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All Lab Experiments

Electromagnetic Theory for NET/Gate Physical Sciences

> Magnetostatics <

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Magnetostatics

Here we have STATIC Magnetic field (not changing with time)

static electric field produces from charge at rest. (Not moving). And its condition is

$$\nabla \times E = 0$$

similarly to produce magnetic field we need current. i.e. To ~~pro~~ static magnetic field can be produced from current.

When Magnetic field not changing with time - Magnetostatic i.e. Current is not a func^n of time.

The current which is not changing with time → Steady Current.

When flow of current is steady then it is steady current

Defination of Steady Current :- The current flowing smoothely without pilling up charge anywhere called Steady Current.

If Charge accumulate then M.f will not be static.

If wire is uncharged. At the point in space, there is mag. field but not electric field will present.

Inside the wire, if current flows then electric field will defenitely present.

If a charge is moving with constant velocity v then there will be static mag field & also electric field elec. field & mag. field in this case do not depend on each other

If v is changing with time then it is accelerated motion. then both elec. & mag. field depend on each other. (both depends on time)

If a wire is connected to a battery then a steady current will flow in wire (dc current)

Magnetic force :- Mag. force on a charge particle q moving with velocity v in mag. field B then

$$\boxed{\vec{F} = q(\vec{v} \times \vec{B})}$$

In Electric field, We have force $\vec{F} = q\vec{E}$

Magnetic force is also called Lorentz force.

Magnetic forces do no work, they only change the dirn of motion of charged particle.

If a charge particle q moving with v in $\vec{B}$ then work done in the distance travelled dr is

$$\boxed{dW = \vec{F} \cdot d\vec{r}}$$

$$dW = \vec{F} \cdot \frac{dr}{dt} dt = q(\vec{v} \times \vec{B}) \cdot \vec{v}\, dt$$

$\vec{v}.dt \rightarrow$ distance travelled

$v \times B$ will be the $\perp$ vector to both $v \times B$.

So $\boxed{dW = 0}$

i.e. Power accociated by mag. force is zero.

These type of forces are called Fictitous forces.

If inany problem, mag. force do work then there-
→by changing the mag. field there gennate mag. force & then In that case mag. field do the work, not the mag. force.

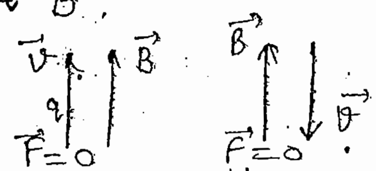
Path of the charged particle in mag. field :- Suppose a charge particle q moving with velocity $\vec{v}$ in $\vec{B}$ then force on it $\vec{F} = q(\vec{v} \times \vec{B})$

then magnitude of force will depend on angle b/w $\vec{v}$ and $\vec{B}$ as $|\vec{F}| = q\, vB\sin\theta$

$\theta \rightarrow$ angle b/w v & B.

Case.1 :- If $\theta = 0, \pi$

$F = 0$

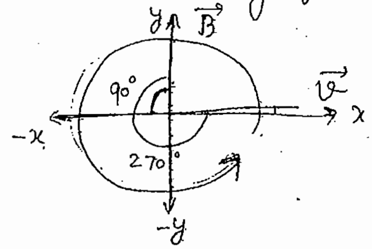


charged particle It will move undeflected if v & B are parallel or antiparallel. [undeflected means straight line]

Case 2. $\theta = \perp (90^\circ)$ $\frac{\pi}{2}, \frac{3\pi}{2}$

then charged particle will follow circular path & rotate in a plane which is perpendicular to the mag. field.

If a charge particle move in a circle i.e. v & B are $\perp$ to each other (angle b/w them is $\frac{\pi}{2}$ or $\frac{3\pi}{2}$) Then its motion is called



<u>Cyclotron motion</u>.

If particle do rotational motion then there will be 2 forces on it. → Lorentz & centripital force.

$$qvB = \frac{mv^2}{r}$$

Momentum $\boxed{p = mv = qBr}$ —— (A)

Now frequency of rotation (Angular freq.):-

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Particle should travell in a circle so

$$T = \frac{2\pi r}{v}$$

$$\therefore \omega = \frac{2\pi v}{2\pi r} \Rightarrow \omega = \frac{v}{r}$$

Compare the value of $\frac{v}{r}$ with eq. (A),

ang. freq. $\boxed{\omega = \frac{qB}{m}}$

freq. $f = \frac{\omega}{2\pi} \Rightarrow \boxed{f = \frac{qB}{2\pi m}}$

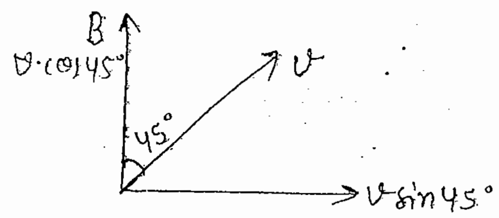
Time $T = \frac{1}{f} \Rightarrow \boxed{T = \frac{2\pi m}{qB}}$

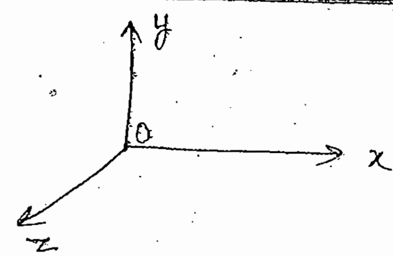
This time is required by a particle to complete one rotation of a circle.

<u>Case 3</u>:- When $\theta \neq 0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}$

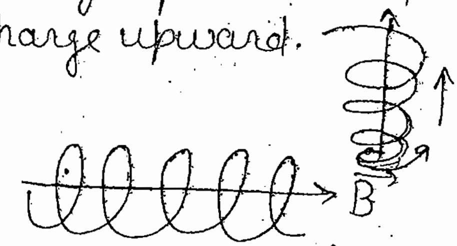
Suppose angle $\theta = 45^\circ$

If a charge particle enter in mag. field $\vec{B}$ so that ~~it~~ $\vec{v}$ makes angle 45° with $\vec{B}$.





$v \sin\theta$ comp. will tend to rotate the charge particle and
$v \cos\theta$ " " " " move this charge upward.
This is called Hellical Path.
$\vec{B}$ will lie along the axis of hellix.

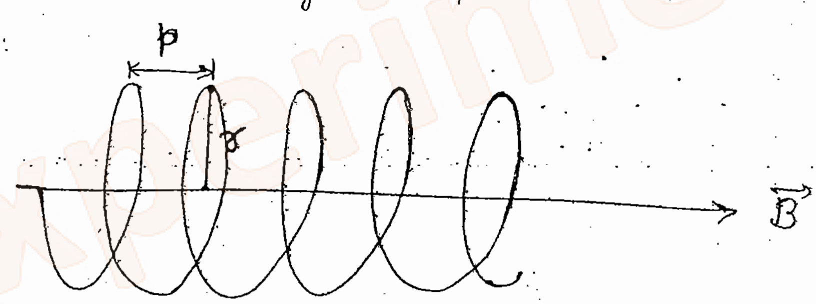


Ques. 1 :- An e^- enter in a uniform mag. field with magnitude 0.3 tesla, at an angle of 45° w.r.to mag. field.

M.K.S. Unit of Mag. field → Tesla — 1 T = 1 weber/m^2
C.G.S. " " " → Gauss

$\boxed{1\,T = 10^4\,G}$ → Relation b/w both units.

Determine the radius r & pitch P of the electron's helical path. Assuming its speed is 2×10^6 m/s.



p → pitch
r → radius

$\theta = 45°$, $v = 2 \times 10^6$ m/sec

$v_\perp = v \sin 45° = 2 \times 10^6 \times \frac{1}{\sqrt{2}} = \sqrt{2} \times 10^6$

$v_\parallel = v \cos 45° = 2 \times 10^6 \times \frac{1}{\sqrt{2}} = \sqrt{2} \times 10^6$

$v_\parallel = v_\perp = \frac{\sqrt{2}}{\sqrt{2}} \times 10^6$ m/sec

$v_\perp$ is responsible for circular motion.
$v_\parallel$ " " " forward "

So Rotational motion is determined by $v_\perp$.
Translational " " " $v_\parallel$.

$$q v_\perp B = \frac{m v_\perp^2}{r}$$

$$\Rightarrow r = \frac{m v_\perp}{qB} = \frac{9.1 \times 10^{-31} \times \sqrt{2} \times 10^{6}}{1.6 \times 10^{-19} \times 0.3}$$

$$r = 26.81087 \times 10^{-6}$$

$$\boxed{r = 26.8\ \mu m}$$

To calculate the pitch, Ist Calculate time for making a circle,

$$\Delta t = \frac{2\pi r}{v_\perp}$$ {bcoz circular motion $v_\perp$ is responsible}

Pitch $p = v_\parallel \Delta t$

$$= v_\parallel \times \frac{2\pi r}{v_\perp}$$ ($V_\perp = V_\parallel$ in this Ques.)

$$p = 2\pi r = 2 \times 3.14 \times 26.8 = 168.304$$

$$\boxed{p = 169\ \mu m}$$

<u>Note</u>:- • pitch of a helical path is constant in uniform magnetic field while it is not constant in Non uniform mag. field.

$$p \propto t \quad \& \quad t \propto \frac{1}{B}$$

$$\text{so} \quad p \propto \frac{1}{B}$$

If non uniform mag. field will increase then pitch decrease.

• force will be $F = q v_\perp B$, $F \neq q v_\parallel B$

<u>Motion of the charge particle in both Electric field & magnetic field</u>:-

We have a charge particle q at rest.

In present case $E \perp B$

$\vec{E}$ is along z-axis.

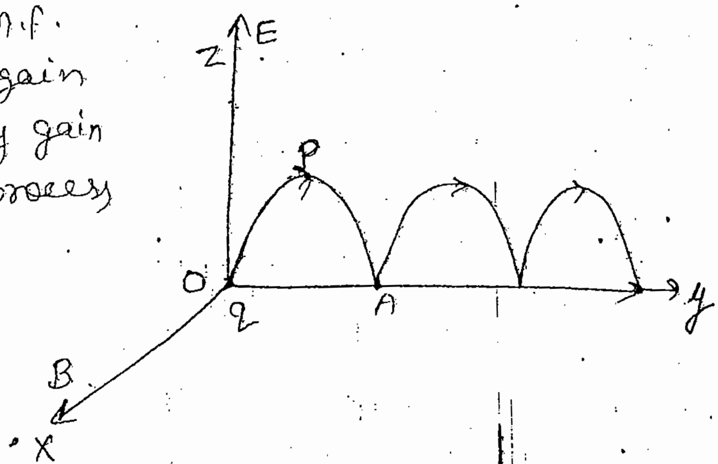
$\vec{B}$ " " X-axis.

<u>charge is at rest placed at origin.</u>

When charge particle is at rest $\vec{E}$ will work on it & it will start motion. As $v \uparrow$, $B \uparrow$

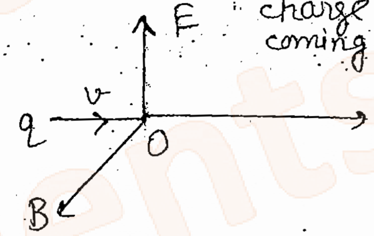
At P, it lose its velocity so M.F. decreases & at A, vel. = 0. Again E.F. will act on it, velocity gain & process will go on, the process is repeated.

The path followed by the charged particle will be <u>Cycloid</u>.



- If charge particle moving with velocity $\vec{v}$ then Electric field push the charge in upward dir^n
  Mag. " " " " downward "
  So both are opposite.
  Generally strength of electric field dominent over speed.

charge q is coming with speed v.

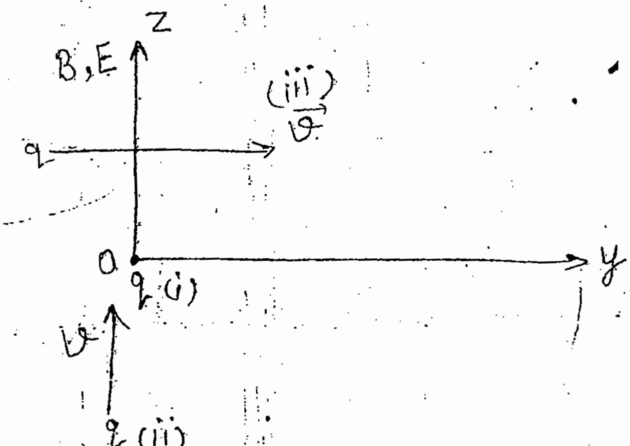


$E \perp B$,

$$qE = qvB$$
$$E = vB$$
$$\boxed{v = \frac{E}{B}}$$

If strength of both $\vec{E}$ & $\vec{B}$ are same then it will move undeflected i.e. in straight line.

- If E & B are parallel



(i) If q is at rest then path will be straight line.

(ii) If q is moving in z-dir^n (∥ dir^n) then path is also straight line.

(iii) If q is moving in x or y dir^n (⊥ dir^n) then path will be spiral or helical.

$v \perp$ to $\vec{E}$ & $\vec{B}$, v along y dir^n & $\vec{E}, \vec{B}$ along z-dir^n

$\vec{E}$ push the charge particle in upward dir^n & $\vec{B}$ make the path helical in upward $\hat{z}$ dir^n.

Currents :-

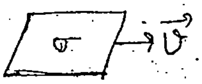
Line Current :- $\boxed{I = \frac{q}{t} = \lambda v}$

$\lambda \rightarrow$ line charge density , $v \rightarrow$ velocity

If any line charge λ moving with v velocity then current in it will be $I = \lambda v$

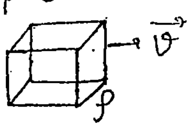
Surface Current :- If a surface charge moving with v then $\vec{K} = \sigma \vec{v}$, $\vec{K} \rightarrow$ surface current
it is a vector quantity.

If any +ve charge particle moving with $\vec{v}$ then the dirn of current will be in the dirn of motion of q



$\boxed{\vec{K} = \sigma \vec{v} = \frac{I}{l_1}}$ (current / unit length)

Volume Current :- If Volume charge ρ move with v then produce vol. current.



$\boxed{\vec{J} = \rho \vec{v} = \frac{I}{a_1}}$ (current / unit area)

Continous force in terms of these Currents :-

$$\vec{F} = q(\vec{v} \times \vec{B})$$
$$= \int dq \left[\frac{d\vec{l}}{dt} \times \vec{B}\right]$$
$$= \int \frac{dq}{dt} \, d\vec{l} \times \vec{B}$$

$I = \frac{dq}{dt}$

$$\boxed{F = I \int d\vec{l} \times \vec{B}}$$

force on a current carring wire,

for a surface current,

$$\vec{F} = \int \sigma da \, (\vec{v} \times \vec{B})$$
$$= \int da \, (\sigma \vec{v} \times \vec{B})$$
$$\boxed{\vec{F} = \int_S (\vec{K} \times \vec{B}) \, da}$$

This is force on a surface element having current density K

Similarly for Volume,

$$\vec{F} = \int \rho\, d\tau\, (\vec{v} \times \vec{B})$$

$$\boxed{\vec{F} = \int_V (\vec{J} \times \vec{B})\, d\tau}$$

This is force on a volume current.

$\vec{K} = \frac{I}{l_\perp}$ current/unit length perpendicular to the flow of current is Surface current.

$\vec{J} = \frac{I}{a_\perp}$ current/unit area perpendicular to the flow of current is Volume current.

Que :- (a) If Current I is uniformly distributed over a wire of circular cross-section with radius a, Find the volume current density $\vec{J}$.

(b) If Current in the wire is proportional to the distance from the axis $J = Ks$. Find the total current. K is any constant.

(a) $J = ?$

$$\vec{J} = \frac{I}{a_\perp} = \frac{I}{\pi a^2}$$

(Area $\perp$ to flow of current $a_\perp$ = area of circular cross section of radius a)

(b) $J = Ks$

$I = ?$ $\quad J = \frac{I}{a_\perp}$

This current density is Non-uniform so

$$I = \int J \cdot da_\perp$$

$$= \int_0^{2\pi}\int_0^a Ks \cdot s\, ds\, d\phi$$

$$= K\left(\frac{s^3}{3}\right)_0^a 2\pi$$

$$\boxed{I = \frac{2\pi K a^3}{3}}$$

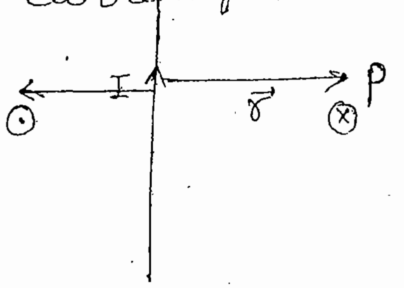
wire is a cylinder so dir^n of flow of current is z dir^n along the length.

Biot - Savert Law :-

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I(d\vec{l} \times \hat{r})}{r^2}$$

The $d\vec{l}$ is a vector along the dir^n of current flow.

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I(d\vec{l} \times \vec{r})}{r^3}$$



• dir^n of $\vec{B}$ = $d\vec{l} \times \vec{r}$

$\otimes$ → into the page
$\odot$ → out of the page

Wires are of cylendrical shape
So dir^n of current is $\hat{z}$ &
dir^n of Mag. field will be $\hat{\phi}$.

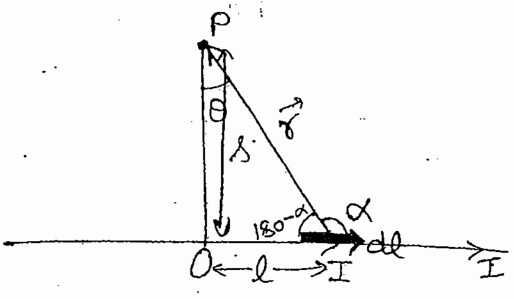
Mag. field curl around the wire.
If ~~is~~ the dir^n of current is $\hat{\phi}$ then dir^n of mag. field will be $\hat{z}$.

In Solanoid → Current is in $\hat{\phi}$ & Mag. field is in $\hat{z}$.

12/8/2012 **Application of Biot-Savert law :-**

Q. Find the mag. field at a distance s from a straight wire carring a steady current I.

Take an small element dl.
dir^n of element dl will be in the dir^n of flow of current.



$$dB = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \vec{r}}{r^3}$$

Complete mag. field for wire

$$B = \int dB = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \vec{r}}{r^3}$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{dl\, r \sin\alpha}{r^3}$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{dl \sin\alpha}{r^2}$$

r, α → unknown

$$\cos\theta = \frac{s}{r}$$

$$r = \frac{s}{\cos\theta}$$

$90 + 180 - \alpha + \theta = 180$ (Total angles in $\Delta = 180$)

$$\boxed{\alpha = 90^\circ + \theta}$$

$\therefore \sin\alpha = \sin(90+\theta) = \cos\theta$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{\cos^2\theta}{s^2} \times \cos\theta\, dl$$

$\tan\theta = \dfrac{l}{S}$

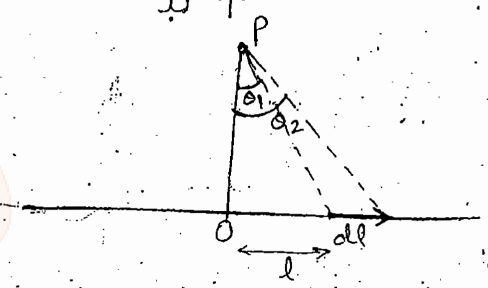
$l = S\tan\theta$

$dl = s\sec^2\theta\, d\theta$

$$= \frac{\mu_0 I}{4\pi}\int \frac{\cancel{\cos^2\theta}}{s^2}\cos\theta\, \cancel{s}\,\cancel{\sec^2\theta}\, d\theta$$

$$= \frac{\mu_0 I}{4\pi s}\int_{\theta_1}^{\theta_2} \cos\theta\, d\theta$$

If length of element is ↑



$$\boxed{B = \frac{\mu_0 I}{4\pi s}\left[\sin\theta_2 - \sin\theta_1\right]}$$ ✓

This is the mag-field of finite wire.

Dir^n :- $\odot$ i.e. $[\hat{\phi}]$

<u>For Infinitly long wire</u> :-

then $\theta_1 = -90^\circ$, $\theta_2 = +90^\circ$

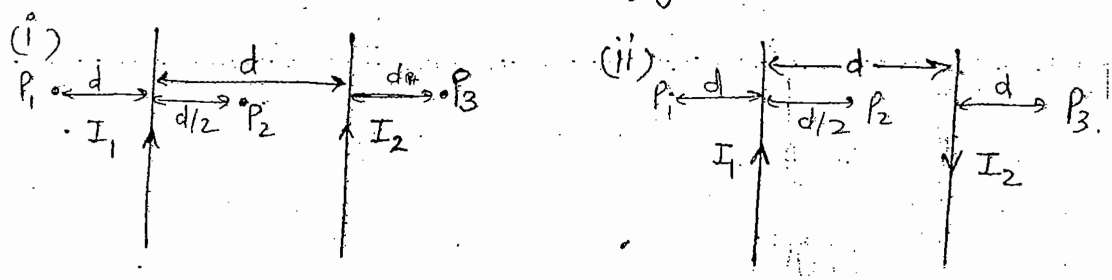
$$\boxed{B = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

$\boxed{\hat{\phi} \rightarrow \text{Circumfrantial vector}}$

for a infinitly long wire, we have electric field $\boxed{E = \dfrac{\lambda}{2\pi\epsilon_0 s}}$

Here $\mu_0 \leftrightarrow \dfrac{1}{\epsilon_0}$ & $I \leftrightarrow \lambda$

<u>Q.</u> Find the force per unit length in two parallel wire arrang-ment as shown in the figure.



Also Find the mag. field at point P_1, P_2 & P_3.

$$B = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

force on wire 2 will be due to mag. field of wire 1.

$$\vec{F_2} = I_2 \int d\vec{l_2} \times \vec{B_1}$$ (force on wire 2)

force per unit length is $f_2 = \vec{I_2} \times \vec{B_1}$

$\vec{B_1} \rightarrow$ mag. field on wire 1 at the position of 2.

$$\vec{B_1} = \frac{\mu_0 I_1}{2\pi d}$$ (into the page) dir^n of ~~wire~~ $\vec{B}$ on wire(2) $\rightarrow \otimes$

in region of wire (1) $\rightarrow \odot$

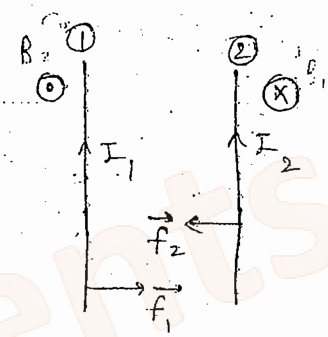
$$\boxed{\vec{f_2} = \frac{\mu_0 I_1 I_2}{2\pi d}}$$ (dir^n $\rightarrow$ to the ~~length~~ left)

force/unit length $\propto I_1$ & I_2
$\propto \frac{1}{d}$

<u>force on wire 1</u>,

$$\vec{F_1} = I_1 \int d\vec{l_1} \times \vec{B_2}$$

$$\vec{f_1} = \vec{I_1} \times \vec{B_2}$$

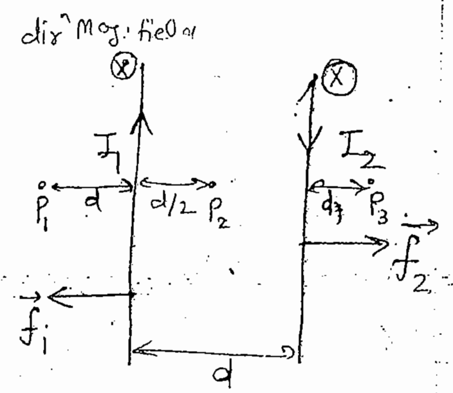


$$B_2 = \frac{\mu_0 I_2}{2\pi d}$$ (out of page)

$$\boxed{\vec{f_1} = \frac{\mu_0 I_1 I_2}{2\pi d}}$$ (to the right)

If current is in same dir^n (for two wires) then force is <u>attractive</u>. This attraction is due to magnetic force.

Q (ii) Magnitude of force per unit length is same. Only dir^n will be different.



dir^n of Mag field on wire (2) due to (1) is $\otimes$

" " " (1) " (2) " $\otimes$

If Currents are in opposite dir^n the force is Repulsive

$$\vec{f_1} = \frac{\mu_0 I_1 I_2}{2\pi d}$$ (to the left)

$$\vec{f_2} = \frac{\mu_0 I_1 I_2}{2\pi d}$$ (to the right)

At P_1, Mag. field at any point is vector sum of $\vec{B} = \vec{B}_1 + \vec{B}_2 + \cdots$

$\vec{B}_1 \rightarrow \odot$, $\vec{B}_2 \rightarrow \otimes$

$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi d}$ (out of the page)

$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi(2d)}$ (into the page)

Out mag. field is more than into mag. field so resultant mag. field will be in out of the page dirn.

$$\vec{B} = \vec{B}_1 + \vec{B}_2 \Rightarrow \vec{B} = \frac{\mu_0}{2\pi d}\left[I_1 - \frac{I_2}{2}\right]$$

If current in 2 wires are same then

$$\boxed{\vec{B}(P_1) = \frac{\mu_0 I}{4\pi d}}$$ (out of page)

At P_2, $\vec{B}_1 \rightarrow \otimes$, $\vec{B}_2 \rightarrow \otimes$

Resultant Mag. field

$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi(d/2)}$

$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi(d/2)}$

$$B = \frac{\mu_0}{2\pi}\left[\frac{1}{d/2}(I_1 + I_2)\right]$$

$$B = \frac{\mu_0}{2\pi}\frac{2}{d}(I_1 + I_1)$$

If $I_1 = I_2 = I$ then $\boxed{B(P_2) = \frac{2\mu_0 I}{\pi d}}$ (into the page)

At P_3, $B_1 \rightarrow \otimes$, $B_2 \rightarrow \odot$

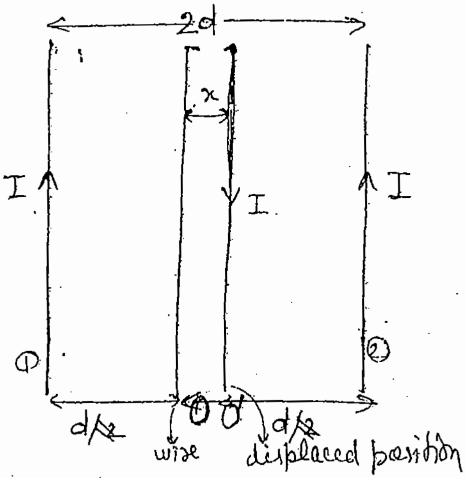
$B_1 = \frac{\mu_0 I_1}{2\pi(2d)}$, $B_2 = \frac{\mu_0 I_2}{2\pi(d)}$

$$B(P_3) = \frac{\mu_0}{2\pi d}\left[-\frac{I_1}{2} + I_2\right]$$

$I_1 = I_2 = I$

$$\boxed{B(P_3) = \frac{\mu_0}{4\pi d}}$$ (out of page)

Q. We have a 3 wire arrangement. Current in each wire is I. Mass per unit length of the middle wire is m. Earlier it was placed at mid point. Now it is displaced by x. Calculate its freq. of oscillation.



$$F \propto x$$

Here, force / unit length so

$$f \propto x$$

If wire is placed at mid point then resultant force is zero.

But when it is displaced then force will be Non-zero.

At Present position,

$$\vec{B_1} = \frac{\mu_0 I}{2\pi(d+x)}, \quad \vec{B_2} = \frac{\mu_0 I}{2\pi(d-x)} \text{ (out of page)}$$

(into the page)

{ B_1 is field at ①. due to mid
B_2 " " ② "

Resultant Mag. field, $\vec{B} = \vec{B_1} + \vec{B_2}$

$$B = \frac{\mu_0 I}{2\pi}\left[\frac{1}{(d-x)} - \frac{1}{(d+x)}\right] \text{ (out of page)}$$

$$B = \frac{\mu_0 I}{2\pi}\left[\frac{d+x-d+x}{d^2-x^2}\right]$$

Here x^2 is very small. This wire will perform Simple harmonic motion (if x is not small then wire will not perform S.H.M.) so neglect x^2

$$\boxed{B = \frac{\mu_0 I x}{\pi d^2}} \text{ (out of page)}$$

After displament the motion of wire will be Simple Harmonic as it starts to oscillate.

force/unit length $f = I \times B$

$$f = \frac{\mu_0 I^2 x}{\pi d^2}$$

$$\boxed{f \propto x} \quad \text{i.e. S.H.M.}$$

$$F = m\frac{d^2x}{dt^2} \quad \& \quad f = \frac{m}{l}\frac{d^2x}{dt^2}$$

Here mass per unit length is m. So $\boxed{f = m\frac{d^2x}{dt^2}}$

$$f = Kx$$

$$\frac{\mu_0 I^2 x}{\pi d^2} = Kx \Rightarrow \boxed{K = \frac{\mu_0 I^2}{\pi d^2}}$$

Both f & K are force per unit length

So freq. of Oscillation, $\omega = \sqrt{\frac{K}{m}}$

$$\omega = \sqrt{\frac{\mu_0 I^2}{\pi d^2 m}}$$

$$\boxed{\omega = \frac{I}{d}\sqrt{\frac{\mu_0}{\pi m}}}$$

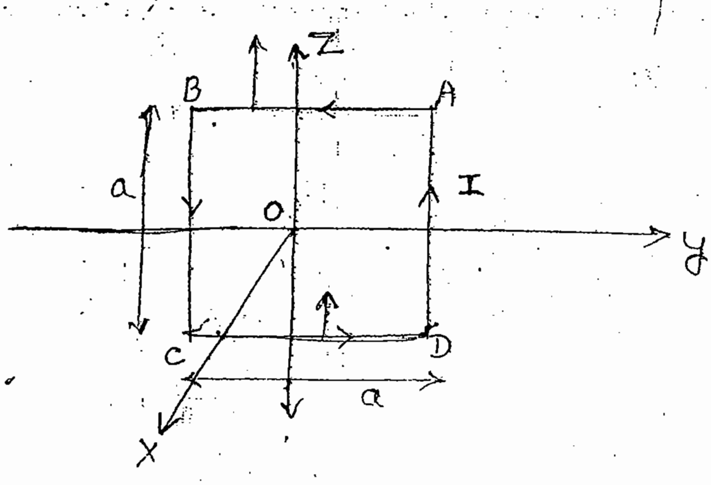
$$\boxed{\text{Linear frequency} = \frac{\omega}{2\pi} = \frac{I}{2\pi d}\sqrt{\frac{\mu_0}{\pi m}}}$$

$$\text{Time} = \frac{1}{\text{freq.}} \Rightarrow \boxed{T = \frac{2\pi d}{I}\frac{1}{\sqrt{\mu_0/\pi m}}}$$

Q. A Mag. field in some region is given by $\boxed{\vec{B} = kz\hat{x}}$ where K is constant. Find the force on a square loop of sides a lying in the y-z plane & centered at the origin if it carries a current I in the counter clockwise dirn when you look down the x-axis.

$$F = \int I d\vec{l} \times \vec{B}$$

$$= I\int d\vec{l} \times \vec{B}$$



for AB wire, Mag. field

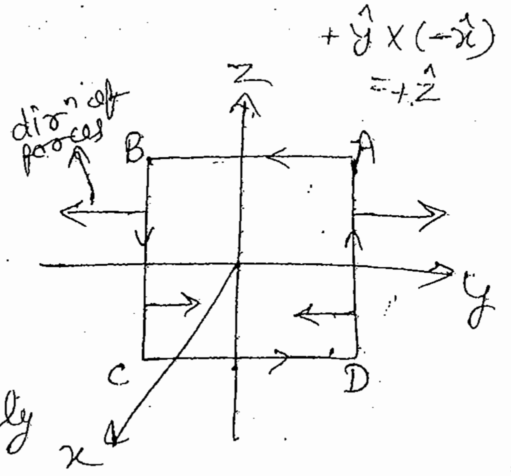
$$B = \frac{ka}{2}\hat{x}$$

$$\boxed{\vec{F}_{AB} = \frac{IKa^2}{2}\hat{z}}$$ $(\because \hat{y}\times\hat{x} = -\hat{z})$

On wire CD, Mag. field $B = \frac{ka}{2}(-\hat{x})$

$$\boxed{\vec{F}_{CD} = \frac{IKa^2}{2}\hat{z}}$$

$+\hat{y} \times (-\hat{x}) = +\hat{z}$



On AD upper half, Dir^n of $\vec{B} \rightarrow \hat{x}$

On BC " " " $\rightarrow \hat{x}$

On AD lower half Dir^n of $\vec{B} \rightarrow -\hat{x}$

BC " " " $\rightarrow -\hat{x}$

forces on AD & BC are oppositely directed so

$$\vec{F}_{AD} = \vec{F}_{BC} = 0$$

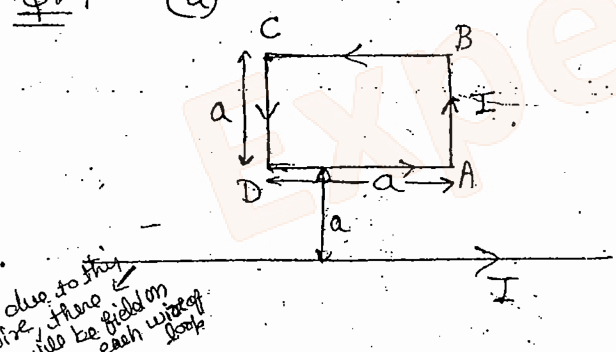
[Dir^n of mag. field is opposite, +z or −z]

Resultant force $\vec{F} = \left(\frac{IKa^2}{2} + \frac{IKa^2}{2}\right)\hat{z} \Rightarrow \boxed{\vec{F} = IKa^2\hat{z}}$

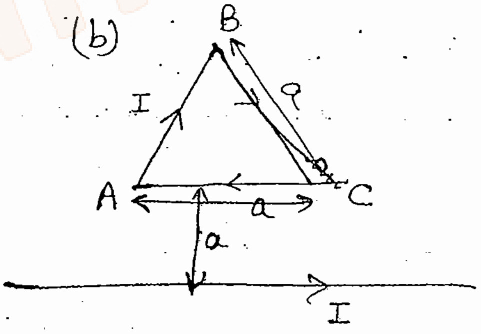
If $\vec{B} = K\hat{x}$ then Resultant $F = 0$

Becz dir^n of of mag. field on upper & lower part is same here & in previous case it is different.

Q:- (a)



(b)



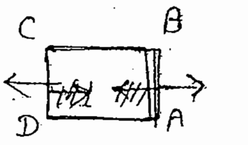
(a) Find the force on a square loop placed near an infinite straight wire. Both carries current I as shown in figure

(b) find the force on a equilateral triangle loop placed near infinite straight wire. Both carry current I.

(a) Dir^n of $\vec{B}$ for AB = ⊙

CD = ⊙

$F_{AB} = F_{CD} = 0$



$\vec{F} = I\int d\vec{l} \times \vec{B}$

$$F = I \int dl \times B$$

$$F_{DA} = I \frac{\mu_0 I a}{2\pi a} \qquad \vec{B} = \frac{\mu_0 I}{2\pi a} \odot$$

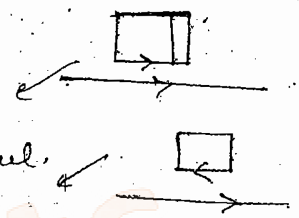
$$f_{DA} = \frac{\mu_0 I^2}{2\pi} \text{ (downward)}$$

$$F_{BC} = \frac{\mu_0 I^2 a}{2\pi(2a)} \Rightarrow F_{BC} = \frac{\mu_0 I^2}{4\pi} \text{ (upward)} \qquad \boxed{B \odot}$$

Resultant force $\boxed{\vec{F} = \frac{\mu_0 I^2}{4\pi}}$ (downward)

If current in DA wire & below wire is same then attractive and if opposite the repulsive.

So This force is <u>Attractive</u>.

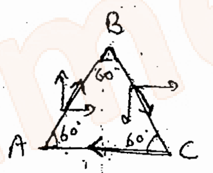


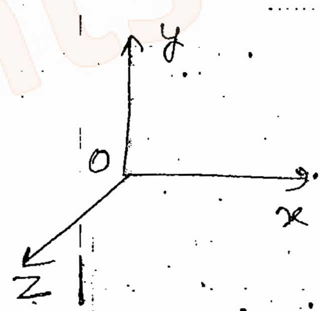
Q.(ii) Now wires are not parallel & perpendicular.
This loop is in x-y plane.

$$\vec{dl}_{AB} = dx\,\hat{x} + dy\,\hat{y}$$

$$\vec{dl}_{BC} = dx\,\hat{x} - dy\,\hat{y}$$

$$\vec{dl}_{CA} = -dx\,\hat{x}$$





$$\vec{dl} = dl_{AB} + dl_{BC} = 2dx\,\hat{x}$$

$$\sin 60 = \frac{h}{a}$$

$$\frac{\sqrt{3}}{2} = \frac{h}{a}$$

$$h = \frac{\sqrt{3}}{2} a$$

Mag. field for part ABC, $\vec{B} = \frac{\mu_0 I}{2\pi y}\hat{z}$

$$\vec{dF}_{ABC} = I(\vec{dl} \times \vec{B})$$

$$\vec{dF}_{ABC} = \frac{\mu_0 I^2}{2\pi y} 2dx(-\hat{y})$$

Total force $\vec{F}_{ABC} = \int \vec{dF}_{ABC} = \frac{\mu_0 I^2}{\pi}\int \frac{dx}{y}(-\hat{y})$

$$\tan\theta = \frac{y}{x} \Rightarrow \tan 60° = \frac{y}{x} \Rightarrow \sqrt{3} = \frac{y}{x} \Rightarrow y = \sqrt{3}\,x$$

Convert y into x bcoz in integration y is variable & integration is over x.

$$\vec{F}_{ABC} = \frac{\mu_0 I^2}{\pi} \int_{a/\sqrt{3}}^{a/\sqrt{3}+a/2} \frac{dx\,(-\hat{y})}{\sqrt{3}\,x}$$

from going $A \to B \to C$
x changes by a distance, a
{ y changes distance a to B.
if we convert in y }

$$= \frac{\mu_0 I^2}{\pi\sqrt{3}} \int_{a/\sqrt{3}}^{\frac{a}{\sqrt{3}}+\frac{1}{2}} \frac{dx}{x} (-\hat{y})$$

$$= \frac{\mu_0 I^2}{\pi\sqrt{3}} \ln\left(\frac{\frac{a}{\sqrt{3}}+\frac{a}{2}}{\frac{a}{\sqrt{3}}}\right)(-\hat{y})$$

As $y = \sqrt{3}x \Rightarrow x = \frac{y}{\sqrt{3}}$
when $y = a$, $x = a/\sqrt{3}$
$y = a + \frac{\sqrt{3}}{2}a$, $x = \frac{(a+\frac{\sqrt{3}}{2}a)}{\sqrt{3}}$

$$\boxed{\vec{F}_{ABC} = \frac{\mu_0 I^2}{\pi\sqrt{3}} \ln\left(1+\frac{\sqrt{3}}{2}\right)(-\hat{y})} \simeq 0.13$$

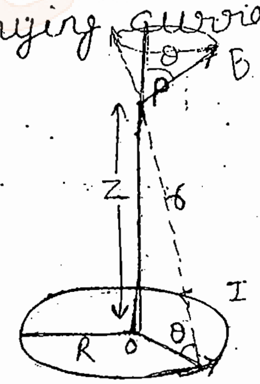
for CA Wire, $\vec{B} = \frac{\mu_0 I}{2\pi y}\hat{z}$ (out of the page)

So force $\boxed{\vec{F}_{CA} = \frac{\mu_0 I^2}{2\pi}(\hat{y})} \simeq 0.5$

{ $\frac{\mu_0 I}{2\pi a}$
$I\int d\vec{l}\times\vec{B}$
$\frac{\mu_0 I^2 a}{2\pi a}\hat{y}$ }

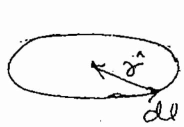
$\vec{F}_{CA}$ is greater than $\vec{F}_{ABC}$ so Resultant force is in $\hat{y}$ dir^n. This force will be <u>Repulsive</u>.

Q. Find the magnetic field at a distance z above the centre of a circular loop of radius R carrying current I.

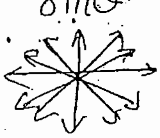


We Use Biot-Savart law.
Take a small element $d\vec{l}$.
dir^n of Mag. field $\to d\vec{l} \times \vec{r}$
Inclination of $\vec{r}$ w.r.to this plane is θ.
Here r is not in this plane while it is tilted by an angle θ
So mag. field will also tilt by an angle θ.
i.e. $\vec{r}$ tilt from horizontal = $\vec{B}$ tilt from vertical

{ if we have to find $\vec{B}$ at centre then $\vec{r}$ will be & dir^n of $\vec{B}$ will be $\hat{z}$ }



If we break the components of B then
$\sin\theta$ comp. will cancel out bcoz $\sin\theta$ comp. is rotating
Only $\cos\theta$ is responsible for $\vec{B}$.



$\cos\theta$ comp. adds up so it is along upward dir^n

So Net Mag. field will be in z-dirⁿ.

$$B_z = B\cos\theta$$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{\vec{dl}\times\vec{r}}{r^3} = \frac{\mu_0 I}{4\pi}\int \frac{dl\, r\sin\theta}{r^3}$$

$\theta = 90°$ { $\vec{r}$ makes angle with horizontal natu.. $\vec{dl}$. It is 90°}

$$B = \frac{\mu_0 I}{4\pi}\int \frac{dl}{r^2}$$

$$\cos\theta = \frac{R}{r} = \frac{R}{(R^2+z^2)^{1/2}}$$

$$\therefore B_z = \frac{\mu_0 I}{4\pi}\int \frac{dl}{(R^2+z^2)}\cdot\frac{R}{(R^2+z^2)^{1/2}} = \frac{\mu_0 I R}{4\pi}\int\frac{dl}{(R^2+z^2)^{3/2}}$$

$$= \frac{\mu_0 I R \times 2\pi R}{4\pi(R^2+z^2)^{3/2}}$$

$$\boxed{B_z = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}}$$ ✓

Limiting Cases :-

(1) At the centre of the loop :- $z = 0$

$$B_{centre} = \frac{\mu_0 I}{2R}$$

(2) $z >> R$. very far from the loop :- neglect R^2 bcoz z is very large

$$B_z = \frac{\mu_0 I R^2}{2z^3} \times \frac{2\pi}{2\pi}$$

$$B_z = \frac{\mu_0 I\, 2\pi R^2}{4\pi z^3}$$

At far distances, Mag. field $\propto \frac{1}{z^3}$ $z \rightarrow$ distance

(3) If loop contains N turns :-

$$B_z = N B_z$$ (bcoz current will become N times)

$$\boxed{B_z = \frac{N\mu_0 I R^2}{2(R^2+z^2)^{3/2}}}$$

Find the value of z at which mag. field is maximum :-

$$\frac{dB}{dz} = 0 \qquad B = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$$

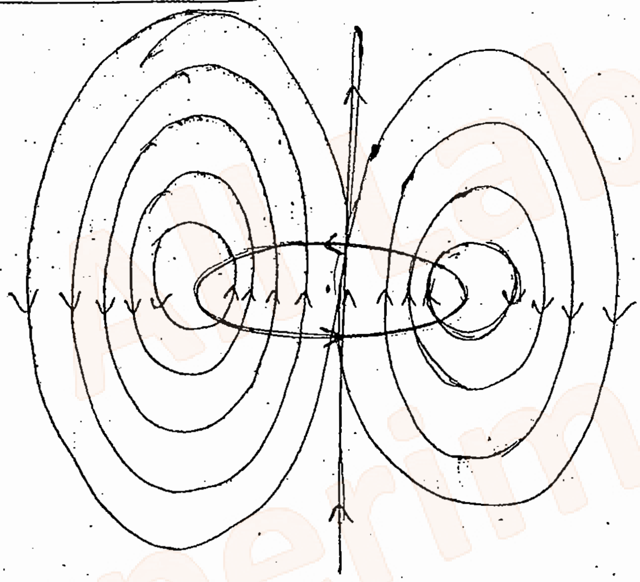
$$\frac{\mu_0 I R^2}{2} \cdot -\frac{3}{\cancel{2}} \frac{\cancel{2}z}{(R^2+z^2)^{5/2}} = 0$$

$$\Rightarrow \quad z = 0$$

i.e. At the centre Mag field will be maximum

Maxi. Field $B = \frac{\mu_0 I R^2}{2R^3} \Rightarrow \boxed{B_{max} = \frac{\mu_0 I}{2R}}$

Magnetic field lines :-



Q.(a) Find the magnetic field at the centre of a square loop which carries steady current I. R be the distance from centre to sides

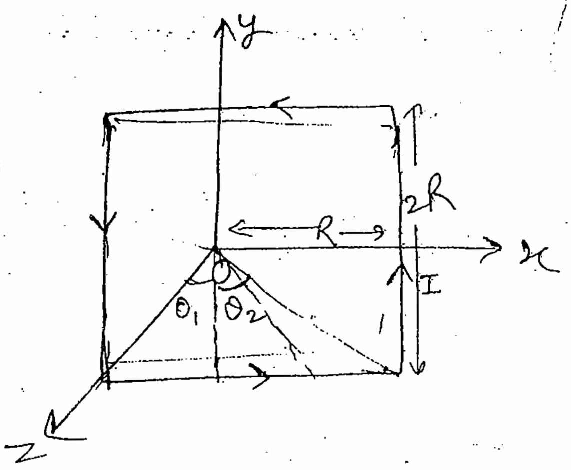
(b) Find the field at the centre of a N side polygon carrying a steady current I.

for all the wires dirn of B is outside.

It is additive so Mag field at centre will be 4 times of mag. field due to one wire.

Mag. field for a finite wire

$$B = \frac{\mu_0 I}{4\pi s}[\sin\theta_2 - \sin\theta_1]$$



for lower wire;

$$B = \frac{\mu_0 I}{4\pi R}\left[\sin 45^\circ - (-\sin 45^\circ)\right]$$

$$= \frac{\mu_0 I}{4\pi R} 2\sin 45^\circ = \frac{\mu_0 I}{4\pi R} 2 \times \frac{1}{\sqrt{2}}$$

$$\vec{B} = \frac{\mu_0 I \sqrt{2}}{4\pi R}\hat{z}$$

for all the sides i.e. for square loop

B = 4 times of (B for one wire)

$$\boxed{\vec{B} = \frac{\sqrt{2}\,\mu_0 I}{\pi R}\hat{z}}$$

(b) <u>general formula</u> :-

Mag. field for n-side polygon is

$$\boxed{B = \frac{n\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{n}\right)}$$

$I \rightarrow$ Current

$n \rightarrow$ sides

$R \rightarrow \perp$ distance from the mid point of wire

Now check it for Square loop

$$\vec{B} = \frac{4\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{4}\right) = \frac{2\mu_0 I}{\pi R}\frac{1}{\sqrt{2}}$$

$$\vec{B} = \frac{\sqrt{2}\,\mu_0 I}{\pi R}\hat{z}$$

<u>Circle</u> is ∞ sides polygon so $n \rightarrow \infty$

$$B = \frac{n\mu_0 I}{2\pi R}\cdot\frac{\pi}{n} \quad \because \theta \rightarrow 0$$

when $\theta = 0$, $\sin\theta \simeq \theta$

$$\boxed{B = \frac{\mu_0 I}{2R}}$$

* We can not apply this formula for ractangle. This is valid only for regular polygon. & ractangle is not a ragular polygon.

Q. Find the mag. field at the centre of a regular hexagon as shown in figure.

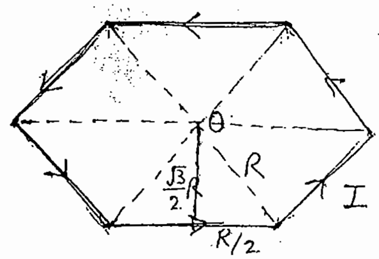
It is placed in y-z plane.

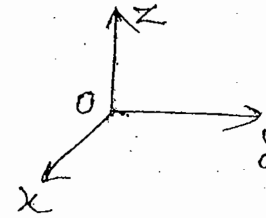
~~for square loop n = 6~~

Mag. field for n side polygon

$$B = \frac{n \mu_0 I}{2\pi R} \sin\left(\frac{\pi}{n}\right)$$

$n = 6$ for Hexagon





$$B = \frac{6 \mu_0 I}{2\pi (\frac{\sqrt{3}}{2} R)} \sin\frac{\pi}{6} = \frac{2\,3 \mu_0 I}{\sqrt{3}\pi R} \frac{1}{2}$$

$$B = \frac{\sqrt{3}}{\cancel{2}} \frac{\mu_0 I}{\pi R} (+\hat{z})$$

~~Rad~~ distance R in the formula is the ⊥ distance from centre to wire

Mag. field due to all the wires is outward so dir^n = $+\hat{z}$

so $\boxed{B = \frac{\sqrt{3} \mu_0 I (+\hat{z})}{\pi R}}$

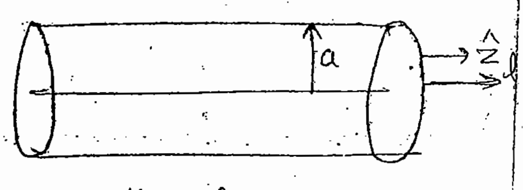
Q. A current I flows down a wire of radius a.

(a) If it is uniformly distributed over the surface. What is surface current density K.

(b) If it is distributed in such a way that the volume current density is inversely proportional to the distance from the axis then find J.

$[\vec{K} = \sigma \vec{v}]$

(a) $K = \frac{I}{l_\perp}$



$l_\perp$ length ⊥ to the flow of current = $2\pi a$ (circumference of cross section of radius a)

$$\boxed{K = \frac{I}{2\pi a} \hat{z}}$$

(b) $J \propto \frac{1}{s}$ (distance from axis → s vector away from axis → $\hat{s}$)

$$J = \frac{K}{s}$$

$$\int \vec{J} \cdot d\vec{S}_\perp = I$$

$$\int_0^{a}\int_0^{2\pi} \frac{K}{s} \cdot s\, ds\, d\phi\, \hat{z} = I$$

area element $\Rightarrow$
$d\vec{S}_\perp = s\, ds\, d\phi\, \hat{z}$

$$\int_0^{2\pi}\int_0^{a} K\, ds\, d\phi = I$$

$$K\,(s)_0^a\,(2\pi) = I \Rightarrow I = 2\pi K a$$

$$K = \frac{I}{2\pi a}$$

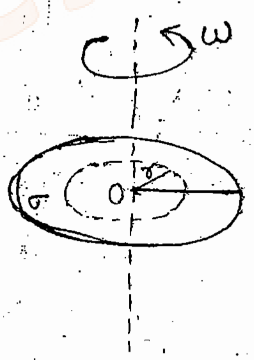
$$\vec{K} = \frac{I}{2\pi a}\hat{z}$$

So $\boxed{J = \dfrac{I}{2\pi a s}\hat{z}}$ Ans

Qu :- If phonograph records carries a uniform density of static electricity is σ. If it rotates at angular velocity ω. What is the current density K at a distance r from the centre.

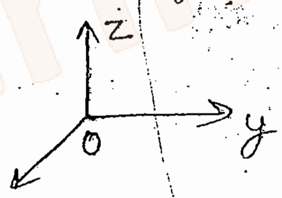
dir^n of $\omega \rightarrow \hat{z}$

Curl the fingers in the dir^n of rotation.



Surface current density,

$\vec{K} = \sigma\vec{v}$ ——(1)



dir^n of $\vec{K}$ will be same as of $\vec{v}$.

$\vec{v} = \vec{\omega} \times \vec{r}$

$= \omega r[\hat{z} \times \hat{r}]$

$(\vec{r} = r\hat{r})$

$\vec{v}$ will be tangential but ω will be $\perp$ to the plane

$\boxed{\vec{v} = \omega r \hat{\phi}}$

dir^n of $\vec{v}$ will be $\hat{\phi}$.

$\theta = 90°$.

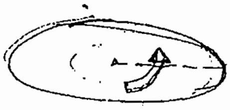
{ If We write $\hat{r}$ in cartisian then
$z \times r = \sin\theta\, \hat{\phi}$ & $\theta = 90°$ so $\hat{\phi}$ }

$$\vec{K} = \sigma \vec{v}$$

$$\Rightarrow \boxed{\vec{K} = \sigma \omega r \hat{\phi}}$$

If R is the radius then current I_T = ?

$$K = \frac{I}{\ell_\perp}$$



dir^n $\perp$ to the flow of current $\rightarrow \hat{r}$
element = dr

Total current $I = \int K \, dl$

$$I = \int_0^R \sigma \omega r \, dr$$

$$\boxed{I = \frac{\sigma \omega R^2}{2}}$$

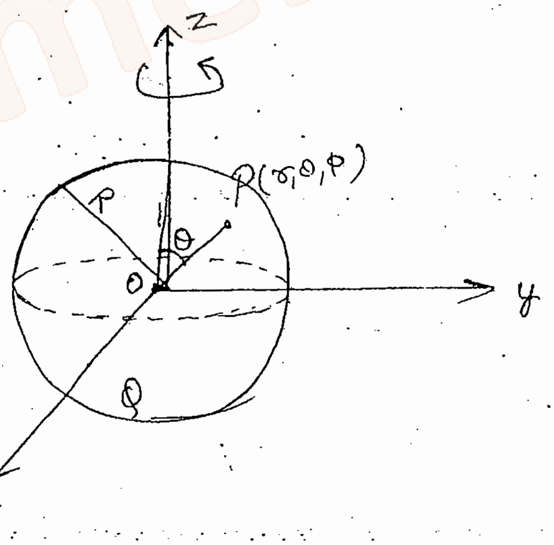
Q.:- A uniformly charged solid sphere of radius R & total charge Q is centred at the origin & spining at a constant angular velocity ω about z-axis. Find the current density J at r, θ, ϕ within the sphere.

$\vec{J}$ at r, θ, ϕ.

$$\vec{J} = \rho \vec{v}$$

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$$

As the sphere is uniformly charged otherwise we can not write like this.



dir^n of $\omega \rightarrow \hat{z}$

$$\vec{\omega} = \omega \hat{z}$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$= \omega r (\hat{z} \times \hat{r})$$

$$= \omega r \sin\theta \, \hat{\phi}$$

$$\hat{z} \times \hat{r} = (\cos\theta \, \hat{r} - \sin\theta \, \hat{\theta}) \times \hat{r}$$

$$= -\sin\theta (\hat{\theta} \times \hat{r})$$

$$= \sin\phi \, \hat{\phi}$$

$$\vec{J} = \rho \omega r \sin\theta \, \hat{\phi}$$

$$\vec{J} = \frac{Q}{\frac{4}{3}\pi R^3} \omega r \sin\theta \, \hat{\phi}$$

Total Current = ?

$$I = \int \vec{J} \cdot d\vec{S}$$

$$dS_\phi = r\,dr\,d\theta$$

$$I = \int_0^R \int_0^\pi \rho \omega r \sin\theta \, r\,dr\,d\theta$$

$$I = \rho\omega \frac{R^3}{3} \left(-\cos\theta\right)_0^\pi = \frac{\rho \omega R^3}{3}(1+1)$$

$$I = \frac{2\omega \rho R^3}{3}$$

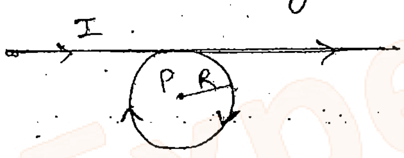
$$I = \frac{2}{3}\omega \cancel{R^3} \frac{Q}{\frac{4}{3}\pi \cancel{R^3}} = \frac{Q\omega}{2\pi}$$

$$\boxed{I = \frac{Q\omega}{2\pi}}$$

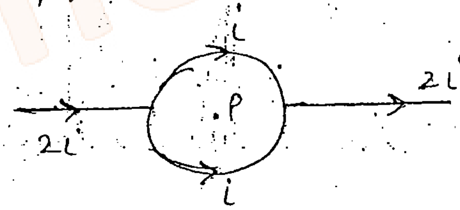
This total current is due to volume charge density. There may be some current at surface also but there is not given σ & A. If given then Total current will be due to surface current density + vol. current density.

Q. Find the mag. field at point P.

(i)

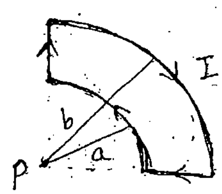


(ii)

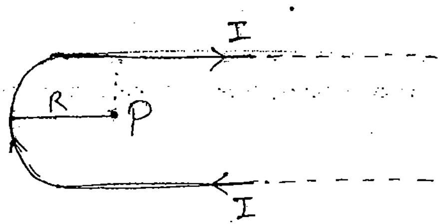


(iii)

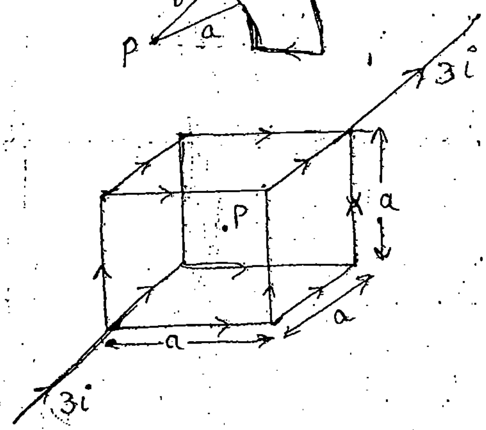
(iv)



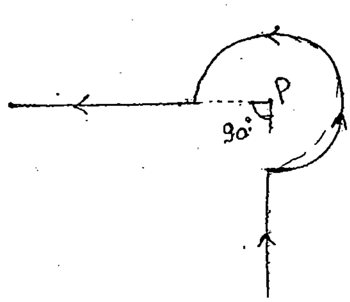
(v)



(vi)



(vii)



(i) Mag. field of circular loop $= \frac{\mu_0 I}{2R}$ (inward) (bcoz of circle)

In the ∞ wire, $\vec{B} = \frac{\mu_0 I}{2\pi R}$ (inward)

Total Mag. field $= \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R}$

$$\boxed{B_P = \frac{\mu_0 I}{2R}\left(1 + \frac{1}{\pi}\right)}$$

Due to $\perp$ bend in wire, Mag. field $\uparrow$ Otherwise in straight wire it is $\mu_0 I/2\pi R$.

(ii) Due to lower half $= \frac{\mu_0 I}{4 \cancel{2\pi} R}$ out of page

upper " $= \frac{\mu_0 I}{4R}$ into the page

for Half circle $\vec{B} = \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right)$

Total $\vec{B}$ at P $= \frac{\mu_0 I}{4R} - \frac{\mu_0 I}{4R}$

$$\boxed{B = 0}$$

Due to the wires no contribution. So Total $B = 0$

$\left(B = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \vec{r}}{r^3}\right)$ $\angle$ b/w dl & $r = 0$

(iii) Dir^n of $\vec{B}$ = outward (for circle)

$\vec{B} = \frac{\mu_0 I}{2R}$

Due to wire $\Rightarrow$ $B = 0$

Total $\boxed{B = \frac{\mu_0 I}{2R}}$

(iv) $\frac{1}{4}$th part of circle

for a radius part $B = \frac{1}{4}\frac{\mu_0 I}{2a} = \frac{\mu_0 I}{8a}$ (out of page)

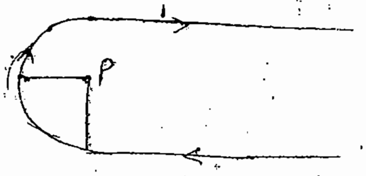
" b " $B = \frac{\mu_0 I}{8b}$ (into ")

Contribution of B is only due to curved path.

due to other 2 path $B = 0$ ($\angle$ b/w $\hat{r}$ & surface $= 0$)

Total $\boxed{B = \frac{\mu_0 I}{8}\left[\frac{1}{a} - \frac{1}{b}\right]}$ (out of page)

(V) It is a combination of half circle & 2 semi ∞ wire.
Due to all 3 paths dir^n of $\vec{B}$ is into the page.



$\theta_2 = 90°$

~~for lower~~ $\theta_1 = 0$

wire, $B = \frac{\mu_0 I}{4\pi R}[\sin\theta_2 - \sin\theta_1]$

$= \frac{\mu_0 I}{4\pi R}$

point P is from mid point of wire

for upper half ∞ wire,

$B = \frac{\mu_0 I}{4\pi R}$

So Total B due to both wires $= \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4\pi R} = \frac{\mu_0 I}{2\pi R}$

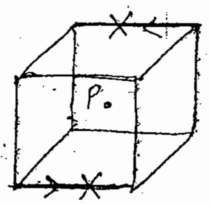
Due to curved wire $\vec{B} = \frac{\mu_0 I}{4R}$

So Total Mag. field at P, $B = \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{2\pi R}$

$$\boxed{B = \frac{\mu_0 I}{2R}\left[\frac{1}{2} + \frac{1}{\pi}\right]}$$

Mag. field Depends on the shape of wire

(Vi) Same Current flow in all the wires of same type as the resistance of wires are same.
So Mag. field $= 0$



(In these 2 surfaces B is same, distance from P is same, only dir^n ~~of~~ of B is different so they cancell out.)

for Digonally opposite sides, $\vec{B}$ have opposite dir^n so B will cancel out.

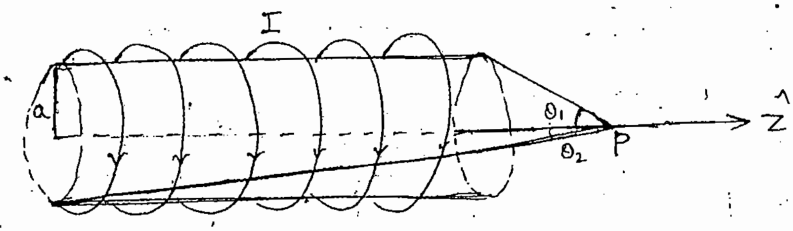
So $\boxed{\text{Total } B = 0}$

(Vii) Circular path → $\frac{3}{4}$ circle

$B = \frac{3}{4} \times \frac{\mu_0 I}{2R} \Rightarrow B = \frac{3\mu_0 I}{8R}$ (out of page)

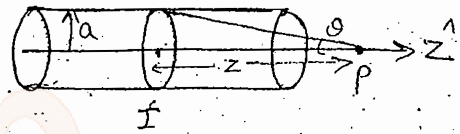
Due to wires $\Rightarrow B = 0$

Q. Find the mag. field at a point on the axis of a tightely wound solanoid consisting of a small n turns/unit length & carrying current I. Express your answer in terms of θ_1 & θ_2 (finite Solanoid).



The mag-field of a single circle is

$$B = \frac{\mu_0 I a^2}{2(a^2+z^2)^{3/2}}$$



There are many circles bounded tightely ~~by~~ with each other.

Total no. of turns $= n$, length dz.

- total no. of turns in length $dz = n\,dz$
- Total current in this thickness $= I n dz$

from Biot - Savart law,

$$dB = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\vec{r}}{r^3}$$

$$\cancel{dB} = \frac{\mu_0 \cancel{I}\, n \cancel{dz}}{\cancel{4\pi}}$$

$$dB = \frac{\mu_0 a^2 n I\, dz}{2(a^2+z^2)^{3/2}} \qquad \text{dir}^n \text{ of } B \rightarrow \hat{z}$$

$$B = \int dB$$

$$= \frac{\mu_0 a^2 n I}{2}\int \frac{dz}{(a^2+z^2)^{3/2}}$$

$$= \frac{\mu_0 a^2 n I}{2}\int \frac{-a\,\operatorname{cosec}^2\theta\, d\theta}{a^3(1+\cot^2\theta)^{3/2}}$$

$$= \frac{\mu_0 \cancel{a^2} n I}{2}\int \frac{-\cancel{a}\,\operatorname{cosec}^2\theta\, d\theta}{\cancel{a^3}\,\operatorname{cosec}^3\theta}$$

$$= \frac{\mu_0 n I}{2}\int_{\theta_1}^{\theta_2} -\sin\theta\, d\theta$$

Rel^n b/w θ & z

$$\tan\theta = \frac{a}{z}$$

$$z = a\cot\theta$$

$$dz = -a\operatorname{cosec}^2\theta\, d\theta$$

As $dz \uparrow$, there will be two ends. & limits from θ_1 to θ_2

$$\therefore B = \frac{\mu_0 n I}{2}\left[\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$\boxed{B = \frac{\mu_0 n I}{2}\left[\cos\theta_2 - \cos\theta_1\right]\hat{z}}$$

Limiting Cases :-

(i) Mag. field of a Infinite Solenoid :-

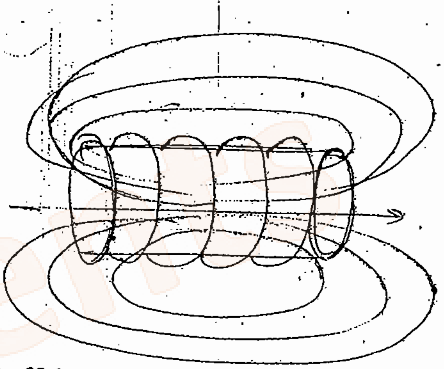
$$\theta_1 = 180^\circ$$
$$\theta_2 = 0^\circ$$

$$\boxed{\vec{B} = \mu_0 n I \hat{z}}$$

Mag. field inside the solenoid is constant.

{ for a finite solanoid ; mag. field inside the solanoid is still $\vec{B} = \mu_0 n I \hat{z}$

(ii) Outside the solanoid :-



$$B_{out} = 0$$

Practically B at pt. P is not zero but very weak. so theoritically we assume it is zero.

Bcoz mag. field lines of upper & lower are opposite.

for a short Solanoid, $\theta_1 = \theta_2$ so $B = 0$

(iii) At End point :- $\theta_2 = 0$; $\theta_1 = 90^\circ$

$$B_{end} = \frac{\mu_0 n I}{2}$$

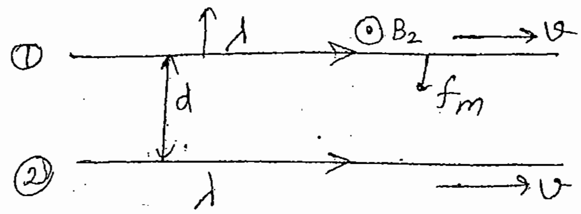
Mag field at the end of solanoid is half of the field ~~at~~ inside the solanoid.

- Dirn of Wire & solanoid are interchangable.

dirn of M.F. $\rightarrow \hat{z}$, current $\rightarrow \hat{\phi}$ } solanoid

$I \rightarrow \hat{z}$, M.F. $\rightarrow \hat{\phi}$ } Wire

Q. Suppose 2 ∞ line charges λ a distance d apart moving at a constant speed v. Calculate the value of v (speed) in order for mag. attraction to balance electrical repulsion.



If λ & λ are +ve charges moving with velocity v, there must be the electrical repulsion.

But current is flowing in same dirⁿ (current will be in same dirⁿ as v) so there will be mag attraction.

Electric force $\vec{F_e} = q\vec{E}$

wires are ∞ line charges so force per unit length

$$f_e = \lambda E$$

force/unit length for wire ~~or~~ (1) $f_{e_1} = \lambda_1 E_2$

$$f_{e_1} = \frac{\lambda_1 \lambda}{2\pi\epsilon_0 d} = \frac{\lambda^2}{2\pi\epsilon_0 d} \quad \text{(upward)} \qquad \text{——(1)}$$

same force will be on wire (2) but in opposite dirⁿ but we are interested only in magnitude.

Magnetic force $F_m = q(\vec{v} \times \vec{B})$

$$f_{m_1} = \lambda_1 (\vec{v_1} \times \vec{B_2}) \quad \text{(downward)}$$

$$B_2 = \frac{\mu_0 I}{2\pi d} \text{ (out of page)} = \frac{\mu_0 \lambda v}{2\pi d}$$

$I = \frac{q}{t}$

$= \lambda$ ~~v~~

$\theta = 90°$

$$f_{m_1} = \lambda_1 v_1 B_2$$

$$f_{m_1} = \frac{\mu_0 \lambda^2 v^2}{2\pi d} \qquad \text{——(2)}$$

These 2 forces must balance each other so

$$\frac{\mu_0 \cancel{\lambda^2} v^2}{\cancel{2\pi} \cancel{d}} = \frac{\cancel{\lambda^2}}{\cancel{2\pi}\epsilon_0 \cancel{d}}$$

$$v^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\mu_0 = 4\pi \times 10^{-7} \Rightarrow \quad v = \frac{1}{\sqrt{4\pi\epsilon_0 \times 10^{-7}}} = \frac{1}{\sqrt{\frac{1}{9\times 10^9} \times 10^{-7}}}$$

$$v = 3 \times 10^8 \ m/c$$

$$v = c \ m/s$$

- If $v < c$ then electric force will dominent.
  $v \uparrow$, $F_m \uparrow$

AMPERE's Law :- This law is valid only in Magnetostatic

In Electrostatics → Coulomb law ⟷ Biot-Savert law
Gauss law ⟷ Ampere's law

$$\boxed{\oint B \cdot dl = \mu_0 I_{enc}} \Rightarrow \text{Integral form of Ampere's law.}$$

By using Stokes theorm, closed line integral can be written as open surface integral.

$$\Rightarrow \int_S (\vec{\nabla} \times \vec{B}) \cdot d\vec{s} = \mu_0 \int_S \vec{J} \cdot d\vec{s}$$

$$\Rightarrow \int_S (\nabla \times B - \mu_0 J)\, ds = 0$$

$$\text{If} \quad \boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}} \Rightarrow \text{Differential form}$$

Compare with Electrostatics :-

$\nabla \cdot E = \rho/\epsilon_0$	$\nabla \cdot B = 0$
$\nabla \times E = 0$	$\nabla \times B = \mu_0 J$

Curl $B \neq 0$

So Mag. field is not a Conservative field, ~~and~~

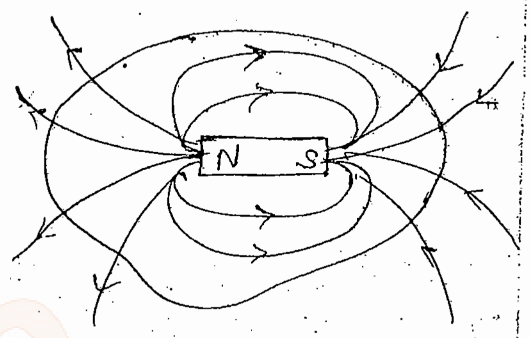
Physical significance of $\nabla \cdot B = 0$ is Non existance of magnetic monopole.

Integral form of $\nabla \cdot \vec{B} = 0$ is

$$\boxed{\oint_S \vec{B} \cdot d\vec{s} = 0}$$ (by using Gauss law)

It is also called
Gauss law in Magnetostatic $\Rightarrow$ $\vec{\nabla} \cdot \vec{B} = 0$ or $\oint_S \vec{B} \cdot d\vec{s} = 0$

If we have a close surface & put a magnet into this closed surface then mag-flux passing through closed surface this $= 0$.



Mag. field lines originates at North pole & terminats at south pole.

$\boxed{\text{line enter in south pole} = \text{line leaving North pole}}$

Net no. of field lines coming or going $= 0$

So Magnetic monopole do not exist.

Theoritically Mag. monopole exist but No experimental proof till now.

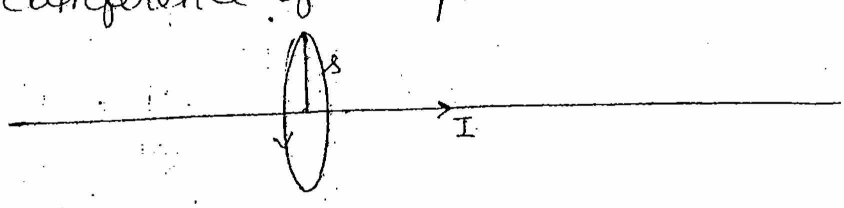
Note: If we want to find out the mag. field from Ampere's law than there should be symmetry in problem.

Que:- Find the magnetic field at a distance s from a long straight wire carrying a steady current I.

(If we have a short wire then the dir^n of $\vec{B}$ will be complex i.e. there will be complexity. dir^n of $\vec{B}$ at P $\rightarrow$ outward
but $\rightarrow$ complexity (at P in this case, we can't define dir^n of $\vec{B}$)
Short wire

So wire should be long or infinite.

Sketch a Amperical loop s.t. point should be lie on the circumference of loop.



$I_{enc} = I$

Ampere's law $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

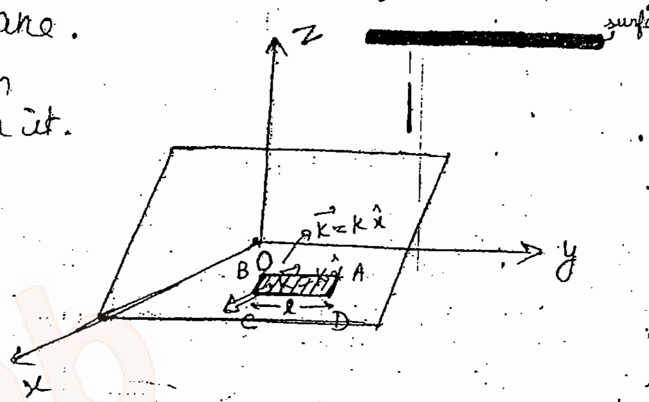
$B \cdot 2\pi s = \mu_0 I$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi s}\,\hat{\phi}}$$

Q. Find the mag. field of an infinite uniform surface current $\vec{K} = K\hat{x}$ flowing over x–y plane.

Amperical loop should be choosen s.t. current must pass through it.

surface is very thin.



$\oint \vec{B}\cdot d\vec{l} = \int_{AB} + \int_{BC} + \int_{CD} + \int_{DA}$

If we put to many wires together then they make a surface.

$\int_{AB} \vec{B}\cdot d\vec{l} = B \cdot l \cos 0 = Bl$

$\int_{BC} \vec{B}\cdot d\vec{l} = B \cdot l \cos 90 = 0$

$\int_{CD} \vec{B}\cdot d\vec{l} = B \cdot l \cdot \cos 0 = Bl$

$\int_{DA} \vec{B}\cdot d\vec{l} = B \cdot l \cos 90 = 0$

Right hand
Palm on surface
Thumb in the dir^n of current
fingers will tell the dir^n of $\vec{B}$

i.e. for upper surface dir^n of $\vec{B} \Rightarrow -\hat{y}$
lower surface $\Rightarrow +\hat{y}$

So $Bl + 0 + Bl + 0 = \mu_0 I_{enc}$

$K = \frac{I}{l_\perp} \Rightarrow \frac{I}{l} = K \Rightarrow I = Kl$

so $2B\cancel{l} = \mu_0 K \cancel{l} \Rightarrow B = \frac{\mu_0 K}{2}$

$$\boxed{\begin{aligned} \vec{B} &= \frac{\mu_0 K}{2}(-\hat{y}), \quad z>0 \\ \vec{B} &= \frac{\mu_0 K}{2}(+\hat{y}), \quad z<0 \end{aligned}}$$

(upper surface)
(lower surface)

This mag. field is <u>independent of distance</u>.

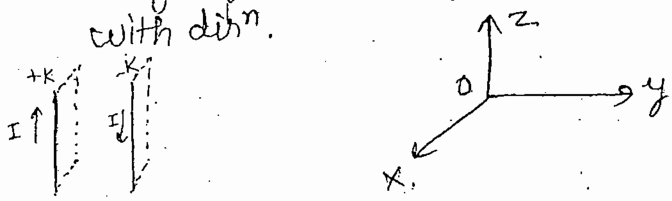
If we replace $\mu_0 \to \frac{1}{\epsilon_0}$ & $K \to \sigma$ then, we get

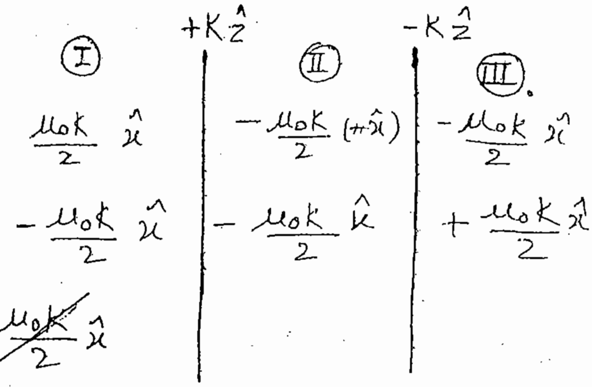
$\vec{E} = \frac{\sigma}{2\epsilon_0}(+\hat{z}), \quad z>0$

$\vec{E} = \frac{\sigma}{2\epsilon_0}(-\hat{z}), \quad z<0$

so $\vec{E}$ & $\vec{B}$ are Equivalent.

Q. If we have 2 parallel surfaces one carries surface current $+K$ & another carries surface current $-K$. find the mag. field in region I, II, III with dirn.





In (I) region, $\vec{B} = \cancel{\frac{\mu_0 K}{2}\hat{x}} - \cancel{\frac{\mu_0 K}{2}\hat{x}}$

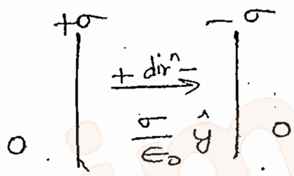
$\vec{B} = 0$

(III) region, $\vec{B} = 0$

(II) region, $\vec{B} = -\frac{\mu_0 K}{2}\hat{x} - \frac{\mu_0 K}{2}\hat{x}$

$$\boxed{\vec{B} = \mu_0 K(-\hat{x})}$$

i.e. If 2 plates have equal & opposite surface current then mag. field in b/w plates is Non-zero. & outside is zero. similar as Ele. field

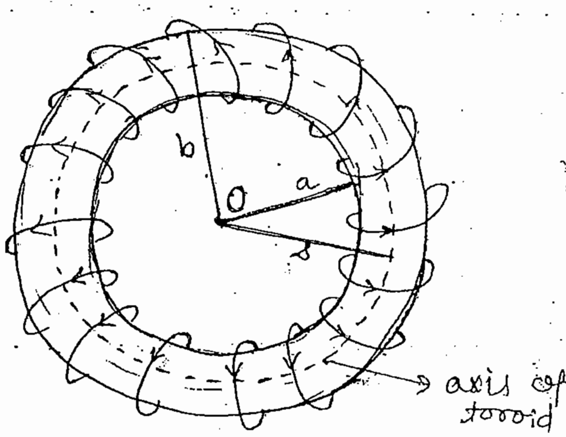


Q. Mag. field of a toroidal coil [Toroid is endless solanoid]

If Toroid contains N no. of total turns then Magnetic field inside the toroidal coil is

$$\boxed{\vec{B}_{in} = \frac{\mu_0 NI}{2\pi s}\hat{\phi}}, \quad a<s<b$$

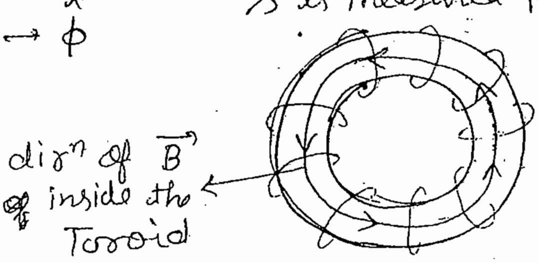
$$\boxed{\vec{B}_{out} = 0}, \quad s>b \text{ or } s<a$$



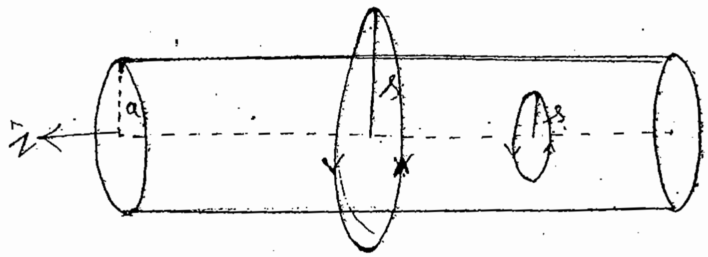
Mag field of a solanoid is along $\rightarrow \hat{z}$

but " " " toroid " $\rightarrow \hat{\phi}$

s is measured from origin



Q. A steady current I flows down a long cylindrical wire of radius a. Find the mag. field both inside & outside the wire if (a) current is uniformly distributed over the outside surface of the wire (b) current is distributed in such a way that $J \propto s$.



(a) $I_{enc} = 0$ (Inside)

$\oint B \cdot dl = \mu_0 I$

$\Rightarrow \boxed{B_{in} = 0}$

Outside :- $I_{enc} = I$

$\oint B_{out} \cdot dl = \mu_0 I \Rightarrow B_{out} \cdot 2\pi s = \mu_0 I$

$$\boxed{\vec{B}_{out} = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

(b) $J \propto s$

$J = Ks$

$I = \int \vec{J} \cdot d\vec{s} = \int_0^{2\pi}\int_0^{a} Ks \, s\,ds\,d\phi$

$I = K\left(\frac{s^3}{3}\right)_0^a (2\pi) = K\frac{a^3}{3}2\pi$

$$\boxed{K = \frac{3I}{2\pi a^3}}$$

$$\boxed{J = \frac{3Is}{2\pi a^3}}$$

Inside : $I_{enc} = \int_0^{2\pi}\int_0^{s} J \cdot ds = \int_0^{2\pi}\int_0^{s} \frac{3I}{2\pi a^3} s \cdot s\,ds\,d\phi$

$I_{enc} = \frac{Is^3}{a^3}$

$\oint B_{in} \cdot dl = \mu_0 I_{enc} \Rightarrow B_{in} \cdot 2\pi s = \mu_0 \frac{Is^3}{a^3}$

$$\boxed{B_{in} = \frac{\mu_0 I s^2}{2\pi a^3}\hat{\phi}}$$

Outside :- $I_{enc} = I$

$$\boxed{\vec{B}_{out} = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

[same as part a]

Note :- If $J \propto s^n$ then

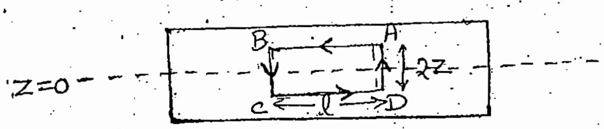
$B_{in} \propto s^{n+1}$, $B_{out} \propto \frac{1}{s}$

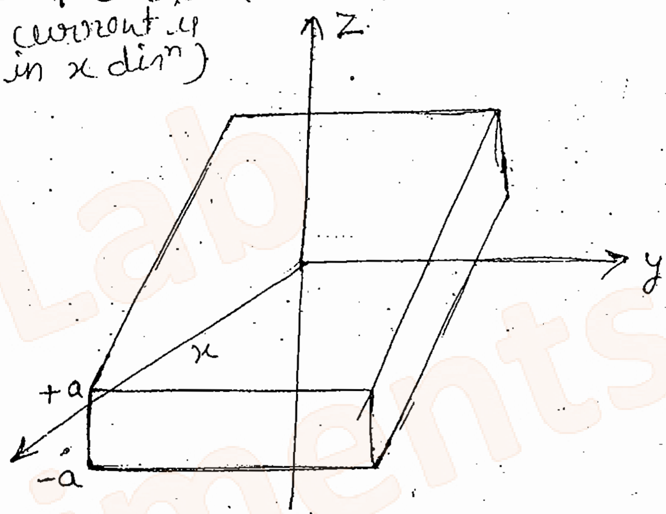
$\rho \propto s^n$

$E_{in} \propto s^{n+1}$, $E_{out} \propto \frac{1}{s}$

Q. A thick slab extending from $z = -a$ to $z = +a$ carries a uniform volume current $\vec{J} = J\hat{x}$. Find the magnetic field as a funcⁿ of z both inside & outside the slab.

$\vec{J} = J\hat{x}$ (i.e. current is in x dirⁿ)





$\vec{B} \rightarrow -\hat{y}$ for $z > 0$

$\vec{B} \rightarrow \hat{y}$ $z < 0$

Inside Current enclosed by the loop

$I = JA_\perp$

$$\boxed{I = J\,2z\,l}$$

Inside Mag. field

$$\oint \vec{B}\cdot d\vec{l} = \int_{AB} B\cdot dl + \int_{BC} B\cdot dl + \int_{CD} B\cdot dl + \int_{DA} B\cdot dl$$

$$= Bl + 0 + Bl + 0 = 2Bl$$

$\oint B\cdot dl = \mu_0 I_{enc}$

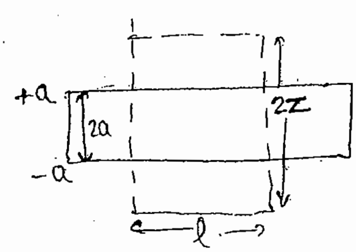
$\cancel{2}Bl = \mu_0 J \cancel{2} z \cancel{l} \Rightarrow B = \mu_0 J z$

$$\boxed{\begin{aligned}\vec{B}_{in} &= \mu_0 J z(-\hat{y}), \quad z > 0 \\ \vec{B}_{in} &= \mu_0 J z(+\hat{y}), \quad z < 0\end{aligned}}$$

So Mag. field inside is linearly ↑ (as it depends on distance z)

Outside $I = J \cdot 2al$

$\oint B\cdot dl = 2Bl$



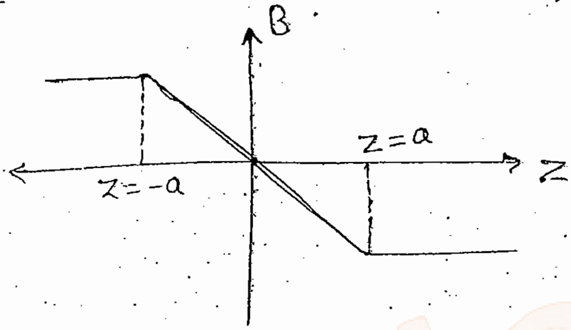
$$2Bl = \mu_0 \cdot J\, 2al$$

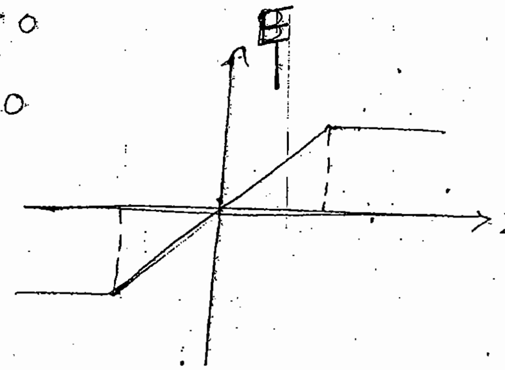
$$B_{out} = \mu_0 J a \quad \text{(constant)}$$

Outside mag. field is constant.

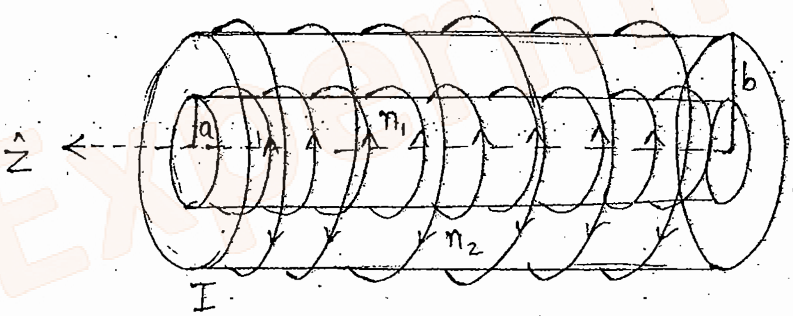
$$\boxed{\begin{aligned} B_{out} &= \mu_0 J a(-\hat{y}) \\ B_{out} &= \mu_0 J a(+\hat{y}) \end{aligned}} \quad \begin{aligned} &, z>0 \\ &, z<0 \end{aligned}$$

Plot :- Similar as electric field.





Q. :- Two long co-axial solenoids each carry current I but in opposite dirn as shown in the figure. The inner solenoid radius a has n_1 turns per unit length and outer one radius b has n_2 turns per unit length. Find mag. field in 3 regions.



(i) $s < a$
(ii) $a < s < b$
(iii) $s > b$

(i) $s < a$, region is inside both the solenoid.

for inner solenoids, $S_1 \rightarrow \quad \vec{B} = \mu_0 n_1 I\, \hat{z}$

outer $\quad S_2 \rightarrow \quad \vec{B} = -\mu_0 n_2 I\, \hat{z}$

$$\boxed{\vec{B} = \mu_0 (n_1 - n_2) I\, \hat{z}}$$

(ii) $a < s < b$, $\quad B = 0$ for S_1

$B = \mu_0 n_2 I(-\hat{z})$ for S_2

$$\boxed{\vec{B} = \mu_0 n_2 I(-\hat{z})}$$

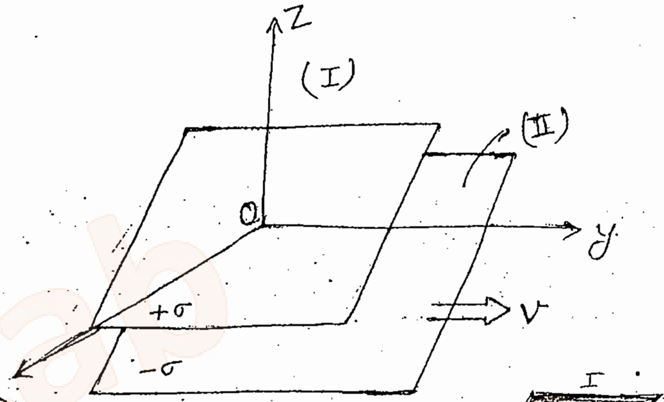
(iii) $s > b$

$B_{total} = 0$ for both S_1 & S_2 as $I_{enc} = 0$

Q. A large parallel plate capacitor with uniform surface charge σ on the upper plate & $-\sigma$ on lower plate moving with constant speed v. (a) find $\vec{B}$ b/w the plates & outside the plates.
(b) find the mag. force per unit area (pressure) on the upper plate with dir^n
(c) What should be the value of v ~~to balance~~ such that magnetic ~~attraction~~ repulsion balances the electrical attraction.

(a) $\vec{K}_{upper} = \sigma v \hat{y}$

$\vec{K}_{lower} = \sigma v(-\hat{y})$

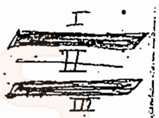


Due to upper plate $\vec{B}$ in region

I $\rightarrow$ $\frac{\mu_0 K}{2}\hat{x}$

II $\rightarrow$ $-\frac{\mu_0 K}{2}\hat{x}$

III $\rightarrow$ $-\frac{\mu_0 K}{2}\hat{x}$



{ for $+\sigma$, dir^n of K is same as v
for $-\sigma$, opposite to v

Due to lower plate, $\vec{B}$ in

I $\rightarrow$ $-\frac{\mu_0 K}{2}\hat{x}$, II $= -\frac{\mu_0 K}{2}\hat{x}$, III $= \frac{\mu_0 K}{2}\hat{x}$

So In region I, $\boxed{\vec{B} = 0}$, In III, $\boxed{\vec{B} = 0}$

In Region II, $\vec{B} = -\frac{\mu_0 K}{2}\hat{x} - \frac{\mu_0 K}{2}\hat{x}$

$\vec{B} = -\mu_0 K\hat{x} = \mu_0 K(-\hat{x})$

$$\boxed{\vec{B} = \mu_0 \sigma v(-\hat{x})}$$

Magnitude of $K = \sigma v$

(b). Magnetic force, $\vec{F} = q(\vec{v} \times \vec{B})$

In terms of surface current $\vec{F}_m = \int (\vec{K} \times \vec{B}) \cdot d\vec{a}$

Mag. force / unit area $\vec{f}_{mu} = (\vec{K}_u \times \vec{B}_L)$

mag. repulsion $\leftarrow$ $\boxed{\vec{f}_{m\,(upper)} = \frac{\mu_0 \sigma^2 v^2}{2}(+\hat{z})}$

dir^n of B due to σ = $\uparrow B$, $\rightarrow x$

dir^n of $\vec{K}_u \rightarrow \hat{y}$, dir^n of $B_L \rightarrow -\hat{x}$

so $f_m \rightarrow$ $y \times (-x) = +\hat{z}$

IIly f_m' for lower plate $\vec{F}_m = \frac{\mu_0 \sigma^2 v^2}{2}(-\hat{z})$.

(c) $\vec{F} = q\vec{E}$

$$f_{e_{\text{upper}}} = \sigma_u \vec{E}_l = \sigma \frac{\sigma}{2\epsilon_0}(-\hat{z})$$

$$\frac{\mu_0 \sigma^2 v^2}{2} = \frac{\sigma^2}{2\epsilon_0}$$

$$\Rightarrow v^2 = \frac{1}{\epsilon_0 \mu_0} \Rightarrow \boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c}$$

The speed for which d remains constant (when attractive force = repulsion forces) (d → distance b/w 2 plates)

Magnetic Vector Potential ($\vec{A}$): As for electrostatic field,

$$\boxed{\vec{\nabla} \times \vec{E} = 0} \Rightarrow \vec{E} = -\vec{\nabla} V$$

$V \rightarrow$ scalar potⁿ.

; If we put the V in $\nabla \times E$ then it still 0 bcz curl (grad) = 0

$$\& \quad \boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}} \Rightarrow \boxed{\nabla^2 V = -\rho/\epsilon_0} \rightarrow \text{Poission's eqⁿ in electrostatics.}$$

But here, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ & $\vec{\nabla} \cdot \vec{B} = 0$

If $\vec{\nabla} \cdot \vec{B} = 0$ than B can be written as

$$\boxed{\vec{B} = \vec{\nabla} \times \vec{A}} \quad \text{as div (curl)} = 0$$

Put the value of B in $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$, we get

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J}$$

In Magnetostatic, We choose a condition that

$$\vec{\nabla} \cdot \vec{A} = 0$$

Hence $\boxed{\nabla^2 A = -\mu_0 \vec{J}}$

This is called <u>Poission's eqⁿ in Magnetostatics</u>

* If $\vec{B}$ is given & we have to find $\vec{A}$ then

$$\boxed{\vec{A} = \frac{1}{2}(\vec{B} \times \vec{r})}$$

This is conditionally true. Its condⁿ is $\vec{B}$ should be uniform i.e. $\boxed{\vec{\nabla} \times \vec{B} = 0}$

i.e. If $\vec{\nabla} \times \vec{B} = 0$ only then we can use this formula of $\vec{A}$

where $\vec{r}$ is a position vector.

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$ in Magnetostatic

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I\,d\vec{l}}{r}$$

$$= \frac{\mu_0}{4\pi} \int \frac{\vec{K}\,da}{r}$$

$$= \frac{\mu_0}{4\pi} \int \frac{\vec{J}\,d\tau}{r}$$

These formulas are applicable only if current is not extended to ∞.

e.g. If we have a ∞ current carrying wire then these formulas can not be applicable.

$$\nabla^2 V = -\rho/\epsilon_0$$ in E.S.

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda\,dl}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \int \frac{\sigma\,da}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \int \frac{\rho\,d\tau}{r}$$

These formulas are valid only if charge is localised i.e. charge is not extended to ∞.

* Dir^n of A is $\perp$ to dir^n of B always. generally it matches with dir^n of current (parallel or antiparallel). If I in $\hat{x}$ dir^n then $\vec{A}$ can be in $\hat{x}$ or $-\hat{x}$. but can never be in $\hat{y}$ or $\hat{z}$.

Q. :- Find the mag. vector potential of an ∞ solenoid with n turns per unit length, radius R & current I

Here current is extended to ∞ i.e. Not localised. So can't use above formula to find $\vec{A}$.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \quad \text{(Ampere's law)}$$

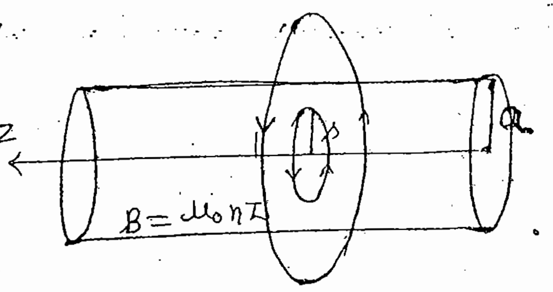
Mag flux passing through any surface S

$$\phi_m = \int_S \vec{B}\cdot d\vec{S} = \int_S (\vec{\nabla}\times\vec{A})\cdot d\vec{S}$$

$$= \oint \vec{A}\cdot d\vec{l} \quad \text{(from Stokes's theorem)}$$

Mag. field inside the solenoid

$$B = \mu_0 n I$$



$$\phi_m = B\cdot\pi s^2$$

$$= \mu_0 n I\cdot \pi s^2 = \oint \vec{A}\cdot d\vec{l}$$

$$\mu_0 n I \pi s^2 = A\cdot 2\pi s$$

$$\Rightarrow \boxed{\vec{A}_{in} = \frac{\mu_0 n I s}{2}\hat{\phi}}$$

Outside :-

$$\therefore \phi_m = B.\pi a^2$$
$$= \mu_0 n I \cdot \pi a^2 = \oint \vec{A}\cdot d\vec{l}$$
$$= A_{out} \cdot 2\pi s$$
$$\Rightarrow \vec{A}_{out} = \frac{\mu_0 n I a^2}{2s}\hat{\phi}$$

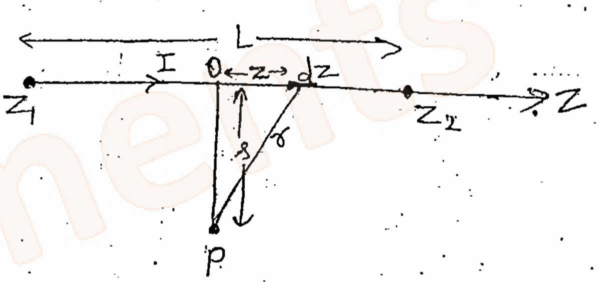
Mag. vector pot^n inside & outside the solenoid = Non-zero
while Mag. field inside $\rightarrow$ Non-zero, outside $\rightarrow$ zero.
& $\vec{A}_{out} \propto \frac{1}{s}$ dir^n matchs with current

Note :- It is Not necessary that if $\vec{B} = 0$ then $\vec{A} = 0$

17/8/2012

Q. Find the magnetic vector potential of a finite segment of straight wire carrying current I. Put the wire along z-axis from z_1 to z_2.

We have to find mag. vector pot^n at point P which is at a distance s from wire.



This is a finite wire

so $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I\,dl}{r}$

Take a small element, its length is dz which is at a distance z from mid point of wire O.

$$\vec{A} = \frac{\mu_0}{4\pi}\int_{z_1}^{z_2} \frac{I\,dz}{(s^2+z^2)^{1/2}} = \frac{\mu_0}{4\pi}\int_{-l/2}^{l/2} \frac{I\,dz}{(s^2+z^2)^{1/2}}$$

This is standard integral. It gives

$$\vec{A} = \frac{\mu_0 I}{4\pi}\ln\left[z + \sqrt{z^2+s^2}\right]_{-l/2}^{l/2}$$

$$\vec{A} = \frac{\mu_0 I}{4\pi}\ln\left[\frac{l/2 + \sqrt{\frac{l^2}{4}+s^2}}{-l/2 + \sqrt{\frac{l^2}{4}+s^2}}\right]$$

$$\vec{A} = \frac{\mu_0 I}{4\pi}\ln\left[\frac{l/2 + (l^2/4+s^2)^{1/2}}{-l/2 + (l^2/4+s^2)^{1/2}}\right]$$

This is mag. vector pot^n at P from mid point of wire.

$\Rightarrow$ If wire is very very long $L >> s$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \ln\left[\frac{\cancel{L/2}\left[1+\left(1+\frac{4s^2}{L^2}\right)^{1/2}\right]}{\cancel{L/2}\left[-1+\left(1+\frac{4s^2}{L^2}\right)^{1/2}\right]}\right]$$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \ln\left[\frac{1+1+\frac{2s^2}{L^2}}{\cancel{-1}+\cancel{1}+\frac{2s^2}{L^2}}\right] = \frac{\mu_0 I}{4\pi}\ln\left[\frac{2+\frac{2s^2}{L^2}}{\frac{2s^2}{L^2}}\right]$$

$$\vec{A} = \frac{\mu_0 I}{4\pi}\ln\left[\frac{L^2}{s^2}+1\right]$$

$L >> s$ then $\frac{L}{s} >> 1$ & $\frac{L^2}{s^2} >>>> 1$ so neglect 1 as compare to L^2/s^2 so

$$\vec{A} = \frac{\mu_0 I}{4\pi}\ln\left(\frac{L}{s}\right)^2$$

$$\boxed{\vec{A} = \frac{\mu_0 I}{2\pi}\ln\left(\frac{L}{s}\right)\hat{z}}$$ (dirⁿ same as current)

This is the mag. vector potⁿ of a very long wire at pt. P which is at a distance s from mid point O of wire.

Q. Find the current density corresponding to a vector potⁿ $\vec{A} = K\hat{\phi}$ where K is constant in cylendrical co-ordinates.

$\vec{A} \to \vec{B} \to J$ | $V \to E \to \rho$

$J = ?$

$\vec{A} = K\hat{\phi}$

$\{\nabla^2\vec{A} = -\mu_0\vec{J}$ (it may give wrong ans)$\}$

$$\cancel{\frac{1}{s^2}\frac{\partial^2}{\partial\phi^2}A_\phi} = \cancel{-\mu_0\vec{J}} \Rightarrow \cancel{\frac{1}{s^2}\frac{\partial^2}{\partial\phi^2}(K\phi)} \cancel{= -\mu_0\vec{J}}$$

$\cancel{\frac{1}{s^2}}$ First find $\vec{B}$ & then $\vec{J}$.

$$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_s & sA_\phi & A_z \end{vmatrix}$$

$$= \frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & sA_\phi & 0 \end{vmatrix} = \frac{1}{s}\left[\frac{\partial}{\partial s}(sA_\phi)\hat{z} - \frac{\partial A_\phi}{\partial z}\hat{s}\right]$$

$$= \frac{1}{s}\left[\frac{\partial}{\partial s}(sK)\hat{z}\right]$$

$$\vec{B} = \frac{1}{s} K \hat{z} \quad \text{or} \quad \boxed{\vec{B} = \frac{K}{s}\hat{z}}$$

We have $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

$$\vec{J} = \frac{1}{\mu_0}(\vec{\nabla} \times \vec{B})$$

$$\vec{J} = \frac{1}{\mu_0}\frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ B_s & sB_\phi & B_z \end{vmatrix} = \frac{1}{\mu_0}\left[-\frac{\partial}{\partial s}(B_z)\hat{\phi} + \frac{1}{s}\frac{\partial}{\partial \phi}(B_z)\hat{s}\right]$$

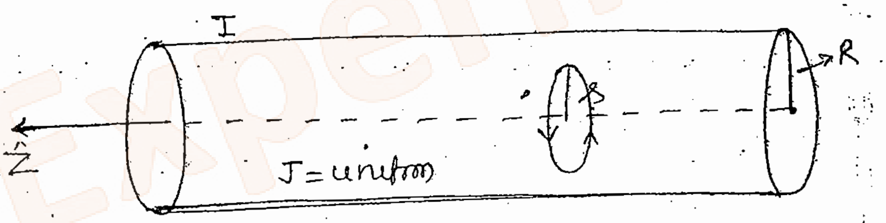
$$= \frac{1}{\mu_0}\left[-\frac{\partial}{\partial s}(B_z)\hat{\phi}\right]$$

$$\vec{J} = -\frac{1}{\mu_0}\frac{\partial}{\partial s}\left(\frac{K}{s}\right)\hat{\phi} = \frac{1}{\mu_0}\frac{K}{s^2}\hat{\phi}$$

$$\boxed{\vec{J} = \frac{K}{\mu_0 s^2}\hat{\phi}}$$

Q. Find the mag. vector potⁿ inside & outside the infinite wire if its radius is R and total current I is uniformly distributed over the cross-section. Assume that vector potⁿ vanishes on the surfaces of the wire.

We have a thick wire of radius R & current I is flowing ∥ in z dirⁿ.



If wire is infinite then we can not apply directly

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I\,dl}{r}$$ (This can be used only if current is localised)

When current extended ~~to~~ itself to ∞ then we'll find A through B.

$$\oint \vec{A}\cdot d\vec{l} = \phi_m = \int_S \vec{B}\cdot d\vec{S}$$

• Inside

To find $\vec{B}_{in}$, take a Ampirical loop inside the wire ($\vec{J}$ = uniform) & By Ampere's law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$J = I/A_{\perp} = I/\pi R^2$ { current is distributed over the whole cross-section

Current enclose by the loop

$I_{enc} = \int_0^s J \cdot ds = \oint_s J \cdot \pi s^2 = \frac{I}{\pi R^2} \pi s^2$

$$\boxed{I_{enc} = \frac{I s^2}{R^2}}$$

So $\oint B \cdot dl = \mu_0 I_{enc}$

$$B \cdot 2\pi s = \frac{\mu_0 I s^2}{R^2}$$

$$\boxed{\vec{B}_{in} = \frac{\mu_0 I s}{2\pi R^2} \hat{\phi}}$$

for a uniformely charged ∞ thick wire, then mag. field inside is $\propto s$. i.e. if $\vec{J}$ = uniform then $\boxed{B_{in} \propto s}$

- Outside :-

$I_{enc} = \int_0^R J \cdot ds = \frac{I}{\pi R^2} \pi R^2$

$I_{enc} = I$

So. $B \cdot 2\pi s = \mu_0 I$

$$\boxed{\vec{B}_{out} = \frac{\mu_0 I}{2\pi s} \hat{\phi}}$$

Now find Mag. Vector potⁿ,

$$\oint \vec{A} \cdot \vec{dl} = \int_s \vec{B} \cdot \vec{ds} = \phi_m$$

We can not use this formula bcoz $\vec{B}$ is in $\hat{\phi}$ dirⁿ & Amperical loop ~~and~~ is also in $\hat{\phi}$ dirⁿ so no flux will pass through the loop. So

We use $\vec{B} = \vec{\nabla} \times \vec{A}$

$$\vec{B} = \frac{1}{s} \begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_s & sA_\phi & A_z \end{vmatrix}$$

$$B\hat{\phi} = \cancel{\tfrac{1}{s}}\left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right) \hat{\phi}$$

$\frac{\partial A_s}{\partial z} \rightarrow$ Mag. vector potⁿ is in s dirⁿ & variation with z.

$\frac{\partial A_z}{\partial s} \rightarrow$ " " " " " "

If current is along z then mag. vector potⁿ must be in z-dirⁿ. $\frac{\partial A_s}{\partial z} = 0$

$$B\hat{\phi} = \left(0 - \frac{\partial A_z}{\partial s}\right)\hat{\phi}$$

$$B_{in} = -\frac{\partial}{\partial s} A_{in} \Rightarrow -\frac{\partial}{\partial s} A_{in} = \frac{\mu_0 I s}{2\pi R^2}$$

$$A_{in} = \int \frac{\mu_0 I s}{2\pi R^2}\, ds + C$$

$$A_{in} = -\frac{\mu_0 I}{2\pi R^2}\frac{s^2}{2} + C$$

Now apply boundary condⁿ's -

At $s = R$ (at surface) $A = 0$

$$\Rightarrow C = \frac{\mu_0 I}{4\pi R^2} R^2$$

So $$\boxed{\vec{A}_{in} = \frac{\mu_0 I}{4\pi R^2}(R^2 - s^2)\,\hat{z}}$$

Now, $$B_{out} = -\frac{\partial}{\partial s} A_{out} = \frac{\mu_0 I}{2\pi s}$$

$$A_{out} = -\frac{\mu_0 I\, ds}{2\pi s}$$

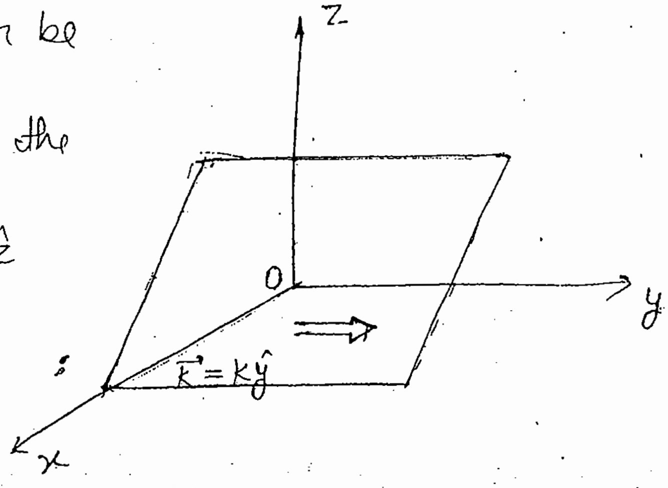
$$A_{out} = -\frac{\mu_0 I}{2\pi}\ln(s) + C$$

At $s = R \Rightarrow A = 0 \Rightarrow C = \frac{\mu_0 I}{2\pi}\ln(R)$

So $$\boxed{\vec{A}_{out} = \frac{\mu_0 I}{2\pi}\ln\left(\frac{R}{s}\right)\hat{z}}$$

Q. Find the mag. vector potⁿ above & below the plane having uniform surface current, $\vec{K} = K\hat{y}$ flowing over the x-y plane & ∞ dimension.

dirⁿ of mag. field can never be ⊥ʳ to the plane (x-y plane)($\hat{z}$)
Also can not be parallel to the dirⁿ of current. ($\hat{y}$)
So dirⁿ of $\vec{B}$ can't be $\hat{z}$ or $\hat{y}$. So it may be $+\hat{x}$ or $-\hat{x}$.



$$\vec{B} = \frac{\mu_0 K}{2}\hat{x} \quad (z>0)$$

$$\vec{B} = \frac{\mu_0 K}{2}(-\hat{x}) \quad (z<0)$$

We have to find $A = ?$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{x}\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]$$

if current is in y-dirⁿ so there will be no variation of A_z with y.

$$\vec{B} = \hat{x}\left(0 - \frac{\partial A_y}{\partial z}\right)$$

$z>0$, $$\frac{\mu_0 K}{2} = -\frac{\partial}{\partial z}A_y$$

$$A = \frac{-\mu_0 K z}{2} + C$$

Assuming the B.C., at $z=0$ (at surface) $A=0$

$$\Rightarrow C = 0$$

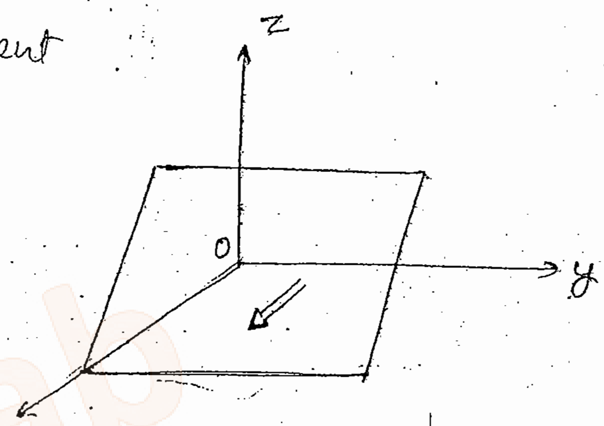
So $$\boxed{\vec{A} = \frac{-\mu_0 K z}{2}\hat{y}} \quad (z>0)$$

$z<0$, $$\boxed{\vec{A} = \frac{\mu_0 K z}{2}\hat{y}} \quad (z<0)$$

Magnetic Boundary Conditions
OR
B.C.s on Mag. field :- Whenever electric field crosses surface charge σ it suffers a discontinuity.

Illy, whenever ~~surfa~~ mag. field crosses surface current K it suffers discontinuity.

We have a surface current
$\vec{K} = K\hat{x}$



$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \Leftarrow \oint \vec{E}\cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0} \Rightarrow$

$E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\epsilon_0}$

$\vec{\nabla}\times\vec{E} = 0 \Leftarrow \oint \vec{E}\cdot d\vec{l} = 0$

$\Rightarrow E^{\parallel}_{above} = E^{\parallel}_{below}$

Similarly, for Mag. field,

$\vec{\nabla}\cdot\vec{B} = 0 \Leftarrow \oint \vec{B}\cdot d\vec{s} = 0 \Rightarrow \boxed{B^{\perp}_{above} = B^{\perp}_{below}}$

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \Leftarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow \boxed{B^{\parallel}_{above} - B^{\parallel}_{below} = \mu_0 K}$

Conclusion - Tangential component of electric field is continous but Normal comp. of Mag. field is Continous. Also, Normal comp. of $\vec{E}$ is discontinous by the amount $\frac{\sigma}{\epsilon_0}$ & tangential " " $\vec{B}$ " " " " " $\mu_0 K$.

In Electrostatic, scalar pot^n V & In Magnetostatic vector pot^n is $\vec{A}$.

$V_{above} = V_{below}$ | $\boxed{A_{above} = A_{below}}$

$\frac{\partial V_{above}}{\partial n} - \frac{\partial V_{below}}{\partial n} = \frac{-\sigma}{\epsilon_0}$ | $\boxed{\frac{\partial A_{above}}{\partial n} - \frac{\partial A_{below}}{\partial n} = -\mu_0 K}$

n → vector normal to the surface

$\frac{\partial V_{above}}{\partial n}$ → E. field normal to the surface.

18/8/2012

Multipole Expansion of $\vec{A}$:-

$$A(r) = \frac{\mu_0 I}{4\pi}\left[\underbrace{\frac{1}{r}\oint dI}_{\text{Monopole}} + \underbrace{\frac{1}{r^2}\oint r'\cos\theta\, dI}_{\text{dipole}} + \underbrace{\frac{1}{r^3}\oint (r')^2\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) dI}_{\text{Quadroupol}} + \cdots\right]$$

$\oint dI = 0$ so Monopole term $= 0$

Dipole term is given by $\boxed{\vec{A}_{dipole} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}}$

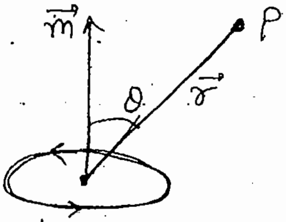
where m is magnetic dipole mom (or magnetic moment) & $\vec{r}$:

If we have a current loop (it will behave like a dipole) & we have to find mag. vector pot^n at point P then $\vec{r}$ be the distance from centre of dipole to the point P

Magnetic Mom. = Current X area

$$\boxed{\vec{m} = I\,\vec{a}}$$

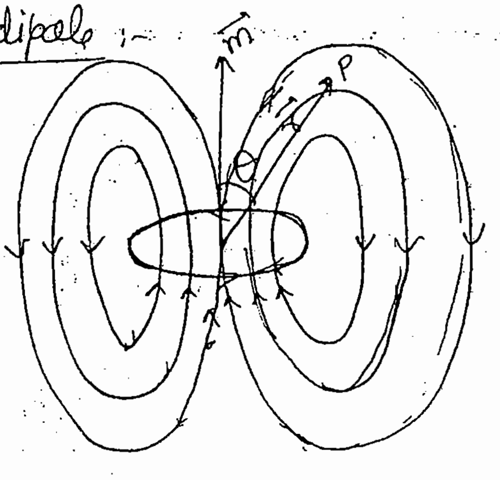
Its dir^n will be the dir^n of $\vec{a}$.
(Curl the fingure in the dir^n of current & thumb will tell the dir^n of mag. moment.)



$$\boxed{\vec{A}_{dip} = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi}}$$

As Electric dipole mom always directed along z-dir^n. Here loop will be in x-y plane & mag. mom. $\vec{m}$ is along z-axis & it will make θ angle with $\vec{r}$,

Field lines of Mag. dipole :-



$\vec{m} = m\hat{z}$

dir^n ⇒ $\vec{m}\times\hat{r}$

$\hat{z}\times\hat{r} = \sin\theta$

i.e. dir^n of $\vec{A}$ $= \hat{\phi}$

dir^n of $\vec{A}$ matches with dir^n of current.

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$B = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

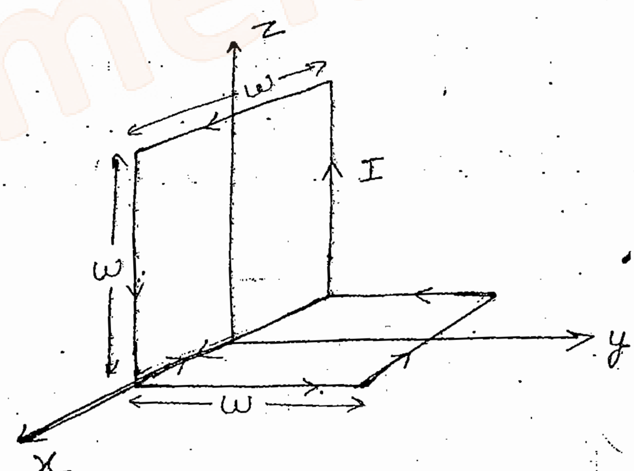
$$\vec{B} = \frac{1}{r^2\sin\theta}\left[\hat{r}\left\{\frac{\partial}{\partial\theta} r\sin\theta\, A_\phi - \underbrace{\frac{\partial}{\partial\phi} rA_\theta}_{0}\right\} - r\hat{\theta}\left\{\frac{\partial}{\partial r} r\sin\theta A_\phi - \underbrace{\frac{\partial}{\partial\phi} A_r}_{0}\right\}\right]$$

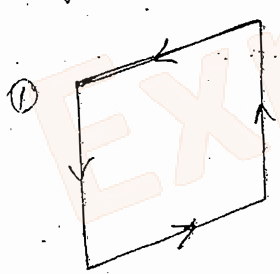
$$\boxed{\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]}$$

Similar as $\vec{E}$, $\vec{E} = \frac{p}{4\pi\epsilon_0 r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$

Q. Find the mag. dipole moment of a bookend-shape loop. All sides have length w and current I.

Seperate both loop. & find the mag mom. of each loop.

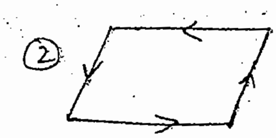




This loop is in x-z plane

So dir^n of mag. mom. is $\hat{y}$.

$$\vec{m}_1 = I w^2 \hat{y}$$



Again for IInd loop, loop is in x-y plane

so dir^n of mag. mom. is $\hat{z}$

$$\vec{m}_2 = I w^2 \hat{z}$$

So total mag. mom., vector sum of $\vec{m}_1$ & $\vec{m}_2$

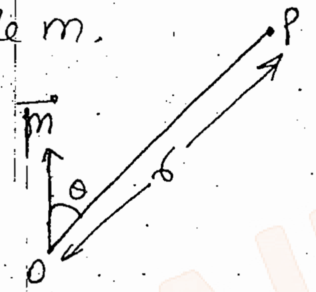
$\vec{m} = \vec{m}_1 + \vec{m}_2 \Rightarrow \boxed{\vec{m} = I w^2 (\hat{y} + \hat{z})}$

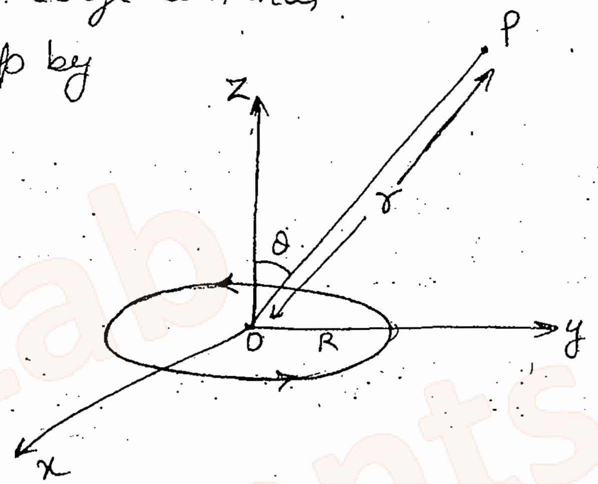
$$\boxed{|\vec{m}| = \sqrt{2}\, I w^2}$$

* Magnetic dipole mom. is always independent on choice of origin bcoz mag. monopole mom. is always 0.

Q. A circular loop of wire with radius R lies in the x-y plane centred at the origin & carries a current I running counter clockwise as viewed from z-axis.
(a) find mag. dipole moment.
(b) Approximate mag. field at large distances

We can replace the whole loop by a single m.





Mag. dipole mom.

$$\vec{m} = I\vec{a}$$

$$\boxed{\vec{m} = I\pi R^2 \hat{z}}$$

Mag. field at point P,

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

$$\boxed{\vec{B} = \frac{\mu_0 I \pi R^2}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]}$$

• Magnetic Vector pot^n at the axis of loop, $\theta = 0$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi}$$

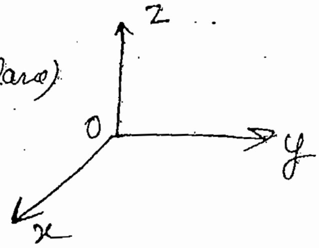
$$\boxed{\vec{A}_{axis} = 0}$$

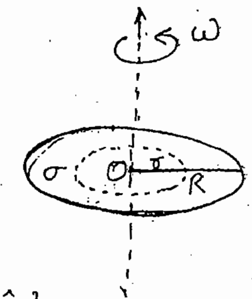
Q. A phonograph record of radius R carrying a uniform surface charge σ is rotating at a constant angular velocity ω. Find its magnetic dipole moment.

$$\vec{V} = \vec{\omega} \times \vec{r}$$
$$= \omega r \sin\theta\ \hat{\phi} \qquad \{\hat{z}\times\hat{r} = \sin\theta\,\hat{\phi}\}$$

$\vec{v} = \omega r \hat{\phi}$

$(\theta = 90^\circ)$ in x-y plane)





$\vec{K} = \sigma \vec{v}$

$\boxed{\vec{K} = \sigma \omega r \hat{\phi}}$

Surface Current

Total current $I = \int_0^R K \cdot dr$ $\quad$ $\{$No change in $\hat{\theta}\}$ $\hat{r}, \hat{\theta}, \hat{\phi}$

Magnetic Mom. $\vec{m} = I\vec{a}$

Area vector, ⊥ to the flow of current (θ is not changing)

$da = r \sin\theta \, d\phi \, dr$

$= r \, d\phi \, dr \qquad (\theta = 90^\circ)$

$= 2\pi r \, dr$

$I = \int_0^R \sigma \omega r \, dr \qquad \left\{ \vec{m} = \frac{\sigma \omega R^2}{2} \times \pi R^2 \text{ is wrong} \right\}$

I is func^n of r. At the differt point of disc, the value of current will be different & corresponding mag mom. will be diffent.

Total current in the ring of radius r,

$dI = \sigma \omega r \, dr$

Mag. Mom. $\vec{m} = I\vec{a}$

$\vec{m} = \int a \cdot dI$

$\vec{m} = \int_0^R \sigma \omega r \, dr \cdot \pi r^2 \, \hat{z}$

$$\boxed{\vec{m} = \frac{\sigma \omega \pi R^4}{4} \hat{z}}$$

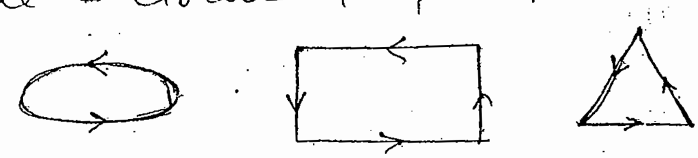
The disc is made up of small rings then mag. mom in ring $= dI \cdot a$

When disc is form tho

Mag. mom. $= \int dI \cdot a$

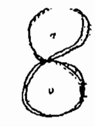
Note- dir^n of length vector is always along the line while the " " area " " perpendicular to the area.

Mag. dipole is a current loop, It may be of any shape - circular, square, ractangular --- etc.



$\oint dI$ = Net displacement $= 0$ $\quad$ So closed loop behaves like Mag. ~~Monopole~~ dipole.

- In 8 (eight shape), there are 2 loops. So we have to seperate these 2 loops & then find $\vec{m}$ of each.



- Definet wire carrying current I not form a loop so not behave like magnetic dipole.

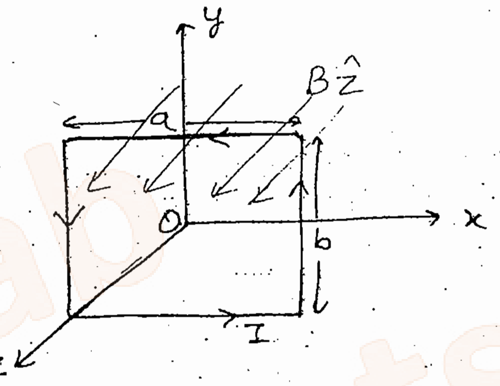
Force & Torque on a magnetic dipole :-

Magnetic dipole experiences no force (0 force) in a uniform magnetic field but it experiences Non-zero torque in uniform magnetic field.

Let us consider a mag. dipole of rectangular shape
Its mag. mom,

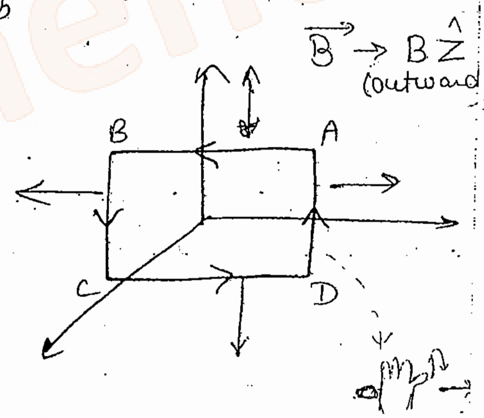
$\vec{m} = I \cdot ab\, \vec{a}$

$$\boxed{\vec{m} = I ab\, \hat{z}}$$



If we apply uniform mag. field $\vec{B}$ along $\hat{z}$, then force on this current loop

$$\vec{F} = I \oint (d\vec{l} \times \vec{B})$$



Sum of total forces for AB, BC, CD, DA will cancel out. So

$$\boxed{\vec{F} = 0}$$

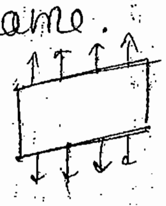
Thus there is No force on the mag. dipole in Uniform mag. field

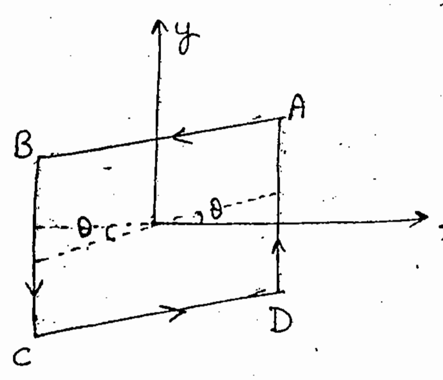
Now Torque :- Rotate the loop s.t. angle b/w $\vec{m}$ & $\vec{B}$ is θ.

(Earlier angle b/w $\vec{m}$ & $\vec{B}$ was zero as $\vec{m} = I ab \hat{z}$ & $\vec{B} = B\hat{z}$)

Now line AD & BC are not at the same value of x.

forces of line AB & CD will cancell out bcoz their line of action is same.





But forces along BC & AD will not cancell out bcoz their line of action is not same, it is different.

$$\boxed{\vec{F}_{AB} + \vec{F}_{CD} = 0}$$

If line of action is different (not same) than they produce Torque.

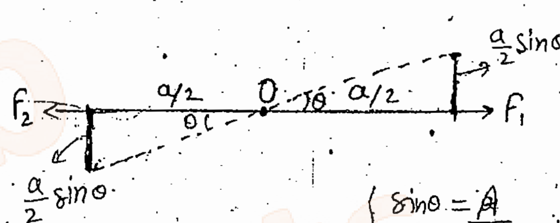
For line BC & ADA, the angle b/w current & mag. field is always 90°.

$$|\vec{F}_{BC}| = |\vec{F}_{DA}| = IbB$$

$$\left\{ \begin{aligned} F &= I \int d\vec{l} \times B \\ &= IB \int dl \\ &= IBb \end{aligned} \right.$$

Now Torque $\vec{N} = \vec{r} \times \vec{F}$

$$N = IbB \frac{a}{2} \sin\theta + IbB \frac{a}{2} \sin\theta$$



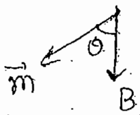
$$N = IabB \sin\theta = mB\sin\theta$$

$$\boxed{\vec{N} = \vec{m} \times \vec{B}} \; \hat{y}$$

$$\left\{ \begin{aligned} \sin\theta &= \frac{A}{a/2} \\ A &= \frac{a}{2}\sin\theta \end{aligned} \right.$$

$$N = \vec{p} \times \vec{E}$$

* $\vec{m}$ & $\vec{B}$ are in x-z plane so dir^n of $\vec{N}$ will be $\hat{y}$.



→ dir^n of torque → $\hat{y}$

Torque will try to rotate the plane s.t. $\vec{B}$ & $\vec{m}$ are ~~parall~~ parallel. & when $\vec{m}$ & $\vec{B}$ ~~are~~ become parallel then

$\vec{m}\uparrow \; \uparrow\vec{B}$ Torque $= 0$.

Force & Torque in Non-Uniforme mag. field :-

$$\boxed{\vec{F}_{Non\text{-}Uniform} = \vec{\nabla}(\vec{m}\cdot\vec{B})} \neq (\vec{m}\cdot\vec{\nabla})\vec{B}$$

$$\left\{ \vec{F}_{Non\text{-}unifm} = (\vec{p}\cdot\vec{\nabla})\vec{E} = \nabla(\vec{p}\cdot\vec{E}) \right.$$

→ conditionally true cond^n if p is not a func^n of position x

bcoz in magnetostatic $\vec{\nabla} \times \vec{B} \neq 0$

Electrostatic $\vec{\nabla} \times \vec{E} = 0$

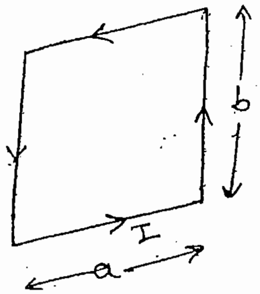
Torque :- $\boxed{\vec{N} = \vec{m} \times \vec{B} + \vec{r} \times \vec{F}_{Non-uniform}}$

Potential Energy of a Magnetic Dipole :-

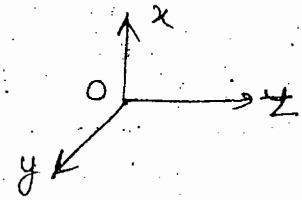
In Electrostatic relation, Replace P by m & E by B.,

$U = -\vec{P} \cdot \vec{E}$

i.e. $\boxed{\vec{U} = -\vec{m} \cdot \vec{B}}$

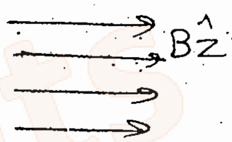


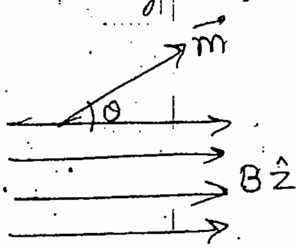
Suppose, we have a loop & current is flowing in this loop is I.
Mag. field is along z-dir^n.



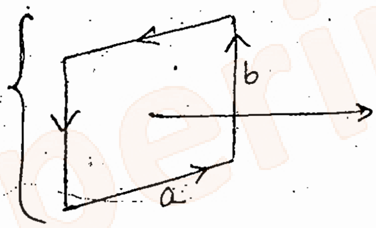
If loop is straight then work done to take the loop from ∞ to in the mag field is zero, as $\vec{m}$ & $\vec{B}$ are in same dir^n.

But Now loop is tilted by the angle θ. So work done is Non-zero as angle b/w m & B is θ.

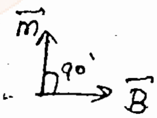




$$dW = N\, d\theta$$



$\vec{m} = Iab\,\hat{z}$



Now angle changes from 90° to θ, So Work done

$$U = W = \int_{\pi/2}^{\theta} mB\sin\theta\, d\theta = -mB\left[\cos\theta\right]_{\pi/2}^{\theta}$$

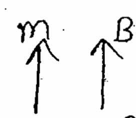
This work will store as P.E.

$$= -mB\cos\theta$$

$$\boxed{U = -\vec{m} \cdot \vec{B}}$$

When ∠ b/w $\vec{m}$ & $\vec{B}$ is 0 i.e. $U = -mB$ then P.E will be minimum.

This will be the stable configuration.



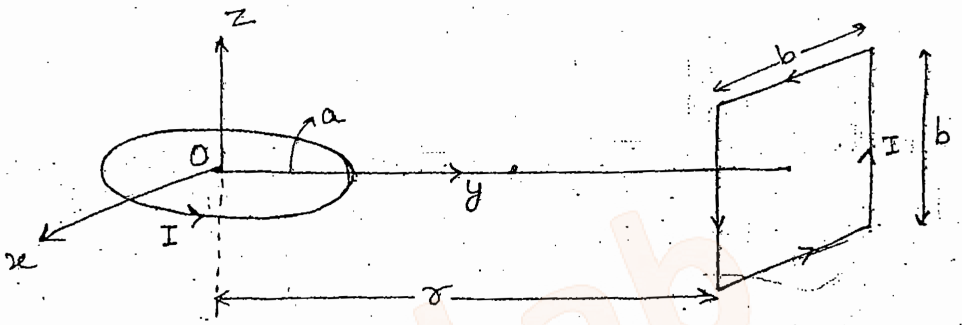
$\cos 0 \xrightarrow{\theta = 0}$ max.
P.E. = min

As torque rotate the plane then flux changes & it will induce electromagnetic field will do work.
(Mag. force does not do work)

Magnetic Dipole Interaction Energy :-

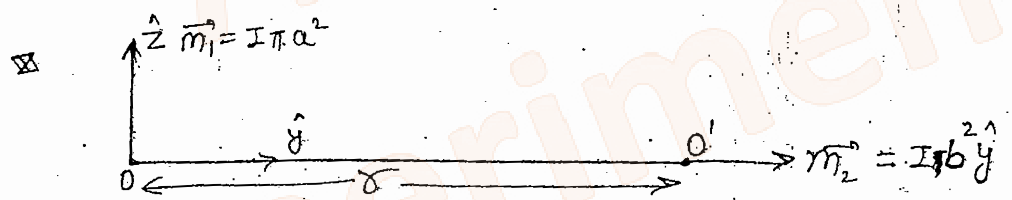
$$\boxed{U = \frac{\mu_0}{4\pi r^3}\left[\vec{m}_1\cdot\vec{m}_2 - 3(\vec{m}_1\cdot\hat{r})(\vec{m}_2\cdot\hat{r})\right]}$$

Q. Calculate the torque & the interaction energy for 2 loops as shown in the figure.



When $r > a, b$ then these 2 loops can be replaced by their mag. moment.

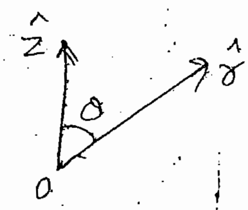
These 2 loops are equilant to 2 dipoles.



Mag. field of loop 2 applys a torque on loop 1.

$$\vec{N}_1 = \vec{m}_1 \times \vec{B}_2$$

Magnetic field on loop (2) at a distance r



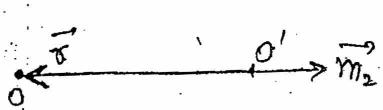
$$\vec{B}_2 = \frac{\mu_0 m_2}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right].$$

$$\vec{B}_2 = \frac{\mu_0 m_2}{4\pi r^3}\left[-2\hat{r}(-\hat{y})\right]$$

$\theta = 180°$

$\hat{r} = -\hat{y}$

$$\vec{B}_2 = \frac{2\mu_0 m_2}{4\pi r^3}\hat{y}$$



$$\boxed{\vec{B}_2 = \frac{2\mu_0 I b^2}{4\pi r^3}\hat{y}}$$

$\vec{r}$ is always away from the origin.

~~On loop 2,~~ $\cancel{\vec{N}_2 = \vec{m}_2 \times \vec{B}_1}$

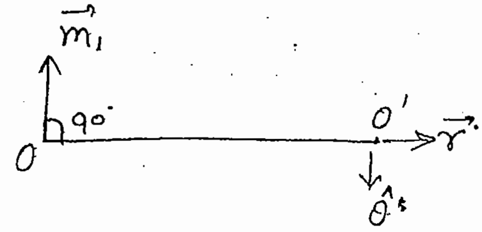
Torque $\vec{N_1} = \vec{m_1} \times \vec{B_2}$

$$\vec{N_1} = I\pi a^2 \, \frac{2\mu_0 I b^2}{4\pi r^3} (\hat{z} \times \hat{y})$$

$$\boxed{\vec{N_1} = \frac{\mu_0 I^2 a^2 b^2}{2 r^3} (-\hat{x})}$$

Torque on loop 2, $\vec{N_2} = \vec{m_2} \times \vec{B_1}$

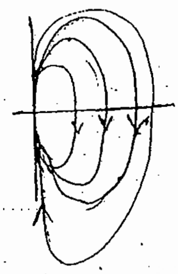
$m_2 = I b^2 \hat{y}$



$$\vec{B_1} = \frac{\mu_0 m_1}{4\pi r^3} \left[2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta} \right]$$

$$\vec{B_1} = \frac{\mu_0 I \cdot \pi a^2}{4\pi r^3} \, \hat{\theta}$$

$$\hat{\theta} = -\hat{z}$$



At equitorial line dirⁿ of B is ↓ (this is $\hat{\theta}$ dirⁿ)

$$\vec{B_1} = \frac{\mu_0 I \pi a^2}{4\pi r^3} (-\hat{z})$$

$$\vec{N_2} = \vec{m_2} \times \vec{B_1}$$

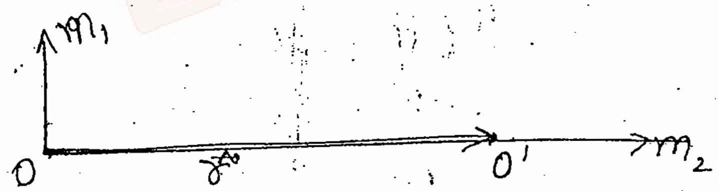
$$\boxed{\vec{N_2} = \frac{\mu_0 I^2 a^2 b^2}{4 r^3} (-\hat{x})}$$

$\{ \hat{y} \times (-\hat{z}) = -\hat{x} \}$

Interaction Energy :-

$$U = \frac{\mu_0}{4\pi r^3} \left[\vec{m_1} \cdot \vec{m_2} - 3(\vec{m_1} \cdot \hat{r})(\vec{m_2} \cdot \hat{r}) \right]$$

Mutual angle b/w $\vec{m_1}$ & $\vec{m_2}$ is 90°.



$\vec{r}$ can be from O to O'
or can be from O' to O

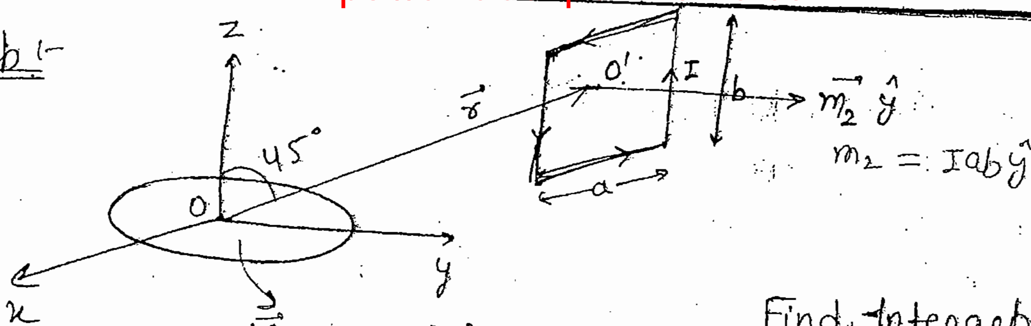
$(\vec{m_1} \cdot \hat{r}) = 0$

$(\vec{m_2} \cdot \hat{r}) \neq 0$

so $U = \frac{\mu_0}{4\pi r^3} \left[m_1 m_2 \cos 90° - 3(0)\, 0 \right]$

$$\boxed{U = 0}$$

Prob:-



$\vec{m}_1 = I\pi a^2 \hat{z}$

Find Interaction Energy ?

$$U = \frac{\mu_0}{4\pi r^3}\left[\vec{m}_1\cdot\vec{m}_2 - 3(\vec{m}_1\cdot\hat{r})(\vec{m}_2\cdot\hat{r})\right]$$

$\vec{m}_1\cdot\vec{m}_2 = m_1 m_2 \cos 90° = 0$ $\quad (\angle$ b/w m_1 & $m_2 = 90°)$

$\vec{m}_1\cdot\hat{r} = m_1 \cos 45° = \frac{m_1}{\sqrt{2}} = \frac{I\pi a^2}{\sqrt{2}}$

$\vec{m}_2\cdot\hat{r} = m_2 \cos 45° = \frac{m_2}{\sqrt{2}} = \frac{Iab}{\sqrt{2}}$

$$\therefore\ U = \frac{\mu_0}{4\pi r^3}\left[0 - \frac{3\pi I^2 a^3 b}{2}\right]$$

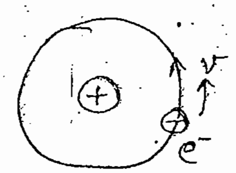
$$U = -\frac{3\mu_0 I^2 a^3 \pi b}{8\pi r^3} \Rightarrow \boxed{U = \frac{-3\mu_0 I^2 a^3 b}{8r^3}}$$

Bound Currents:- There are 2 types of currents:-

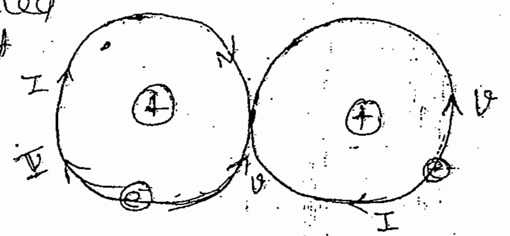
Free currents - are due to motion of free charges. If inside the metal free charges are moving under path then current arises.

Bound currents - arises due to the motion of bound charges.

e^- perform a orbital motion & this motion is equivalent to magnetic dipole. If a e^- is moving with velocity v around nuclei & correspond to a current, Now it 'll act as a current dipole. It is the bound current.



If another atom is placed near this atom. The current b/w them cancel out & Net current will be on surface.



Then it is called Surface bound Current &

If any atom is missing in b/w, there will be current Non-zero then net current will exist in volume. It is called Volume bound current.

$\vec{K_b} \rightarrow$ Surface bound current

$\vec{J_b} \rightarrow$ Volume bound current

As $\vec{P}$ is polarisation i.e. dipole mom. per unit volume.

Similarly in magnetostatic,

Magnetisation (M):- Magnetic mom. $\vec{m}$ per unit volume.

As $\sigma_b = \vec{P}\cdot\hat{n}$, ρ_b &

$\rho_b = -\vec{\nabla}\cdot\vec{P}$

if P is uniform then $\rho_b = 0$

P is Non uniform then $\rho_b \neq 0$

In Magnetostatics, Total current will be surface current if magnetisation is uniform.

$$\boxed{\vec{K_b} = \vec{M} \times \hat{n}}$$

$K_b \rightarrow$ surface bound current

If Magnetisation is Non-uniform then K_b & J_b both contribute, volume bound current

$$\boxed{\vec{J_b} = \vec{\nabla} \times \vec{M}}$$

Div of Curl is zero so $\boxed{\vec{\nabla}\cdot\vec{J_b} = 0}$

Que:- An infinitely long circular cylinder carries a uniform magnetization M parallel to its axis. Find surface & volume bounds current & also find mag. field inside & outside the cylinder.

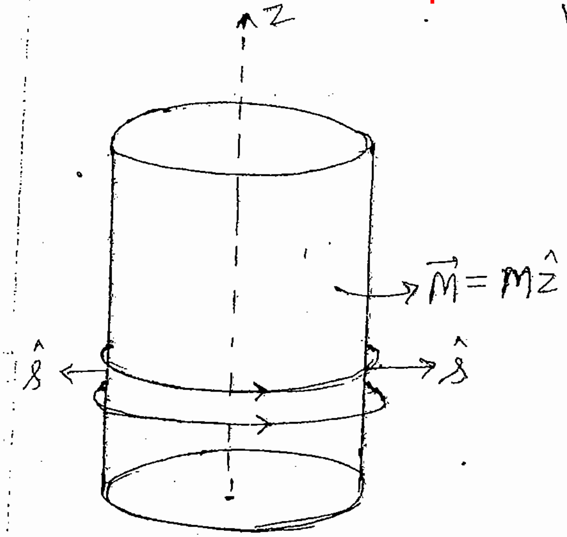
$\vec{M}$ is uniform, not depending on space co-ordinate

No free current is present.

First find bound Current, Using Ampere's law

$$\boxed{I_{enc} = I_b + I_f}$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$$



Vol. bound current, $J_b = 0$ bcoz $\vec{M}$ is uniform

Surface bound Current,

$$\vec{K_b} = \vec{M} \times \hat{n}$$

$\perp$ vector to $\vec{M}$ will be $\hat{s}$ ..

$$\vec{K_b} = M(\hat{z} \times \hat{s})$$

$$\vec{K_b} = M\hat{\phi}$$

Magnetisation is like mag. field
(arries from bound current) — arriues from current (bound as well as free)

$B \xrightarrow{\text{determine from}} I_b + I_f$

$M \rightarrow I_b$

$H \rightarrow I_f$ (free current)

Mag. field Intensity

Note :- Curl the finger in the dirn of bound current, thumb will tell the dirn of Magnetisation.

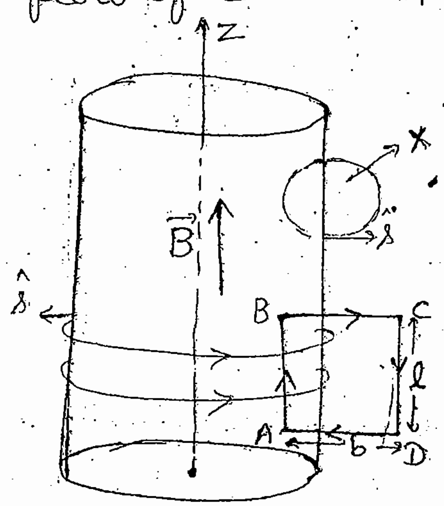
There is no free current so total current will be bound current.

It is like a solanoid. So to find the current of this infinitely long cylender we must draw a Ampirical loop. Ampirical loop should be s.t. current passes through the loop. So loop should be $\perp$ to the flow of current.

Take a rectangle as amperical loop.

(If we take a ~~of~~ circular loop then dl will be different at each point of loop so B·dl will be variable. So we can not take a circle as ampirical loop).

Dirn of loop will be according to the dirn of mag. field $\vec{B}$.



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$\oint \vec{B}\cdot d\vec{l} = \int_{AB} \vec{B}\cdot d\vec{l} + \underbrace{\int_{BC} \vec{B}\cdot d\vec{l}}_{\substack{\angle \text{ b/w them is } 90^\circ \\ = 0}} + \underbrace{\int_{CD} \vec{B}\cdot d\vec{l}}_{\substack{\text{out of cylender} \\ \text{No current} \\ = 0}} + \underbrace{\int_{DA} \vec{B}\cdot d\vec{l}}_{\substack{\angle \text{ b/w them} = 90^\circ \\ = 0}}$$

Note :- Bound Current can never be in the dirn of Magnetisation.

$$\oint \vec{B}\cdot d\vec{l} = \int_{AB} \vec{B}\cdot d\vec{l} = Bl$$

Surface current $= \dfrac{\text{Total current}}{\text{length} \perp \text{to the flow}} \Rightarrow \dfrac{I}{l_\perp} = M$

$$I = M\cdot l$$

$$B\cancel{l} = \mu_0 M \cancel{l}$$

$$\boxed{\vec{B} = \mu_0 \vec{M}\ \hat{z}}$$

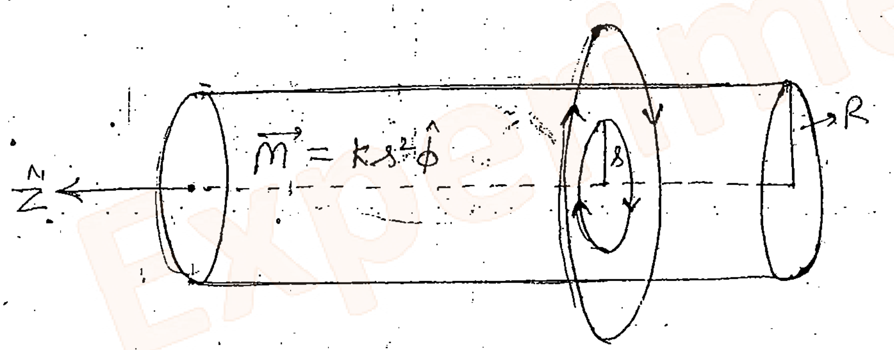
Mag. field through bound current.
In terms of surface current (K) is

$$\boxed{B = \mu_0 K}$$

for n no. of loops ; $\vec{B} = \mu_0 n I$

$B = \mu_0 \underline{nI}$ — Total current / unit length i.e. surface current

Que:- A long circular cylinder of radius R carries a magnetisation $\vec{M} = Ks^2\hat{\phi}$ where K is the constant & s is the distance from the axis, $\hat{\phi}$ is the azimuthal vector. Find the mag. field due to $\vec{M}$ inside & outside the cylinder.



If $\vec{M}$ is in the $\hat{\phi}$ then bound current will be in $\hat{z}$ &
" $\vec{M}$ " " $\hat{z}$ " " " " $\hat{\phi}$.

Vol. bound current, $\vec{J}_b = \vec{\nabla}\times\vec{M}$

$$= \frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ M_s & sM_\phi & M_z \end{vmatrix}$$

$$= \frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & sKs^2 & 0 \end{vmatrix} \qquad (M_\phi = Ks^2)$$

$$= \frac{1}{s}\hat{z}\left[\frac{\partial}{\partial s}(Ks^3)\right] = \frac{1}{s}K3s^2\hat{z}$$

$$\boxed{\vec{J}_b = 3Ks\hat{z}}$$

Surface bound current,

$$\vec{K}_b = \vec{M}\times\hat{n}$$

(unit vector normal surface $\Rightarrow \hat{n} = \hat{s}$)

$$= Ks^2(\hat{\phi}\times\hat{s})\Big|_{s=R}$$

$$= KR^2(-\hat{z})$$

$(\hat{\phi}\times\hat{s} = -\hat{z})$

$$\boxed{\vec{K}_b = -KR^2\hat{z}}$$

Surface bound current & vol. bound current are opositelly directed.

Magnetic field Inside :-

Only volume bound curret will contribute. Take a circular Ampirical loop.

Current Enclose by the circular loop,

$$I_b(\vec{J}_b) = \int \vec{J}_b\cdot d\vec{S}_\perp$$

$dS_\perp = s\,ds\,d\phi$ (z is not changing)

$$= \int_0^s\int_0^{2\pi} 3Ks\; s\,ds\,d\phi$$

$$= 3K\frac{s^3}{3}\cdot 2\pi$$

$\boxed{I_b(\vec{J}_b) = 2\pi Ks^3}$ The Ampirical loop will enclose this current.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$B\,\cancel{2\pi}s = \cancel{2\pi}Ks^3\mu_0$$

$$\boxed{\vec{B}_{in} = \mu_0 Ks^2\hat{\phi}}$$

Magnetic field Outside :- both volume & surface bound currents will contribute.

$$\rightarrow I_b(\vec{J}_b) = \int_0^R\int_0^{2\pi} 3Ks\; s\,ds\,d\phi$$

$$I_b(\vec{J}_b) = 2\pi KR^3$$

$$\rightarrow I_b(K_b) = K_b \times \text{Length}$$

$$= -KR^2 2\pi R\hat{z} \Rightarrow I_b(K_b) = -2\pi KR^3$$

Total bound current $= 2\pi K R^3 - 2\pi K R^3$

$$\boxed{I_b(\text{Total}) = 0}$$

So $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$$\Rightarrow \boxed{B_{out} = 0}$$

So Mag field outside

Ampere's law in Magnetised Material :- The division of materials is based on how they respond to external ~~elect~~ magnetic field.

If a material is placed in mag field & there is no change in its property then this material is called Non-magnetic material.

Because Every magnetic material respond to mag field i.e. On placing it in mag field its properties changes.

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

Total $\vec{J} = \vec{J}_b + \vec{J}_f$

$$\therefore \vec{\nabla}\times\vec{B} = \mu_0(\vec{J}_b + \vec{J}_f)$$

$$\Rightarrow \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\times\vec{M} + \vec{J}_f$$

$$\nabla\times\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = \vec{J}_f$$

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J}_f}$$

$\left\{ \frac{B}{\mu_0} - M = H \right.$

$B = \mu_0(M+H)$

This is the differential form of Ampere's law in magnetisation material,

$H \rightarrow$ Magnetic field intensity

Its M.K.S. Unit $\rightarrow$ A/m (ampere/meter)

$\vec{J}_f \rightarrow$ free current density

Now, Take Surface integral of both the sides,

$$\int_S (\nabla \times \cancel{B} H)\cdot ds = \int_S J\cdot ds$$

$$\boxed{\oint \vec{H}\cdot d\vec{l} = I_{f\,enc.}} \rightarrow \text{Integral form}$$

In Magnetostatics → H, determining only by the free current. It includes bound current in itself. Similar as D in electrostatics;

Que :- In previous Question, $M = K s^2 \hat{\phi}$

$$\vec{B} = \mu_0(\vec{H}+\vec{M})$$

$\vec{H}_{in} = 0$, $\vec{H}_{out} = 0$ } bcoz there is no free ~~current~~ current inside & outside. (only bound current)

So $\vec{B}_{in} = \mu_0 \vec{M}_{in}$

$$\boxed{\vec{B}_{in} = \mu_0 K s^2 \hat{\phi}}$$

$$\vec{B}_{out} = \mu_0 \vec{M}_{out} = \mu_0 \times 0$$

$$\boxed{\vec{B}_{out} = 0}$$

$$\vec{B} = \mu \vec{H}$$

$\mu \rightarrow$ permiability of the medium.

Permiability :- It is defined as

Relative permiability $\mu_r = \frac{\mu}{\mu_0}$ as $\epsilon_r = \frac{\epsilon}{\epsilon_0}$

μ_r decides, the type of material.

$\mu_0 \rightarrow$ permiablity in free space.

For Diamagnetic Material, $\mu_r < 1$

Paramag. " $\mu_r > 1$

ferromag. " $\mu_r >> 1$.

Physical Meaning - If we have a hollow cylinder, wrape conducting wires over it. Current flow is constant. It'll become a solanoid. Current flowing in conductor is free current. Mag. field intensity H inside will be Non-zero.

It is hollow before. Now if we place a Iron rod inside it then, (Iron is ferromag. material)

$\vec{H}$ → No change bcoz it totally determine by free current

& $\vec{B}$ increases. bcoz of bound current

In free space $\vec{B} = \mu_0 \vec{H}$

If we place a rod then The magnetisation will be in the dirⁿ of mag field then

$$\vec{B} = \mu_0(\vec{H}+\vec{M})$$

$$\vec{B} = \mu\vec{H} \Rightarrow B = \mu_r \mu_0 \vec{H}$$

⇒ If $\mu_r < 1$ then $B \uparrow$

$\mu_r > 1$ then $\underset{\text{can be}}{B \uparrow}$ or $\underset{\text{can be}}{B \downarrow}$ depanding upon material

while $\vec{E} \downarrow$.

⟹ Polarisation is always in the dirⁿ of external E-field

$E_{ext} \uparrow$ - $P \uparrow$

But for a Mag. Material

We have a dia mag. matrial & apply H then

$$H_{ext} \uparrow \quad M \downarrow$$

for para mag. $H_{ext} \uparrow \quad M \uparrow$

Magnetic Succeptability :-

$$\vec{B} = \mu_0(\vec{H}+\vec{M})$$

$$\chi_m = \frac{M}{H} \quad \Rightarrow \quad \mu H = \mu_0(H + \chi_m H)$$

$$\frac{\mu}{\mu_0} = 1+\chi_m$$

$$\boxed{\mu_r = 1+\chi_m}$$

$$\epsilon_r = 1+\chi_e$$

$\epsilon_r > 1$ always but $\mu_r > 1$ or may be $\mu_r < 1$

$\mu_r < 1$ i.e. $\chi_m < 0$ then Diamag. material.

$\mu_r > 1$ i.e. $\chi_m > 0$ " Para " "

If Magnetisation is proportional to first power of H then these types of materials are called Linear Magnetic Materials. i.e.

$$M \propto H$$

defination of susceptability $\chi_m = \frac{M}{H}$ is true only for linear mag. material

Dia, para are linear mag. material but ferro mag. material is not linear.

$$\chi_m \neq \frac{M}{H} \quad \text{(for ferro)}$$

for ferro mag. material $\boxed{\chi_m = \frac{\partial M}{\partial H}}$

for Ex :- $M = \tanh\left(\frac{\mu H}{KT}\right)$

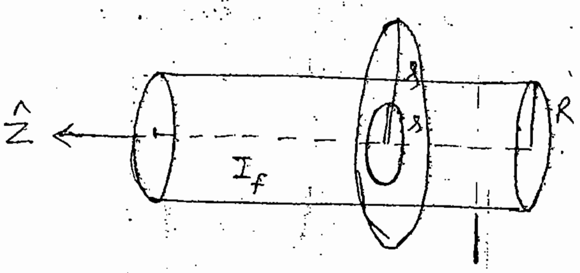
M is not linearly depending on H, bcoz $\tan x \simeq x$ only for small value of x. [i.e. $x<1$]

It is ferro mag. material.

Que :- A long Copper rod of radius R carries a uniformly distributed free current I. find the H inside & outside the rod.

Current $I = J \times (\text{Area})$

$$I = J \pi R^2$$



$$J = \frac{I}{\pi R^2}$$

Inside :-

$$I_{enc} = \int \vec{J} \cdot d\vec{s} = J.\text{area} \; (J \text{ is uniform})$$

$$= \frac{I}{\cancel{\pi} R^2} \cancel{\pi} s^2$$

$$\boxed{I_{enc} = \frac{I s^2}{R^2}}$$

$$\oint \vec{H} \cdot d\vec{l} = I_{f\,enc}$$

$$H \cdot 2\pi s = \frac{I s^2}{R^2}$$

$$\Rightarrow \boxed{\vec{H} = \frac{I s}{2\pi R^2} \hat{\phi}}$$

dir^n of H is same as B.

So $\boxed{H_{in} \propto s}$

Outside :- $I_{enc} = I$

$H \cdot 2\pi s = I$

$$\boxed{\vec{H} = \frac{I}{2\pi s}\hat{\phi}}$$

If permiablity of that medium is μ then mag field will be $B = \mu H \Rightarrow \boxed{\vec{B}_{in} = \frac{\mu I s}{2\pi R^2}\hat{\phi}}$

(but can't determine B_{in} Here bcoz μ is not given)

We can determine B_{out} bcoz outside permiability is μ_0

So $\boxed{B_{out} = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$

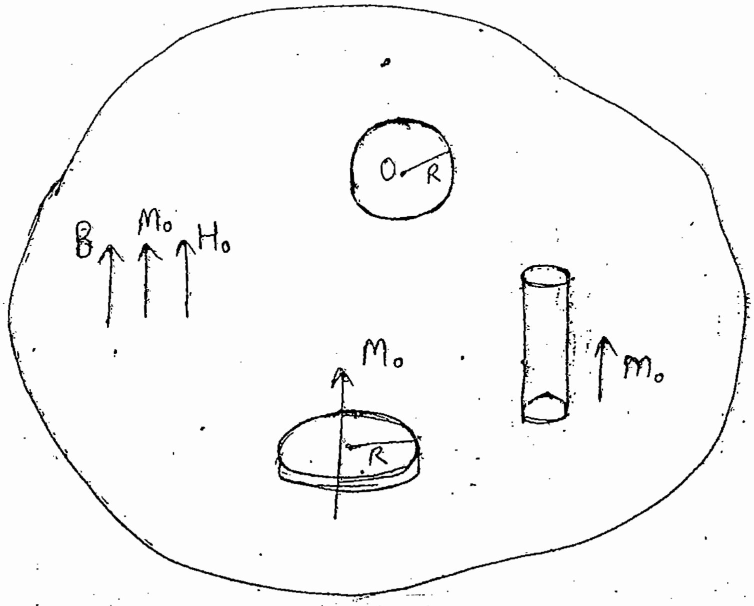
Ques* :- A infinitely long cylender of radius R carries a magnetisation $\vec{M} = ks\hat{z}$ parallel to its axis where s is the distance from the axis of the cylender. There is no free charge anywhere. Find surface & volume bound currents. Also find mag field inside & outside the cylender by using 2 different methods → (i) by bound currents & (ii) by H.

Ques :- Suppose the field inside a large piece of magnetic material is B_0 such that $H_0 = \frac{B_0}{\mu_0} - M$.

(a) Now a small spherical cavity is hollowed out of the material, find the mag. field at the centre of the cavity.

(b) Do the same for a long niddle shaped cavity parallel to M.

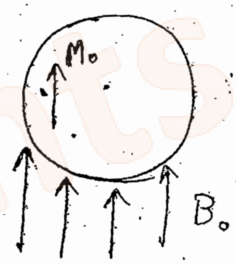
(c) Do the same for a ~~waffour~~ wafer shaped cavity ⊥ to $\vec{M}$.



When B & M are in same dir^n then mag. field inside the material will decrease –(Para & ferro mag. material B & M are in same dir^n but in dia dir^n of B & M are opposite)

<u>sphere</u>: In free space of ~~Mate~~ sphere

$$\vec{B} = \vec{B_0} - \frac{2}{3}\mu_0 \vec{M_0} \quad \text{ie. } B_{in} = -\frac{2}{3}\mu_0 \vec{M_0}$$



~~This is~~ the mag. field of uniformly magnetised sphere,

$$B_{in} = \frac{2}{3}\mu_0 \vec{M_0}$$ (vol. current in + = surface current in

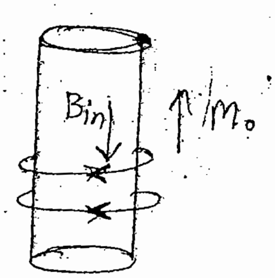
$$E_{in} = -\frac{1}{3}\frac{P_0}{\epsilon_0}$$

So Mag. field inside the spherical cavity

$$\vec{B_0} - \frac{2}{3}\mu_0 \vec{M_0}$$

<u>Niddle shape</u> :-

$$\vec{B} = \vec{B_0} + \vec{B_{in}}$$



In free space, Magnetisation = 0
Outside Magnetisation M_0 then we have to place $-M$ mag. inside the cavity to make the total mag. zero. (Magnetisation → by superposition)

Mag. field inside the cavity :

$$\boxed{\vec{B_{in}} = \vec{B_0} - \mu_0 \vec{M_0}}$$

{ If Thumb → $\vec{B}$, fingers → tell the dir^n of current

* If the length of rod is long → more effect of surface current (in magnetostatics)

If length of rod is long (in Electrostatic) → surface charges are far apart & effect will be less.

<u>Disc</u> (~~Waffer~~ Wafer shape) :- Current will flow on the surface of disc → Total current = surface current X ⊥ length

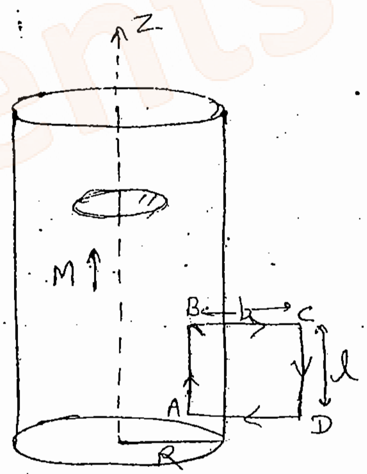
In disc, amount of bound ~~charge~~ current is very less bcoz ⊥ length → 0

so bound current → 0, Mag. field $\vec{B} = \vec{B}_0$ $\left\{ E = E_0 + \frac{P}{\epsilon_0} \right\}$

In case of disc, induce mag. field = 0 bcoz length is very small, $\vec{B}_{in} = 0$

so total mag. field $\boxed{\vec{B} = \vec{B}_0}$

<u>Ques</u> * :- $\vec{M} = Ks\,\hat{z}$



Volume bound current

$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$

$$= \frac{1}{s} \begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & Ks \end{vmatrix}$$

$$= -\frac{1}{s}\, s\hat{\phi}\, \frac{\partial}{\partial s}(Ks)$$

$$\boxed{\vec{J}_b = -K\hat{\phi}}$$

Surface bound Current

$$\vec{K}_b = \vec{M} \times \hat{n}$$

$$= Ks(\hat{z} \times \hat{s}) = Ks\hat{\phi}$$

$$\boxed{\vec{K}_b = Ks\hat{\phi}} \Rightarrow \boxed{\vec{K}_b = KR\hat{\phi}}$$

$\int ds \cdot \int dz$

• <u>Magnetic field Inside by bound current</u> :-

$$I_b(\vec{J}_b) = \int \vec{J}_b \cdot d\vec{S}_\perp = \int_0^s \int_0^l -K\, ds\, dz = -K\, s\, l$$

$$\oint B\cdot dl = \int_{AB} B\cdot dl + \int_{BC} B\cdot dl + \int_{CA} B\cdot dl + \int_{DA} B\cdot dl$$

$$= Bl + 0 + 0 + 0 = Bl$$

So $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$$B\cdot l = \mu_0 (+Ksl)$$

$$\boxed{B = +\mu_0 K s\,\hat{z}}$$

by H, There is No free charge. so $\vec{H}_{in} = 0$

$$\vec{B}_{in} = \mu_0(\vec{H}_{in} + \vec{M}_{in})$$

$$\vec{B}_{in} = \mu_0(0 + Ks\hat{z})$$

$$\boxed{\vec{B}_{in} = \mu_0 K s\,\hat{z}}$$

Magnetic field Outside by bound current,

$$I_b(\vec{J}_b) = \int_0^R\int_0^l -K\,ds\,dz = -KRl$$

$$I_b(\vec{K}_b) = K_b \times l_\perp = KR \times l = KRl$$

$$I_{enc} = 0$$

So $\boxed{B_{out} = 0}$

By H, $H_{out} = 0$

so $B_{out} = \mu_0 M_{out}$

$$B_{out} = \mu_0 \times 0 \Rightarrow \boxed{B_{out} = 0}$$

Boundry Conditions on $\vec{H}$:-

$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \boxed{B^{\perp}_{above} = B^{\perp}_{below}}$

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \Rightarrow \boxed{B^{\parallel}_{above} - B^{\parallel}_{below} = \mu_0 K}$

Tangential comp. of mag. field is discontinous by the amount $\mu_0 K$.

$\vec{\nabla}\cdot\vec{H} = -\vec{\nabla}\cdot\vec{M}$

$B = \mu_0(\vec{H}+\vec{M})$
$\nabla\cdot B = 0$
$\Rightarrow \mu_0\nabla\cdot(H+M) = 0$
$\nabla\cdot H = -\nabla\cdot M$

$$\boxed{H^{\perp}_{above} - H^{\perp}_{below} = M^{\perp}_{below} - M^{\perp}_{above}}$$

$\nabla\cdot H$ can be 0 only if $\vec{H}$ is not uniform.

$\vec{\nabla}\times\vec{H} = \vec{J}_f$ $\quad J_f \rightarrow$ free vol. charge density

$$\boxed{H^{\parallel}_{above} - H^{\parallel}_{below} = K_f}$$

Tangential comp. of H is discontinous by the amount K_f (free current)

If there exist a boundries whose permiability are different (i.e. mag. properties are different) for such a boundries

① μ_1
dielectric
② μ_2

$$K_f = 0$$

then tangential comp. of H i.e. $H^{\parallel}_{above} = H^{\parallel}_{below} = 0$

Que :- At the Interface b/w two linear magnetic materials the mag. field lines bend. Show that

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{\mu_2}{\mu_1}$$

Assuming that, there is No free current at the boundry.

If no free current then boundry cond's are $H^{\parallel}$ & $B^{\perp}$ are continous.

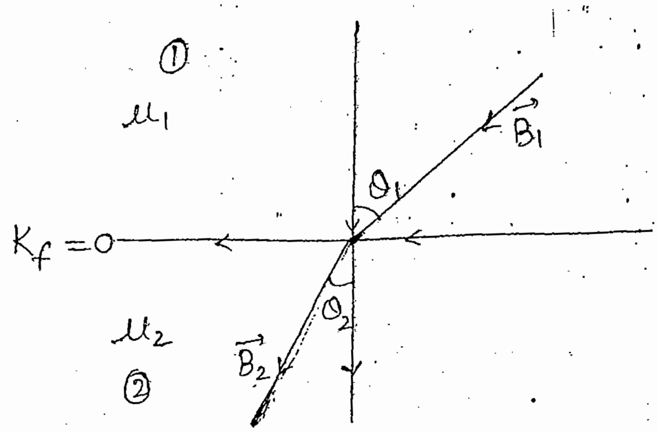
$B^{\perp} \rightarrow$ always continous
$H^{\parallel} \rightarrow$ conditionally "

i.e. B.C's are $\quad H_1^{\parallel} = H_2^{\parallel}$

$$B_1^{\perp} = B_2^{\perp}$$

$B_1^{\parallel} = \mu_1 H_1^{\parallel}$

$B_2^{\parallel} = \mu_2 H_2^{\parallel}$



Now $B_1^{\perp} = B_2^{\perp}$

$\Rightarrow \boxed{B_1 \cos\theta_1 = B_2 \cos\theta_2}$ —— (1)

& $H_1^{\parallel} = H_2^{\parallel}$

$\Rightarrow H_1 \sin\theta_1 = H_2 \sin\theta_2$

$\Rightarrow \boxed{\frac{B_1}{\mu_1} \sin\theta_1 = \frac{B_2}{\mu_2} \sin\theta_2}$ —— (2)

Deviding (2) by (1) $\Rightarrow$

$$\frac{B_1}{\mu_1} \Big/ B_1 \tan\theta_1 = \frac{B_2}{\mu_2} \Big/ B_2 \tan\theta_2$$

$$\frac{1}{\mu_1} \tan\theta_1 = \frac{1}{\mu_2} \tan\theta_2$$

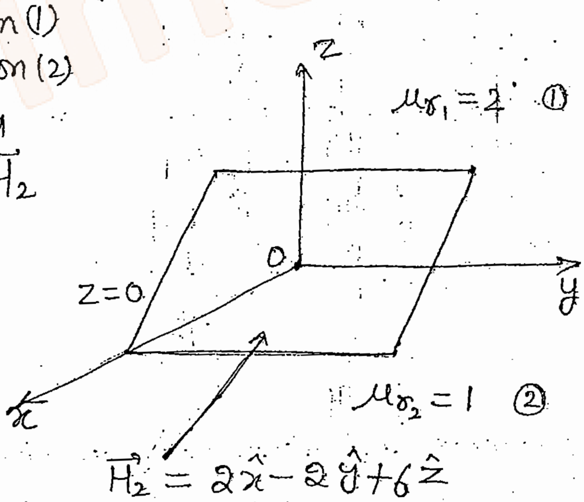
$$\Rightarrow \boxed{\frac{\tan\theta_2}{\tan\theta_1} = \frac{\mu_2}{\mu_1}} = \frac{\epsilon_2}{\epsilon_1} \text{ (for electric lines)}$$

for magnetic lines.

<u>Ques</u> :- Given $\mu_r = 2$ in region (1)
$\mu_r = 1$ in region (2)

Apply a mag. field which is tilted by some angle & $\vec{H_2}$ exist in region (2)

$\vec{H_2} = 2\hat{x} - 2\hat{y} + 6\hat{z}$



Find (i) $\vec{B_1}$ (ii) $\vec{H_1}$

No free current at the surface i.e. $K_f = 0$

If $K_f = 0$ then B.C. are

$H_1^{\parallel} = H_2^{\parallel}$ & $B_1^{\perp} = B_2^{\perp}$

$H_2 = 2\hat{x} - 2\hat{y} + 6\hat{z}$

$\Rightarrow H_{2n} = 6\hat{z}$ & $H_{2t} = 2\hat{x} - 2\hat{y}$

$\cancel{B_1^{\perp} = B_2^{\perp}} \cancel{\Rightarrow} \cancel{B_{1n} = B_{2n}} \Rightarrow \cancel{B_{1n}}$

① $\boxed{H_1^{\parallel} = H_2^{\parallel}} \Rightarrow H_{1t} = H_{2t} = 2\hat{x} - 2\hat{y} = H_1^{\parallel}$

$B_1^{\perp} = B_2^{\perp} \Rightarrow B_{1n} = B_{2n}$

$\mu_1 H_{1n} = \mu_2 H_{2n}$ — ①

$B_2^{\perp} = \mu_2 H_2^{\perp}$

$= \mu_0 \mu_{r_2} H_2^{\perp}$

$B_2^{\perp} = \boxed{6 \mu_0 \hat{z} = B_1^{\perp}}$

$B_1^{\perp} = \mu_1 H_1^{\perp} = \mu_0 2 H_1^{\perp} \Rightarrow H_1^{\perp} = 3\hat{z}$

so $\vec{H}_1 = H_1^{\parallel} + H_1^{\perp}$

$$\boxed{\vec{H}_1 = 2\hat{x} - 2\hat{y} + 3\hat{z}}$$

① $\Rightarrow \dfrac{B_1^{\parallel}}{\mu_1} = H_2^{\parallel} = H_1^{\parallel}$

$\dfrac{B_1^{\parallel}}{\mu_0 \mu_{r_1}} = H_2^{\parallel}$

$B_1^{\parallel} = 2\mu_0 H_2^{\parallel}$

$B_1^{\parallel} = \mu_0(4\hat{x} - 4\hat{y})$

$$\boxed{\vec{B}_1 = \mu_0(4\hat{x} - 4\hat{y} + 6\hat{z})}$$

Note :-

$\vec{B}_1 = \mu_1 \vec{H}_1$

$= \mu_0 \mu_{r_1} \vec{H}_1$

$\vec{B}_1 = \mu_0(4\hat{x} - 4\hat{y} + 6\hat{z})$

Ques ① A long wire has a circular cross-section with radius a the current density in the wire is $J(r) = J_0\left(\dfrac{a^2 - r^2}{a^2}\right)$ where r is the distance from the axis. Calculate :-

(i) Total current in the wire

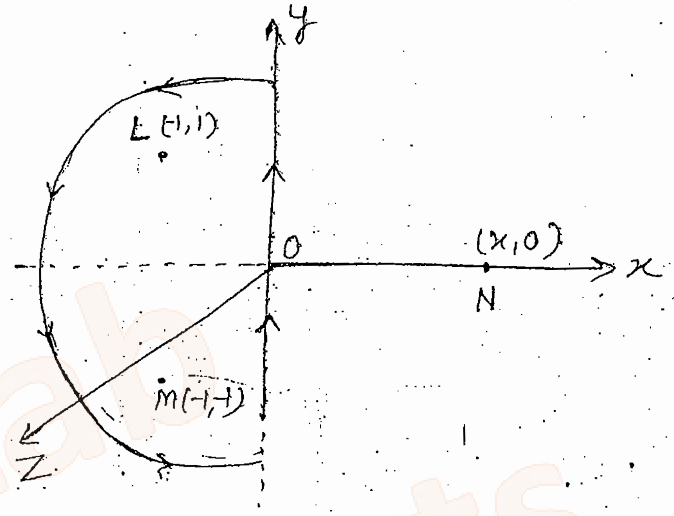
(ii) Mag. field inside & outside the wire

##

Q.2 :- Consider 2 infinitely long wires parallel to z-axis carrying the same current I. One of the wire passes through the point L with co-ordinates (-1, 1) and other through point M with co-ordinates (-1, -1) in the x-y plane as shown in the figure. The dirn of the current in both the wires is in +ve z-dirn.

Find :

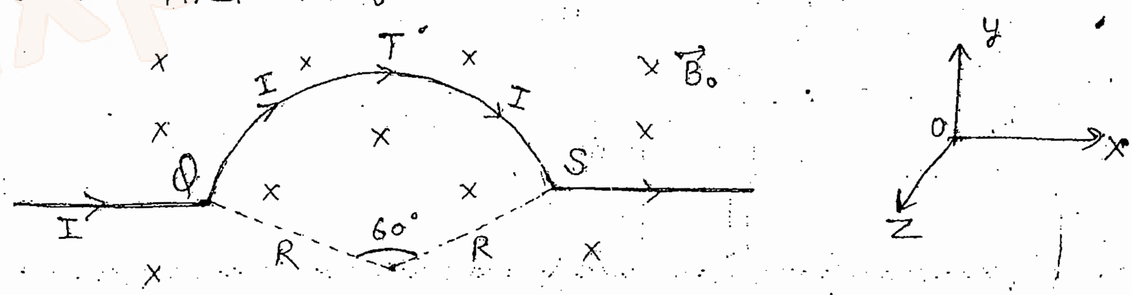
(i) the value of $\oint \vec{B} \cdot \vec{dl}$ for a loop shown in the figure.

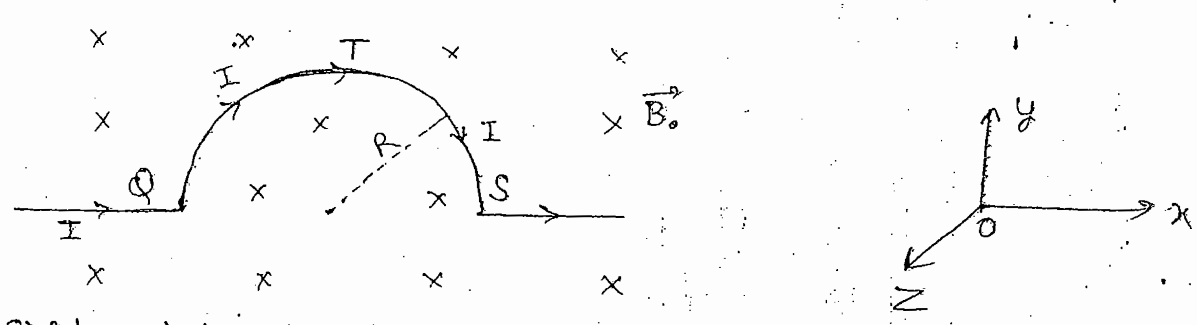


(ii) A IIIrd long wire carrying current I also perpendicular to the x-y plane is placed at the points N with co-ordinate (x, 0) s.t. mag. field at the origin is doubled.

Find the value of x & dirn of the current in IIIrd wire.

Q.3 :- A circular Arc QTS is kept in an external magnetic field $\vec{B_0}$ as shown in the figure. The arc carries a current I. Find the force on the ~~earth~~ arc.



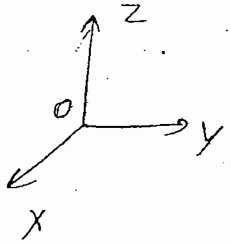


mag. field → into the page

Q.3 :- (i) $\vec{dl_1} = dy\,\hat{y} + dz\,\hat{z}$

$$\vec{dl_2} = dy\,\hat{y} + (-dz\,\hat{z})$$

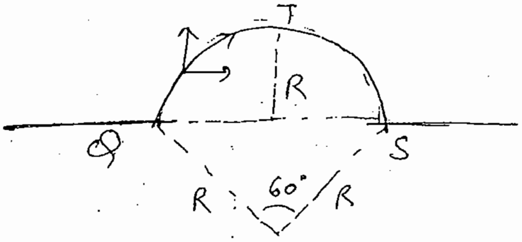
$$\vec{dl_1} + \vec{dl_2} = 2dy\,\hat{y}$$



$$\vec{F} = I\int \vec{dl} \times \vec{B}$$

$$= I\,2B\int_0^{R/2} dy\,\hat{z}$$

$$\boxed{\vec{F} = IBR\,\hat{Z}}$$



(ii) $\vec{F} = I\int \vec{dl} \times \vec{B}$

$$= I\,2B\int_0^{R} dy\,\hat{z}$$

$$\boxed{\vec{F} = 2IBR\,\hat{z}}$$