

Free Study Material from All Lab Experiments



**Electromagnetic Theory
for NET/Gate Physical Sciences
> Electrodynamics Part-1 <**

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Electrodynamics

In Electrodynamics,

changing electric & magnetic field with time.

Now, the fields are time variable.

Static mag. field is produced by steady current.

" electric " " " " charge at rest.

Steady Current - charges are not accumulating anywhere.

If Current I is the fcn of time then Magnetic field will also be the fcn of time.

$$I(t) \rightarrow B(t)$$

& If Mag. field is time dependent then Mag. field & elec. field are dependent on each other.

Time dependent mag. field can produce time dependent Elec. field

" " Elec. " " " " Mag. "

$$I(t) \rightarrow B(t) \rightarrow E(t)$$

Electromotive force (emf) :-

Electromotive force is work done per unit charge.

emf is denoted by $\mathcal{E}$,

$$\mathcal{E} = - \int \vec{E} \cdot d\vec{l}$$

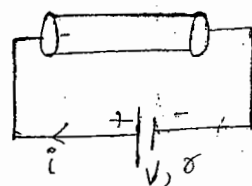
$\vec{E} \cdot d\vec{l}$ is work done per unit charge

$\vec{E} \cdot d\vec{l}$ Work done per unit charge

If we have a wire connected to battery then it is conducting & battery supply the force then free charge in wire are in motion & produce work. This work is done by battery. This work done is emf.

If battery potⁿ is $V - iR$

Internal resistance is r



then By Battery do work by this potⁿ $V - iR$. So emf or work done per unit charge is $\Rightarrow$ emf $\mathcal{E} = V - iR$

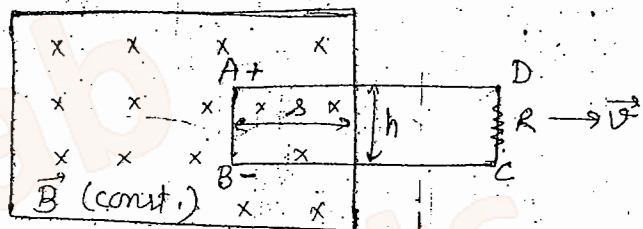
Motional Electromotive force (emf) :-

Generators uses motional emf concept to generate power. This type of emf arises when a conducting loop moves a constant mag. field i.e. $\vec{B}$ remains constant with time.

Let us consider a wooden block, in this we have a closed loop. A resistance is connected with loop.

If someone is pulling the loop then mag. flux changes.

Mag. flux passes through a closed surface is always zero.



$$\Phi_m = \int_s \vec{B} \cdot d\vec{s}$$

$$\left\{ \Phi_m = \oint \vec{B} \cdot d\vec{s} = 0 \right.$$

$$\mathcal{E} = - \frac{d\Phi_m}{dt}$$

This is closed loop but open surface bcoz closed surface bound some volume.

It works on Faraday's law.

Acc. to Faraday law,

Changing flux always induces the emf & dirⁿ of emf is s.t. it oppose the change.

If flux is $\uparrow$ then dirⁿ of emf is s.t. try to $\downarrow$ it. i.e. we take -ve sign.

i.e.

$$\boxed{\mathcal{E} = - \frac{d\Phi_m}{dt}}$$

$\rightarrow$ Faraday's law

$\Rightarrow$ Change in flux can be of 3 types.

(i) either change $\vec{B}$, or (ii) change area ds or (iii) change the angle b/w $\vec{B}$ & $d\vec{s}$.

$$\Phi_m = \int_s B ds \cos \theta$$

In Case of Motional emf,

B is always constant.

So flux can be changed due to either by changing area or by change angle b/w them.

If we pull the loop out then s will vary so area will change.

To find motional emf, we use formula,

$$\mathcal{E} = \oint \vec{f}_{\text{mag}} \cdot d\vec{l}$$

$\vec{f}_{\text{mag}} \rightarrow$ Magnetic force per unit charge.

$\Rightarrow$ In the present Case, when loop is pulled with vel. $\vec{v}$ then amount of emf = ? Current in resistance & dir = ?

If a charge is moving with vel. $\vec{v}$ in mag. field $\vec{B}$ then force on charge

$$\vec{f}_{\text{mag}} = \vec{v} \times \vec{B} \quad \text{if charge is +ve}$$

$$\vec{f}_{\text{mag}} = \vec{B} \times \vec{v} \quad \text{if " " -ve}$$

Here Consider charge is +ve. for +ve charge the dirⁿ of $\vec{f}_{\text{mag}}$ will be upward, $A \rightarrow +ve$
& force on e^- will be downward, $B \rightarrow -ve$

Current will flow due to motion of charges.

Now part AB will behave like a battery. & Current will flow from +ve to -ve terminal. i.e. $A \rightarrow B$.

If AB is perfect conducting wire then its resistance = 0.

Hence AB part of loop is only contributing to the emf.

$$\mathcal{E} = \oint \vec{f}_{\text{mag}} \cdot d\vec{l}$$

$$= v B \int dl$$

$$V = \mathcal{E} = v B h$$

& current $V = i R \Rightarrow i = \frac{V}{R}$

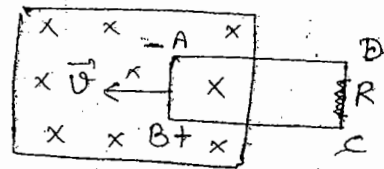
$$i = \frac{v B h}{R}$$

& dirⁿ of flow is from D $\rightarrow$ C

- If loop is pulling in opposite dirⁿ force on +ve charge downward
" " e⁻ upward,

then A $\rightarrow$ -ve & B $\rightarrow$ +ve

& Now Current will flow from C $\rightarrow$ D.

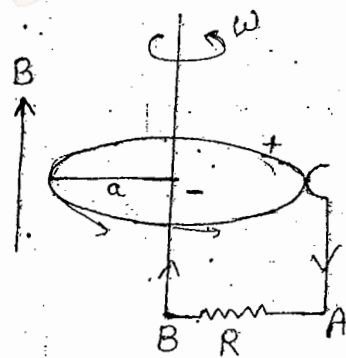


Ques:- A metal disc of radius 'a' rotates with angular velocity ω about vertical axis through a uniform mag. field $\vec{B}$ pointing in the upward dirⁿ. A circuit is made by connecting one end of the resistor to the axle & other two to the sliding contact which touches the outer edge of the disc. Find the current in the resistor with dirⁿ.

If charges are not in motion then no change in flux & there will be No emf.

Emf induces due to motion of charges.

In present case due to the motion of charges, their area is changing bcoz disc is rotating & charges are displacing from their position.



$$\vec{\omega} = \omega \hat{z}$$

$$\vec{v} = \vec{\omega} \times \vec{r} = \omega r (\hat{z} \times \hat{r})$$

$$\vec{v} = \omega r \hat{\phi}$$

$\vec{r}$ $\rightarrow$ radius vector of the disc having dirⁿ $\hat{r}$.

for charges on the circumference the speed v is constant (same) but for all other charges speed is different.

force per unit charge,

$$\begin{aligned}\vec{f}_{\text{mag}} &= \vec{v} \times \vec{B} \\ &= \omega r \hat{\phi} \times B \hat{z} \\ &= \omega r B (\hat{\phi} \times \hat{z})\end{aligned}$$

$$\boxed{\vec{f}_{\text{mag}} = \omega r B \hat{r}}$$

Beoz of this force, charges move towards outer edge
so outer edge become +ve terminal of battery &
centre " " -ve " "

$$\mathcal{E} = \int_0^a \omega B r dr$$

$$\boxed{\mathcal{E} = \frac{\omega B a^2}{2}}$$

The Current flow, $I = \frac{\mathcal{E}}{R}$

$$I = \frac{\omega B a^2}{2R}$$

Dirⁿ of the current will be $A \rightarrow B$

- If disc rotate oppositely then dirⁿ of $\vec{v}$ will be opposite & dirⁿ of force opposite & then +ve charge on centre & -ve on surface so terminal changes.
- The Current flow on External Circuit is from +ve to -ve terminal.

Ques 1 - A metal bar of mass m slides frictionlessly on two parallel conducting rails. A distance l apart a resistor R is connected across the rails & a uniform mag. field $\vec{B}$ is pointing into the page.

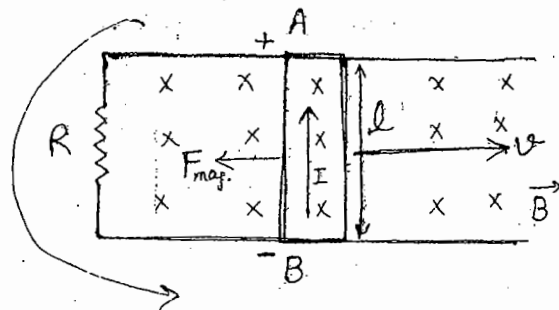
- find dirⁿ & magnitude of the current in the resistor.
- Magnetic force on the metal bar with dirⁿ.
- If v_0 is the speed at $t=0$ time. find the speed at later time t of the conducting bar.

(A)

For +ve charges push upward
-ve " " down

So $A \rightarrow +ve$
 $B \rightarrow -ve$

Current will flow towards
resistance as this is open
circuit from one side.



emf, $\mathcal{E} = \int \vec{F}_{mag} \cdot d\vec{l}$

$$\mathcal{E} = vBl$$

Current $I = \frac{\mathcal{E}}{R} \Rightarrow I = \frac{vBl}{R}$

dirⁿ $\rightarrow$ downwards

Inside the rod, current from $B \rightarrow A$

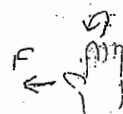
Metal Bar does not have any resistance. So total resistance is R on whole circuit.

(B) If a wire of length l & current flowing in it is I , then Magnetic force will be

$$\begin{aligned} \vec{F}_{mag} &= I \int d\vec{l} \times \vec{B} \\ &= \frac{vBl}{R} \cdot lB \end{aligned}$$

$$\vec{F}_{mag} = \frac{vB^2l^2}{R}$$

dirⁿ $\rightarrow$ to the left



(C) Bar is moving in dirⁿ $\rightarrow$

& $\vec{F}_{mag}$ is in dirⁿ $\leftarrow$

Both will try to stop the bar.

If m be the mass of bar & v be the vel, then mechanical force

$$m \frac{dv}{dt} = - \frac{vB^2l^2}{R} \Rightarrow \frac{dv}{dt} = - \frac{vB^2l^2}{mR}$$

$$\frac{dv}{v} = - \frac{B^2l^2}{mR} dt$$

$$\ln v = -\frac{B^2 l^2}{Rm} t + C$$

At $t=0$, $v=v_0$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{B^2 l^2}{Rm} t$$

$$v = v_0 e^{-\frac{B^2 l^2}{Rm} t}$$

velocity is exponentially $\downarrow$ with time.
As $B \uparrow$, the decrease in v will be more.

Que:- A square loop of wire of side 'a' lies on a table at a distance 's' from a very long straight wire which carries a current I . If someone pulls the loop away from the wire at speed v What is the induced emf in the loop. Also find the dirⁿ of induced current,

Square loop is pulled upward dirⁿ.

Flux passes through the square loop

$$\phi_m = \int B \cdot ds$$

$B \rightarrow$ mag. field of wire
 $ds \rightarrow$ area of loop

Mag. field of a wire is

$$B = \frac{\mu_0 I}{2\pi s} \hat{\phi}$$

dirⁿ of Mag. field $\rightarrow$ It is coming out.

Area, $ds = dx dy$

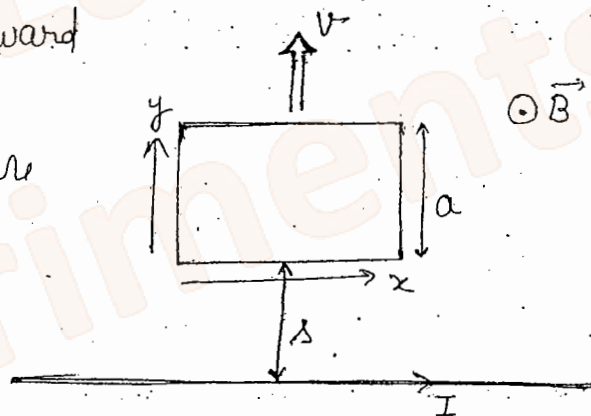
$$\phi_m = \int_0^a \int_s^{s+a} \frac{\mu_0 I}{2\pi y} dx dy$$

{limits of $y \rightarrow s$ to $s+a$ }

Here variation of mag. field is along y dirⁿ

$$\phi_m = \frac{\mu_0 I}{2\pi} a \ln\left(\frac{s+a}{s}\right)$$

If flux $\rightarrow$ const then No emf.



s is variable. If loop is away from wire then $s \uparrow$
 & then flux $\downarrow$. (area of loop & angle $\rightarrow$ constant & mag. field changes)
 So Here flux is not constant. $\downarrow$ depending on s

Now, $\mathcal{E} = \text{mf}$, $\mathcal{E} = -\frac{d\Phi_m}{dt}$

$$= -\frac{\mu_0 I a^2}{2\pi} \left(\frac{s}{s+a} \right) \left(-\frac{a}{s^2} \right) \frac{ds}{dt}$$

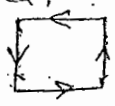
$\frac{ds}{dt} \rightarrow$ Rate of change of distance i.e. velocity v .

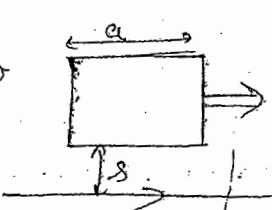
$$\mathcal{E} = \frac{\mu_0 I a^2}{2\pi s(s+a)} v$$

This much emf will induce in the loop.

Dirⁿ of the induce current: As loop is moving away from wire $\rightarrow s \uparrow$.
 Here flux is decreasing. So induce current will try to increase it, & flux is decreasing due to dec. in mag. field so induce current will try to inc. the mag. field.
 To do so, it will develop a mag. field opposite to original.

So Dirⁿ of current - Anticlockwise

• If we are pulling the loop towards the wire, 
 then flux $\uparrow$, $\vec{B} \uparrow$, so current try to dec. the $\vec{B}$.
 Dirⁿ of current - Clockwise

• If we pull the loop parallel to wire then there will be No change in flux. Now flux will pass through the loop but change in flux $= 0$ so $\mathcal{E} = 0$ 

* If wire is finite then on the ends of wire, mag. field will be complicated.

* If loop pulling away from wire, dirⁿ of $I \rightarrow$ Anti
 " " " towards " " " " $\rightarrow$ clockwise

* Induced emf try to dec. the flux

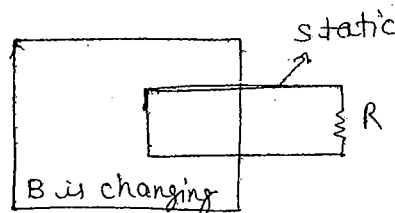
Magnetic Flux with changing Magnetic field

(II case)

Loop is static but B is changing.

emf,

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_m}{dt}$$



This is the integral form of Faraday's law.

$\vec{E} \rightarrow$ This is not constant electrostatic field.
 so $\oint \vec{E} \cdot d\vec{l} \neq 0$ Here. (i.e. Not static)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S}$$

$$\Phi_m = \int \vec{B} \cdot d\vec{S}$$

Loop is static i.e. area is not changing

$$\text{so } \frac{\partial S}{\partial t} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\int \frac{d\vec{B}}{dt} \cdot d\vec{S}$$

$$\Rightarrow \int_S (\nabla \times \vec{E}) \cdot d\vec{S} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\Rightarrow \int_S (\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t}) \cdot d\vec{S} = 0$$

$$\Rightarrow \boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

so we have induced electric field only if $\vec{B}$ is changing with time.

& $\vec{E}$ is not electrostatic here bcoz for electrostatic $\vec{E}$, $(\nabla \times \vec{E} = 0)$

so $\vec{E}$ is not conservative as well,
 $\vec{E} \rightarrow$ Non Conservative & Non electrostatic

No scalar potⁿ is defined for that electric field.
 This ele. field is induced electric field, & The properties of induced elec. field are same as prop of mag. field as it arises from mag. field.

E - field lines $\rightarrow$ Open Curve

B - field lines $\rightarrow$ Close Curve

Field lines of Induced Electric field - Closed Curve

& $\boxed{\vec{\nabla} \cdot \vec{E} = 0}$

bcz there is no charge correspond to it otherwise $\vec{\nabla} \cdot \vec{E}$

$\nabla \cdot \vec{E} \neq 0$ Induced Elec. field is $\rightarrow$

$\rightarrow$ Non Conservative

$\rightarrow$ Non electrostatic

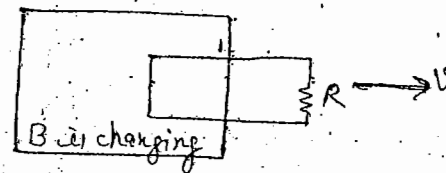
$\rightarrow$ Closed Curve

$\rightarrow \text{Div } \vec{E} = 0$

$$\begin{aligned} \text{emf, } \mathcal{E} &= \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_m}{dt} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S} \\ &= -\int \frac{d\vec{B}}{dt} \cdot d\vec{S} \quad (\text{if } B \text{ is not changing w.r.t } t) \end{aligned}$$

Case III:- If loop & ~~mag~~ mag. field both are changing then

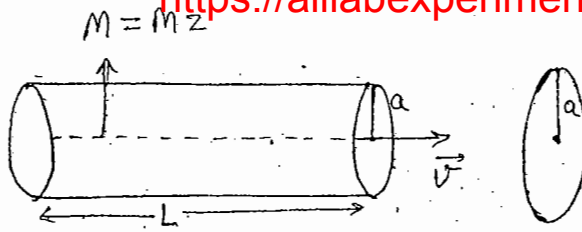
$$\begin{aligned} \mathcal{E} &= \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S} \\ &= -\int \frac{d\vec{B}}{dt} \cdot d\vec{S} + \oint \vec{F}_{\text{mag}} \cdot d\vec{l} \\ &= -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} + \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} \end{aligned}$$



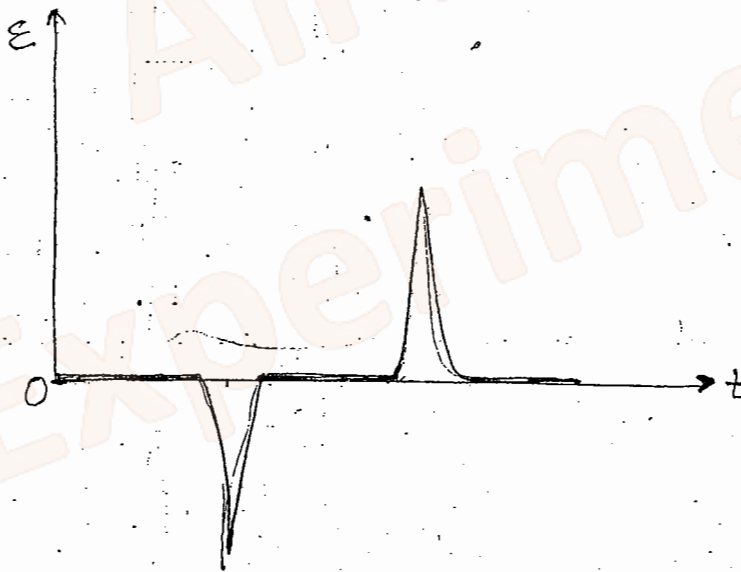
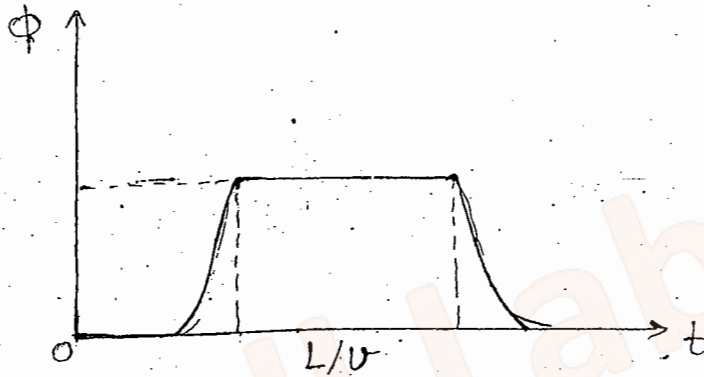
Que:- A long cylindrical magnet of length 'L' & radius 'a' carries a uniform magnetization $\vec{M}$ parallel to its axis. It passes at a constant velocity v through a circular ring of slightly larger diameter. Graph the induced emf in the ring as a funcⁿ of time.

If Magnetization of ~~cylinder~~ ^{magnet} is M then mag. field of a magnet is

$$\boxed{\vec{B} = \mu_0 M \hat{z}}$$



When magnet come close to ring then flux $\uparrow$. When it is in the ring then flux is constant through the passing of the magnet & then it will dec. coz it leave the ring. So emf occur when it enters & leaves the ring.



Ques :- A long solenoid of radius a is driven by an alternating current s.t. field inside the solenoid is $\vec{B}(t) = B_0 \cos \omega t \hat{z}$

A circular loop of wire of radius $a/2$ & resistance R is placed inside the solenoid & coaxial with it. Find the current induce in the loop as a fun of time.

$$\vec{B}(t) = B_0 \cos \omega t \hat{z}$$

B is not a funⁿ of distance i.e.
B uniform. So flux

$$\phi = B \cdot \pi \left(\frac{a}{2}\right)^2$$

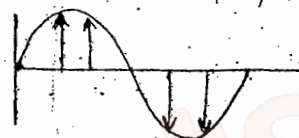
$$= B \frac{\pi a^2}{4}$$

$$\boxed{\phi = \frac{B_0 \pi a^2}{4} \cos \omega t}$$

$$\mathcal{E} = \frac{d\phi}{dt} = \omega B_0 \frac{\pi a^2}{4} \sin \omega t \Rightarrow \boxed{\mathcal{E} = \frac{B_0 \pi a^2 \omega}{4} \sin \omega t}$$

$$\text{Current } I = \frac{\mathcal{E}}{R} \Rightarrow \boxed{I = \frac{B_0 \omega \pi a^2}{4R} \sin \omega t}$$

Dirⁿ of current $\rightarrow$ changing accordingly $\sin \omega t$. For upper half circle & lower half circle dirⁿ will be different,



Que 1 - A square loop of wire with side 'a' lies in the 1st quadrant of x-y plane with one corner at the origin. In this region there is Non-uniform time dependent magnetic field $\vec{B}(y,t) = K y^3 t^2 \hat{z}$ is coming out of the page. Where K is some constant. Find the induced emf in the loop.

$$\vec{B}(y,t) = K y^3 t^2 \hat{z}$$

$\downarrow$
Non uniform & time dependent.

$$\text{Flux } \phi = \int B \cdot d\vec{s}$$

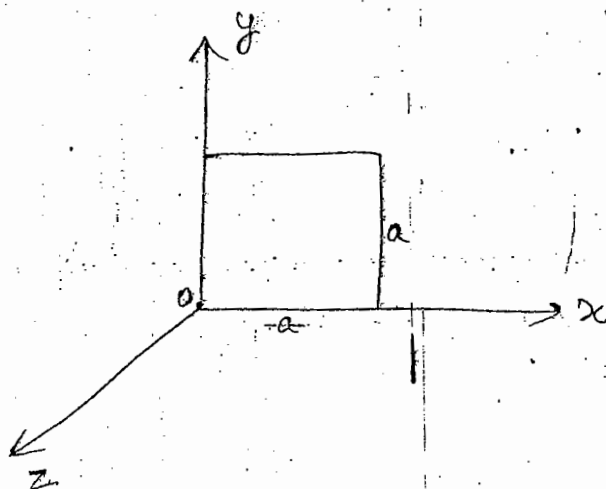
$$= \int_0^a \int_0^a K y^3 t^2 dx dy$$

$$= K t^2 [x]_0^a \left[\frac{y^4}{4} \right]_0^a$$

$$= K t^2 a \cdot \frac{a^4}{4}$$

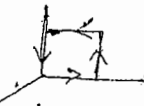
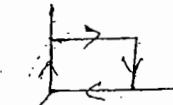
$$\boxed{\phi = \frac{K t^2 a^5}{4}}$$

flux is changing with time.



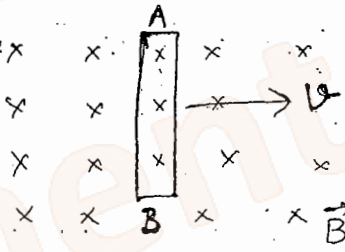
$$\text{Emf } \epsilon = -\frac{d\phi}{dt} = -\frac{d}{dt} \left[\frac{kt^2a^5}{4} \right] = \frac{-ka^5}{4} \cdot 2t$$

$$\boxed{\epsilon = -\frac{kt a^5}{2}}$$

Current producing such mag. field will be Anticlockwise. 
 B is $\uparrow$ with time, so induced current will try to $\downarrow$ it.
 So Dirⁿ of induced current for induced ~~ele.~~ mag. field will be clockwise. 

Note: Induced current always less than the original current except the case of Superconductors ($I_{in} = I$).

Q.1: A ^{metallic} rod AB moves with a uniform velocity v in a uniform mag. field B as shown in the figure.

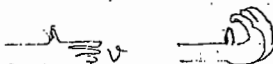
- (a) The rod becomes electrically charged.
 (b) The end A becomes $+$ vely charged.
 (c) " B " " "
 (d) The rod becomes hot because of joule heating.
- 

Q.2: A rod of length l rotates with a uniform angular velocity ω about its $\perp$ bisector. A uniform mag. field B exist parallel to the axis of rotation. The potⁿ difference b/w the centre of the rod & an end is

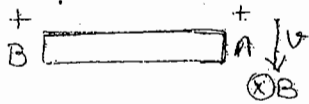
(a) 0 (b) $\frac{1}{8} \omega B l^2$ (c) $\frac{1}{2} \omega B l^2$ (d) $B \omega l^2$

Q.3: The potⁿ diff. b/w two ends of the rod is

(a) 0 (b) $\frac{1}{2} B l^2 \omega$ (c) $B l^2 \omega$ (d) $2 B l^2 \omega$

① (b) A $\rightarrow +ve$, B $\rightarrow -ve$
 B $\rightarrow \otimes$ 
 so upward $\rightarrow +ve$

(2) Rod is rotating about mid point.



End A & B $\rightarrow +ve$
Centre O $\rightarrow -ve$

It will become a battery of length l .

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$= \omega r$$

The charges in the rod are different. The linear velocity is 0 at the centre & increased away from origin & max at A.

$$\mathcal{E} = \int_0^{l/2} \omega r B dr = \frac{\omega B}{2} \left(\frac{l}{2} \right)^2 = \frac{1}{8} \omega B l^2$$

$$\boxed{\mathcal{E} = \frac{1}{8} \omega B l^2}$$

(3) A & B both have +ve charge so Potⁿ diff b/w them.
i) 0

$$\mathcal{E} = \int_{-l/2}^{l/2} \omega r B dr = \frac{\omega B}{2} \left(\frac{l^2}{2} \right)_{-l/2}^{l/2} = \frac{\omega B}{2} \left[\frac{l^2}{4} - \frac{l^2}{4} \right] = 0$$

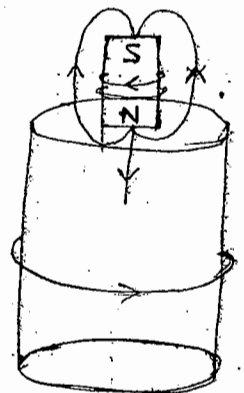
$$\boxed{\mathcal{E} = 0}$$

Q. :- A bar magnet is released from the rest along the axis of a very long vertical copper tube. After some time the magnet

- (i) will stop in the tube.
- (ii) will move with almost constant speed
- (iii) will move with an acceleration g .
- (iv) will oscillate.

[(ii) is correct]

If it is in the copper tube then it will move with 0 acceleration & constant speed, (almost constant). It depends on how strong the magnet is.

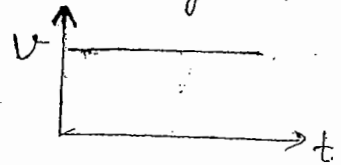


If copper ~~rod~~ ^{magnet} move downward inside the tube. As it pass through the rings (cylinder is made by rings) ^{again & again} change in the flux, & induces mag. field which oppose it.

As magnet enters in one ring then flux constant & again it enters in the next ring so flux will change, & give rise to induce mag. field.

When magnet enters into the tube then current will be opposite to the actual current,

& When leaving the tube then dirⁿ of current will be same as actual current.



Ques:- A uniform mag. field $B(t)$ pointing in the up dirⁿ fills the circular region as shown in the figure. If B is changing with time. What is the induce electric field,

Induce electric field;

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt}$$

$$= -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \quad \text{--- (1)}$$

Sketch a Amperical loop of radius s .

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 \int \vec{J} \cdot d\vec{S}$$

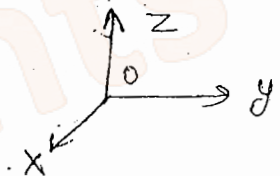
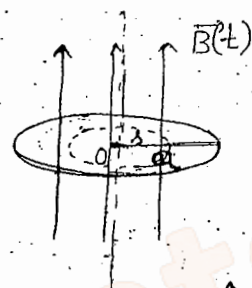
flux passing through the loop,

$$(1) \Rightarrow \phi_m = B \pi s^2$$

$$\text{Now, } \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt} \Rightarrow E \cdot 2\pi s = -\pi s^2 \frac{\partial B(t)}{\partial t}$$

$$\boxed{E_{in} = -\frac{s}{2} \frac{\partial B(t)}{\partial t} \hat{\phi}}$$

If mag. field is in z dirⁿ then E_{in} will be in ϕ dirⁿ.



• If $\vec{B} = at^2 \hat{z}$

Here B is $\uparrow$ with t so Flux $\uparrow$.

So induce current or induce ele. field will try to $\downarrow$ the flux. [So its dirⁿ will be opposite to the actual cur^t (anti c.)]
so dirⁿ of E_{in} will be Clockwise by watching up from up

• If B is $\downarrow$ with t then dirⁿ of E_{in} will be Anticlockwise

• If we switch off the mag. field, i.e. $B=0$ then flux $\downarrow$ & induce current will try to $\uparrow$ the flux then dirⁿ of E_{in} will be Anticlockwise.

Q. A line charge λ is uniformly distributed on the rim of a wheel of radius b which is then suspended horizontally as shown in the figure, & it is free to rotate. In the central region upto radius a there is a uniform mag. field B pointing in the up dirⁿ. Now if someone turn off the magnetic field then calculate the angular momentum acquired by the wheel.

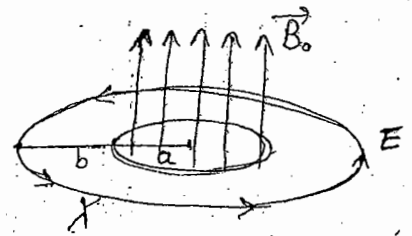
$B_0 \rightarrow \text{const.}$ Not depending on time

Right & now flux pass

$$\Phi_m = B_0 \pi a^2$$

Now switch off B i.e. $B = 0$

$$\Phi_m = 0$$



Flux changes then there is a induced electric field. dirⁿ of induce electric field is s.t. it will try to restore the flux.

Each charge is fix in ring. On applying force, charges do motion & due to this whole rim will do motion so this rim have angular mom.

$$\oint E \cdot d\vec{l} = - \frac{d\Phi_m}{dt}$$

$$= - \pi a^2 \frac{dB_0}{dt}$$

mag. field is lying in the region of radius a .

So Torque $N = \vec{r} \times \vec{F}$

$\vec{F} \rightarrow$ force required to rotate the charge, electric field will do work for rotation of charges - Electric force.

force/unit length $= f = \lambda E$

So total force $= \lambda \oint \vec{E} \cdot d\vec{l}$

$\therefore N = b \lambda \oint \vec{E} \cdot d\vec{l}$

dirⁿ of torque $\rightarrow$ upward

$$N = \frac{dL}{dt} = - \pi a^2 b \lambda \frac{dB_0}{dt}$$

$$\left\{ \begin{aligned} q &= \oint \lambda \cdot d\vec{l} \\ F &= qE \\ &= \int \lambda d\vec{l} \cdot \vec{E} \\ &= \lambda \oint \vec{E} \cdot d\vec{l} \end{aligned} \right.$$

$$\left\{ \begin{aligned} \vec{N} &= \oint \vec{r} \times \vec{F} \\ &= \oint \vec{r} \times \lambda \vec{E} \cdot d\vec{l} \end{aligned} \right.$$

$L \rightarrow$ angular Mom.

$$dL = -\pi a^2 b \lambda dB_0$$

When $B = B_0$; $L = 0$
 $B = 0$; $L = L$

$$\text{So } \int_0^L dL = -\pi a^2 b \lambda \int_{B_0}^0 dB_0$$

$$\Rightarrow \boxed{L = \pi a^2 \lambda b B_0}$$

This much angular mom. will be acquired by the rim.

We know $L = m v r$

If mass of the rim is m ,

$r = b$, $L = m v b$

$$L = m \omega b^2$$

$$(v = \omega r = \omega b)$$

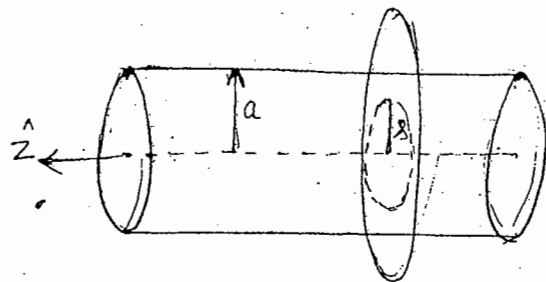
So $m \omega b^2 = \pi a^2 b \lambda B_0$

$$\boxed{\omega = \frac{\pi a^2 \lambda B_0}{m b}}$$

Q. 1 - A long solenoid with radius a & n turns per unit length carries a time dependent current $I(t)$ in the $\hat{z}$ dirⁿ. Find the electric field both dirⁿ & magnitude at a distance s from the axis both inside & outside the solenoid.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_m}{dt}$$

Mag. field inside the solenoid $\Rightarrow B = \mu_0 n I(t)$



$$E \cdot 2\pi s = -\mu_0 n \cdot \pi s^2 \frac{dI}{dt}$$

$$\boxed{\vec{E}_{in} = -\frac{\mu_0 n s}{2} \frac{dI}{dt} \hat{\phi}}$$

If B is in $\hat{z}$ dirⁿ & $\vec{E}_{in}$ (induced) will be in $\hat{\phi}$ dirⁿ

Outside:- Flux = $\mu_0 n I \pi a^2$

$$E \cdot 2\pi s = -\mu_0 n \pi a^2 \frac{dI}{dt}$$

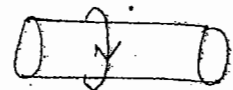
$$\vec{E}_{\text{out}} = -\frac{\mu_0 n a^2}{2s} \frac{dI}{dt} \hat{\phi}$$

Although mag. field outside the solenoid is 0 but induced electric field is not $\vec{E}_{\text{out}} \neq 0$

$$(E_{\text{induc}})_{\text{inside}} \propto s$$

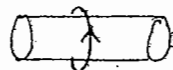
$$(E_{\text{induc}})_{\text{outside}} \propto \frac{1}{s}$$

If I (current) is $\uparrow$ with time then



induce current opposite the flux

If $I \downarrow$ with time



create the flux

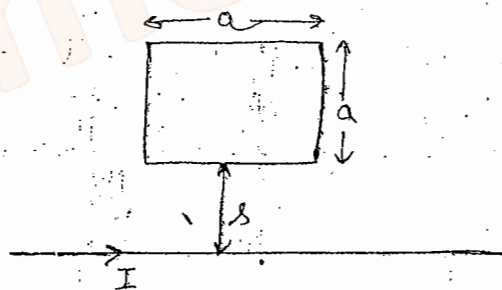
Q. A square loop of side a & resistance R lies at a distance s from an infinite straight wire that carries a current I . Now someone cut the wire s.t. I drops to zero. In what dirⁿ induce current in loop will flow. And calculate the total charge passes a given point in the loop during this time of current flow.

emf

$$\mathcal{E} = -\frac{d\phi}{dt}$$

$$\mathcal{E} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{s}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi s} \text{ (out of page)}$$



$$ds = dx dy$$

$$\mathcal{E} = -\frac{d}{dt} \int_s^{s+a} \frac{\mu_0 I}{2\pi s} \cdot a ds$$

$$= -\frac{d}{dt} \left[\frac{\mu_0 I a}{2\pi} (\ln s) \Big|_s^{s+a} \right] = -\frac{\mu_0 a}{2\pi} \ln\left(\frac{s+a}{s}\right) \frac{dI}{dt}$$

$$\text{emf} = I_{\text{loop}} \cdot R$$

$$I_{\text{loop}} \cdot R = -\frac{\mu_0 a}{2\pi} \ln\left(\frac{s+a}{s}\right) \frac{dI}{dt}$$

$$\Rightarrow I_{\text{loop}} = -\frac{\mu_0 a}{2\pi R} \ln\left(\frac{s+a}{s}\right) \frac{dI}{dt}$$

We know $I = dQ/dt$

$$\frac{d\phi_{\text{loop}}}{dt} = -\frac{\mu_0 a}{2\pi R} \ln\left(\frac{s+a}{s}\right) \frac{dI}{dt}$$

$$\int_0^{\phi} d\phi_{\text{loop}} = -\int_I^0 \frac{\mu_0 a}{2\pi R} \ln\left(\frac{s+a}{s}\right) dI$$

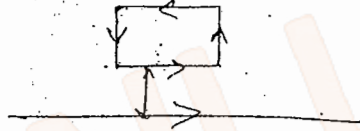
$$\Rightarrow \phi_{\text{loop}} = \frac{\mu_0 I a}{2\pi R} \ln\left(\frac{s+a}{s}\right)$$

$$\Rightarrow \phi_{\text{loop}} = \frac{1}{R} \left[\frac{\mu_0 I a}{2\pi} \ln\left(\frac{s+a}{s}\right) \right]$$

$$\Rightarrow \boxed{\phi_{\text{loop}} = \frac{\Phi}{R}} \checkmark$$

Dirⁿ of Induce current $\rightarrow$ Anticlockwise

$I \downarrow$, $B \downarrow$, $\phi \downarrow$ so induce current will try to $\uparrow$ the flux.



Q. A square loop of edge 'a' having 'n' turns is rotated with a uniform angular velocity ω about one of its diagonals which is kept fixed in horizontal position as shown in the figure. A uniform mag. field exist in the vertical dirⁿ. Find the induce emf in the coil. Also plot the graph b/w induce current & phase wt.

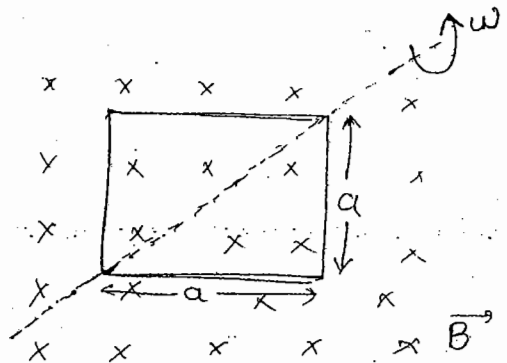
Area is fixed. B is fixed but angle is changing.

So flux changes due to angle change

$$\text{flux } \phi = B a^2 \cos \theta$$

If loop is rotating with ang. velocity ω then in time t , loop rotate by angle $\theta = \omega t$

$$\text{So } \phi = B a^2 \cos \omega t$$

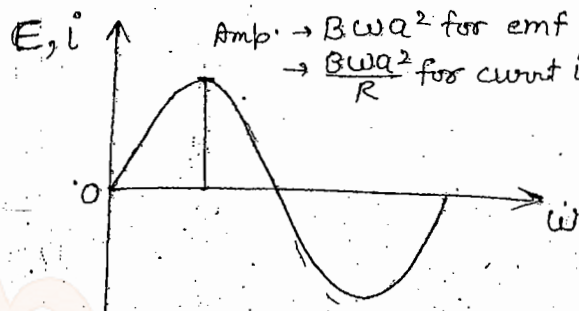
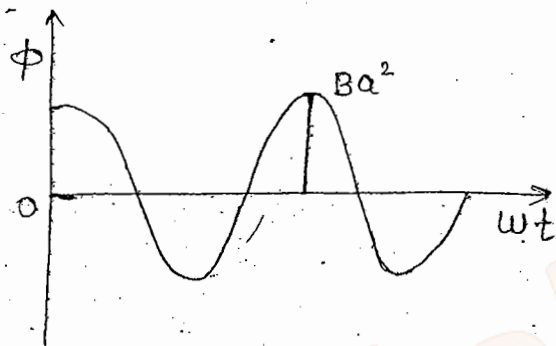


emf $\mathcal{E} = -\frac{d\phi}{dt} = B\omega a^2 \sin \omega t$

If Resistance of the loop is R then current flow through the loop will be

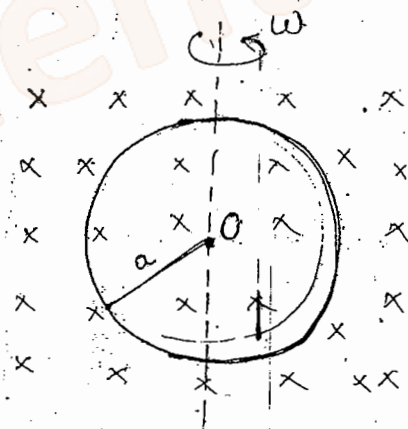
$$i = \frac{\mathcal{E}}{R} \Rightarrow \boxed{i = \frac{B\omega a^2 \sin \omega t}{R}}$$

Plot:-



Plot of emf & current will be same. Only difference is of amplitudes.

Q. A conducting circular loop of radius a is rotated about its diameter at a constant angular velocity ω . In a mag. field B , $\perp$ to the axis of rotation, in what position of the loop, induced emf will be zero.



flux passes through the loop

$$\phi = B \pi a^2 \cos \theta$$

$$\because \theta = \omega t$$

$$\boxed{\phi = B \pi a^2 \cos \omega t}$$

emf $\boxed{\mathcal{E} = -\frac{d\phi}{dt} = B \pi a^2 \omega \sin \omega t}$

When $\theta = \omega t = \frac{\pi}{2}$ then emf will be maximum & flux = 0
(phase diff. b/w emf & flux is of 90°)
flux is leading in phase than emf.

$$\text{as } \sin(\omega t + \frac{\pi}{2}) = \cos \omega t$$

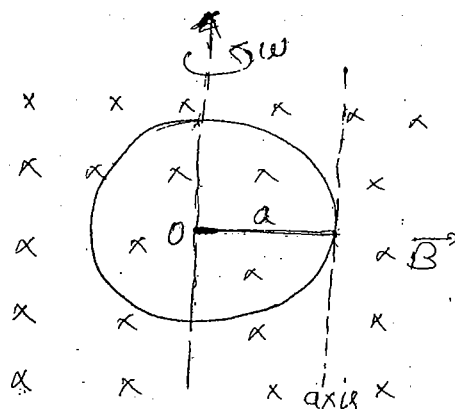
As compare to $\cos$ & $\sin$, we get $\sin$ is $\pi/2$ ahead flux is ahead in phase.

- In this case

$$\text{flux } \phi = B \pi a^2 \cos \omega t$$

$$\text{emf } \varepsilon = B \pi a^2 \omega \sin \omega t$$

Now the whole loop is rotating about the axis. There will be No difference in ϕ & ε .

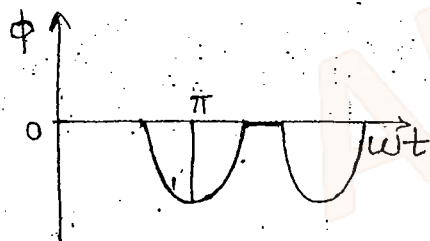


- But if there is no mag field on the R.H.S. of axis then:

flux will exist only from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$

$$\text{i.e. } -\frac{\pi}{2} < \omega t < \frac{\pi}{2} \quad (\theta = \omega t)$$

then wave of flux will be Half wave rectifier.



If flux wave is half $\cos$ wave then emf wave will be half sine wave.

$\varepsilon \rightarrow$ half wave sinusoidal.

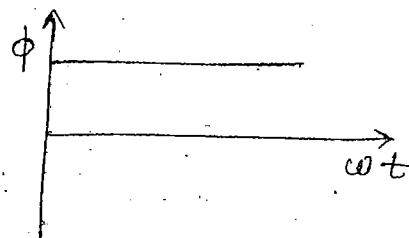
The wave can be on +ve or -ve axis depends upon the rotation.

The Axis of rotation is parallel to the axis of loop.

flux pass through loop $\phi = B \pi a^2$

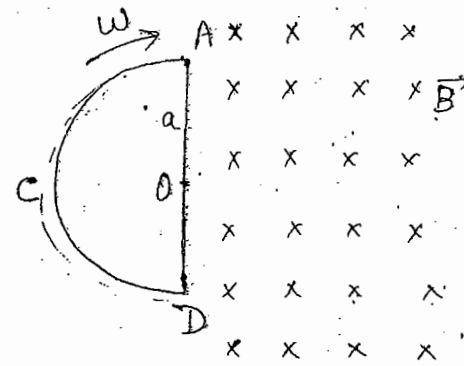
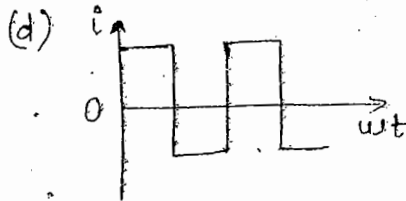
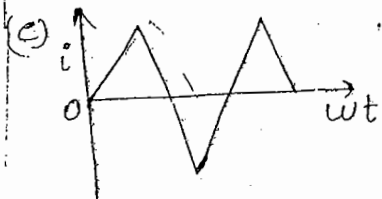
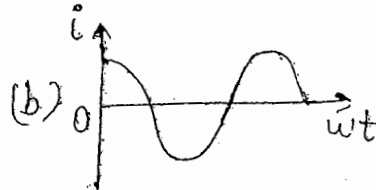
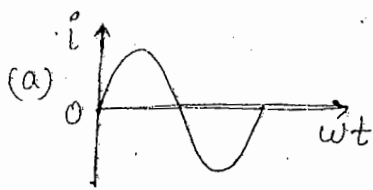
change in flux = 0

so Induced emf = 0



Q. The mag. field B shown in the figure is directed into the plane of the paper; ACDA is a semicircular loop of radius 'a' with the centre at O. The loop is now made to rotate clockwise with a constant angular velocity ω about axis passing through O & $\perp$ to the plane of the paper. The resistance of the loop is R . Obtain an expression for the magnitude of the induced current.

of the loop. Plot a graph b/w induced current & time.

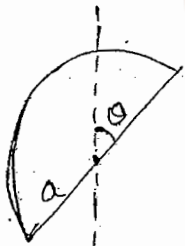


axis of rotation is O.

Axis of Rotation is O i.e. loop is rotating about O. So flux passing through the loop is variable.

Maximum area limit = $\frac{\pi a^2}{2}$

It is rotating so after some time position of loop will be



→ Angle b/w A (area vector) & B is Not changing

→ B is Not changing

→ Area is changing.

If loop rotate by 180° then it comes in mag. field i.e. area A is completely ins the $\vec{B}$.

$\theta = \omega t$, { If $\theta = \pi$ }

then area

$A = \left(\frac{\pi a^2}{2} \right) \left(\frac{\theta}{\pi} \right) = \frac{a^2 \theta}{2}$

If $0 < \theta < \pi$ then flux $\uparrow$ (area linearly $\uparrow$ with θ)

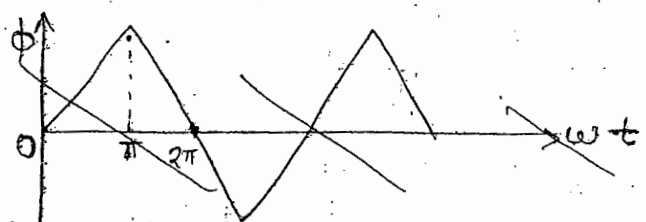
If $\pi < \theta < 2\pi$ then flux $\downarrow$

So flux ($0 < \theta < \pi$)

$\phi = B \frac{a^2 \theta}{2} \Rightarrow \phi = \frac{B a^2 (\omega t)}{2}$

flux $\uparrow$ linearly with θ (wt) up to 0 to π . then from π to 2π flux linearly $\downarrow$ with θ bcoz loop is out of $\vec{B}$ now. At

$\theta = 2\pi$, ϕ again zero bcoz (2π same 0)



1 wave complete $0 \rightarrow 2\pi$

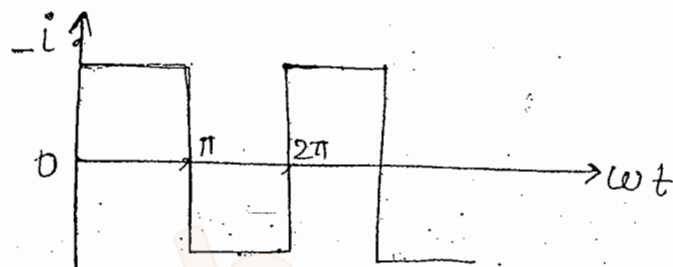
Now emf, $\mathcal{E} = -\frac{d\phi}{dt}$

flux linear with t so on differentiating we get constant. so we get square wave.

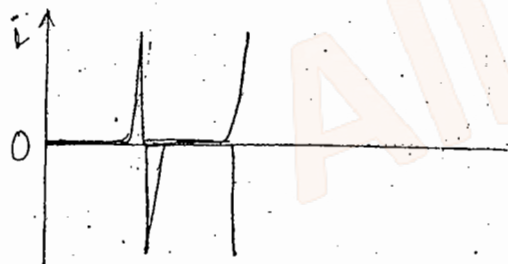
Slope of OA is +ve so current or emf will be -ve.

$\mathcal{E} = -\frac{d\phi}{dt}$ (here -ve sign present so take $-i$ on axis)

* Differentiation of linear wave is always a square wave.



If we apply differentiator of square wave - then we get 'Spikes.'



If sharp change; then spikes are narrow.

If broad change, then spikes are spread.

If we apply Integrator on spikes - then we get square wave again.

Wave of $\phi \rightarrow$ triangular
Wave of $i \rightarrow$ square wave

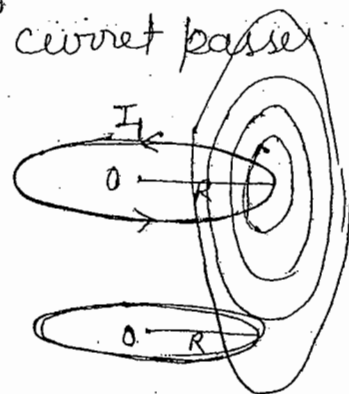
Mutual Inductance :-

If we have 2 current carrying coils of radius R .

Put these coils close to each other. If current passes through coil (1) is I_1 & coil (2) is zero.

Due to current I_1 of loop (1),

Magnetic field lines of loop (1) will pass through the loop (2), so there will be a magnetic flux through loop (2).



So there will be a induced emf

If we change the current I , then mag. field B , will change as $B \propto I$.

as $B \propto I$

∴ flux changed which is linked to loop (2).

so there will be induced emf.

The dirⁿ of

flux & dirⁿ of current in loop (1) & (2) are same
" " " " " opposite



11

4

•

„ opposite

So flux pass through loop (2) is due to the mag. field of loop (1) & area of loop (2)

of loop (1) & area of loop (2)

$\phi_2 \propto I_1$ } as $B \propto \phi \propto B$ & $B \propto I$ so
 $\phi \propto I$

$$\phi \propto I$$

And constant of proportionality is called Mutual Inductance, (M)

It called Mutual Inductance, (M)

$$\phi_2 = M_{21} I_1$$

$M_2 \rightarrow$ Mutual Inductance b/w 2 & 1.

⇒ Now ^{index} change the current, (I_1 & $I_2 \rightarrow$ same magnitude)
Now current is flowing through loop (2)

Now current is flowing through loop (2)

And mag. flux pass through loop (1)

will depend upon due to the mag. field B of loop (2).

field B of loop (2).

then $\phi_1 \propto I_2$

$$I_1 = I_2 = I \text{ (same mag.)}$$

So both flux ϕ_1 & ϕ_2 will be same.

$$\phi_1 = M_{12} I_2$$

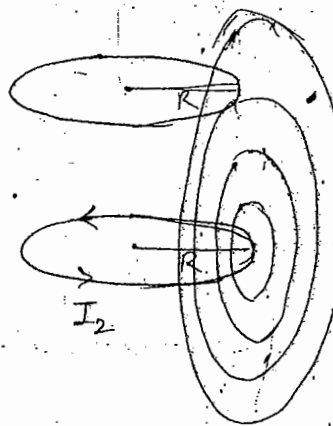
$$\phi_1 = \phi_2$$

flex $\propto$ furore

On Interchanging current, flux will be interexchange irrespective of their areas.

irrespective of their areas

Mutual Inductance is purely geometrical quantity. This depends



on size, shape & positions of the loop.

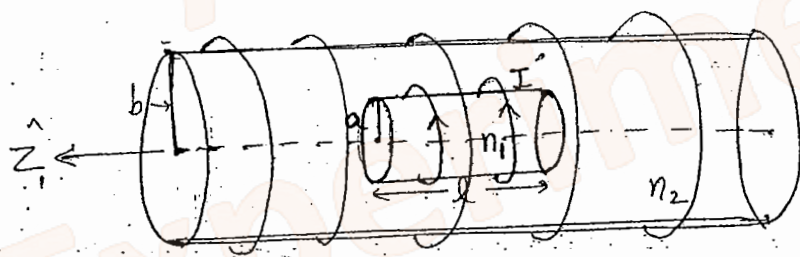
⇒ If Radius of both loops are different still flux is same.

$$\left. \begin{array}{l} \text{loop(1)} \rightarrow 2R \\ \text{loop(2)} \rightarrow R \end{array} \right\} \text{ then } \phi_1 = \phi_2$$

⇒ On changing size of loop, M will change.

⇒ To calculate flux of any loop, we have to fix the value of M (i.e. in the process, size, shape & distance b/w both loops can not be changed)

Q. A short solenoid of length L & radius a with n_1 turns per unit length lies on the axis of a very long solenoid of radius b & n_2 turns per unit length. Current I flows in the short solenoid. What is the flux through the long solenoid.



No current on large solenoid.

Mag. field of small solenoid passing through the large one.

but Mag. field of a short solenoid is complicated.

Inside $\rightarrow B = \mu_0 n_1 I$ but outside, Not exactly zero at the ends of short loop.

Beoz mag. field of short solenoid is complicated.

Hence interchange the current.

Now assume, current I is flowing in the long solenoid & we have to find the mag. flux through short solenoid.

Mag. field of large solenoid

$$\underline{B} = \mu_0 n_2 I \hat{z}$$

By single ring of a ~~short~~ solenoid, flux pass out

$$\phi_{\text{single}} = \mu_0 n_2 I \cdot \pi a^2$$

And in short solenoid, n_1 are the no. of turns & l is the length.

So $n_1 l \rightarrow$ turns per unit length.
then flux

$$\phi_{\text{Total}} = n_1 l \phi_{\text{single}}$$

$$\phi_{\text{Total}} = \mu_0 n_1 n_2 l \cdot \pi a^2 I$$

So same flux will pass through the short & long's.

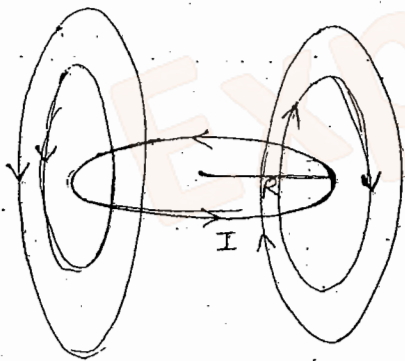
Now Mutual Inductance of this configuration

$$\phi = MI \Rightarrow M = \mu_0 n_1 n_2 l \cdot \pi a^2$$

M depends on medium b/w both solenoids.

Self Inductance (L) :-

Suppose we have a circular coil. Current flowing in this is I & its radius is R . Then there will be flux passing through the coil due to its own magnetic field.



$$\text{flux } \phi \propto B \text{ \& } B \propto I$$

So flux passing through the coil is proportional to the current

$$\phi \propto I$$

And Constant of proportionality is Self Inductance.

Self $\rightarrow$ bcoz it is due to self current.

$$\boxed{\phi = LI}$$

If we are changing current with time then B is changing with time & hence ϕ is changing with time. Area is const. L depends on size & shape of loop.

So there will be an induced emf

$$\text{emf } \varepsilon = -\frac{d\phi}{dt} = -L \frac{dI}{dt}$$

If $I \uparrow$ with t then $\frac{dI}{dt} \rightarrow +ve$ but $\varepsilon = -ve$

So induced current will oppose the change in current.

This is called Self Inductance.

✶

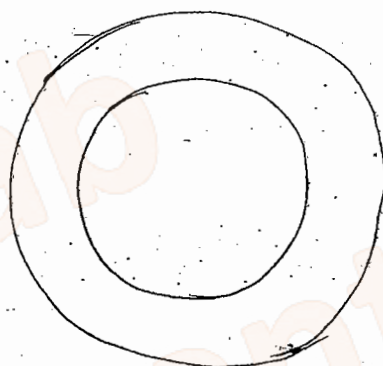
Q. Find the self inductance of a toroidal coil with rectangular cross-section inner radius a & outer radius b & height h which carries a total N turns.

Toroid $\rightarrow$ Endless solenoid.

Mag. field of toroidal coil is

$$\vec{B} = \frac{\mu_0 N I}{2\pi s} \hat{\phi}$$

(Inside the toroidal coil) where $a < s < b$ [Non-uniform]



There are N no. of turns.

Flux passing through a single turn,

$$\phi_{\text{single}} = \frac{\mu_0 N I}{2\pi} \int_a^b \frac{1}{s} h ds$$

$$\phi_{\text{single}} = \frac{\mu_0 N I h}{2\pi} \ln(b/a)$$

Total Flux passing through N turns of toroid, will be

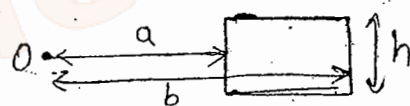
$$\phi_{\text{Total}} = N \phi_{\text{single}}$$

$$\phi_{\text{Total}} = \frac{\mu_0 N^2 h}{2\pi} \ln(b/a) I$$

We know $\phi = L I$

So self inductance will be

$$L = \frac{\mu_0 N^2 h}{2\pi} \ln(b/a)$$



$$\text{area} = (b-a)h$$

$$\int_a^b \int_0^h dx dy$$

Mag. field varying with b
 $ds = dx dy$. but B is
 not changing with y
 so $dy = h$

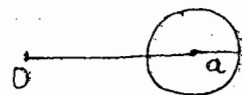
This is the self inductance of toroidal coil with rectangular cross-section.

If cross-section is circular instead of rectangular:-

$$\text{Area element} = s d\phi ds$$

$$= 2\pi s ds$$

(no variation with ϕ , only with s)



$$\text{Now, } \phi_{\text{single}} = \frac{\mu_0 N I}{2\pi} \int_0^a \frac{1}{s} s ds 2\pi$$

$$\phi_{\text{single}} = \mu_0 N I a$$

$$\text{Total flux } \phi_{\text{Total}} = N \cdot \phi_{\text{single}}$$

$$\boxed{\phi_{\text{Total}} = \mu_0 N^2 I a}$$

$$\text{So Self Inductance } \boxed{L = \mu_0 N^2 a}$$

$$\int_0^{2\pi} \int_0^a s ds d\phi$$

$$\int_0^a s ds 2\pi$$

$$\Rightarrow \pi a^2$$

Analogy b/w Mechanics & Electromagnetism:-

In Mechanics, K.E. is $K = \frac{1}{2} m v^2$

& P.E. is $U = \frac{1}{2} k x^2$

If we have a coil & current is flowing in it is I & self inductance of that coil is L then magnetic energy stored by that coil is $U_m = \frac{1}{2} L I^2$

Capacitor stores the electrical energy & Inductor stores magnetic energy.

↳ in the form of charge q or field in eff.

current → dynamic
charge → static

Inductor → L → I

Capacitor → C → q

$$U_e = \frac{q^2}{2C} \quad \& \quad \text{Also } U_e = \frac{\epsilon_0}{2} \int E^2 d\tau$$

$$K = \frac{1}{2} m v^2 \longleftrightarrow U_m = \frac{1}{2} L I^2$$

$$U = \frac{1}{2} k x^2 \longleftrightarrow U_e = \frac{q^2}{2C}$$

Electromag. Waves carry energy but not carry charge.
With E.M. Wave, both energies are associated in ^{or current} electrical & magnetic. So magnetic energy resides in the magnetic field & electric energy resides in El. field. bcoz there is No charge (or current)

In Magnetostatics,

$$U_m = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 d\tau$$

Mechanics		Electromag.
Magnetostatic		
m	$\longrightarrow$	L
v	$\longrightarrow$	I
K	$\longrightarrow$	$\frac{1}{C}$
x	$\longrightarrow$	q

$\Rightarrow$ If we have 2 bodies of mass m & $2m$ then body of mass m will accelerate more.

Mass $\rightarrow$ less, More $\rightarrow$ acceleration

If M is more it will be difficult to $\uparrow$ the velocity.

$\Rightarrow$ If we have 2 coils of self inductance L & $2L$ then it is easy to $\uparrow$ the current of coil of S.I. L .

To $\uparrow$ current, have to fight against back emf.

If L is more, it will be difficult to $\uparrow$ the current.

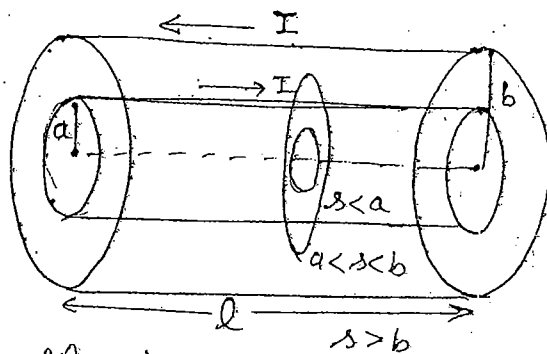
Q. 1 - A long co-axial cable carries a current I , the current flows down the surface of inner cylinder of radius a & back along the cylinder of radius b . Find the magnetic energy stored in the section of length l .

Magnetic energy stored

$$U_m = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 d\tau$$

$$U_e = \frac{1}{2} \int_{\text{all space}} \vec{D} \cdot \vec{E} d\tau$$

Also
$$U_m = \frac{1}{2} \int_{\text{all space}} \vec{H} \cdot \vec{B} d\tau$$



Current is flowing on the surface of both inner & outer cylinder

for $s < a$, $I_{enc} = 0$

$$\text{So } \boxed{B = 0}$$

for $a < s < b$, $I_{enc} = I$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi s} \hat{\phi}}$$

dirⁿ of I_{enc} is $\hat{z}$ Hence dirⁿ of mag. field is $\hat{\phi}$

for $s > b$, $I_{enc} = 0$

bcoz current on both cylinders are equal & opposite
so cancel out $I_{enc} = +I - I = 0$

$$\text{So } \boxed{B = 0}$$

$$\text{So } U_m = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 d\tau$$

$$U = \frac{\mu_0^2 I^2}{2 \mu_0 4\pi^2} \int_a^b \frac{1}{s^2} s ds d\phi dz$$

{ limits on $s \rightarrow 0$ to ∞ but from $\int_0^a (B=0)$ & $\int_b^\infty (B=0)$ }

$$U = \frac{\mu_0 I^2}{8\pi^2} 2\pi \cdot l \ln(b/a)$$

$$\boxed{U = \frac{\mu_0 l}{4\pi} \ln(b/a) I^2}$$

This much energy will be stored in this configuration.

Self Inductance of this confi.

$$U = \frac{1}{2} L I^2$$

$$\boxed{L = \frac{\mu_0 l}{2\pi} \ln(b/a)}$$

Electrodynamics before Maxwell :-

(Gauss Law in electrostatics) (i) $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$ (ii) $\vec{\nabla} \cdot \vec{B} = 0$ (Gauss law in magnetostatics)

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday's law)

(iv) $\vec{\nabla} \times \vec{B} = +\mu_0 \vec{J}$ (Ampere's law)

These 4 eqⁿ deriving the electrodynamics before Maxwell.
Are these consistent in Electrodynamics :-

Physical significance of $\vec{\nabla} \cdot \vec{B} = 0$ (Sometimes No name)

→ Non existence of magnetic monopoles.

⇒ Taking ^{div} curl of eqⁿ (iii),

L.H.S: $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{E}) = 0$ (div of curl is always zero)

R.H.S: $-\frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{B}) = 0$

$\vec{\nabla} \cdot \vec{B} = 0$ always, No restriction.

So Faraday's law i.e. this eqⁿ is valid in electrodynamics.

⇒ Taking div. of eqⁿ (iv),

$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = 0$

$\vec{\nabla} \cdot \vec{J} = 0$

but $\vec{\nabla} \cdot \vec{J} = 0$ only in magnetostatics.

So this Ampere's law is valid only in Magnetostatics
i.e. where there is No changing fields & current must be steady.

If accumulation of charge is present then this is not valid.

So this law must be modified -

Modification of Ampere's Law by Maxwell :-

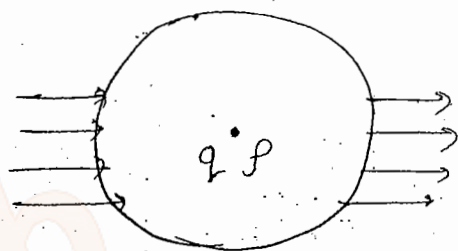
After modification, eqⁿ (iv) becomes Modified Ampere's Law. All 4 eqⁿ together called Maxwell's eqⁿs.

In Magnetostatics $\vec{\nabla} \cdot \vec{J} = 0$

But if charge is changing with time i.e. ρ " " " " then $\vec{\nabla} \cdot \vec{J} \neq 0$

If we have a close surface & charge q is placed in this close surface & its charge density is ρ . This ρ is Not changing with time i.e. charge is conserved, i.e. No charge is flowing in or out.

OR 2nd possibility is that - charge coming in = charge coming out.



Flux of the current passing through the boundary of sphere = 0

then $\vec{\nabla} \cdot \vec{J} = 0$

In 2nd case, still flux = 0

then $\vec{\nabla} \cdot \vec{J} = 0$

i.e. $\vec{\nabla} \cdot \vec{J} = 0$ if ρ is not depending on time.

But if ρ or q is a funcⁿ of time.

then flux pass through boundary = 0

→ If the flux is dec. with time then current will flow outward (in the dirⁿ of flow of +ve charge)

→ If charge ↑ with time then current will flow into the sphere.

→ If charge is not change with time then current flow = 0

If Electrodynamics,

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

If ρ dec. with time then $\rho = -ve$ then $\vec{\nabla} \cdot \vec{J} = \frac{\partial \rho}{\partial t}$ current flow outward.

If ρ ↑ with time $\rho = +ve$

$$\vec{\nabla} \cdot \vec{J} = - \frac{\partial \rho}{\partial t} \quad \text{then current flow into}$$

This eqⁿ is called eqⁿ of Continuity & its physical significance is local conservation of charge.

$$\boxed{\vec{\nabla} \cdot \vec{J} = - \frac{\partial \rho}{\partial t}}$$

→ charge is conserved globally, neither be created nor be destroyed.

Here charge is conserved on a local body that's why it is called Local conservation of charge.

$$\begin{aligned} \text{eqⁿ (1)} &\Rightarrow \vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0 \\ &\Rightarrow \rho = \epsilon_0 (\vec{\nabla} \cdot \vec{E}) \end{aligned}$$

Put it into Continuity eqⁿ,

$$\vec{\nabla} \cdot \vec{J} = - \epsilon_0 \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{E})$$

change the order then, (bcz both operator are independent on each other)

$$\vec{\nabla} \cdot \vec{J} = - \epsilon_0 \vec{\nabla} \left(\frac{\partial \vec{E}}{\partial t} \right)$$

$$\Rightarrow \boxed{\vec{\nabla} \cdot \left[\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right] = 0}$$

In electrodynamics, We need to replace $\vec{J}$ by $\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}$. then we get Modified Ampere's law.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \Rightarrow \boxed{\vec{\nabla} \times \vec{B} = \mu_0 \left[\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right]}$$

Now L.H.S = 0 & R.H.S = 0

(A)

So this eqⁿ is consistent with electrodynamics.

This eqⁿ is called Modified Ampere's law.

Also called 4th Maxwell eqⁿ.

$$\frac{\partial E}{\partial t} = 0$$

$$\text{do } \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$\epsilon_0 \frac{\partial \vec{E}}{\partial t} \rightarrow$ called displacement current density in vacuum.

Displacement Current - follow no other property of current. This is being called current only bcoz it is producing mag. field. There are no charges corresponding to this displacement current. Corresponding to $\vec{J}$ there are charges.

4 legs together $\rightarrow$

3rd eqⁿ say $\rightarrow$ change in Mag. field producing Ele. field.
4th " " " " Ele. " " Mag. " "

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \chi|_E = -\frac{\partial B}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Differential forms

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{s} = 0$$

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{S}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{A}$$

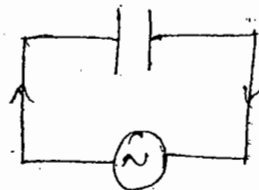
Integral forms

Practical Example of Displacement Current :- is found
* in air-filled capacitor.

If we connect it with source then current flow.

In Air gage, No current.

→ Current is flowing — displacement current:

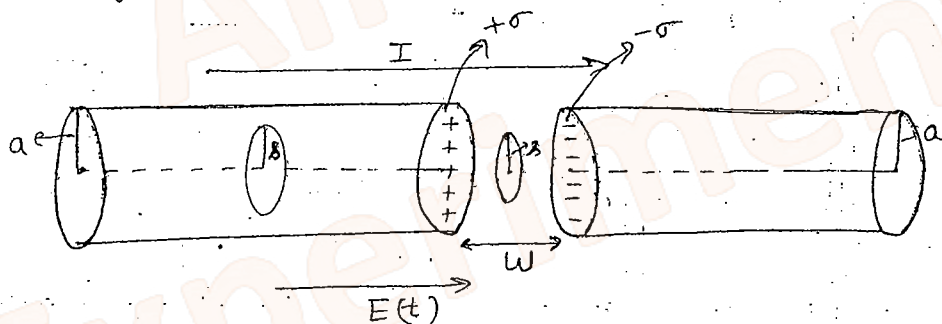


If source is time dependent $\rightarrow$ charge accumulate on plate of capacitor $\rightarrow$ E-field produce. Here $\sigma = \sigma(t)$ bcoz charge is built up with time. So E-field will also be the funⁿ of time. So a current produces due to the change in Elec. field called Displacement Current.

Current produces due to motion of free electron called Conduction Current.

{ bound current $\rightarrow$ due to motion of bound charge }.

Q. A thick wire of radius a carries a constant current I uniformly distributed over its cross-section. A narrow gap in the wire of width $w \ll a$ forms a parallel plate capacitor as shown in the figure. Find the mag. field in the gap at a distance $s < a$ from the axis.



$$E(t) = \frac{\sigma(t)}{\epsilon_0}$$

Current density $\vec{J}_a = \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \epsilon_0 \frac{1}{\epsilon_0} \frac{\partial \sigma}{\partial t} = \frac{\partial \sigma}{\partial t}$

dirⁿ of current is the dirⁿ of elec. field

Current Encluse by the s loop,

$$I_{enc} = \int \vec{J}_d \cdot d\vec{s}$$

$$= \epsilon_0 \frac{\partial \sigma(t)}{\partial t} \cdot \frac{\pi s^2}{\epsilon_0}$$

$$= \frac{\partial \sigma(t)}{\partial t} \pi s^2 \times \frac{\pi a^2}{\pi a^2}$$

$$I_{enc} = \frac{1}{\pi a^2} \frac{\partial q(t)}{\partial t} \pi s^2 \quad \left\{ \begin{array}{l} \text{charge / unit time} \\ \text{— current} \end{array} \right\}$$

$\downarrow$
current

$$I_{enc} = \frac{I s^2}{a^2}$$

Ampere's law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_c + \mu_0 \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{s}$$

$\downarrow$
conduction current

$I_c = 0$ bcoz no free e^- in the gap
so Total current will be due to Ind term.

$$B \cdot 2\pi s = \mu_0 \frac{I s^2}{a^2}$$

$$\vec{B} = \frac{\mu_0 I s}{2\pi a^2} \hat{\phi}$$

$$\left\{ \begin{array}{l} I_{enc} = \int J_d \cdot d\vec{s} \\ = \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{s} \end{array} \right\}$$

Displacement current is necessary to make the current continuous across the capacitor.

This mag. field is same as $\rightarrow$ in case of uniform mag. field inside the wire.

Maxwell's Eqⁿ in Matter:-

- (i) $\vec{\nabla} \cdot \vec{D} = \rho_f$
- (ii) $\vec{\nabla} \cdot \vec{B} = 0$ ($\vec{\nabla} \cdot \vec{H} \neq 0$ as Magnetization is non-zero)
may or may not (depending on condⁿ)
 $\left\{ \begin{array}{l} \text{If } M \text{ not depend on } t \text{ then } \vec{\nabla} \cdot \vec{H} = 0 \end{array} \right\}$
- (iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- (iv) $\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$

Before Maxwell $\rightarrow$ 4th eqⁿ $\rightarrow \vec{\nabla} \times \vec{H} = \vec{J}_f$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

find in terms of $H = ?$

In Inside the matter, there will be polarisation & Magnetisation.

$$\vec{J} = \vec{J}_b + \vec{J}_f$$

$$\vec{J} = \vec{J}_f + \underbrace{\vec{\nabla} \times \vec{M}}_{\text{Magnetisation Current}}$$

$$\frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) = \underbrace{\vec{J}_f}_{\downarrow} + \underbrace{\vec{\nabla} \times \vec{M}}_{\text{Polarisation Current}} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t}$$

(i) Due to the motion of free charges

(ii) Due to the bound charges - bound or Magnetisation current.

(iii) Due to the change in Elec. field - Displacement current.

(iv) If E is changing with time then P also changing with time & current produce - Polarisation current.

In free space E field is E_0



but if space is not free

fill the matter in it then elec. field will be changed.

$$\epsilon E = \epsilon_0 E_0 + P$$

$$E = \frac{E_0}{\epsilon_r} + \frac{P}{\epsilon}$$



Or Due to E field, charges become polarised & began to oscillate so polarisation current produce.

$$\frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) = \vec{J}_f + \vec{\nabla} \times \vec{M} + \frac{\partial}{\partial t}(\epsilon_0 \vec{E} + \vec{P})$$

$$= \vec{J}_f + \vec{\nabla} \times \vec{M} + \frac{\partial \vec{D}}{\partial t}$$

density
displacement current in matter

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}}$$

→ called Maxwell's ϵ_2 in Matter.

Maxwell's Eqⁿ. in Free Space :-

In free space - there is no charge & no current.

$$(i) \nabla \cdot \vec{D} = 0 \Rightarrow \boxed{\nabla \cdot \vec{E} = 0} \quad (D = \epsilon E)$$

$$(ii) \nabla \cdot \vec{B} = 0$$

$$(iii) \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$(iv) \nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\left\{ \begin{array}{l} \nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \\ \parallel \\ 0 \\ \text{(free space)} \end{array} \right\}$$

Maxwell's Eqⁿ for Static Fields :-

$$(i) \nabla \cdot \vec{E} = \rho / \epsilon_0$$

$$(ii) \nabla \cdot \vec{B} = 0$$

$$(iii) \nabla \times \vec{E} = 0$$

$$(iv) \nabla \times \vec{B} = \mu_0 \vec{J}$$

{ static }

$$\left\{ \frac{\partial \vec{D}}{\partial t} = 0 \right\}$$

Maxwell's Eqⁿ for Isotropic Linear Dielectric :-

In dielectric $\rightarrow$ free charges = 0

$$\rho_f = 0$$

for Linear dielectric, $\vec{D} = \epsilon \vec{E}$

Isotropic means ϵ is not depend on space co-ordinate

i.e. ϵ is not a funⁿ of position.

$$(i) \nabla \cdot \vec{D} = 0 \Rightarrow \boxed{\nabla \cdot \vec{E} = 0}$$

$$\left\{ \begin{array}{l} \nabla \cdot (\epsilon \vec{E}) = 0 \\ \Rightarrow \epsilon (\nabla \cdot \vec{E}) = 0 \end{array} \right.$$

(for Anisotropic $\nabla \cdot \vec{D} = 0$ but $\nabla \cdot \vec{E} \neq 0$
bcoz ϵ is a funⁿ of position)

$$(ii) \nabla \cdot \vec{B} = 0$$

$$(iii) \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$(iv) \nabla \times \vec{B} = \vec{J}_f + \frac{\partial \epsilon \vec{E}}{\partial t}$$

$$\Rightarrow \boxed{\nabla \times \vec{B} = \mu \epsilon \frac{\partial \vec{E}}{\partial t}}$$

$\vec{J}_f = 0$ (No free current)
for linear $\rightarrow \vec{B} = \mu \vec{H}$
 $\nabla \times \vec{H} = \epsilon (\nabla \times \vec{B})$

Solution of Maxwell's eqⁿ in free space :-

OR
Electromagnetic Waves in Vacuum :-

Maxwell's eqⁿ in free space,

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \text{--- (1)}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{--- (2)}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (3)}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{--- (4)}$$

Maxwell's eqⁿ are four first order differential eqⁿ.
We need to convert these first " " " into 2nd order diff. eqⁿ.

Take the curl of eqⁿ (iii),

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B})$$

Put the value of $\vec{\nabla} \times \vec{B}$ from eqⁿ (4) into this eqⁿ,

$$\vec{\nabla} (\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$\nabla \cdot \vec{E} = 0$ from eqⁿ (1),

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0} \quad \text{--- (5)}$$

Now, Take the curl of eqⁿ (4),

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \mu_0 \epsilon_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E})$$

(from (3))

$$\vec{\nabla} (\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

($\nabla \cdot \vec{B} = 0$)

$$\boxed{\nabla^2 \vec{B} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0} \quad \text{--- (6)}$$

eqⁿ (5) & (6) are the 2nd order diff. eqⁿ of electric & mag. field.

All the information of 4 Maxwell eqⁿ contained in two eq^s (5 & 6)
eqⁿ (5) & (6) are the Wave eq^s of electric & mag. field.
General Wave eqⁿ,

$$\boxed{\nabla^2 f - \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} = 0} \quad (7)$$

Compare (5) & (6) with (7) $\Rightarrow$ We get

$$\boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c} \quad (8)$$

$c \rightarrow$ speed of light.

Hence, Light waves are Electromagnetic Waves.

They don't require any medium to propagate.

[This can't be proved if Maxwell didn't give Displacement current term]

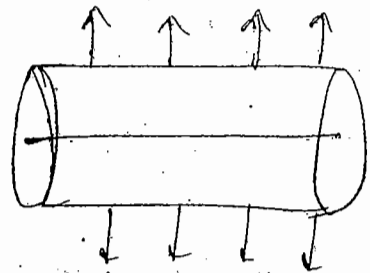
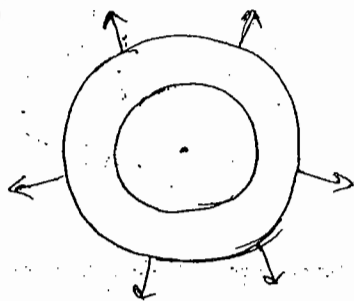
- Solution of IInd order diff. eqⁿ gives $\rightarrow$ one give Ele. field & another give mag. field. And both field depend on each other. They can not exist without each other.

Solution may be of many kind.

In Cartesian co-ordinate $\rightarrow$ we get plane waves

↳ spherical-polar $\rightarrow$ spherical waves

↳ cylindrical $\rightarrow$ cylindrical waves



Small portion of spherical wave front & cylinder
is a plane. (large radius sphere)

We are interested in plane wave solⁿ.

Plane Wave solⁿ of $\vec{E}$ & $\vec{B}$

Suppose $\vec{E}$ & $\vec{B}$ are propagate in x, y, z dirⁿ

$$\vec{E} = E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\vec{B} = B_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

These solⁿs must satisfy the maxwell's eqⁿ as they are the solution of Maxwell's eqⁿ.

$$\vec{\nabla} \cdot \vec{E} = 0 \Rightarrow \boxed{\vec{k} \cdot \vec{E} = 0} \quad \text{--- (9)}$$

∇ -operator apply on plane wave. (it operates on space part)
After operation it gives $i\vec{k}$.

& time operator gives $-i\omega$.

$$\boxed{\begin{array}{l} \nabla \rightarrow i\vec{k} \\ \frac{\partial}{\partial t} \rightarrow -i\omega \end{array}}$$

Vibration of $\vec{E}$ & $\vec{B}$ are the $\perp^r$ to the wave propagation. $\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow i(\vec{k} \cdot \vec{B}) = 0 \Rightarrow \boxed{\vec{k} \cdot \vec{B} = 0}$ --- (10) from (9) & (10)

Wave vector tells the dirⁿ of propagation.

So $\boxed{\vec{E} \text{ \& \> } \vec{B} \text{ are Transvers in Nature}}$

Put the value of $\vec{E}$ & $\vec{B}$ in eqⁿ (3) & (4),

$$\text{eqⁿ (3)} \Rightarrow \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\begin{aligned} i(\vec{k} \times \vec{E}) &= -(-i\omega)\vec{B} \\ &= +i\omega\vec{B} \end{aligned}$$

$$\vec{k} \times \vec{E} = \omega\vec{B}$$

$$\Rightarrow \boxed{\vec{B} = \left(\frac{\vec{k} \times \vec{E}}{\omega} \right)} \quad \text{--- (11)}$$

As $E \perp^r k$ & $B \perp^r k$ so

$\vec{E}$, $\vec{B}$ & $\vec{k}$ are mutually perpendicular.

$$\text{dirⁿ of } \vec{B} = \vec{k} \times \vec{E}$$

$$\begin{aligned} \text{Eqn (4)} \Rightarrow \nabla \times \vec{B} &= \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \\ i(\vec{k} \times \vec{B}) &= \mu_0 \epsilon_0 (-i\omega) \vec{E} \\ \vec{k} \times \vec{B} &= -\mu_0 \epsilon_0 \omega \vec{E} \\ \boxed{\vec{E} = \frac{-c^2 (\vec{k} \times \vec{B})}{\omega}} &\text{--- (12)} \end{aligned}$$

When Wave vector $\vec{k}$ & mag. field $\vec{B}$ are given & find $\vec{E}$ field $\vec{E}$ then use above relation directly.

dirⁿ of $\vec{E} \rightarrow -(\vec{k} \times \vec{B})$

from (12) $\rightarrow \vec{E}, \vec{B}, \vec{k}$ are mutually $\perp^r$.

In free space, Electromag. waves are plane waves.

$$\begin{aligned} \text{Let } \vec{E} &= E \hat{x} \\ \vec{B} &= B \hat{y} \end{aligned}$$

$$\text{Hence } \vec{k} = k \hat{z}$$

$$\begin{aligned} \text{Now, } \vec{E} &= E_0 e^{i(kz - \omega t)} \hat{x} \\ \& \vec{B} &= B_0 e^{i(kz - \omega t)} \hat{y} \end{aligned} \quad (\vec{k} \cdot \vec{r} = kz)$$

$$E = E_0 e^{i(kz - \omega t)} \hat{x} \rightarrow \begin{array}{l} \text{amplitude of wave} \downarrow \\ \text{dir of propagation} \downarrow \end{array} \begin{array}{l} \text{freq. of Vibration is } \omega \\ \text{of field} \end{array}$$

$$\text{If } E = E_0 e^{i(\omega t - kz)} \hat{x}$$

Still dirⁿ of field is $+\hat{z}$
propagation

Either $(\omega t - kz)$ or $(kz - \omega t)$ then dirⁿ of prop. is $+\hat{z}$
i.e. b/w ωt & kz , one plus, one minus then $+\hat{z}$.

If sign of ωt & kz is same then dirⁿ of propagation $-\hat{z}$

$$\begin{aligned} \text{i.e. } \vec{E} &= E_0 e^{i(-\omega t - kz)} \hat{x} \rightarrow -\hat{z} \\ &= E_0 e^{i(\omega t + kz)} \hat{x} \rightarrow -\hat{z} \end{aligned}$$

* Among ω & kz

same sign $\rightarrow -z$

opposite sign $\rightarrow +z$

Energy Density - (u) Energy per unit volume

Electric field vector gives the Electric energy density
& magnetic " " " " magnetic " " "

$$u_e = \frac{\epsilon_0}{2} \int_{\text{all space}} E^2 d\tau$$

$E \rightarrow$ Total elec. field

$$u_e = \frac{\epsilon_0}{2} E^2$$

$$\& \quad u_m = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 d\tau$$

$$u_m = \frac{B^2}{2\mu_0}$$

We have, $\vec{B} = \frac{\vec{K} \times \vec{E}}{\omega}$

$$\begin{aligned} \vec{K} &\rightarrow K \hat{z} \\ \vec{E} &\rightarrow E \hat{x} \end{aligned}$$

$$\{\hat{z} \times \hat{x} = \hat{y}\}$$

$$\vec{B} = \frac{KE}{\omega} \hat{y}$$

$$\boxed{v = \frac{\omega}{K}} = \text{wave velocity (in free space)}$$

$$\text{So } \vec{B} = \frac{E}{c} \hat{y}$$

$$\therefore u_m = \frac{B^2}{2\mu_0} = \frac{E^2}{2c^2\mu_0} = \frac{E^2 \mu_0 \epsilon_0}{2\mu_0} = \frac{\epsilon_0 E^2}{2}$$

$$\boxed{u_m = u_e}$$

Mag. field energy = Electric field energy.

In free space, Mag. field & Ele. field carry equal energy. This energy remain in field, i.e. mag. energy remain in mag. field & e. energy remain in ele. field.

Total Energy density $\boxed{u = u_e + u_m = \epsilon_0 E^2}$

Energy Flux or Poynting Vector ($\vec{S}$) :-

Poynting Vector is defined as Energy per unit area per unit time carried by the electromagnetic wave.

OR . Power per unit area is called Poynting Vector.

Unit :- $\text{J/m}^2\text{-sec}$ or W/m^2 (watt/m²)

$$\vec{S} = \vec{E} \times \vec{H}$$

$$= \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\vec{S} = \frac{EB}{\mu_0} \hat{z}$$

$$\begin{aligned} \text{dir}^n \text{ of } E &\rightarrow \hat{x} \\ B &\rightarrow \hat{y} \\ (\hat{x} \times \hat{y} &\rightarrow \hat{z}) \end{aligned}$$

$$B = \frac{E}{c} \Rightarrow \vec{S} = \frac{E^2}{\mu_0 c} \hat{z}$$

$$\vec{S} = \frac{E^2}{\mu_0 c} \times \frac{c}{c} \hat{z} = \frac{E^2 c}{\mu_0 c^2} = \frac{E^2 \mu_0 \epsilon_0 c}{\mu_0}$$

$$\vec{S} = c \epsilon_0 E^2 \hat{z}$$

$$\boxed{\vec{S} = cu \hat{z}} \quad (u = \epsilon_0 E^2)$$

This is Relation b/w Poynting vector & energy density.

$\vec{S} \rightarrow$ tells the dirⁿ of Energy propagation.

$\vec{K} \rightarrow$ " " " " " wave " "

Generally, dirⁿ of $\vec{S}$ & $\vec{K}$ matches but not every time.

In present case, $\text{dir}^n \text{ of } \vec{K} = \hat{z}$
 $\text{dir}^n \text{ of } \vec{S} = \hat{z}$

In free space, dirⁿ of Wave propagation is same as dirⁿ of energy flow.

Electromag. wave not only carry the energy but also carry the momentum.

Momentum Density of EM Waves :-

Total mom. per unit volume called Mom. density.
denoted by $\vec{p}$

In free space, $\vec{p} = \mu_0 \epsilon_0 \vec{S}$

$$\vec{p} = \frac{\vec{S}}{c^2} = \frac{u}{c} \hat{z}$$

$$(\vec{S} = cu)$$

This is called Electromag. mom. density.

Average Value of these quantities :-

We have, Energy density $u = \epsilon_0 E^2$

Poynting vector $\vec{S} = c \epsilon_0 E^2 \hat{z}$

Momentum density $\vec{p} = \frac{\epsilon_0 E^2}{c} \hat{z}$

Now, we calculate average energy density, Average poynting vector & average mom. density over a cycle.

We know, the solutions are

$$\vec{E} = E_0 e^{i(Kz - \omega t)} \hat{x}$$

$$\vec{B} = B_0 e^{i(Kz - \omega t)} \hat{y}$$

We can write,

$$\vec{E} = E_0 \left[\underbrace{\cos(Kz - \omega t)}_{\text{Real Part}} + i \underbrace{\sin(Kz - \omega t)}_{\text{Imaginary Part}} \right] \hat{x}$$

Only Real part carries the energy.

$$E^2 = \vec{E} \cdot \vec{E} = E_0^2 \cos^2(Kz - \omega t)$$

Average Value of energy density

$$\langle u \rangle = \epsilon_0 E_0^2 \langle \cos^2(Kz - \omega t) \rangle$$

Average value of $\cos^2$ over a cycle of 2π gives $\frac{1}{2}$.

$$\text{so } \langle u \rangle = \frac{1}{2} \epsilon_0 E_0^2$$

→ we can also write it in terms of mag. field (B_0) amp of mag. field

E_0 → amplitude

This is the combination of electric & magnetic energy density

So separately,

$$\langle U_e \rangle = \frac{1}{4} \epsilon_0 E_0^2$$

$$\langle U_m \rangle = \frac{1}{4} \epsilon_0 E_0^2$$

Relation b/w elec. & mag. field, $B_0 = \frac{E_0}{c}$
amplitude of

$$\Rightarrow E_0 = B_0 c$$

$$\Rightarrow E_0^2 = B_0^2 c^2$$

$$\Rightarrow E_0^2 = \frac{B_0^2}{\mu_0 \epsilon_0}$$

$$\text{So } \boxed{\langle U \rangle = \frac{1}{2} \epsilon_0 E_0^2 = \frac{B_0^2}{4 \mu_0}}$$

Average Value of Poynting vector also called Intensity
it is the average energy per unit area per unit time associated with the EM wave.

$$\langle \vec{S} \rangle = I$$

8. An EM wave of given intensity, incident on 1 m^2 area for 10 sec then calculate energy transferred to the surface.

$$\rightarrow I = 2 \text{ J/m}^2 \text{ sec}$$

energy/unit area/unit time

$$\text{So Energy} = 2 \times 10 = 20 \text{ Joule}$$

Now,

$$\langle \vec{S} \rangle \equiv I \text{ (intensity)} \\ = c \langle U \rangle \hat{z}$$

$$\boxed{\langle \vec{S} \rangle = \frac{1}{2} \epsilon_0 c E_0^2 \hat{z}}$$

→ This much intensity is transferred to the surface.

This can be in another form (in terms of mag. field)

Average Value of Mom. density

Mom. density → mom./unit volume,

If Mom. density of EM wave is given per unit volume & calculate mom. transfer to the given volume then

$$\langle \vec{p} \rangle = \frac{1}{2} \frac{1}{c} \epsilon_0 \langle E_0^2 \rangle$$

$$\langle \vec{p} \rangle = \frac{\epsilon_0}{2} \frac{E_0^2}{c}$$

This much avg. mom./unit volume will transfer to the volume.

Radiation pressure (P)

Pressure $\rightarrow$ force per unit area

$$P = \frac{F}{A}$$

If a surface is made of atoms contain charges & e^- revolve in the orbits. If EM wave incident on the surface of atom. Then the force applied per unit area is called the Radiation pressure.

force $\rightarrow$ Rate of change of momentum
i.e. mom. per unit time

We know the mom. density which is mom./unit volume.

i.e. If Mom. density $\times$ volume = ~~force~~ mom.
& ~~force~~ ^{mom.} per unit ^{time} area gives pressure.

$$\langle \vec{p} \rangle = \frac{1}{2} \frac{\epsilon_0 E_0^2}{c}$$

force & force/unit gives pressure

Let Mom. transferred in time Δt is Δp

$$\Delta p = \langle \vec{p} \rangle A c \Delta t$$

{ If EM wave travels with speed of light then in time Δt it will travel distance $c \Delta t$. }

Now force

$$F = \frac{\Delta p}{\Delta t} = \frac{1}{2} \frac{\epsilon_0 E_0^2}{c} \times c A$$

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{c \Delta t}{c}$$

$$\text{Pressure } P = \frac{F}{A} \Rightarrow \boxed{P = \frac{c}{2c} \epsilon_0 E_0^2 = \frac{I}{c}}$$

This much pressure will be on the surface due to electromag. wave.

So Radiation pressure is Intensity divided by speed of (light) EM wave.

→ This is the radiation pressure if the ~~maxten~~ medium is perfectly Absorbing ($P = \frac{I}{c}$)

→ If surface is perfectly reflecting then pressure will be doubled.

$$P = \frac{2I}{c}$$

Note:

• Metal reflects EM Waves (Mirrors are reflectors) bcoz metal contains large no. of free e^- s. So free e^- s are responsible for this reflection.

→ If we incident EM wave on a mirror then free e^- start to vibrate with the freq. of EM wave then it will oscillate & emit radiation. (i.e. e^- Re-radiate the EM wave) in this phenomenon No time lag.

→ When EM wave incident on surface then (one) pressure ^{produce} & when it re-radiate then again there will be pressure. So pressure will be doubled.

→ If for perfectly Absorbing medium, when wave re-radiate then there will be no pressure. So in this case there is single pressure.

Microwaves & EM Wave they can travel with Insulators.

Wave Impedence of free Space :-

It is denoted by Z_0 & defined as

$$Z_0 = \left| \frac{E_0}{H_0} \right|$$

E_0 → Amp. of E-field

H_0 → Amp. of mag. field intensity.

$$B_0 = \mu H_0 \Rightarrow H_0 = \frac{B_0}{\mu}$$

$$\text{So } Z_0 = \left| \frac{E_0}{H_0} \right| = \frac{\mu_0 E_0}{B_0} = \mu_0 c = \mu_0 \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120 \pi$$

This is the impedence of free space.

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120 \pi = 377 \Omega$$

In free space, Z_0 is a real quantity.

Hence $\vec{E}$ & $\vec{B}$ vibrates in same phase. (in free space)

$$\vec{E} = E_0 \cos(kz - \omega t) \hat{x}$$

$$\vec{B} = B_0 \cos(kz - \omega t) \hat{y}$$

There is No phase diff.

Important points of free space :-

→ EM waves are transverse in free space.

i.e. $E \perp K$ & $B \perp K$

$$E \perp B \perp K$$

They are moving with speed of light c .

→ $\vec{E}$ & $\vec{B}$ are vibrating in same phase

→ Same energy lies in $\vec{E}$ & $\vec{B}$ field.

Electromagnetic waves in isotropic linear Dielectric

Medium :- for linear isotropic (dielectric medium), $\vec{D} = \epsilon \vec{E}$

$$\vec{B} = \mu \vec{H}$$

Maxwell's eqs

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

for dielectric medium, No free charge, No free charge density.

$$\rho_f = 0$$

$$\vec{J}_f = 0$$

$$\Rightarrow \nabla \cdot (\epsilon E) = 0 \Rightarrow \boxed{\vec{\nabla} \cdot \vec{E} = 0} \quad (1)$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0} \quad (2)$$

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad (3)$$

$$\boxed{\vec{\nabla} \times \vec{B} = \mu \epsilon \frac{\partial \vec{E}}{\partial t}} \quad (4)$$

Take curl of eqⁿ (3),

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B})$$

$$\Rightarrow \nabla^2 \vec{E} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \text{--- (5)}$$

Take curl of eqⁿ (4),

$$\nabla^2 \vec{B} - \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \quad \text{--- (6)}$$

If we compare these (2) 2nd order eqⁿ with original wave eqⁿ, we get

$$\nabla^2 f - \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} = 0 \quad \text{--- (7)}$$

We get $v = \frac{1}{\sqrt{\mu\epsilon}} \quad \text{--- (8)}$

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} \geq \frac{c}{\sqrt{\mu_r \epsilon_r}}$$

$$v < c$$

So In linear isotropic dielectric, speed of EM wave in that medium is less than speed of light in free space. for Any Transparent medium,

$$v = \frac{c}{n} \quad \text{--- (9)}$$

$n \rightarrow$ refractive index of medium,

$v \rightarrow$ wave speed

Comparing (8) & (9), We get

$$\boxed{n = \sqrt{\mu_r \epsilon_r}}$$

So, Refractive index of the medium can be calculated by Relative permittivity & Relative permeability.

If medium is Non magnetic $\rightarrow \mu_r = 1$

then

$$\boxed{n = \sqrt{\epsilon_r}}$$

Now, Maxwell's eqⁿ implies the solutions,

$$\left. \begin{aligned} \vec{k} \cdot \vec{E} &= 0 \\ \vec{k} \cdot \vec{B} &= 0 \end{aligned} \right\} \text{ These eqⁿs implies that } \rightarrow$$

→ Inside isotropic dielectric, EM waves are transverse wave.

If E_x is given & we have to find B then

$$\boxed{\vec{B} = \frac{\vec{k} \times \vec{E}}{\omega}}$$

$$\Rightarrow \nabla (\vec{k} \times \vec{B}) = -\nabla \omega \mu \epsilon \vec{E}$$

$$\vec{E} = -\frac{1}{\mu \epsilon \omega} (\vec{k} \times \vec{B})$$

$$\boxed{\vec{E} = -\frac{v^2}{\omega} (\vec{k} \times \vec{B})}$$

$$\left\{ v = \frac{1}{\sqrt{\mu \epsilon}} \right.$$

Now, Magnitudes of $\vec{B}$,

$$\boxed{|\vec{B}| = \frac{|\vec{E}|}{v}}$$

where $\frac{\omega}{k} = v$

If wave is travelling in isotropic dielectric medium then $\left[\frac{\omega}{k} = v \right]$

Conclusion :- $\vec{E} \perp \vec{B} \perp \vec{k}$ i.e. mutually $\perp$ to each other.

→ Energy density :-

Electric energy density $u_e = \frac{\epsilon}{2} E^2$

Magnetic " " " $u_m = \frac{B^2}{2\mu} = \frac{E^2 \mu \epsilon}{2\mu} = \frac{E^2 \epsilon}{2}$

$$u_m = \frac{1}{2} \epsilon E^2$$

⇒ $\boxed{u_e = u_m}$, Total Energy density $u = u_e + u_m$

$$\boxed{u = \epsilon E^2}$$

→ Poynting Vector :- $\vec{k} \cdot \vec{E} = 0$, $\vec{k} \cdot \vec{B} = 0$

$$\vec{E} = E_0 e^{i(kz - \omega t)} \hat{x}$$

$$\vec{B} = B_0 e^{i(kz - \omega t)} \hat{y}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$= \frac{\vec{E} \times \vec{B}}{\mu} = \frac{1}{\mu v} E^2 = \frac{v}{\mu v^2} E^2$$

$$(B = E \sqrt{\mu \epsilon} = E/v)$$

$$\boxed{\vec{S} = \epsilon v E^2 \hat{z} = v u \hat{z}}$$

$$v^2 = \frac{1}{\mu \epsilon}$$

So Energy flow is in the dirⁿ of Wave propagation.
(dirⁿ of wave propagation is $\hat{z}$)

On Comparing With free space, We get if we replace

$$\mu_0 \rightarrow \mu$$

$$\epsilon_0 \rightarrow \epsilon$$

$$c \rightarrow v$$

then we get, expressions for isotropic dielectric

Momentum density $\vec{p} = \mu \epsilon \vec{S} = \frac{\mu \epsilon}{v} \hat{z}$

Wave Impedance $Z = \left| \frac{E}{H} \right| = \sqrt{\frac{\mu}{\epsilon}}$

$$Z = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = 120 \pi \sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$\left. \begin{array}{l} \mu_r \geq 1 \\ \epsilon_r \geq 1 \end{array} \right\} Z = 120 \pi \sqrt{\frac{\mu_r}{\epsilon_r}} = \text{Real Value}$$

If Z is Real that means $\vec{E}$ & $\vec{B}$ are vibrating in same phase.

Conclusion :-

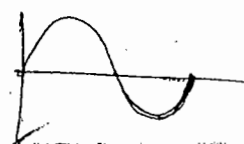
→ speed of medium v is less than c . & $\rightarrow n = \sqrt{\mu_r \epsilon_r}$

→ for a linear isotropic dielectric medium

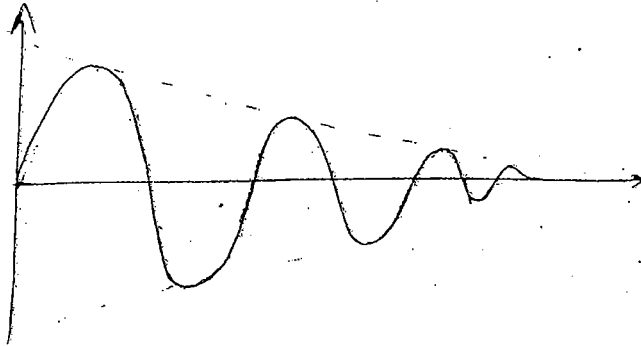
$$\mu_r \geq 1 \quad \& \quad \epsilon_r \geq 1$$

(i) If $\epsilon_r = 1$, $\mu_r = 1$ that means free space is also a dielectric medium of dielectric constant one.

(ii) If $\epsilon_r > 1$, $\mu_r > 1$ then n is real then solⁿ of EM will be Oscillatory



(iii) If $\epsilon_r < 0$ (-ve), $\mu_r \geq 1 \Rightarrow n \rightarrow \text{imaginary}$
If n is imaginary or complex, then solution of EM wave will be damped.



If $\epsilon_r \geq 0$, $\mu_r < 0$, Again solⁿ $\rightarrow$ damped
(inside the conducting medium)

- In all these cases, dirⁿ of $\vec{S}$ & $\vec{K}$ will be same.
Energy flow $\downarrow$ wave prop.

Here if ϵ_r & μ_r are not the funcⁿ of position then medium will be isotropic.

Note :- If ϵ_r & μ_r are funcⁿ of position then dirⁿ of $\vec{S}$ & $\vec{K}$ are different.

Case (iv) :- If ϵ_r & μ_r are simultaneously negative,
then $\vec{S}$ & $\vec{K}$ are antiparallel.

$$\begin{cases} \mu = \mu_0 \mu_r \\ \epsilon = \epsilon_0 \epsilon_r \end{cases}$$

No medium exist in nature, in which both ϵ_r & μ_r are simultaneously -ve.

For Left Handed Material, $\vec{S}$ & $\vec{K}$ are antiparallel.
i.e. This type of medium is called L.H. material.

ES

Electromagnetic Waves inside an Anisotropic Linear Dielectric Medium :-

Generally, for dielectrics, permeability $\mu \approx \mu_0$

This medium is Anisotropic w.r. to permittivity ϵ only.

Maxwell's Eqⁿ,

Linear $\rightarrow \vec{D} = \epsilon \vec{E}$

$\vec{\nabla} \cdot \vec{D} = \rho_f \Rightarrow \boxed{\vec{\nabla} \cdot \vec{D} = 0}$ (1) Dielectric $\rightarrow \rho_f = 0$

$\Rightarrow \nabla \cdot (\epsilon \vec{E}) = 0$

Here ϵ is not const.

but $\nabla \cdot \vec{E} \neq 0$

$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$ (2)

$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$ (3)

$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$

for dielectric $\vec{J}_f = 0$

$\Rightarrow \boxed{\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}}$ (4)

$\vec{\nabla} \cdot \vec{D} = 0 \Rightarrow \vec{k} \cdot \vec{D} = 0$

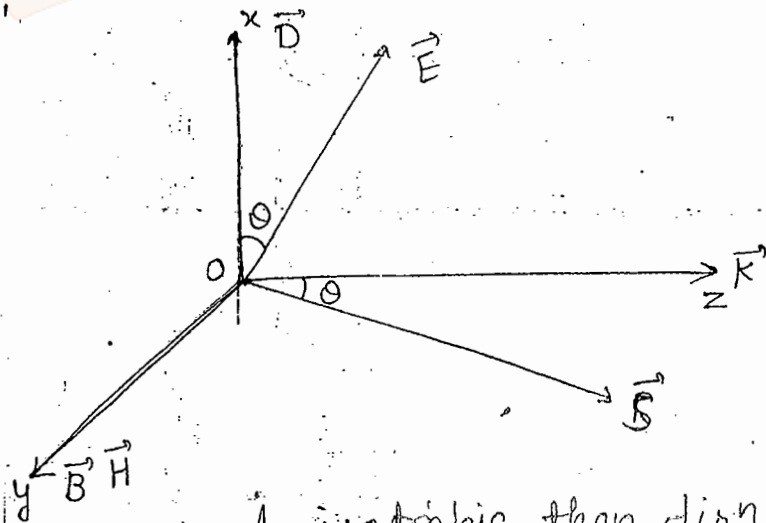
$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \vec{k} \cdot \vec{B} = 0$

i.e. $\vec{B}$ & $\vec{D}$ are $\perp$ to wave propagation i.e.

$\vec{B} \perp \vec{D} \perp \vec{k}$

$\vec{S} = \vec{E} \times \vec{H}$

& $\vec{B} = \mu_0 \vec{H}$

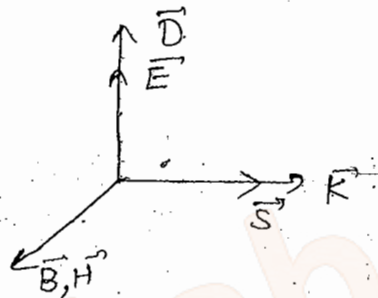


i.e. If medium is Anisotropic then dirⁿ of energy flow is not same as wave prop.

Main Points :-

- EM waves in Anisotropic medium are transverse w.r. to $\vec{B}$ & $\vec{H}$. Not w.r. to $\vec{E}$ & $\vec{D}$.
- Dirⁿ of Energy flow is not same as the dirⁿ of wave propagation.
- The Electric field is making angle θ with $\vec{D}$. Hence $\vec{S}$ is also making angle θ with wave propagation $\vec{k}$.

for Isotropic :-



→ Doubly reflecting Systems ($n_x \neq n_y$) [ref. index is not same in all dirⁿ] are the examples of Anisotropic medium.

Electromag. Waves in Conducting Medium:-

Maxwell's ϵ^n ,

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

for a conducting medium, $\vec{D} = \epsilon \vec{E}$ & $\vec{B} = \mu \vec{H}$

volume free charge $\rho_f = 0$ & $\boxed{\vec{J}_f = \sigma \vec{E}}$

If we put any free charge in conducting medium then free charge will be on surface. No free charge can reside inside the conducting medium.

$$\vec{\nabla} \cdot \vec{D} = 0 \Rightarrow$$

$$\boxed{\vec{\nabla} \cdot \vec{E} = 0} \quad \text{--- (1)}$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0} \quad \text{--- (2)}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{E} + \mu_0 \epsilon \frac{\partial \vec{E}}{\partial t} \quad (4)$$

Take curl of eqⁿ (3),

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B})$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t} [\mu_0 \vec{E} + \mu_0 \epsilon \frac{\partial \vec{E}}{\partial t}]$$

$$\nabla^2 \vec{E} = \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 \vec{E} - \mu_0 \frac{\partial \vec{E}}{\partial t} - \mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0} \quad (5)$$

Take curl of eqⁿ (4),

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \mu_0 (\vec{\nabla} \times \vec{E}) + \mu_0 \epsilon \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E})$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B} = \mu_0 \left(-\frac{\partial \vec{B}}{\partial t}\right) + \mu_0 \epsilon \left(-\frac{\partial^2 \vec{B}}{\partial t^2}\right)$$

$$\boxed{\nabla^2 \vec{B} - \mu_0 \frac{\partial \vec{B}}{\partial t} - \mu_0 \epsilon \frac{\partial^2 \vec{B}}{\partial t^2} = 0} \quad (6)$$

eqⁿ (5) & (6) are IInd order differential eqⁿs.

IInd term, of eqⁿ (5) & (6) are damping terms, comes from damped harmonic oscillator.

$$\frac{d^2 x}{dt^2} - \omega^2 x = \frac{dx}{dt}$$

Bcoz of this term, amplitude of damped harmonic oscillator is decaying.

Now, we take wave vector K is a complex quantity in the solutions of wave eqⁿ.

$$\vec{E} = E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\vec{B} = B_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

These solutions must satisfy the given diff. eqⁿ.

$$(5) \Rightarrow (i\vec{k})^2 \vec{E} - (-i\omega) \mu_0 \vec{E} - (-i\omega)^2 \mu_0 \epsilon \vec{E} = 0$$

$$-(\vec{k})^2 \vec{E} + i\omega \mu_0 \vec{E} + \mu_0 \epsilon \omega^2 \vec{E} = 0$$

(from (6) → We get same eqⁿ)

$$(K^*)^2 = \mu \epsilon \omega^2 + i \omega \mu \sigma$$

$$K^* = [\mu \epsilon \omega^2 + i \omega \mu \sigma]^{1/2} \quad (7)$$

Wave vector is Complex.

Let $K^* = \alpha + i\beta$

$\alpha \rightarrow$ real part
 $\beta \rightarrow$ imaginary part

$$(K^*)^2 = \alpha^2 - \beta^2 + 2i\alpha\beta \quad (8)$$

Compare (7) & (8) $\Rightarrow$

$$\alpha^2 - \beta^2 = \mu \epsilon \omega^2 \quad (9)$$

$$2\alpha\beta = \omega \mu \sigma \quad (10)$$

We have 2 unknowns α & β .

$$(10) \Rightarrow \beta = \frac{\omega \mu \sigma}{2\alpha}$$

$$(9) \Rightarrow \alpha^2 - \frac{\omega^2 \mu^2 \sigma^2}{4\alpha^2} = \mu \epsilon \omega^2$$

$$4\alpha^4 - \omega^2 \mu^2 \sigma^2 = 4\mu \epsilon \omega^2 \alpha^2$$

$$4\alpha^4 - 4\mu \epsilon \omega^2 \alpha^2 - \frac{1}{4}\omega^2 \mu^2 \sigma^2 = 0$$

$$\alpha^2 = \frac{\mu \epsilon \omega^2 \pm \sqrt{(\mu \epsilon \omega^2)^2 + \omega^2 \mu^2 \sigma^2}}{2}$$

$$\alpha^2 = \frac{\mu \epsilon \omega^2 \pm \mu \epsilon \omega^2 \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}}{2}$$

$$\alpha^2 = \frac{\mu \epsilon \omega^2}{2} \left[1 + \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} \right]$$

$$\Rightarrow \alpha = \omega \sqrt{\frac{\epsilon \mu}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} + 1 \right]^{1/2} \quad (11)$$

Similarly for β , $\alpha = \frac{\omega \mu \sigma}{2\beta}$, We get

$$4\beta^4 - 4\mu \epsilon \omega^2 \beta^2 - \frac{1}{4}\omega^2 \mu^2 \sigma^2 = 0$$

$$\Rightarrow \beta = \omega \sqrt{\frac{\epsilon \mu}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} - 1 \right]^{1/2} \quad (12)$$

$$\vec{K}^* = k^* \hat{n}$$

$$= (\alpha + i\beta) \hat{n}$$

$$\vec{E} = E_0 e^{i[(\alpha + i\beta)(\hat{n} \cdot \vec{r}) - \omega t]}$$

$$\vec{E} = E_0 e^{-\beta(\hat{n} \cdot \vec{r})} e^{i[\alpha(\hat{n} \cdot \vec{r}) - \omega t]} \quad (13)$$

$$\vec{B} = B_0 e^{-\beta(\hat{n} \cdot \vec{r})} e^{i[\alpha(\hat{n} \cdot \vec{r}) - \omega t]} \quad (14)$$

Now Amplitude is exponentially $\downarrow$ with space (Not with time).

Decay will be fast or slow, it will depend upon β .

$\rightarrow \beta$ is called Attenuation const. or coefficient.

$\rightarrow \alpha$ is propagation coefficient.

$\rightarrow \beta$ depends upon certain properties of medium,

depends upon $\rightarrow \epsilon, \mu, \sigma$

If medium is dielectric $\rightarrow \sigma = 0 \rightarrow \beta \rightarrow 0$

then no attenuation in wave.

$\rightarrow$ Attenuation in wave is due to σ & conductivity σ is due to free e^- s. { for dielectric $\rightarrow$ No free e^- so $\sigma = 0$ }

$\rightarrow$ Here α is like k (wave vector)

Wave Velocity $v = \frac{\omega}{\alpha}$

$$v = \sqrt{\frac{2}{\epsilon\mu}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1 \right]^{-1/2}$$

This is the wave velocity inside the conducting medium

If we put $\sigma = 0$ then we get expression for dielectric.

$$v = \sqrt{\frac{2}{\epsilon\mu}} \frac{1}{\sqrt{2}}$$

$$v = \frac{1}{\sqrt{\mu\epsilon}}$$

Wave velocity is decreased as compared to dielectric.