Free Study Material
from
All Lab Experiments

Electromagnetic Theory
for NET/Gate Physical Sciences
> Dielectrics <

6/8/2012

Dielectrics :- Dielectrics are the perfect Insulators & they have the property of polarization. If we apply electric field to a dielectric, it will be polarised.

Dielectrics are mainly of 2 types :-

1) Polar Dielectrics
2) Nonpolar Dielectrics

Polar dielectrics are those dielectrics which are made up of polar molecules & Non-polar dielectrics are those which made up of nonpolar molecules.

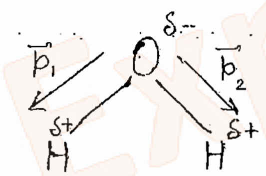
Polar Molecules - Those have permanent dipole moment more precisely - permanent electric dipole moment

{ electric dipole Moment → dipole moment
  Magnetic " " → magnetic moment }

Non-polar Molecule - Those don't have permanent dipole moment.

Ex :-

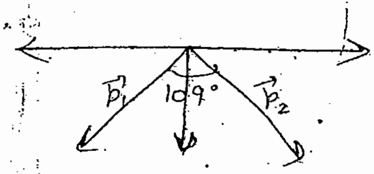
Polar Molecules	Non Polar molecules
NaCl	N_2
HCl	CO_2
H_2O	H_2
	O_2

(1) 

O → oxygen is highly electronegative
O share 1-1 e^- with H to ~~com~~ achieve stability & make covalent bond.

Magnitude of dipole mom. depends upon charge & distance

If we resolve these two dipole mom. in components then It has a permanent dipole Moment.



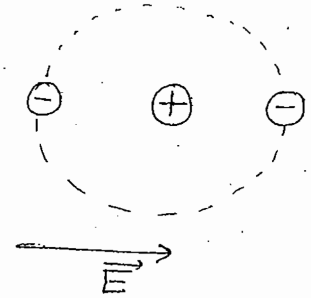
(2) N ..... N both have same electronegativity So shared e^- pair is b/w N and N

So Net dipole Mom = 0

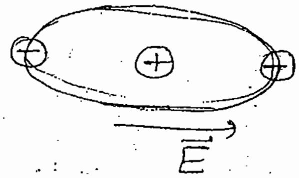
(3) $O^{\delta-} \text{—} C^{\delta+} \text{—} O^{\delta-}$ both dipole mom. are in opposite dir^n So Net dipole mom = 0

Homonuclear Molecules are Non-polar.
Hetero nuclear ..

Induced Dipole Moment :-



Net dipole mom. of an atom is 0 bcoz centre of -ve charges & +ve charges coinside.
If we apply electric field then shape will change & e^- will shift from their position while nucleus will be fix. Now centre of +ve & -ve charge will not coinside. It will accuire some dipole Mom.



This dipole mom. is not permanent dipole mom
It is called Induced dipole mom.

And $\vec{p}_{in} \propto \vec{E}$

$$\boxed{\vec{p}_{in} = \alpha \vec{E}_{loc}}$$

α is polarisability of the atom.

Polarisability :- Abality of polarisation is called polarisability.

This electric field is local electric field.

$$\vec{E} = \vec{E}_{local}$$

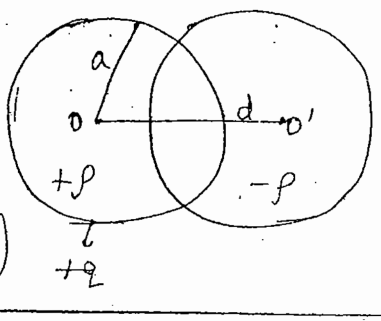
Que 1 :- A primitive model of an atom consist of a point charge +q surrounded by a uniformely charge spherical cloud (-q) of radius a. Calculate the atomic polarisability of the atom.

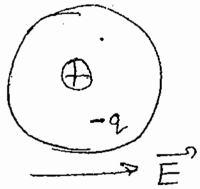
$$\vec{E} = \frac{\rho \vec{d}}{3 \epsilon_0}$$

convert it in form of q.

$$\vec{E} = \frac{q \vec{d}}{4\pi \epsilon_0 a^3}$$

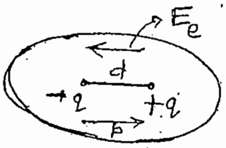
$$\left(\begin{array}{l} q = \rho V \\ \rho = \frac{q}{V} = \frac{q}{\frac{4}{3}\pi a^3} \end{array} \right)$$





When we apply electric field then $-ve$ charge displaced in the opposite dirⁿ of $\vec{E}$.

The distortion of shape of sphere is very small. The order of d is very small (less than $\mathring{A}$).



The shape will change untill External ele. field E becomes equal to the internal equilibrium elec. field E_e.

$$\boxed{E \doteq E_e = \frac{qd}{4\pi\epsilon_0 a^3}} \quad \text{——(1)}$$

Dipole mom.

$$p = qd$$

Dirⁿ of $\vec{p}$ will be in the dirⁿ of externally aplied elec. field. So dipole mom. accurie by the atom is

Induced dipole mom. $\leftarrow$ $$\boxed{p = 4\pi\epsilon_0 a^3 E} \quad \text{——(2)}$$

We have $p_{in} = \alpha E$ ——(3)

On Comparing, We get $\boxed{\alpha = 4\pi\epsilon_0 a^3}$ or $\boxed{\alpha = 3\epsilon_0 V}$ ——(4) ——(5)

Polarisability

$V \to$ volume of the atom

\# In C.G.S. Unit :- $\epsilon_0 = \frac{1}{4\pi}$

So $\alpha = a^3$, $\boxed{\text{unit} = cm^3}$ (volume)

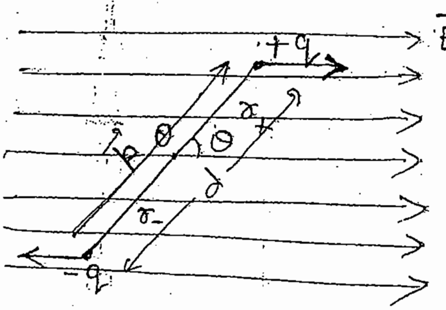
\# In M.K.S. Unit :- $\frac{N}{C} = \frac{1}{\epsilon_0} \times \frac{C}{m^2} \Rightarrow \epsilon_0 = \frac{C^2}{N\text{-}m^2}$

Hence $\boxed{\text{Unit of } \alpha = \frac{C^2\text{-}m}{N}}$

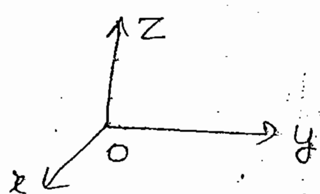
• Force & Torque on a permanent dipole in uniform Electric field :-

A dipole is placed in uniform electric field.

Electric field is uniform i.e. does not depand on space part.



$$\vec{E} = E_0 \hat{y}$$



force on the dipole,

$$\vec{F}_+ = q\vec{E} \quad ; \quad \vec{F}_- = -q\vec{E}$$

Total force on dipole $\vec{F} = \vec{F}_+ + \vec{F}_-$

$$\boxed{\vec{F} = 0}$$

In uniform electric field, force on dipole is zero.

<u>Note</u> :- If elec. field depend on space part → Non uniform then dipole will experience some force.

for a ideal dipole (have equal & opposite charges), ~~so~~ the point of rotation will be centre of mass O. Force due to rotation is <u>Torque</u> :-

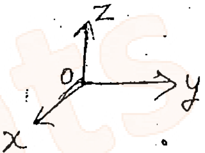
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_-$$

$$\vec{\tau} = \frac{\vec{d}}{2} \times (q\vec{E}) + \left(-\frac{\vec{d}}{2}\right) \times (-q\vec{E})$$

$$\vec{\tau} = q(\vec{d} \times \vec{E})$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$



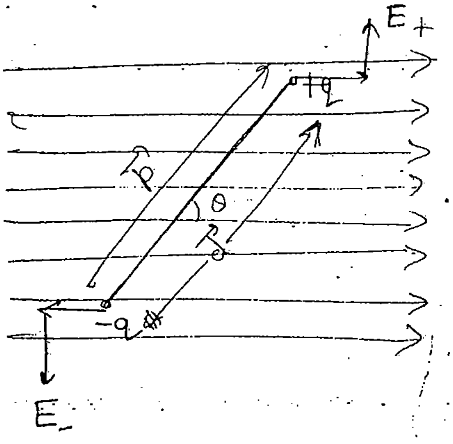
Dirⁿ of Torque → $(-\hat{x})$

<u>Force & Torque in Non-Uniform Electric field</u> :-

E_+ is different from E_-.

$$\vec{F}_{net} = q\vec{E}_+ - q\vec{E}_-$$

$$= q(\vec{E}_+ - \vec{E}_-)$$



d is very small so change in elec. field is not drastic (small change)

So $\boxed{\vec{F}_{net} = q\,\Delta\vec{E}}$

$$\rightarrow \Delta\vec{E} = (\vec{d}\cdot\vec{\nabla})\vec{E}$$

So $\vec{F}_{net} = q(\vec{d}\cdot\vec{\nabla})\vec{E}$

$$\boxed{\vec{F}_{net} = (\vec{p}\cdot\vec{\nabla})\vec{E}} \text{ — Imp}$$

If $\vec{F}_{net} = \vec{\nabla}(\vec{p}\cdot\vec{E})$ → This is conditionally True, if p is not a funⁿ of position only.

from Gradient theorem

$$dA = (\vec{\nabla}A)\cdot d\vec{r}$$

change in A

$$\vec{\nabla}A = \frac{\partial}{\partial x}A\hat{i} + \frac{\partial}{\partial y}A\hat{j} + \frac{\partial}{\partial z}A\hat{k}$$

$$d\vec{A} = (d\vec{r}\cdot\vec{\nabla})\vec{A}$$

$$\vec{\nabla}(\vec{p}\cdot\vec{E}) = \vec{p}\times\underbrace{(\vec{\nabla}\times\vec{E})}_{=0} + \vec{E}\times\underbrace{(\vec{\nabla}\times\vec{p})}_{=0} + (\vec{p}\cdot\vec{\nabla})\vec{E} + \underbrace{(\vec{E}\cdot\vec{\nabla})\vec{p}}_{=0}$$

In electrostatic field   p is not func^n of position ⟶ ″

So $\boxed{\vec{\nabla}(\vec{p}\cdot\vec{E}) = (\vec{p}\cdot\vec{\nabla})\vec{E}}$

In Non Uniform elec. field both force & torque are Non-zero

<u>Torque</u> :- $\vec{\tau} = \underbrace{(\vec{p}\times\vec{E})}_{\text{uniform}} + \underbrace{(\vec{r}\times\vec{F})}_{\text{Non-uniform}}$

where r → distance from axis of rotation.

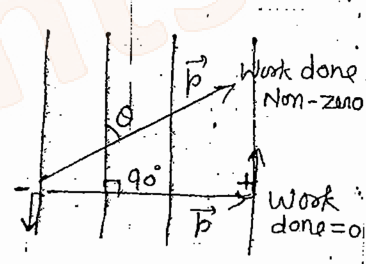
$$\boxed{\vec{\tau} = (\vec{p}\times\vec{E}) + \vec{r}\times[(\vec{p}\cdot\vec{\nabla})\vec{E}]}$$

For a Pure (point or perfect) dipole $\boxed{\vec{\tau} = \vec{p}\times\vec{E}}$ for uniform & Non-uniform elec. field both.

<u>Potential Energy of a dipole</u> :-

Work done under torque τ, will store in form of pot^n energy.

$$dU = dW = \tau\, d\theta$$
$$= |\vec{p}\times\vec{E}|\, d\theta$$
$$= pE\sin\theta\, d\theta$$



$$U = W = \int_{90^\circ}^{\theta} pE\sin\theta\, d\theta = pE(-\cos\theta)_{90^\circ}^{\theta}$$

$$U = -pE\cos\theta \Rightarrow \boxed{U = -\vec{p}\cdot\vec{E}}$$

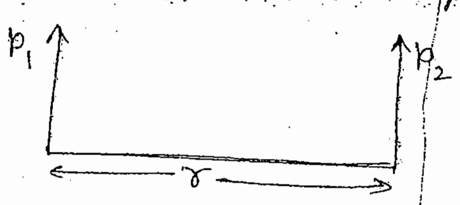
−ve means work done itself.

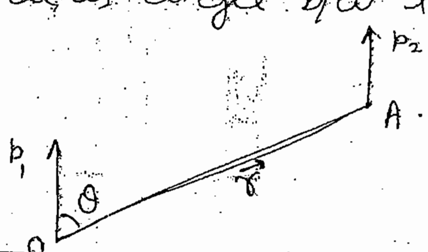
Minimum pot^n energy signifies the stability.

If p & E are in same dir^n then $\theta = 0$

<u>Interaction Energy of 2 Dipoles</u> :-

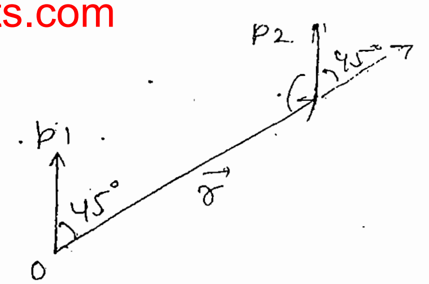
depends upon both the distance b/w the dipoles r as well as angle b/w them.





$$\boxed{U = \frac{1}{4\pi\epsilon_0 r^3}\left[\vec{p}_1\cdot\vec{p}_2 - 3(\vec{p}_1\cdot\hat{r})(\vec{p}_2\cdot\hat{r})\right]}$$

Ques :- find U ?



$$U = \frac{1}{4\pi\epsilon_0 r^3}\left[\vec{p}_1\cdot\vec{p}_2 - 3(\vec{p}_1\cdot\hat{r})(\vec{p}_2\cdot\hat{r})\right]$$

$$= \frac{1}{4\pi\epsilon_0 r^3}\left[p_1 p_2 - 3 p_1\cos 45 \cdot p_2 \cos 45\right]$$

$$= \frac{1}{4\pi\epsilon_0 r^3}\, p_1 p_2 \left(1-\frac{3}{2}\right)$$

angle b/w p_1 & p_2 $= 0$

$$\boxed{U = \frac{-1}{4\pi\epsilon_0 r^3}\,\frac{p_1 p_2}{2}}$$

Ques 1- A dipole $\vec{p}$ is situated at origin & points in z-dirⁿ

(a) What is the force on point charge q at (a,0,0)

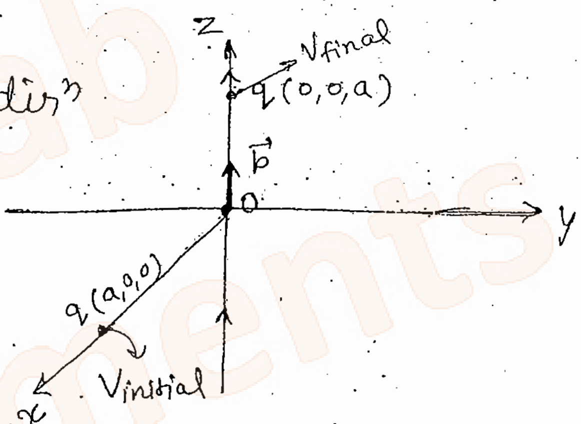
(b) " " " " " at (0,0,a)

(c) How much work done required to move q from (a,0,0) to (0,0,a)

$$\vec{F} = q\,\vec{E}_{dipole}$$

Electric field of a dipole in z-dirⁿ

$$\vec{E} = \frac{p}{4\pi\epsilon_0 r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$



(a) for q at (a,0,0) → $\theta = 90°$

$$\vec{E} = \frac{p}{4\pi\epsilon_0 a^3}\left[0+\hat{\theta}\right] \quad (r=a)$$

$$\boxed{\vec{E} = \frac{p}{4\pi\epsilon_0 a^3}\,\hat{\theta} = \frac{-p}{4\pi\epsilon_0 a^3}\,\hat{z}}$$

$\hat{\theta} = 0 + 0 - \sin\theta\,\hat{z}$

$\hat{\theta} = -\hat{z}$ $\{\theta = 90°\}$

So $$\boxed{\vec{F} = \frac{qp}{4\pi\epsilon_0 a^3}\,\hat{\theta} = \frac{pq}{4\pi\epsilon_0 a^3}(-\hat{z})}$$

(b) for q at (0,0,a) → $\theta = 0°$

$$\vec{E} = \frac{p}{4\pi\epsilon_0 r^3}\left[2\hat{r}\right] \qquad [\hat{r} = \hat{z}]$$

$$\vec{F} = \frac{2pq}{4\pi\epsilon_0 a^3}\,\hat{z}$$

(c) Work done

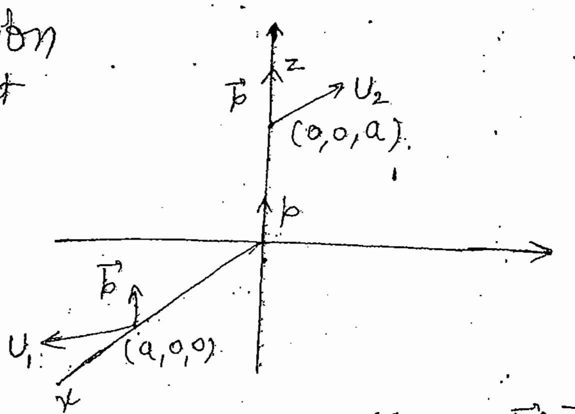
$$\Delta U = \Delta W = q\left(V_{final} - V_{initial}\right)$$

$$V_{final} = \frac{1}{4\pi\epsilon_0}\,\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0}\,\frac{p}{a^2} \qquad (\theta = 0) \text{ at } (0,0,a)$$

$$V_{initial} = 0 \qquad (\theta = 90°) \text{ at } (a,0,0)$$

$$\boxed{W = \frac{1}{4\pi\epsilon_0}\,\frac{pq}{a^2}}$$

Without changing the orientation dipole is shifted from point $(a,0,0)$ to $(0,0,a)$



Angle b/w $\overset{(a,0,0)}{\vec{p}_1}$ & $\overset{(0,0,0)}{\vec{p}_2} = 0°$

$$\vec{p}_1 \cdot \vec{p}_2 = p^2$$

$\vec{p}_1 \cdot \hat{r} = p_i \cos 90° = 0$

$\vec{p}_2 \cdot \hat{r} = p \cdot \cos 90° = 0$

$$U = \frac{1}{4\pi\epsilon_0 r^3}\left[\vec{p}_1 \cdot \vec{p}_2 - 3(\vec{p}_1 \cdot \hat{r})(\vec{p}_2 \cdot \hat{r})\right]$$

$U_1 = -\vec{p} \cdot \vec{E}$
$U_2 = -\vec{p} \cdot \vec{E}$

$$\boxed{U_1 = \frac{p^2}{4\pi\epsilon_0 a^3}}$$

Angle b/w p_1 at $(0,0,0)$ & p_2 at $(0,0,a) = 0°$ so $\vec{p}_1 \cdot \vec{p}_2 = p^2$

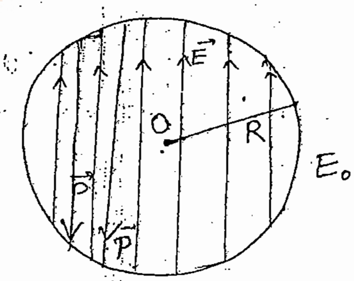
$\vec{p}_1 \cdot \hat{r} = p\hat{z}$ & $\vec{p}_2 \cdot \hat{r} = p\hat{z}$

$$U_2 = \frac{1}{4\pi\epsilon_0 a^3}\left[p^2 - 3p^2\right] \Rightarrow \boxed{U_2 = \frac{-2p^2}{4\pi\epsilon_0 a^3}}$$

- <u>Electric field of a uniformly polarised sphere</u> :-

$\vec{P}$ does not depend on $r, \theta, \phi, s, \phi, z$.

We have to find ele. field inside the sphere. $\boxed{E_{out} = 0}$



<u>Polarisation is dipole moment per unit volume</u>. This is a <u>Macroscopic</u> quantity & dipole mom. is <u>Microscopic</u>.

$$\vec{E} = \frac{\rho \vec{d}}{3\epsilon_0} = \frac{q d}{4\pi\epsilon_0 R^3} = \frac{\vec{p}}{3 \times \frac{4}{3}\pi\epsilon_0 R^3}$$

$$\boxed{\vec{E} = -\frac{\vec{P}}{3\epsilon_0}}$$

Polarisation = dipole mom. per unit volume

Dir^n of $\vec{E}_{in}$ will be opposite to dir^n of $\vec{p}$ inside i.e. polarisation

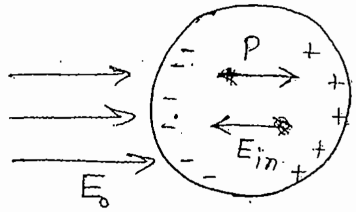
If $\vec{P}$ is uniform then $\vec{E}$ will be uniform.

* $\vec{E}$ is uniform
* opposite to polarisation
* Magnitude is $\frac{P}{3\epsilon_0}$.

· If we apply external ele. field E_0 then it will polarised the sphere then total elec. field will be less than E_0.

$E_t < E_0$

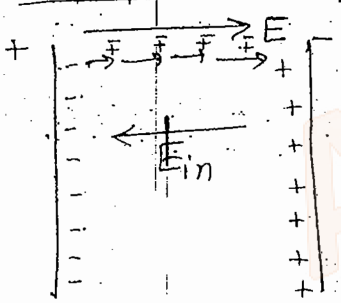
$$\boxed{\vec{E}_t = \vec{E}_0 - \frac{\vec{P}}{3\epsilon_0}}$$



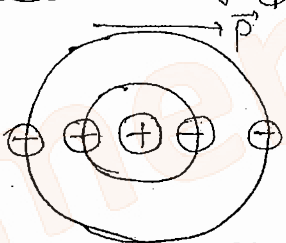
In free space polarisation $\vec{P}$ will be zero & in case of conducting sphere the internal elec. field will be equal to external ele. field.

Bound Charges :- bound charge is bound to the nuclei while free charge move freely.

Example :- Capacitor, filled with dielectric



When Elec. field is applied then polarisation will occur & Dielectric will be polarised. & dipole is formed



{ In solid there is a series of atoms

· Here an internal ele. field will produce due to bound charges. This is in opposite dirn of external applied $\vec{E}$. & E_{in} is less than E.

* Bounds charges are of 2 types :-
→ Surface bound charges
→ Volume " "

⇒ if polarisation is uniform then ~~total effect of E.F.~~ net charge is only on surface then this is called Surface bound charge.

⇒ if polarisation is not uniform & Net charge is on surface & volume also then this is called Volume ~~ch~~ bound charge.

Surface bound charge density

$$\boxed{\sigma_b = \vec{P}\cdot\hat{n}}$$

{ vol. charge density → occurs only in non-uniform field } but surface for both

$\hat{n} \rightarrow$ vector normal to the surface
$P \rightarrow$ Polarisation
Volume bound charge density

$$\boxed{\rho_b = -\vec{\nabla}\cdot\vec{P}}$$

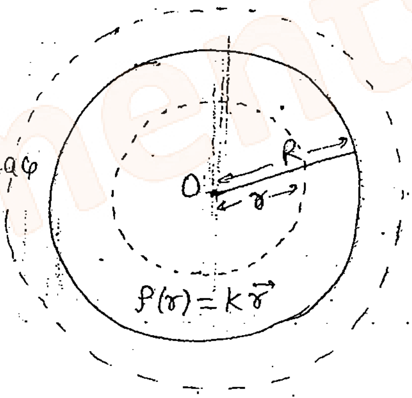
Net Electric field, inside the dielectric the electfield is $E_0 - E$ less than E_0 by the amount E_0/ϵ_r. $\left\{\epsilon_r = \frac{E_0}{E}\right.$

$$\boxed{E_{Net} = \frac{E_0}{\epsilon_r}}$$

$\epsilon_r \rightarrow$ dielectric constant

Que. 1:- A sphere of radius R carries a polarisation $\vec{P}$ $P(r) = k\,\vec{r}$ where k is a constant & $\vec{r}$ is the vector from the centre.
(a) Calculate the bound charges σ_b & ρ_b
(b) Calculate the electric field inside & outside the sphere

(a) $\sigma_b = \vec{P}\cdot\hat{n}$
$= k\,\vec{r}\cdot\hat{n} = kr\,\hat{r}\cdot\hat{r}$
$= kr = \underline{kR}$ $\left[\begin{array}{l}\text{at surface}\\ r=R\end{array}\right.$



$\rho_b = -\vec{\nabla}\cdot\vec{P}$
$= -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, k\cdot r\right)$
$= -\frac{k}{r^2}\cdot 3r^2 = \underline{\underline{-3k}}$

(b) Inside $q_{enc} = \int \rho_b\, d\tau = -3k\cdot\frac{4\pi}{3}r^3$ (q_{enc} due to volume only) {bcoz inside sphere is not enclosing its surface}

$$\boxed{q_{enc} = -4\pi k r^3}$$

By Gauss law, $\oint E\cdot ds = \frac{q_{enc}}{\epsilon_0}$

$E\cdot 4\pi r^2 = \frac{-4\pi k r^3}{\epsilon_0}$

$$\boxed{E_{in} = -\frac{kr}{\epsilon_0}\hat{r}}$$

If polarisation $\vec{P}$ is radially outward then elec. field will be opposite to it (i.e. towards inside $(-\hat{r})$)

Outside the sphere :- q_{enc} will be due to surface & volume both.

$$q_b(\sigma_b) = KR \cdot 4\pi R^2$$

$$q_b(\sigma_b) = 4\pi K R^3$$

$$\& \quad q(\rho_b) = -4\pi K R^3$$

$$\text{Total } q = 4\pi K R^3 - 4\pi K R^3$$

$$\boxed{q = 0}$$

$$\text{So} \quad \boxed{E_{out} = 0}$$

- Gauss Law in Dielectrics :- We have 2 type of bound charge density σ_b & ρ_b.

Gauss law deals with ρ_b. There are 2 types of ρ_b, ρ_b & ρ_F are bound volume charge density & free volume charge density respectively

$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

$$\text{Here} \quad \boxed{\rho = \rho_b + \rho_F}$$

$$\Rightarrow \quad \epsilon_0 (\vec{\nabla} \cdot \vec{E}) = \rho_b + \rho_F$$

$$\epsilon_0 (\vec{\nabla} \cdot \vec{E}) = \rho_F - \vec{\nabla} \cdot \vec{P}$$

$$\nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_F$$

$$\boxed{\vec{\nabla} \cdot \vec{D} = \rho_F}$$

D → displacement vector

D, includes $\vec{E}$ & $\vec{P}$ both.

The is the differential form of Gauss law of Dielectrics

Integral form :- $\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$

$$\boxed{\oint \vec{D} \cdot d\vec{s} = q_{enc}}$$

D is totally determined by the free charges.

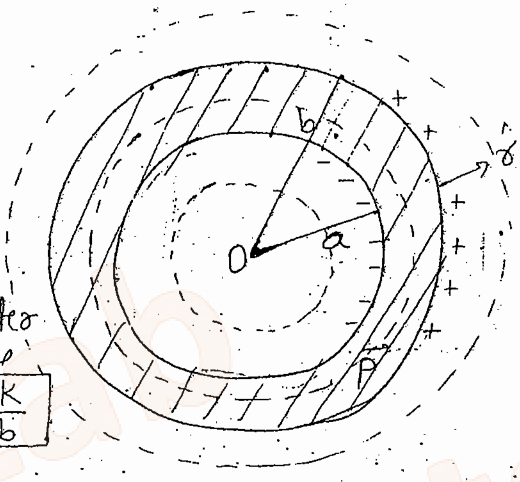
$\vec{E}$ " " " , free & bound both

bcoz In D, bound charge is Included in it.

charge on the surface of metal or conductor → free charge

Que :- A thick spherical shell inner radius a & outer radius b is made of dielectric material with polarizat[ion] $\vec{P}(r) = \frac{K}{r}\hat{r}$ where K is constant & r is the distan[ce] from the centre. There is no free charge in the region. Find the electric field in all the 3 regions :-

(i) $r < a$
(ii) $a < r < b$
(iii) $r > b$



$\sigma_b = \vec{P}\cdot\hat{n}$

$= \frac{K}{r}\hat{r}\cdot\hat{r} = \frac{K}{r}$

inner surface $\boxed{\sigma_b = -\frac{K}{a}}$ & on outer surface $\boxed{\sigma_b = +\frac{K}{b}}$

$\rho_b = -\vec{\nabla}\cdot\vec{P}$

$= -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{K}{r}\right) = -\frac{1}{r^2}\frac{\partial}{\partial r}(Kr)$

$\boxed{\rho_b = -\frac{K}{r^2}}$

In $r<a$, $q_{enc} = 0$

$E = 0$

$a<r<b$

due to volume $q_{enc} = \int_a^r \rho_b\, d\tau = \int_a^r \frac{-K}{r^2}\, r^2 \sin\theta\, d\theta\, d\phi$

$= \int_a^r -\frac{K}{\cancel{r^2}}\, 4\pi \cancel{r^2}\, dr = -4\pi K[r]_a^r$

$= -4\pi K(r-a)$

due to inner surface $q_{enc} = \oint -\frac{K}{a}\cdot 4\pi r^2 = -\frac{K 4\pi}{a}[a^2]$

$= -4\pi K a$

$q = -4\pi K r + \cancel{4\pi K a} - \cancel{4\pi K a}$

$\boxed{q_{enc} = -4\pi K r}$

$E\cdot 4\pi r^2 = -\frac{4\pi K r}{\epsilon_0} \Rightarrow \boxed{E = -\frac{K}{\epsilon_0 r}\hat{r}}$

$r>b$

$q_{enc} = \underset{+4\pi K a}{-4\pi K b} + \underset{-4\pi K a}{4\pi K b} = 0$

$\boxed{E = 0}$

II Method :- (i) $r < a$

$$\oint D \cdot dS = q_{enc}$$

$q_{enc} = 0$ So $\boxed{D = 0}$ $\Rightarrow$ $\epsilon_0 E + P = 0$

$\Rightarrow \boxed{E = 0}$

(ii) $a < r < b$

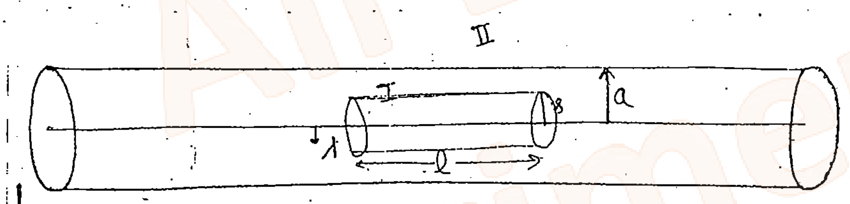
$D = 0$ (No free charge in this region)

$\epsilon_0 E + P = 0 \Rightarrow \epsilon_0 E + \frac{k}{r}\hat{r} = 0$

$$\boxed{E = -\frac{k}{\epsilon_0 r}\hat{r}}$$

(iii) $r > b$ $D = 0 \Rightarrow \epsilon_0 E + P = 0 \Rightarrow \boxed{E = 0}$

Q.2 :- A long straight wire carrying a line charge λ is surrounded by rubber insulation upto radius a. Find electric displacement D & Electric field inside & outside the rubber insulation.



Here polarisation is not given & dont know about bound charge $q_{enc} = q_f + q_b$

Use Gauss law in Dielectrics

Inside :- $\oint D \cdot ds = q_{enc}$

$q_{enc} = \lambda l$

$\therefore D \cdot 2\pi s l = \lambda l$

$$\boxed{D = \frac{\lambda}{2\pi s}\hat{s}}$$

$D = \epsilon_0 E + P$. Also $\vec{D} = \epsilon \vec{E}$. This ϵ includes P.

$\epsilon E = \frac{\lambda}{2\pi s}\hat{s}$

$E = \frac{D}{\epsilon}$

$= \frac{D}{\epsilon_0 \epsilon_r}$

$$\boxed{E_{in} = \frac{D}{\epsilon_0 \epsilon_r} = \frac{\lambda}{2\pi \epsilon_0 \epsilon_r s}\hat{s}}$$

Outside :- q free enclosed $q_{enc} = \lambda l$

$D \cdot 2\pi s l = \lambda l \Rightarrow \boxed{D = \frac{\lambda}{2\pi s}\hat{s}}$

In free space $\epsilon_r = 1$

so $$\boxed{E_{out} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}}$$

Linear Dielectrics :- are those dielectrics in which polarisation is proportional to the electric field.

$$\vec{P} \propto \vec{E}$$

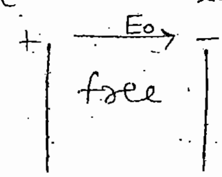
$$\Rightarrow \boxed{\vec{P} = \epsilon_0 \chi_e \vec{E}_{Macro}}$$

$\epsilon_0 \rightarrow$ permiability

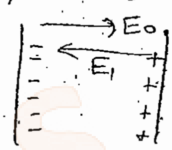
$\chi_e \rightarrow$ Electric Suceptability

$E \rightarrow$ Macroscopic elec. field

$\{ \vec{P} = \alpha \vec{E}_{local} \}$



When applied elec. field is E_0. This is local ele. field, & if we filled dielectric b/w these plates



There are many atoms in dielectric

Then a inside elec field E_1 which is in opposite dir^n to E_0. Then total Elec field $= E_0 + E_1$ (vector)

This Ele. field is Macroscopic. $= E_0 - E_1$ (magnitude)

In Maxwell eq^n, the Macroscopic E-field on a individual atom is diffrent from average field on plates called Local E.F.

We have $\vec{D} = \epsilon_0\vec{E} + \vec{P}$ Also $D = \epsilon E$

so $$\epsilon \cancel{\vec{E}} = \epsilon_0 \cancel{\vec{E}} + \epsilon_0 \chi_e \cancel{\vec{E}}$$

$$\epsilon = \epsilon_0(1+\chi_e)$$

$$\boxed{\epsilon_r = 1 + \chi_e}$$

{ If we apply external E.F. on a material then How much it oppose that E.F. will be susceptability. }

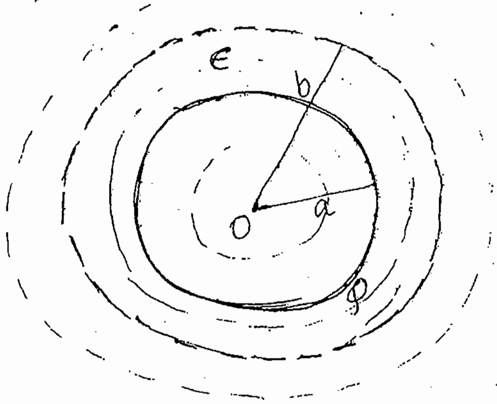
Ques :- A metalic sphere of radius 'a' carries a charge Q. It is surrounded by a linear dielectric of permitivity ϵ upto radius b. find

(i) Polarisation (ii) bound charge densities

(iii) Displacement Vector (iv) Electric field

(v) Pot^n at the centre of the sphere
(vi) Energy of this configuration.

$$P = \epsilon_0 \chi_e E$$



$r<a$, $q_{enc} = Q \Rightarrow \boxed{D=0}$

$a<r<b$, $\oint D \cdot ds = q_{enc}$

$$D \cdot 4\pi r^2 = Q$$

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2}\hat{r}}$$

$b<r$, $\boxed{\vec{D} = \frac{Q}{4\pi r^2}\hat{r}}$

Now electric field, $\vec{D} = \epsilon \vec{E}$

$r<a$, $D=0 \Rightarrow \boxed{E=0}$

$a<r<b$, $\boxed{E = \frac{Q}{4\pi \epsilon r^2}\hat{r}}$

$r>b$, $\boxed{E = \frac{Q}{4\pi \epsilon_0 r^2}\hat{r}}$ Outside the sphere permitivity is ϵ_0.

Polarisation, $\vec{P} = \epsilon_0 \chi_e \vec{E}$

$r<a$, $\boxed{P=0}$

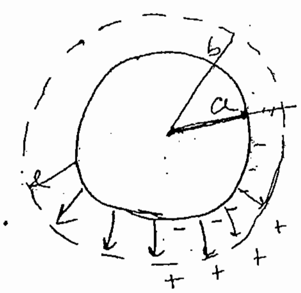
$a<r<b$, $\vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E}$

$$\boxed{\vec{P} = \frac{(\epsilon - \epsilon_0)Q}{4\pi \epsilon r^2}\hat{r}}$$

$r>b$, $\boxed{\vec{P} = 0}$ $\left\{ \begin{array}{l} \vec{P} = \epsilon_0(1-1)\vec{E} \\ = 0 \quad [\epsilon_r = 1] \end{array} \right.$

Bound charge density

$\sigma_b = \vec{P} \cdot \hat{n} \Rightarrow \sigma_b\big|_{r=a} = -\frac{(\epsilon - \epsilon_0)Q}{4\pi \epsilon a^2}$



$$\sigma_b\big|_{r=b} = \frac{+(\epsilon - \epsilon_0)Q}{4\pi \epsilon b^2}$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} \Rightarrow \rho_b = -\frac{(\epsilon-\epsilon_0)Q}{4\pi\epsilon r^2}\cdot 4\pi\,\delta^3(r)$$

$$\boxed{\rho_b = -\frac{(\epsilon-\epsilon_0)Q}{\epsilon}\,\delta^3(r)}$$

$\boxed{\rho_b = 0}$ $a<r<b$, .. Not enclosing the origin.

<u>Potⁿ at the centre</u>

$$Q_a = Q + \sigma_b\big|_{r=a} \times 4\pi a^2$$

$$= Q - \frac{(\epsilon-\epsilon_0)Q}{4\pi\epsilon a^2}\,4\pi a^2$$

$$= Q - \frac{(\epsilon-\epsilon_0)Q}{\epsilon} = Q - Q + \frac{\epsilon_0}{\epsilon}Q$$

$$\boxed{Q_a = \frac{Q}{\epsilon_r}}$$

That's why elec. field inside the dielectric becomes $\frac{1}{\epsilon_r}$ of its value bcoz charge inside " " is Q/ϵ_r.

$$Q_b = \cancel{Q} \cancel{+} \sigma_b\big|_{r=b} \times 4\pi b^2$$

$$= \cancel{Q} \cancel{+} \frac{(\epsilon-\epsilon_0)Q}{\epsilon} = \cancel{Q} + \cancel{Q} - \cancel{\frac{\epsilon_0}{\epsilon}Q}$$

$$\boxed{\cancel{Q_b = 2Q \mp \frac{Q}{\epsilon_r}}}$$

Potⁿ at centre $V = \frac{1}{4\pi\epsilon_0}\frac{Q_a}{a} + \frac{1}{4\pi\epsilon_0}\frac{Q_b}{b}$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{\epsilon_r a} + \left(\frac{\epsilon-\epsilon_0}{\epsilon}\right)\frac{Q}{b}\right]$$

$$\boxed{V_{centre} = \frac{Q}{4\pi}\left[\frac{1}{\epsilon a} + \frac{1}{\epsilon_0 b} - \frac{1}{\epsilon b}\right]}$$

By Another Method, $V_{centre} = -\int_\infty^0 \vec{E}\cdot d\vec{r}$

$$= -\int_\infty^b \vec{E}\cdot d\vec{r} - \int_b^a \vec{E}\cdot d\vec{r} - \int_a^0 \vec{E}\cdot d\vec{r}$$

$$V = \frac{-Q}{4\pi\epsilon_0}\left[\frac{-1}{r}\right]_\infty^b - \frac{Q}{4\pi\epsilon}\left[\frac{-1}{r}\right]_b^a - 0 = \frac{Q}{4\pi\epsilon_0}\frac{1}{b} + \frac{Q}{4\pi\epsilon}\frac{1}{a} - \frac{Q}{4\pi\epsilon}\frac{1}{b}$$

Energy. $\cancel{E}$ $U = \frac{\epsilon_0}{2} \int_{\text{all space}} E^2 d\tau$

$$U = \frac{\epsilon_0}{2}\left[\int_0^a E_1^2 4\pi r^2 dr + \int_a^b E_1^2 4\pi r^2 dr + \int_b^\infty E_1^2 4\pi r^2 dr\right]$$

$$U = 0 + \frac{\epsilon_0}{2}\int_a^b \frac{Q^2}{(4\pi\epsilon)^2 r^4} 4\pi r^2 dr + \frac{\epsilon_0}{2}\int_b^\infty \frac{Q^2}{(4\pi\epsilon_0)^2 r^4} 4\pi r^2 dr\Big]$$

$$= \frac{\epsilon_0}{2}\frac{Q^2}{4\pi\epsilon^2}\left[-\frac{1}{r}\right]_a^b + \frac{\epsilon_0}{2}\frac{Q^2}{4\pi\epsilon_0^2}\left[\frac{-1}{r}\right]_b^\infty$$

$$= \frac{\epsilon_0}{2}\frac{Q^2}{4\pi}\left[\frac{-1}{\epsilon^2}\left(\frac{1}{b}-\frac{1}{a}\right) - \frac{1}{\epsilon_0^2}\left[-\frac{1}{b}\right]\right]$$

$$= \frac{\epsilon_0}{2}\frac{Q^2}{4\pi}\left[-\frac{1}{\epsilon^2}\left(\frac{1}{b}-\frac{1}{a}\right) + \frac{1}{\epsilon_0^2 b}\right]$$

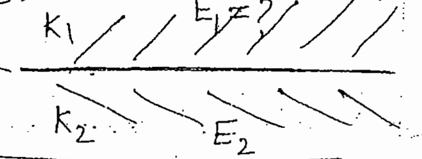
$$\boxed{U = \frac{\epsilon_0 Q^2}{8\pi}\left[\frac{1}{\epsilon^2 a} - \frac{1}{\epsilon^2 b} + \frac{1}{\epsilon_0^2 b}\right]}$$

* Energy inside the Dielectric is

$$\boxed{U = \frac{1}{2}\int_{\text{all space}} \vec{D}\cdot\vec{E}\, d\tau}$$

Boundry conditions at dielectric interface :-

Boundry comes when medium changes.
If we have 2 dielectrics having dielectric constants K_1 & K_2 ~~then~~ & electric field is given in one region suppose E_2 & we have to find out E field in another region E_1.
This can be done by Boundry Conditions.



Gauss law in free space

$$\oint \vec{E}\cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$$

(I) $$\boxed{E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\epsilon_0}}$$

Gauss law in Dielectric

$$\oint \vec{D}\cdot d\vec{S} = q_{enc}$$

(II) $$\boxed{D^{\perp}_{above} - D^{\perp}_{below} = \sigma_f}$$ → free charge density

D is discontinous by the amount σ_f. $D^{\perp}$ → Normal Comp.

(2) $\oint E \cdot dl = 0$

$$\boxed{E^{\parallel}_{above} = E^{\parallel}_{below}}$$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

$\vec{E} = \frac{1}{\epsilon_0}[\vec{D} \pm \vec{P}]$

So $\oint E \cdot dl = \frac{1}{\epsilon_0} \oint [D - P] \cdot dl = 0$

$\oint \vec{D} \cdot d\vec{l} = \oint \vec{P} \cdot d\vec{l}$

$$\boxed{D^{\parallel}_{above} - D^{\parallel}_{below} = P^{\parallel}_{above} - P^{\parallel}_{below}}$$

The tangencial comp. of $\vec{D}$ is discontinuous by the difference b/w Polarisation.

Case

1. Air dielectric interface :- for air dielectric interface

air ①

σ

ϵ_r dielectric ②

$\sigma_f = 0$

$D^{\perp}_{above} = D^{\perp}_{below}$

$$\boxed{D_{1n} = D_{2n}}$$

$D_n \Rightarrow$ normal comp. of ~~the~~ D

Normal comp. of displacement vector & tangential comp. of E is continous but normal comp. of $\vec{E}$, & tangential comp. of D is discontinous bcoz it contain both charge density (free & bound) σ_b & σ_f

$$\boxed{E_{1n} - E_{2n} = \frac{\sigma}{\epsilon}}$$

$\sigma = \sigma_b + \sigma_f$

Tangential Component :-

$E^{\parallel}_{above} = E^{\parallel}_{below}$

$$\boxed{E_{1t} = E_{2t}}$$

& $D^{\parallel}_{above} = D^{\parallel}_{below}$

$$\boxed{D_{1t} = D_{2t}}$$

still $\sigma_f = 0$

2. Dielectric - Dielectric Interface :- $\sigma_f = 0$

$E_{1t} = E_{2t}$

$D_{1n} = D_{2n}$ (σ_f should be zero)

ϵ_{r_1} ① dielectric

σ

ϵ_{r_2} ② dielectric

3. Air - Conductor Interface :-

This boundary contain free charge Then

$\boxed{\sigma_b \neq 0}$

air ①

σ_f

Conductor ②

$$D^{\perp}_{above} - D^{\perp}_{below} = \sigma_f$$

$$\boxed{D_{1n} - D_{2n} = \sigma_f}$$

& $\boxed{E_{1n} - E_{2n} = \dfrac{\sigma_f}{\epsilon_0}}$

In region ② the normal & tangential comp. of Electric field is zero. $E_{1t} = E_{2t}$ $\{E_{2t} = 0\}$

$$\boxed{E_{1t} = 0}$$

$\boxed{E_{1n} = \dfrac{\sigma_f}{\epsilon_0}}$ & $\boxed{D_{1t} - D_{2t} = P_{1t} - P_{2t}}$

4. Dielectric - Conductor Interface :-

Here charge density is σ.

σ_f bcoz of conductor &

σ_b " " dielectric

dielectric ①

σ

Conductor ②

$$\sigma = \sigma_b + \sigma_f$$

$$\boxed{E_{1n} - E_{2n} = \frac{\sigma}{\epsilon_0}}$$

$\boxed{E_{2n} = 0}$ $\boxed{D_{1n} - D_{2n} = \sigma_f}$

$\boxed{E_{1n} = \dfrac{\sigma}{\epsilon_0}}$ $\boxed{D_{1t} - D_{2t} = P_{1t} - P_{2t}}$

$$\boxed{E_{1t} = 0}$$

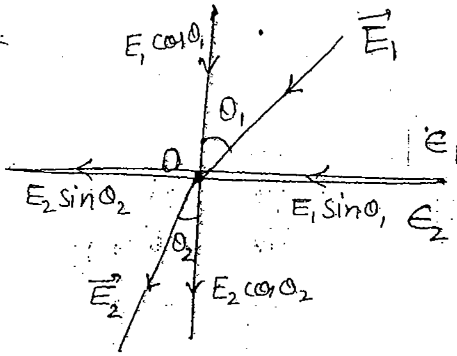
Que :- At the interface b/w 2 linear dielectrics & electric field lines bend as shown in the figure. Show that $\dfrac{\tan\theta_2}{\tan\theta_1} = \dfrac{\epsilon_2}{\epsilon_1}$. Assuming no free charge at the boundary.

For linear dielectrics

$$\vec{D} = \epsilon \vec{E}$$

This rel^n is possible only for linear dielectrics.

In boundary b/w dielectric & dielectric, there is No free charge.



$$D_{1n} = D_{2n} \quad \text{—— (i)}$$
$$E_{1t} = E_{2t} \quad \text{—— (ii)}$$

elec. field in region ① is $\vec{E_1}$ & ② is $\vec{E_2}$ and permitivity in ① & ② are ϵ_1 & ϵ_2 then

(i) $\rightarrow$ $\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$

$\Rightarrow \epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2$ —— (1)

(ii) $\rightarrow$ $E_1 \sin\theta_1 = E_2 \sin\theta_2$ —— (2)

Devide $\frac{(2)}{(1)} \Rightarrow \frac{\tan\theta_1}{\epsilon_1} = \frac{\tan\theta_2}{\epsilon_2}$

$$\Rightarrow \boxed{\frac{\tan\theta_2}{\tan\theta_1} = \frac{\epsilon_2}{\epsilon_1}}$$

Here permitivity was given but if dielectric constant ϵ_{r_1} & ϵ_{r_2} are given then

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{\epsilon_2}{\epsilon_1} = \frac{\epsilon_0\epsilon_{r_2}}{\epsilon_0\epsilon_{r_1}}$$

$$\Rightarrow \boxed{\frac{\tan\theta_2}{\tan\theta_1} = \frac{\epsilon_{r_2}}{\epsilon_{r_1}}}$$

OR $\boxed{\frac{\cot\theta_2}{\cot\theta_1} = \frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{\epsilon_1}{\epsilon_2}}$

If an electric field comes from lower to higher dielectric region then it will bend away from the normal.

If $\epsilon_{r_1} = 2$, $\epsilon_{r_2} = 3$ $\Rightarrow$ $\frac{\epsilon_{r_2}}{\epsilon_{r_1}} > 1$

$\left\{ \frac{\epsilon_{r_2}}{\epsilon_{r_1}} = \frac{3}{2} = 1.5 > 1 \right\}$

so $\tan\theta_2 > \tan\theta_1$

$$\boxed{\theta_2 > \theta_1}$$

Que:- Two extensive homogenous isotropic dielectric meet on $z=0$ plane. for $z>0$, $\epsilon_{r_1}=4$ & for $z<0$, $\epsilon_{r_2}=3$. A uniform electric field E_1

$$\vec{E_1} = 5\hat{x} - 2\hat{y} + 3\hat{z} \text{ KV/m}$$

exist for $z\geq 0$. find

(a) E_2 for $z<0$ (b) D_1, $z>0$ & D_2, $z<0$

(c) The Angles, E_1 & E_2 making with the normal.

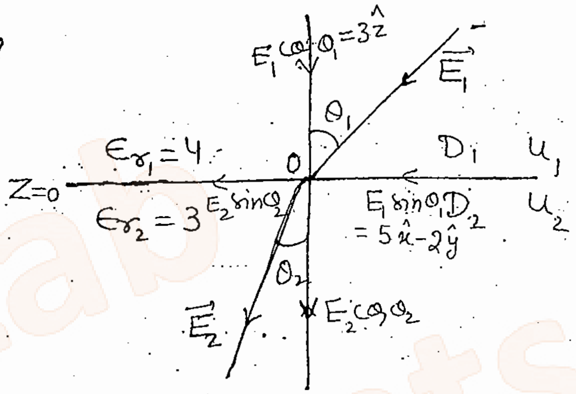
(d) Energy densities Joule/m^3 in both dielectrics.

Both are dielectrics - so there is no free charge. $\therefore \sigma_f = 0$

$D_{1n} = D_{2n}$ ——— ①

$E_{1t} = E_{2t}$ ——— ②

$\Rightarrow \epsilon_1 E_{1n} = \epsilon_2 E_{2n}$ ——— ③



vector normal to the surface is z.

then $E_{1t} = 5\hat{x} - 2\hat{y}$, $E_{1n} = 3\hat{z}$

(a). from ② $\Rightarrow$ $E_{1t} = E_{2t} \Rightarrow \boxed{E_{2t} = 5\hat{x} - 2\hat{y}}$

③ $\Rightarrow$ $\cancel{\epsilon_0}\epsilon_{r_1} E_{1n} = \cancel{\epsilon_0}\epsilon_{r_2} E_{2n}$

$$E_{2n} = \frac{\epsilon_{r_1}}{\epsilon_{r_2}} E_{1n} = \frac{4}{3} \times 3\hat{z}$$

$$E_{2n} = 4\hat{z}$$

So $E_2 = E_{1t} + E_{2n}$

$$\boxed{\vec{E_2} = 5\hat{x} - 2\hat{y} + 4\hat{z}} \text{ Ans}$$

(b) $\vec{D} = \epsilon\vec{E}$

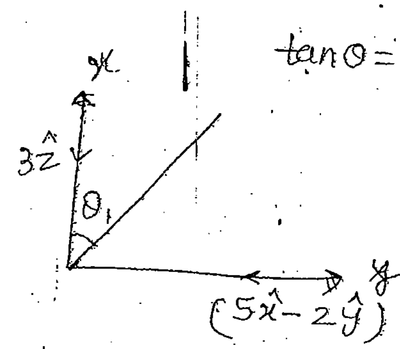
$\vec{D_1} = \epsilon_{r_1}\epsilon_0 \vec{E_1}$

$$\boxed{\vec{D_1} = 4\epsilon_0 \vec{E_1}}$$

& $$\boxed{\vec{D_2} = 3\epsilon_0 \vec{E_2}} \text{ Ans}$$

(c). $\frac{\tan\theta_2}{\tan\theta_1} = \frac{\epsilon_{r_2}}{\epsilon_{r_1}} = \frac{3}{4}$

$\tan\theta =$



$\tan\theta_1 = \frac{|5\hat{x}-2\hat{y}|}{|3\hat{z}|} = \frac{\sqrt{29}}{\sqrt{9}}$

$\boxed{\tan\theta_1 = \frac{\sqrt{29}}{3}}$

So $\tan\theta_2 = \tan\theta_1 \times \frac{3}{4} = \frac{\sqrt{29}}{3}\times\frac{3}{4}$

$\boxed{\tan\theta_2 = \frac{\sqrt{29}}{4}}$

(d) $U = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 d\tau$

{ Energy density = energy per unit volume $u = \frac{U}{V}$ }

$u_1 = \frac{\epsilon_0}{2} E_1^2$

$u_2 = \frac{\epsilon_0}{2} E_2^2$

$\Rightarrow u_1 = \frac{\epsilon_0}{2}\left[\sqrt{25+4+9}\right]^2 = \frac{\epsilon_0}{2}(38) = 19\,\epsilon_0$

$\boxed{u_1 = 19\,\epsilon_0}$

$\Rightarrow u_2 = \frac{\epsilon_0}{2}\left[\sqrt{25+4+16}\right]^2 = \frac{\epsilon_0}{2}(90)$

{ $E = kV$ }
{ $E = 10^3\,V$ }

$\boxed{u_2 = 45\,\epsilon_0}$

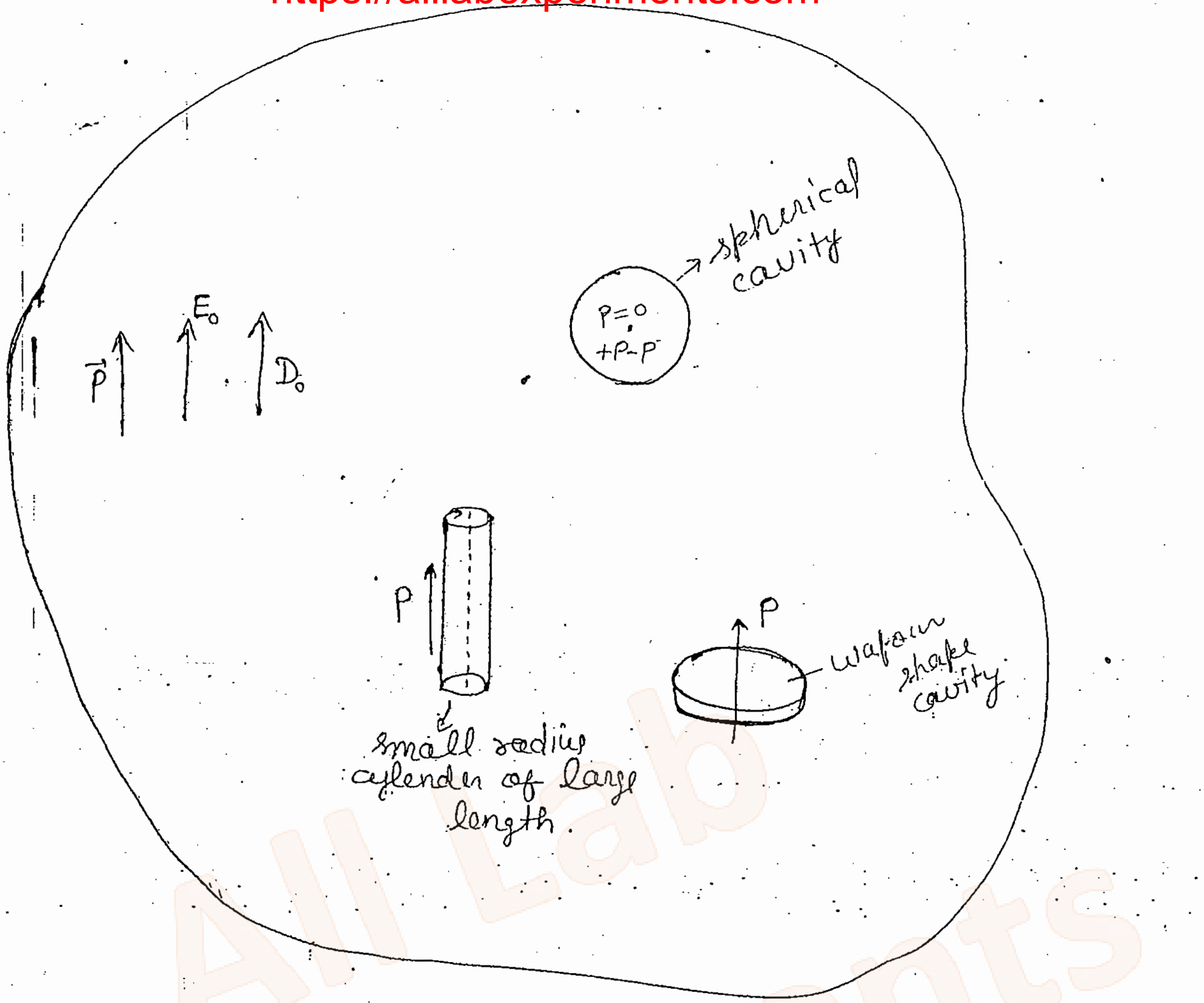
$\Rightarrow \boxed{u_1 = 19\epsilon_0 \times 10^6\ J/m^3 \quad \& \quad u_2 = 45\epsilon_0 \times 10^6\ J/m^3}$

Que :- Suppose the field inside a large piece of dielectric is E_0 such that electric displacement is D_0 s.t. $\vec{D}_0 = \epsilon_0 \vec{E} + \vec{P}$,

(a) Now a spherical cavity is hollowed out of the material. Find electric field & displacement vector inside the cavity.

(b) To the same for a long niddle shape cavity running parallel to $\vec{P}$.

(c) To the same for a thin wavoup shaped cavity $\perp$ to $\vec{P}$.



Electric field in spherical cavity is more as compare to Elec. field in dielectrics. As we know that elec-field in free space is greater than elec. field in dielectric.

$D_0 = \epsilon_0 \vec{E} + \vec{P}$

<u>Inside the spherical cavity</u>

Polarisation, $P = 0$

$+P - P = 0$

bcoz of $+P$ the elec. field will be E_0

& " " $-P$ " " " " $-\frac{\vec{P}}{3\epsilon_0}$

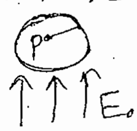
$$\vec{E} = -\frac{\vec{P}}{3\epsilon_0}$$

So total Elec. field in spherical cavity is

$$= \vec{E_0} + \frac{\vec{P}}{3\epsilon_0}$$

We have a uniformly polarised sphere & E_0 is external applied Elec. field then due to external field, internal elec. field will be produced $P/3\epsilon_0$.

Total $= E_0 - \frac{P}{3\epsilon_0} < E_0$



$$\vec{D_0} = \epsilon_0 \vec{E_0} + \vec{P} \quad \Rightarrow (\epsilon_0 E_0 = D_0 - P)$$

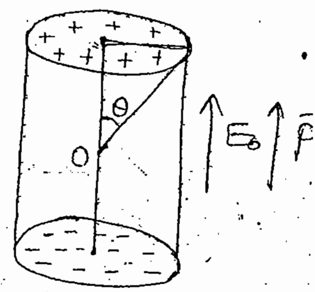
$$\vec{D} = \epsilon_0 \vec{E} \quad (\text{Put } \vec{E})$$

$$\vec{D} = \epsilon_0 \vec{E_0} + \frac{\vec{P}}{3}$$

$$\vec{D} = D_0 - \vec{P} + \frac{\vec{P}}{3} \Rightarrow \boxed{\vec{D} = \vec{D_0} - \frac{2\vec{P}}{3}}$$

<u>Niddle shape Cavity</u>

When external elec. field E_0 is applied then it will produce polarisation $\vec{P}$ inside it then total elec. field



$$\boxed{\vec{E} = \vec{E_0} - \frac{\vec{P}}{\epsilon_0}(1 - \cos\theta)}$$

Elec. field inside the dielectric is less than E_0

for Niddle shape cavity

$$\boxed{\theta = 0^\circ}$$

So $\vec{E} = \vec{E_0} - \frac{\vec{P}}{\epsilon_0}(1-1)$

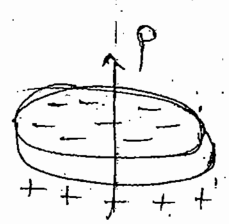
$$\Rightarrow \boxed{\vec{E} = \vec{E_0}}$$

$$\vec{D} = \epsilon_0 \vec{E}$$

$$= \epsilon_0 \vec{E_0} - P + P$$

$$\boxed{\vec{D} = \vec{D_0} - \vec{P}}$$

<u>Wafeaur shape</u> $\boxed{\theta = \pi/2}$



$$\vec{E} = \vec{E_0} + \frac{\vec{P}}{\epsilon_0}$$

$\{ P = 0 \Rightarrow +P - P = 0$ → by superposition Principal

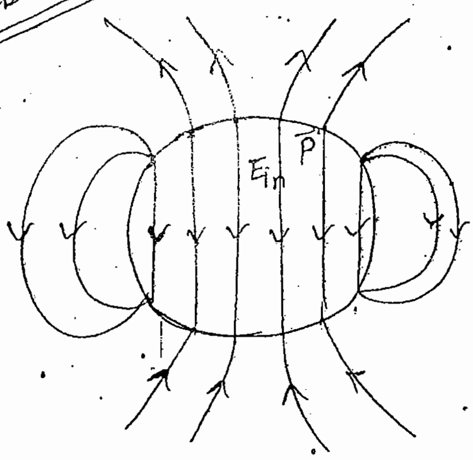
$+P \Downarrow E_0$, $-P \Downarrow -\frac{P}{\epsilon_0}$

$$\vec{D} = \epsilon_0 \vec{E}$$

$$= \epsilon_0 \vec{E_0} + P$$

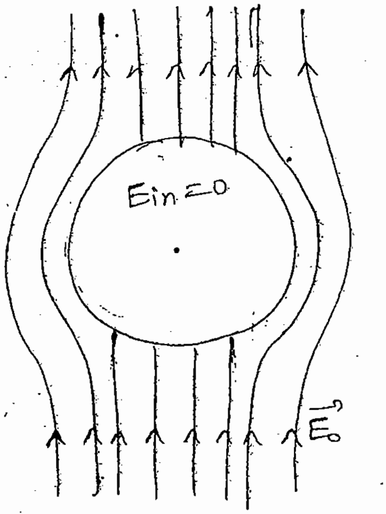
$$\boxed{\vec{D} = \vec{D_0}}$$

Problem :-



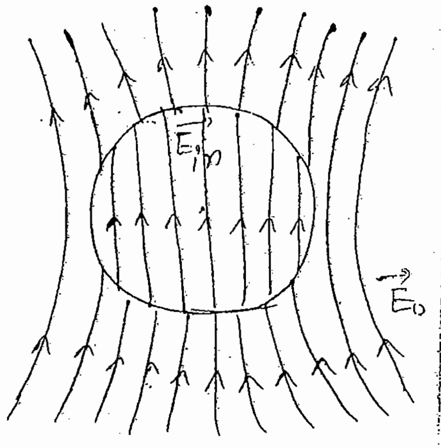
(i) $\vec{P}$ uniformly polarised sphere

$\vec{E}_{in} = -\dfrac{\vec{P}}{3\epsilon_0}$



(ii) Conducting sphere in uniform elec. field

$E_{in} = 0$



(iii) sphere fielled with homogeneous linear dielectric

$\vec{E}_{in} = \dfrac{3}{\epsilon_r + 2}\vec{E}_0$

Ques :- The space b/w plates of parallel plate capacitor is fielled with two slabs of linear dielectric material. Each slab has thickness a. slab 1 has dielectric const. of 2. slab 2 has a dielectric const. of 1.5. Free charge density on the top plate is $+\sigma$ & the bottom plate is $-\sigma$. Find

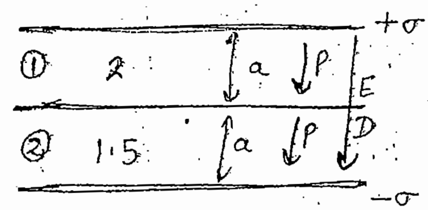
(a) $\vec{D}$ in each slab
(b) $\vec{E}$ in each slab
(c) $\vec{P}$ in each slab
(d) V b/w the plates
(e) location & amount of bound charges

$+\sigma$ & $-\sigma$ are free charge density bcoz plates of parallel plates capacitor is made up of metal. & on metal surface free charges are found & bound charge density is zero.

①, $\oint D \cdot dS = q_{enc}$.

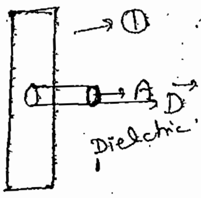
$D \cdot A = \sigma A$

$\boxed{D_1 = \sigma}$



Amount of bound charges in slab (1) & (2) are different bcz dielectric constant is diffrnt. But amount of free charges is same. & D is determined by free charges. so

$\boxed{D_1 = D_2 = \sigma}$



flux will pass throug only by the cross-sectio which is in dielectric

Electric field :- It is determined by both bound & free charges.

$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$

$\boxed{\vec{E_1} = \frac{\sigma}{2\epsilon_0}(-\hat{z})}$

$\boxed{\vec{E_2} = \frac{2\sigma}{3\epsilon_0}(-\hat{z})}$

Polarisation :- $\vec{P} = \epsilon_0 \chi_e \vec{E}$

$\vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E}$

$\boxed{\vec{P_1} = \frac{\sigma}{2}(-\hat{z})}$

$\boxed{P_2 = \frac{\sigma}{3}(-\hat{z})}$

D_1 & D_2 are same but E_1 & E_2 and P_1 & P_2 are different. so E_1 & P_1 and E_2 & P_2 adjest so that D_1 & D_2 same.

Potn diff :-

$V = V_1 + V_2$

$V = \frac{\sigma}{2\epsilon_0} a + \frac{2\sigma}{3\epsilon_0} a$

$= \frac{\sigma a}{\epsilon_0}\left[\frac{1}{2} + \frac{2}{3}\right]$

$\boxed{V = \frac{7\sigma a}{6\epsilon_0}}$

(e) There are 4 surfaces :- Upper & lower surfaces of dielectric (1) & (2).

Upper surface of Upper dielectric $\sigma_b = -\vec{P}\cdot\hat{n} = -\frac{\sigma}{2}$

lower " " " " $\sigma_b = +\frac{\sigma}{2}$

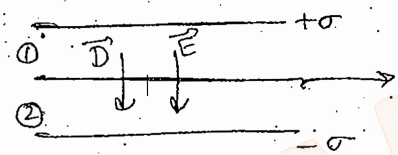
Upper surface of lower dielectric $\sigma_b = -\frac{\sigma}{3}$

lower " " " " $\sigma_b = +\frac{\sigma}{3}$

In dielectric (1), $\rho_{b_1} = 0$. { bcoz P is uniform }

" (2), $\rho_{b_2} = 0$ { " " " " }

*



At this interface :-

(a) both E & D are continous

(b) " " " " dis "

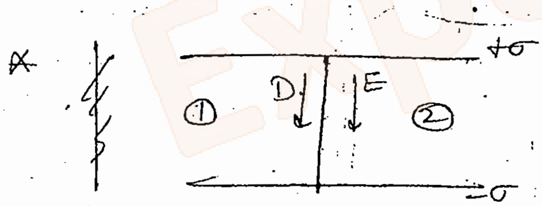
(C) E is continous, D is discontinous

✓(d) E is discontinous, D is continous

Dir^n & $\vec{D}$ & $\vec{E}$ → Normal to the surface. Normal comp. of D is continous

In Dielectric Interface → free charges are zero.

*

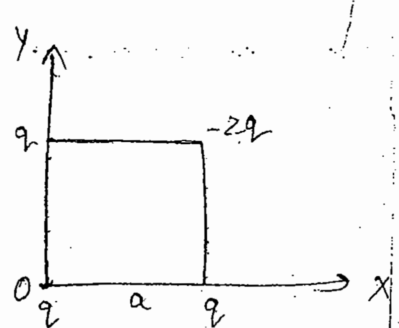


Dir^n of $\vec{D}$ & $\vec{E}$ → tangential to the surface then

✓(C) → correct.

tangential comp. of E is continous.

(34) Taking O → origin



$\vec{p} = 0 + q(a)\hat{x} + qa\hat{y} + (-2q)(a)\hat{x} + (-2q)(a)\hat{y}$

$= -qa\hat{i} - qa\hat{j}$