$

This will happen When reflacted light wave & transmitted light wave are $\perp^r$ to each other.

$$\boxed{\theta_r + \theta_T = \frac{\pi}{2}}$$

$\theta_i = \theta_r$ (angle of incident = angle of reflection)

$$\therefore \quad \theta_i + \theta_T = \frac{\pi}{2}$$

By Snell's law, $\dfrac{\sin\theta_i}{\sin\theta_T} = \dfrac{n_2}{n_1}$

$n_1 \rightarrow$ refractive index of medium 1.
$n_2 \rightarrow$ " " " " 2

$$\Rightarrow \frac{\sin\theta_B}{\sin\theta_T} = \frac{n_2}{n_1} \quad \Rightarrow \tan\theta_B = \frac{n_2}{n_1}$$

$$\left\{ \begin{aligned} &\sin\theta_T \\ &= \sin\left(\tfrac{\pi}{2} - \theta_r\right) \\ &= \sin\left(\tfrac{\pi}{2} - \theta_B\right) \\ &= \cos\theta_B \end{aligned} \right.$$

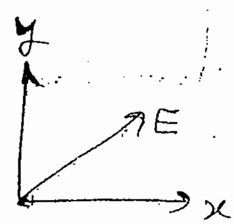
$$\Rightarrow \boxed{\theta_B = \theta_P = \tan^{-1}\left(\frac{n_2}{n_1}\right)}$$

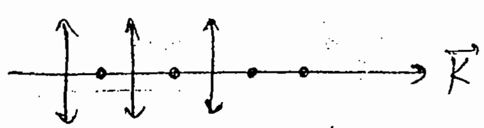
This is Brewstor angle & also called polarisation angle i.e. angle at which unpolarised light is completely ~~polar~~ plane polarised.

Note :- for air-glass interface and glass-air interface, this angle θ_B or θ_P is different as their ref. index has been inverted.

(ii) Refraction method :-

We can break any vector of unpolarised light into 2 components - horizontal & vertical.

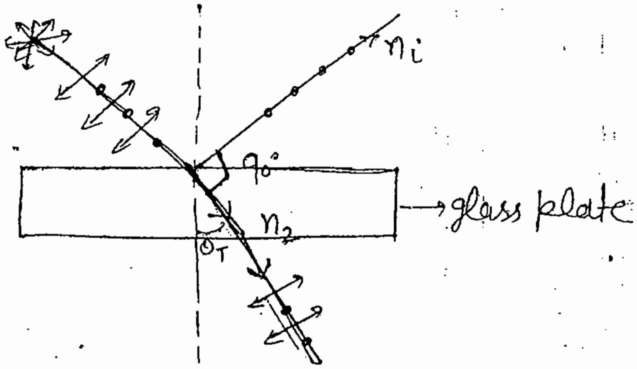


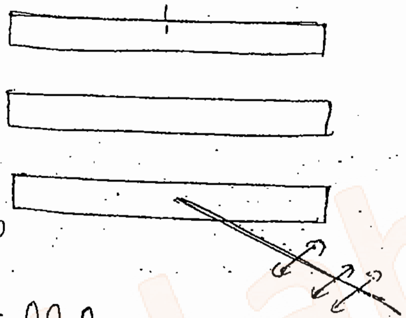


$\updownarrow \rightarrow$ parallel to the plane of incidence

$\bullet \rightarrow \perp^r$ " " "

Interface plane → plane which seperates 2 medium.





If we place a larger no. of plates then there will be polarisation by refraction. & this will be parallel to the plane of incidence.

⇒ After passing through large no. of plates, ⊥r wave will filter & only parallel wave will remain

- This law is valid for both rare to dencer & dencer to rare

air → glass (rare to dencer) $n_{glass} > n_{air}$

glass → air

Polarisation by Scattering :- When a EM wave is incident on atom or molecule then reflacted light will be partially plane polarised.

Sky is blue due to the scattering of blue colour — Raleigh scattering (it is $\propto \frac{1}{\lambda^4}$).
Red colour sca. least as it wavelegth is large

When unpolarised light is scattered by small particles, the scattered light is partially polarised. The blue light recived from the sky is accordingly partially polarised light.

(iv) <u>By phase retardation</u> :- In nature there are certain types of crystals which splitt the electromagnetic waves into 2 parts.

(i) E - ray (extraordinary ray)
(ii) O - ray (Ordinary ray)

→ E-ray does not follows, the laws of refraction while O-ray follows the laws of refraction.

→ E-ray travells with different speeds in different directions ~~will~~ while O-ray travells with same speed in different dirn.

→ Along a perticular dirn in a crystal E-ray & O-ray travells with same speed, this dirn is called <u>optic axis</u> of the crystal.

If a medium is s.t. in which a ray split into E-ray & O-ray after entering in it. then there must be some angle at which E-ray & O-ray travelle with same speed. The crystals in which this phenomenon occur are known as <u>Doubly Reflecting Crystals</u>. These are <u>Anisotropic</u> medium as they have diff. properties in diff. dirn.
as Refractive index is different in different dirn.

• <u>Doubly refleeting Crystals are of 2 types</u> -

(i) Uniaxial → one optic axis
(ii) Biaxial → two optic axis

uniaxial
→ negative
→ positive

<u>Negative uniaxial crystal</u> - In -ve crystal,

$\boxed{n_o > n_e}$

Hence $\boxed{v_e > v_o}$

n_o → Ref. index of ordinary ray
n_e → " extra " "
v_e → speed of ~~or~~ extraordinary ray
v_o → " " ordinary ray

example → Calsite

Positive Uniaxial crystal

$$n_e > n_o$$

Hence $\boxed{v_o > v_e}$

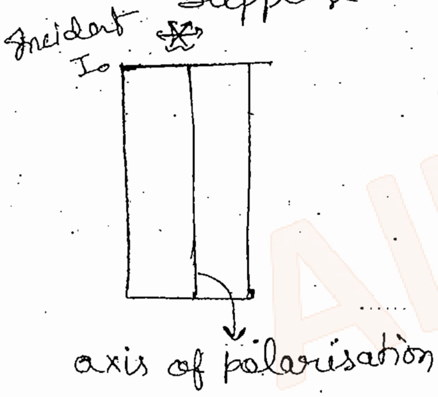
example - Quartz.

Polarisers - Those produces plane polarised light.

Nikol prism is a device which convert unpolarised light into plane polarised light.

Nikol prism is made up of doubly reflecting crystal

Suppose we have a polariser,



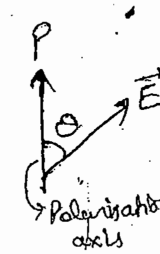
If we incident some unpolarised light on polariser then we get vibrations $\parallel$ to the polarisation axis of polariser.

If intensity of unpolarised light is I_o then transmitted intensity of polarised light is always half of I_o. $\left(\frac{I_o}{2}\right)$

Transmitted intensity is always found by the law of Malus.

Transmitted intensity is given by

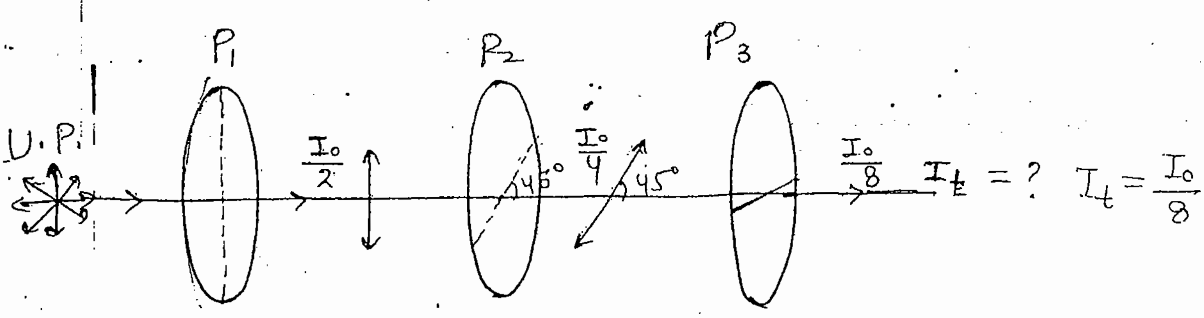
$$\boxed{I_t = I_o \cos^2\theta}$$



where, θ is the angle b/w dirn of electric field & plarisation ~~angle~~ axis.

For unpolarised light we need to take average value of $\cos^2\theta$ (i.e. $\frac{1}{2}$) so $\boxed{I_t = \frac{I_o}{2}}$

Polarisation axis is also called Transmission Axis.



$I_t = \frac{I_0}{8}$

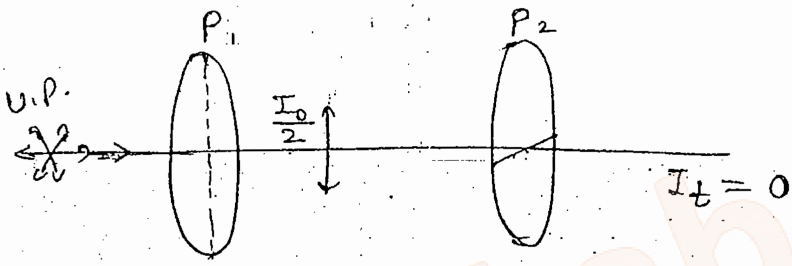
$I_t = \frac{I_0}{2}$

$I_t = \frac{I_0}{2}\cos^2 45°$

$I_t = \frac{I_0}{4}$

$I_t = \frac{I_0}{4}\cos^2 45°$

$I_t = \frac{I_0}{8}$



$I_t = \frac{I_0}{2}\cos 90°$

$I_t = 0$

How to produce Circularly & elliptically polarised light :- We use Quarter wave plate & quater wave Plates is made up of doubly reflecting ~~waves~~ crystals. s.t. it produce a path difference of $\lambda/4$ i.e. phase diff of $\pi/2$ b/w E-ray & O-ray. That's why it is called $\frac{\lambda}{4}$ - Plate (Quarter wave plate)

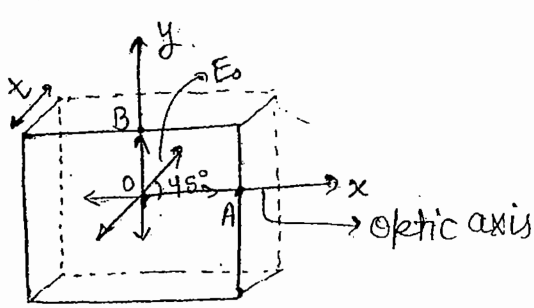
$$\Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}$$

$$\boxed{\Delta\phi = \frac{\pi}{2}}$$

Vibrations of E-ray & O-rays are $\perp^r$ to each other

Suppose we have a plate of doubly reflecting crystal, of thickness t is placed in x-y plane.

If we incident an EM wave on it which is in -z dir^n. & Elec field



$$\vec{E} = E_0 e^{i(kz+\omega t)} \left(\frac{\hat{x}+\hat{y}}{\sqrt{2}} \right)$$

As ele. field enter in crystal EM wave split into 2 part - one is along x, $E_0 \cos 45°$ & one is along y - $E_0 \sin 45°$

Vibrations of Electric field parallel to optic axis is E-Ray & Vibration " " perpendicular " " " O-Ray.

$OA = E_0 \cos 45°$ (E-ray)
$OB = E_0 \sin 45°$ (O-ray)

As E-rays travell with diff. speed so when these ray come ~~or~~ out of this crystal then there must be some path difference in these ~~ways~~. The path travelled by the wave is called optical path,

Optical path = ref. index of that medium × distance travelled

optical path $= nt$

for E-ray, $n_e t$
for O-ray, $n_o t$

So path diff. $= (n_e t - n_o t)$ (+ve)

for circularly polarised light

Phase diff → $\pi/2$
Path → $\lambda/4$

$$\therefore \boxed{t = \frac{\lambda}{4(n_o - n_e)}}$$

for plane polarised light

Phase diff → π
Path → $\lambda/2$

$$\boxed{t = \frac{\lambda}{2(n_o - n_e)}}$$

Now this is known as half wave plate or $\frac{\lambda}{2}$-plate.

It depens upon thickness t, that how much path difference is produced by wave.

If t is in b/w these two values then phase diff is not π

i.e. phase diff. $\Delta\phi \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

Then Elliptically Polarised & thickness t lies b/w these 2.

Detection of Light Wave :-

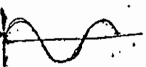
If a given light is passing through a rotating Nicol prism then, there are 3 posibilities,

given light

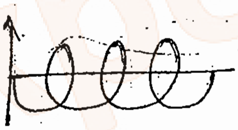
rotating Nicol Prism

There may be 3 posibilities

- variation in intensity with 0 minima then Light wave is surly plane polarised. $I \propto E_0^2$
- Variation in intensity with Non-zero minima then 2 posibilities
  - → Elliptical
  - → Partially plane P. i.e. mixture of unpolarised & plane polarised.
- No variation in intensity. then
  - → Circularlly P.
  - → Unpolarised



When Elec. field $= 0$, then Intensity $= 0$ (I can't be −ve as it is square of amp. E_0)

For Elliptical   In ellipse radius can't be zero.

for Circular 

In (b) case, there is some doubt, so ~~we~~ ~~have~~ use a λ/4 plate.

(b) Partially P.

λ/4

No effect

variation int. with non-zero minima

(b) elliptically light

λ/4 plate

P.P.

rotating Crystal

variation int. with 0 minima

If elliptically polarised light passes through $\lambda/4$ plate then it will produce a phase diff of $\pi/2$ b/w E-ray & O-ray. It'll become Plane polarised & →

If partially plane polarised ligh passes through $\lambda/4$ plate then it will remain partially plane polarii (no effect) & after appassing through nicol crystal there will be variation intensity with non-zero minima. It'll become Partially Plane polarised.

(→ & after passing through rotating crystal, there will be variating in int. with 0 minima i.e. it was elliptical)

If C.P. light pases through $\lambda/4$ plate then it'll become ~~circul~~ plane polarised. If we get variation in intensity with zero minima. This confirms it was circularly P. light.

C.P. → $\boxed{\lambda/4}$ → P.P. → $\boxed{\text{rotating c}}$ → variation int. with 0 m

If unpolarised light passes through $\lambda/4$ plate. It remains ". If un.P. light passes through rotating crystal then No. variation in intensity. This confirms it was Unpolarised.

Un.P. → $\boxed{\lambda/4}$ → $\boxed{\text{rotating C.}}$ → No variation in intensity.

e.g. P.P. → $\boxed{\lambda/2}$ → P.P.

C.P. → $\boxed{\lambda/2}$ → C.P.

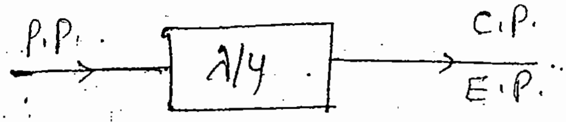
R.C.P. → $\boxed{\lambda/2}$ → L.C.P.

L.C.P. → $\boxed{\lambda/2}$ → R.C.P.

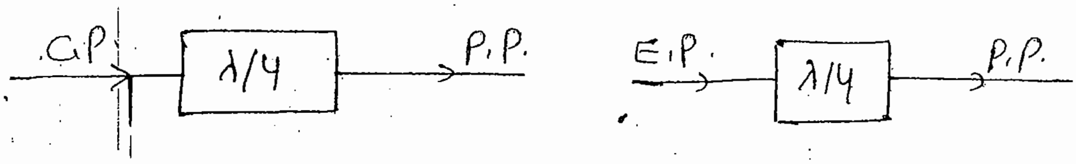
E.P. → $\boxed{\lambda/2}$ → E.P.

R.E.P. → $\boxed{\lambda/2}$ → ~~R~~ L.E.P.

L.E.P. → $\boxed{\lambda/2}$ → R.E.P



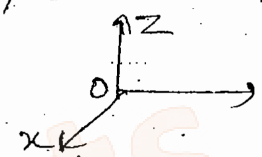
depends on amp. of wave
if amp. is same then C.P.
" " not " " E.P.

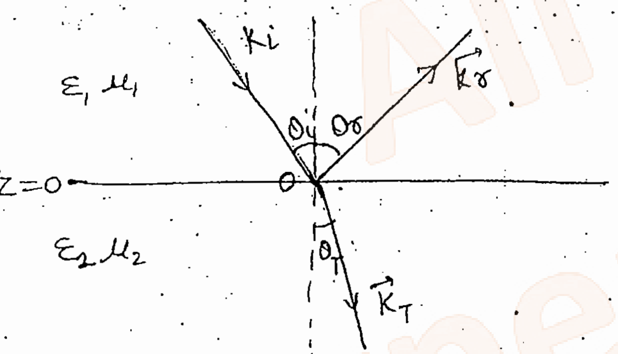


- Plane polarised light or Linearly Polarised light is the combination of L.C.P & R.C.P.

Reflection & Refraction of EM Waves at the Dielectric Interface :-

If we have a (dielectric) interface having different dielectric above & below the interface.





(interface is in x-y plane & z dir^n is normal to the surface)

EM waves carries energy & when EM waves fall on an interface then there will be redistribution of energy & some part will reflect & some part will Transmit.

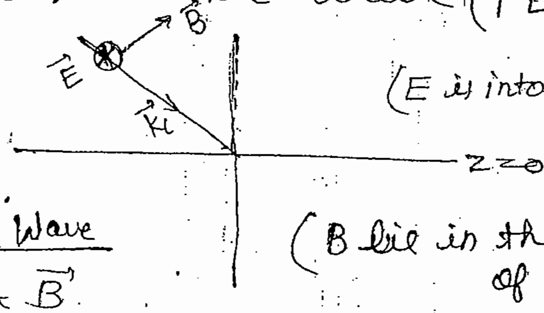
Interface Plane :- The plane dividing the 2 mediums.
In present case → x-y plane

Plane of Incidence :- The plane that contains propagation vector & normal to the plane
In present case → y-z plane of incidence

(i) $\vec{E}$ (polarisation Vector) is $\perp$ to plane of incident.
This is also called Transverse electric wave (TE wa

- $\vec{B}$ & $\vec{K_i}$ both lie in x-y plane
- $\vec{E}$ is lying in −x plane.
- $\vec{E}$ is $\perp^r$ to both $\vec{K}$ & $\vec{B}$.

So $\vec{E}$ is transverse electric wave
i.e. $\vec{E}$ is transverse to $\vec{K}$ & $\vec{B}$.

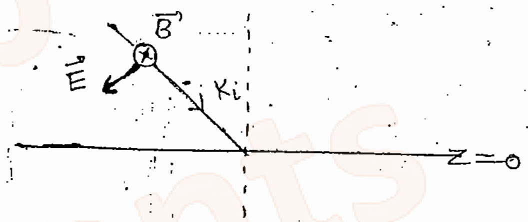


(E is into the pag

(B lie in the plan of k.)

& Normal waves are Transverse electromagnetic wave
i.e. transverse electric & transverse magnetic wave bot

(ii) $\vec{E}$ ~~vec~~ parallel to plane of incident
Here $\vec{E}$ is not transvers.
& $\vec{B}$ is into the page.
$\vec{E}$ vector is in y-z plane i.e. ∥ to the plane of incidence.
It is called Transverse Magnetic Wave
$\vec{B}$ is $\perp$ to the plane of $\vec{E}$ & $\vec{K}$



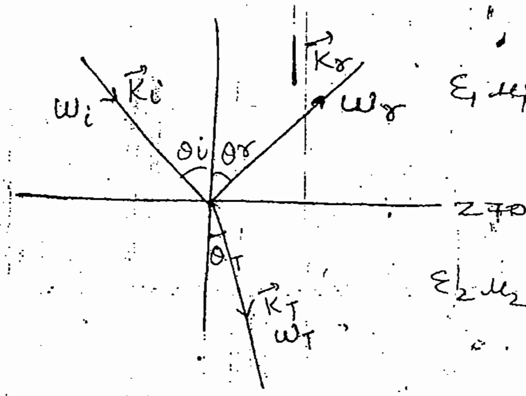
Static properties :- Same for TE & TM waves.

(i) $\omega_i = \omega_r = \omega_t$

(ii) Wavelength & wave velocity changes s.t. freq. remains same.

(ii) $\theta_i = \theta_r$

(iii) $\dfrac{\sin\theta_i}{\sin\theta_t} = \dfrac{n_2}{n_1} = \dfrac{v_1}{v_2} = \dfrac{\sqrt{\varepsilon_2\mu_2}}{\sqrt{\varepsilon_1\mu_1}}$



~~Dynamical properties~~ :-

Boundry Cond^n :- (i) $E_1^{\parallel} = E_2^{\parallel}$ (On interface, no free charge so $\varepsilon_1^f = \varepsilon_2^f$)

(ii) $\varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp}$.

(iii) $B_1^{\perp} = B_2^{\perp}$; (iv) $\dfrac{B_1^{\parallel}}{\mu_1} = \dfrac{B_2^{\parallel}}{\mu_2}$

On the basis of these 4 B.C., we can derive the Dynamical properties.

Dynamical Properties :- (Fresnel's Relations) :-

For TE Waves $(\perp)$:-

$$\boxed{\left(\frac{E_{or}}{E_{oi}}\right)_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

$E_{or} \rightarrow$ Amp. of E-field of reflected wave

$E_{oi} \rightarrow$ " " " incident "

$\perp$ means electric field is $\perp^r$ to the plane of incidence.

where $\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$ & $\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$

$$\boxed{\left(\frac{E_{ot}}{E_{oi}}\right)_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

$E_{ot} \rightarrow$ amp. of E-field of transmitted wave.

$\frac{H_{or}}{H_{oi}} \Rightarrow$ $$\boxed{\left(\frac{E_{or}}{E_{oi}}\right)_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = -\left(\frac{H_{or}}{H_{oi}}\right)_{\perp}}$$

(-ve sign indicate phase change)

$H_{or} \rightarrow$ Amp. of Mag. field of reflected wave

$H_{oi} \rightarrow$ " " " " incident "

$\perp \rightarrow$ decides from electric field i.e. E-vector is $\perp$ to the plane of incidence.

When reflected vector of electric field do not change phase then " " Mag. " changes the phase of π.

& $$\boxed{\left(\frac{H_{ot}}{H_{oi}}\right)_{\perp} = \frac{2\eta_1 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

2. For TM Wave $(\parallel)$:-

$$\left(\frac{E_{or}}{E_{oi}}\right)_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = -\left(\frac{H_{or}}{H_{oi}}\right)_{\parallel}$$

$$\left(\frac{E_{ot}}{E_{oi}}\right)_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\left(\frac{H_{ot}}{H_{oi}}\right)_{\parallel} = \frac{2\eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

D) For $\perp^r$ components :-

When medium is Non-magnetic,

$$\mu_1 = \mu_2 = \mu_0$$

$$-\left(\frac{H_{or}}{H_{oi}}\right) = \left(\frac{E_{or}}{E_{oi}}\right)_{\perp} = \frac{-\sin(\theta_i - \theta_T)}{\sin(\theta_i + \theta_T)}$$

$$\left(\frac{E_{ot}}{E_{oi}}\right)_{\perp} = \frac{2\sin\theta_T \cos\theta_i}{\sin(\theta_i + \theta_T)}$$

$$\left(\frac{H_{ot}}{H_{oi}}\right)_{\perp} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \cdot \left(\frac{E_{ot}}{E_{oi}}\right)_{\perp}$$

If $\varepsilon_1 < \varepsilon_2$ then $\boxed{n_2 > n_1}$ $(n = \sqrt{\varepsilon_r})$

if EM wave going from Rare to dencer (air-glass) then phase change will be

$$\frac{\sin\theta_i}{\sin\theta_T} > 1 \qquad (\because n_2 > n_1)$$

i.e. $\boxed{\theta_i > \theta_T}$

If EM wave going from Rare-dencer then it will bend toward normal & reflected component of $E_{\perp}$ will suffer a phase change of π. And Reflected comp. of mag. field will not suffer a phase change.

Transmitted Components of electric & mag. field suffer No phase change.

If $\varepsilon_1 > \varepsilon_2$ then $\boxed{n_1 > n_2}$

$\boxed{\theta_i < \theta_T}$

When a EM wave is going from dencer to rare (glass-air) or (water-air) then reflected component of mag. field will suffer a phase change of π & Reflected comp. of Electric field vector will not suffer any phase change.

Transmitted components in this case suffer no phase change either go from rare to dencer or dencer to rare.

) For Parallel Components :-

$$-\left(\frac{H_{or}}{H_{oi}}\right)_{\parallel} = \left(\frac{E_{or}}{E_{oi}}\right)_{\parallel} = \frac{-\tan(\theta_i - \theta_T)}{\tan(\theta_i + \theta_T)}$$

$$\left(\frac{E_{oT}}{E_{oi}}\right)_{\parallel} = \frac{2\sin\theta_T \cdot \cos\theta_i}{\sin(\theta_i+\theta_T)\cdot\cos(\theta_i-\theta_T)}$$

$$\left(\frac{H_{oT}}{H_{oi}}\right)_{\parallel} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\left(\frac{E_{oT}}{E_{oi}}\right)_{\parallel}$$

Transmitted components never suffers a phase change in any case [if $\theta_i > \theta_T$ or $\theta_T > \theta_i$ then $\cos(\theta_i - \theta_T) \Rightarrow \cos(-\theta) = \cos\theta$]
i.e. either go from rare to dencer or dencer to rare.

below 90°, $\tan = +ve$

If $(\theta_i + \theta_T) < 90^\circ$

$\tan(\theta_i + \theta_T) > 0$

$\theta_i > \theta_T \Rightarrow \varepsilon_1 < \varepsilon_2 \Rightarrow n_2 > n_1$

EM wave is going from Rare $\rightarrow$ dencer & $(\theta_i + \theta_T) < 90^\circ$ then reflected vector of electric vector will suffer a phase change of π, & Mag. field will Not and for $(\theta_i + \theta_T) < 90^\circ$

$\varepsilon_1 > \varepsilon_2 \Rightarrow n_1 > n_2$

$\theta_T > \theta_i$

EM wave is going from dencer to rare & ref. comp. of Mag. field H will suffer a phase change of π.

No component of Electric or mag. field will be reflected which is parallel to the plane of incidence.

$\tan 90^\circ = \infty$ then $\left(\frac{E_{0r}}{E_{0i}}\right) = 0$

Hence reflected light will contain only $\perp^r$ comp. of electric field which is normal to the plane of incidence.

$$\theta_i + \theta_T = 90^\circ$$
$$\theta_i = \theta_B$$

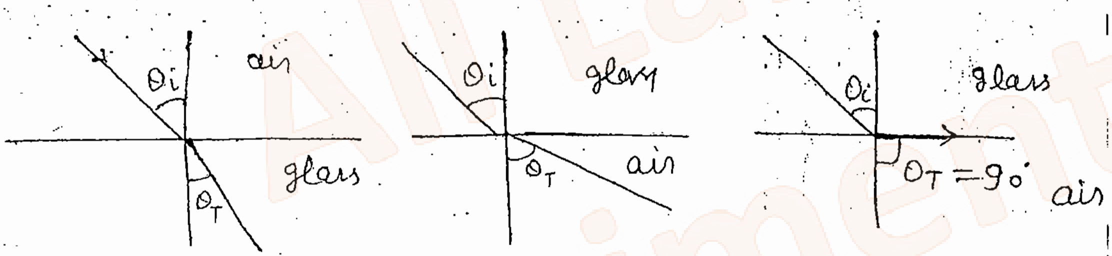
$$\boxed{\theta_P = \theta_B = \tan^{-1}\frac{n_2}{n_1} = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}} \longrightarrow \text{Brewster's law}$$

So Reflected comp. of elec. field will contain only $\perp^r$ vibrations of ele. field.

This is valid for both rare to denser & denser to rare medium.

denser to rare $\longrightarrow$ away from normal

rare to denser $\longrightarrow$ towards the normal

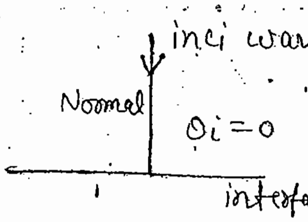


for any angle of incidence ($\theta_i > \theta_c$) then there will be Total internal reflection. It can only observe for denser to rare medium.

$$\frac{\sin\theta_i}{\sin\theta_T} = \frac{n_2}{n_1}$$
$$\sin\theta_i = \frac{n_2}{n_1}$$
$$\boxed{\theta_i = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

Relation b/w the amplitude of mag. & electric field for Normal Incidence :-

for Normal incidence $\theta_i = 0$



$$\mu_1 = \mu_2 = \mu_0$$

then $-\left(\frac{H_{0r}}{H_{0i}}\right) = \left(\frac{E_{0r}}{E_{0i}}\right)_{\perp \text{ or } \parallel} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)$

If EM wave is incident from Rare → dencer then reflected comp of E-field will suffer a phase change & If wave is inci from dencer to rare then reflected comp. of mag. field will suffer a phase change.

$\frac{E_{or}}{E_{oi}}$ ⇒ This indicate → if E_{oi} is the amp. of incident wave then how much part of incident wave has been reflect

$\frac{E_{ot}}{E_{oi}}$ → " " transmit

$$\left(\frac{E_{oT}}{E_{oi}}\right) = \frac{2n_1}{(n_1+n_2)} \quad \& \quad \left(\frac{H_{oT}}{H_{oi}}\right) = \frac{2n_2}{(n_1+n_2)}$$

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Reflection & Transmission coefficients for normal incidence :-

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

It indicate How much part of energy / power / Intensity will be reflected or transmitted.

Intensity is,

$$\langle \vec{S} \rangle = I = \vec{E} \times \vec{H}$$
$$= E_o H_o$$

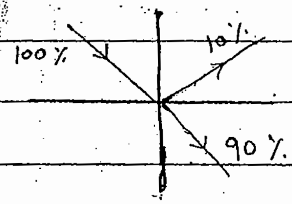
At Interface, energy conservation must be followed.

If 100% energy is incident & 10% is reflected then 90% energy should be transmitted.

$R = 10\%$

$T = 90\%$

$$\boxed{R + T = 1}$$



Q. :- A EM wave $\vec{E} = 4\cos(\omega t - Kz)\,\hat{x}$ incident from air to glass ($n = 1.5$) interface normally. Calculate reflected & transmitted amplitudes of electric field & also their phase relation. Calculate % of energy reflected & transmitted at the interface. θ_B for air-glass interface = ?

$$\vec{E} = 4\cos(\omega t - Kz)\,\hat{x}$$

$$n = 1.5$$

$E_{0i} = 4$

$$\left(\frac{E_{0T}}{E_{0i}}\right) = \frac{2n_1}{n_1+n_2} = 0.8 \qquad \begin{bmatrix} n_1 = 1 \\ n_2 = 1.5 \end{bmatrix}$$

$$\frac{E_{0T}}{4} = 0.8 \Rightarrow E_{0T} = 0.8 \times 4$$

$$\boxed{E_{0T} = 3.2}$$

$$\left(\frac{E_{0r}}{E_{0i}}\right) = \frac{n_1-n_2}{n_1+n_2} = \frac{-1}{5} = -0.2$$

$$E_{0r} = -0.2 \times 4$$

$$\boxed{E_{0r} = -0.8}$$

$$R = \left(\frac{n_1-n_2}{n_1+n_2}\right)^2 = (-0.2)^2 = 0.04$$

$$\boxed{R = 4\%}$$

$R+T = 1 \Rightarrow \boxed{T = 96\%}$

Brewstor angle = ?

$$\theta_P = \theta_B = \tan^{-1}\left(\frac{1.5}{1}\right)$$

$$\boxed{\theta_B = 56.30^\circ}$$

* Brewstor angle for air-glass interface is 56.30°.

If we incident a EM wave at this angle then reflected comp. of electric field will be plane polarised.

$\theta_T = ?$

$$\theta_T = \frac{\pi}{2} - \theta_B$$

$$\theta_T = 90^\circ - 56.30^\circ$$

$$\boxed{\theta_T = 33.7^\circ}$$

Q. Do the same for air-water ($n = 1.33$) interface. Calculate critical angle θ_c for water-air interface.

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for air-water interface, $n_1 = 1$, $n_2 = 1.33$

$$\left(\frac{E_{oT}}{E_{oi}}\right) = \frac{2n_1}{n_1+n_2} = \frac{2}{1+1.33} = \frac{2}{2.33}$$

$$E_{oT} = 0.85837 \times 4$$

$$\boxed{E_{oT} = 3.43348}$$

$$\left(\frac{E_{or}}{E_{oi}}\right) = \frac{n_1-n_2}{n_1+n_2} = \frac{1-1.33}{1+1.33} = \frac{-0.33}{2.33} = -0.14163$$

$$E_{or} = -0.14163 \times 4$$

$$\boxed{E_{or} = -0.56652}$$

$$R = \left(\frac{n_1-n_2}{n_1+n_2}\right)^2 = (-0.14163)^2 = 0.02006$$

$$\boxed{R = 2.006\ \%}$$

$\because R + T = 1 \Rightarrow \boxed{T = 97.994\ \%}$

$$\theta_p = \theta_B = \tan^{-1}\frac{n_2}{n_1}$$

$$\theta_B = \tan^{-1}\left(\frac{1.33}{1}\right) = 53.06 \Rightarrow \boxed{\theta_B = 53.06^\circ}$$

$$\theta_T = \frac{\pi}{2} - \theta_B = 90^\circ - 53.06$$

$$\boxed{\theta_T = 36.94}$$

Critical angle for water-air interface,

$$\theta_c = \sin^{-1}\frac{n_2}{n_1}$$

water $n_1 = 1.33$

air $n_2 = 1$

$$\theta_c = \sin^{-1}\left(\frac{1}{1.33}\right)$$

$$\boxed{\theta_c = 48.75^\circ}$$

Dielectric - Conductor

$\sigma_1 = 0$ always wheather for air or dielectric.

For perfect conductor $\sigma_2 \to \infty$

$\varepsilon_1, \mu_1, \sigma_1 = 0$
dielectric
conductor
$\varepsilon_2, \mu_2, \sigma_2$

For Normal Incidence

$$\boxed{E_{or} = -E_{oi}}$$
$$\boxed{E_{oT} = 0}$$

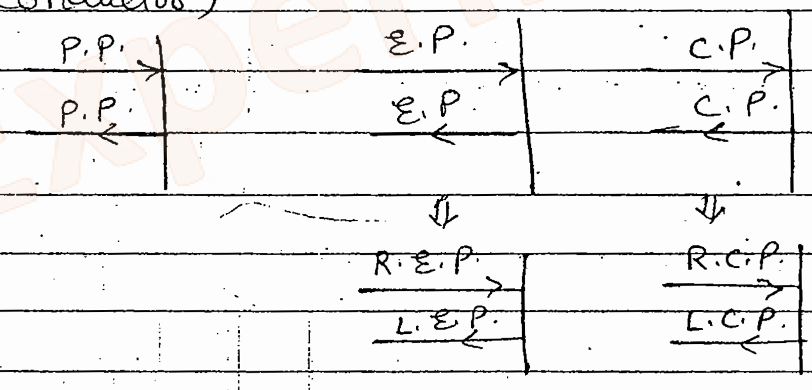
$-ve \Rightarrow$ Phase change

Inside the conductor Ele. field $= 0$

Bcoz $E_{oT} = 0$ i.e. all the components will be reflected back.

Reflected comp. of Ele. field will suffer a phase change of π. And Reflected comp. of mag. field will not suffer a phase change.

For Any Rare to Denser Medium, (air to conductor)



When Conductivity is finite then

$$\left(\frac{E_{or}}{E_{oi}}\right) = \frac{1 - (1+i)\Delta}{1 + (1+i)\Delta}$$

$$\left(\frac{E_{oT}}{E_{oi}}\right) = \frac{2}{1+(1+i)\Delta}$$

where

$$\Delta = \left(\frac{\sigma_2 \mu_1}{2 \mu_2 \varepsilon_1 \omega}\right)^{1/2}$$

For perfect Conductor, $\sigma_2 \to \infty$ i.e. $\Delta \to \infty$

$$\left(\frac{E_{or}}{E_{oi}}\right) = -1$$

Reflectance :- How much part of energy will be reflected back

$$R = \left|\frac{E_{or}}{E_{oi}}\right|^2 = 1 \quad \text{(for perfect conductor)}$$

$$T = 0$$

$$\boxed{R + T = 1}$$

No energy can transmit in perfect conducting medium, all energy will be reflected

Reflectance if conductivity is finite

$$\boxed{R = 1 - \frac{2}{\Delta}}$$

$$T = 1 - R$$

Q. Calculate the reflectance coefficient for light at an air-silver interface $\mu_1 = \mu_2 = \mu_0$, $\varepsilon_1 = \varepsilon_0$, $\sigma = 6 \times 10^7 \, \Omega^{-1}\text{-}m^{-1}$ & $\omega = 4 \times 10^{15}$.

$$\Delta = \left(\frac{\sigma_2 \mu_1}{2 \mu_2 \varepsilon_1 \omega}\right)^{1/2}$$

$$= \left(\frac{\cancel{6}3 \times 10^7 \, \cancel{\mu}}{\cancel{2} \, \cancel{\mu} \, \varepsilon_0 \times 4 \times 10^{15}}\right)^{1/2} = \left(\frac{3}{4\varepsilon_0} \times 10^{-8}\right)^{1/2}$$