Free Study Material from

All Lab Experiments

Electromagnetic Theory for NET/Gate Physical Sciences

> Electrostatics Part-1 <

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Electrostatics

* Constant electric field which is not changing with time.

(i) $\vec{E} = r\cos\theta\,\hat{r}$

(ii) $\vec{E} = r\cos\omega t\,\hat{r}$

(iii) $\vec{E} = rt\,\hat{r}$

(iv) $\vec{E} = ct\,\hat{r}$

(v) $\vec{E} = c\,\hat{r}$

(vi) $\vec{E}$ $c\,\hat{\theta}$

Here (i), (v) & (vi) are constant electric field.

* If ~~space~~ electric field is not changing with space – uniform.

Changing with space – Non uniform.
$(x, y, z, r, \theta, \phi)$

(i), (ii), (iii) $\rightarrow$ Non-uniform

(iv), (v), (vi) $\rightarrow$ uniform

* If a func is constant with space & time (x, y, z, t) then it will be uniform & constant.

Coulomb force :-

4 fundamental forces

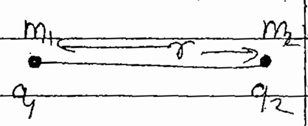
EM (∞), gravitational (∞), strong, weak

$\rightarrow$ Coulomb force comes under Electromag force.

Its range is ∞.

Note :- magnitude of gravitational force is very smaller than magnitude of EM force. That's why effect of gravitational force can be seen on massive bodies.

$$F_G \propto \frac{M_1 M_2}{r^2} \quad \& \quad F_c = \frac{q_1 q_2}{r^2}$$



Force → Vector Quantity

$F_G = G\dfrac{m_1 m_2}{r^2}$ This is primarily attractive

$F_C = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r^2}$ It is attractive as well as repulsive depending on the nature of charge

If q_1, q_2 are of same sign then repulsive.
" " " opposite " " attractive.

Note:- Strong & weak forces are of short range, so these forces found in nucleus.

Energy quanta of EM force is photon. In b/w two charged bodies there will be exchange of photons. & energy quanta of gravitational force is graviton.

Ques 1- Two free charges $+4q$ & $+q$ are placed at a distance r apart. find the coulomb force b/w them. Also find magnitude, sign & location of 3rd charge q' such that

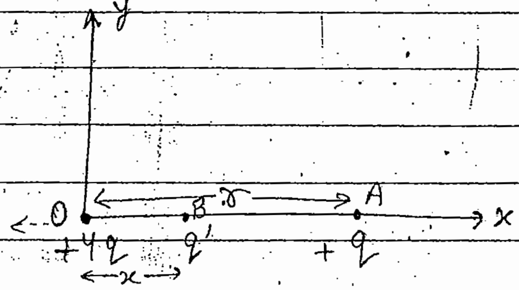
(i) No force on q'

(ii) No force on either of the charge $+4q, +q$ & q'

This is called Entire system is in equilibrium.

$\vec{F}_{4q} = \dfrac{1}{4\pi\epsilon_0}\dfrac{4q\cdot q}{r^2}\hat{r}$

$= \dfrac{1}{4\pi\epsilon_0}\dfrac{4q^2}{r^2}(-\hat{x})$



$\vec{F}_q = \dfrac{1}{4\pi\epsilon_0}\dfrac{4q^2}{r^2}(+\hat{x})$

(i) Suppose $q' = +ve$

$\vec{F}_q{}' = \dfrac{1}{4\pi\epsilon_0}\dfrac{4qq'}{x^2}(+\hat{x})$ —(1)

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$$\vec{F_{q'}} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{(r-x)^2}(-\hat{x}) \quad \text{---(2)}$$

There will be no force on q' if magnitude of the forces ((1) & (2)) are same bcz dirn are already opposite.

Note :- If there are more than 1 force on a point then total force at this point will be vector sum of these forces. $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots \vec{F_n}$

(1) + (2)

$$\Rightarrow \vec{F_{q'}} + \vec{F_{q'}} = 0$$

$$\frac{1}{4\pi\epsilon_0}\frac{4qq'}{x^2}\hat{x} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{(r-x)^2}(-\hat{x})$$

$$\Rightarrow 4(r-x)^2 = x^2$$

$$4r^2 + 4x^2 - 8rx = x^2$$

$$3x^2 - 8rx + 4r^2 = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{8r \pm \sqrt{64r^2 - 48r^2}}{6} = \frac{8r \pm 4r}{6}$$

$$= 2r \text{ or } \frac{2r}{3}$$

$x = 2r$ is not acceptable.

So $x = \frac{2r}{3}$ distance of q' from +4q

i.e. if $x = \frac{2r}{3}$ then there will be no force on q'

Now if q' is -ve

then force b/w 4q & q' and also b/w q' & q is attractive.

$$\vec{F_{q'}} = \frac{1}{4\pi\epsilon_0}\frac{4qq'}{x^2}(-\hat{x})$$

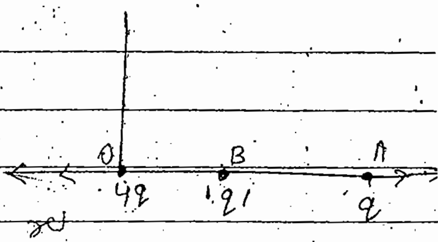
$$\vec{F_{q'}} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{(r-x)^2}(\hat{x})$$

In this case distance will be same

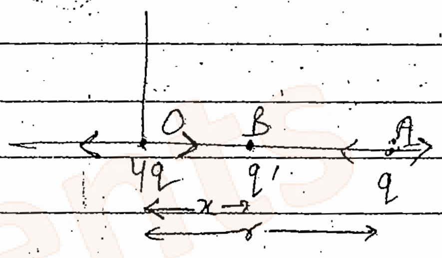
$$x = \frac{2r}{3}$$

Zero force cond^n can be achieved by either +ve or −ve or any other magnitude charge.

(ii) If q' = +ve, equilibrium can not be achieved in entire system. So zero force cond^n can not be achieved.



If q' = −ve then zero force cond^n can be achieved & entire system will be in equilibrium.



$$\vec{F}_{4q} = \frac{1}{4\pi\epsilon_0}\frac{4qq'}{x^2}(+\hat{x})$$

$$\vec{F}_{4q} = \frac{1}{4\pi\epsilon_0}\frac{4qq}{r^2}(-\hat{x})$$

$$\vec{F}_{4q} + \vec{F}_{4q} = 0$$

$$\frac{1}{4\pi\epsilon_0}\frac{4qq'}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{4qq}{r^2}$$

$$\frac{q'}{x^2} = \frac{q}{r^2}$$

$$q' = \frac{q}{r^2} \times x^2 = \frac{q}{r^2} \times \frac{4r^2}{9}$$

$$\boxed{q' = -\frac{4q}{9}}$$

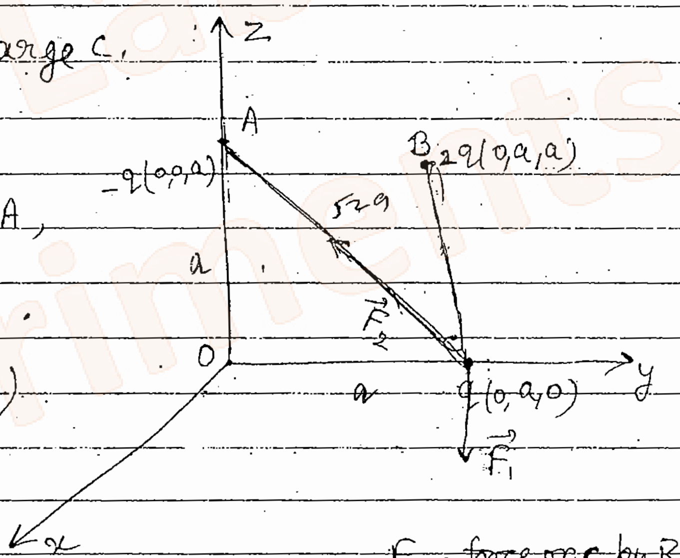
if we put q' at $\frac{4q}{9}$ distance then there will be no force on entire system.

~~Note~~ :- ~~If 4q~~ 4

Note:- If both charges are of same sign than ~~force b~~ equilibrium point (point at which net force = 0) is in ~~the~~ middle between these two charges.
But if both charges are of opposite sign then equ. point is outside of this system
i.e. Equilibrium point does not exist in the system for dissimilar charges.

Similar charges → within the system
dissimilar charges → outside the system

Que:- Find the force on charge c.



$\vec{F_c} = ?$

~~force on charge c by charge A,~~

$$\vec{F_0} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^3}\vec{r}$$

$$\vec{F_1} = \frac{1}{4\pi\epsilon_0}\frac{2q\,q}{a^2}(-\hat{z}) \quad \text{---(1)}$$

$$\vec{F_2} = \frac{1}{4\pi\epsilon_0}\frac{q^2(a\hat{z}-a\hat{y})}{(\sqrt{2}a)^3} \quad \left\{\begin{array}{l}\vec{r} = -a\hat{z}\\ |\vec{r}| = a\end{array}\right\}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2(\hat{z}-\hat{y})}{2\sqrt{2}a^2} \quad \left\{\begin{array}{l}\vec{r} = a\hat{z} - a\hat{y}\\ |\vec{r}| = \sqrt{2}a\end{array}\right.$$

—(2)

$F_1 \rightarrow$ force on c by B
$F_2 \rightarrow$ force on c by A

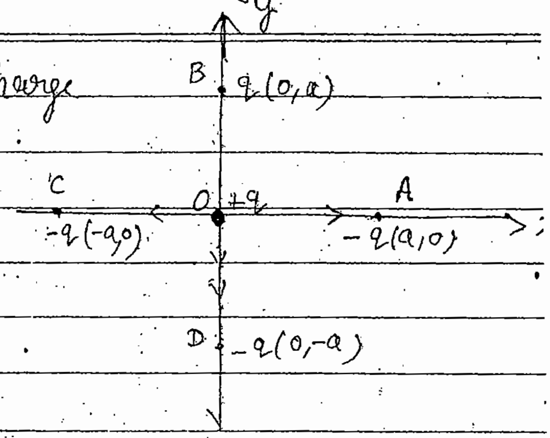
Resultant force on C is

$$\vec{F_c} = \vec{F_1} + \vec{F_2}$$

$$\boxed{\vec{F_c} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\left[\left(-2 + \frac{1}{2\sqrt{2}}\right)\hat{z} - \frac{1}{2\sqrt{2}}\hat{y}\right]}$$

This is the resultant force on C charge.

Ques:- Find the force on a fifth charge $+q$ placed at O.



$$\vec{F}_{OA} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^3} a\hat{x}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\hat{x}$$

$$F_{OB} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}(-\hat{y})$$

$$F_{OC} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}(-\hat{x})$$

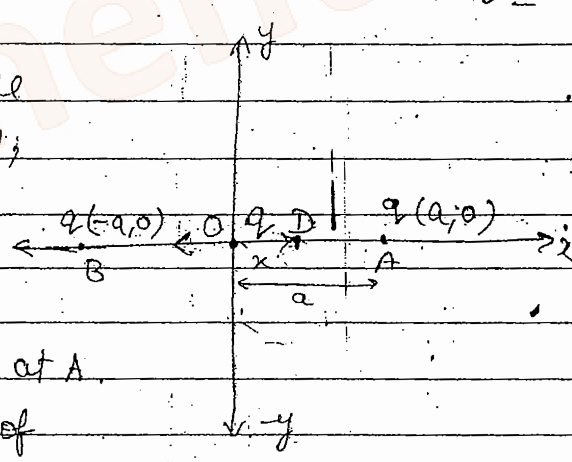
$$F_{OD} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}(-\hat{y})$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\left[-2\hat{y}\right] \Rightarrow \vec{F} = -\frac{1}{4\pi\epsilon_0}\frac{2q^2}{a^2}\hat{y}$$

Ans

Ques:- If a third charge $+q$ place at O, What is the force on the charge $\rightarrow$ zero.

If charge is displaced a small distance x towards the charge placed at A. Find the freq. of oscillation of the charge placed at O.



Note:- $-$ve charge will oscillate in b/w $-$ve charge & $+$ve " " " " $+$ve "

oscillating charge is in stable equilibrium.

If $-$ve charge is placed b/w $+$ve charges then there will be unstable equilibrium.

In Simple harmonic motion:

$$F \propto +x$$

$$F = kx \quad \text{--- (1)} \qquad \omega = \sqrt{\frac{K}{m}}$$

If the mass of +q charge is m then resultant force on D,

$$\vec{F}_D = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(a-x)^2}(-\hat{x}) + \frac{1}{4\pi\epsilon_0}\frac{q^2}{(a+x)^2}(+\hat{x})$$

$$|\vec{F}_D| = \frac{-q^2}{4\pi\epsilon_0}\left[\frac{1}{(a-x)^2} - \frac{1}{(a+x)^2}\right]$$

$$= -\frac{q^2}{4\pi\epsilon_0}\left[\frac{(a+x)^2-(a-x)^2}{(a-x)^2(a+x)^2}\right]$$

$x << a$

$$= -\frac{q^2}{4\pi\epsilon_0}\left[\frac{4ax}{(a)^2(a)^2}\right]$$

$$\boxed{|\vec{F}_D| = \frac{-q^2 x}{\pi\epsilon_0 a^3}} \quad \text{--- (2)}$$

$$(1) \Rightarrow \quad \frac{+q^2 x}{\pi\epsilon_0 a^3} = Kx$$

$$\boxed{K = \frac{q^2}{\pi\epsilon_0 a^3}}$$

ang. freq. $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{q^2}{m\pi\epsilon_0 a^3}}$

$$\boxed{\omega = \sqrt{\frac{q^2}{m\pi\epsilon_0 a^3}}}$$

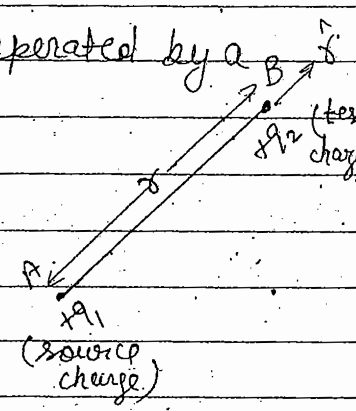
linear freq. $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{q^2}{m\pi\epsilon_0 a^3}}$

$$\boxed{f = \frac{q}{2\pi a}\sqrt{\frac{1}{m\pi\epsilon_0 a}}}$$

Electric Field :- It is defined as (coulomb) force per unit charge

$$\vec{E} = \frac{\vec{F}}{q}$$

Suppose we have 2 charge, $+q_1$ & $+q_2$ seperated by a distance r.



force on q_2 due to q_1

$$\vec{F}_B = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

electric field at B,

$$\vec{E}_B = \frac{\vec{F}_B}{q_2}$$

souce charge → due to which Ele. field is find out
Test " → at which electric field is find out

$$\boxed{q_2 = +1\,C}$$

Test charge is always of unity magnitude & it is always +ve.

* Find electric field at point B means find the force of test charge placed at B.

Direction of electric field depand on source charge

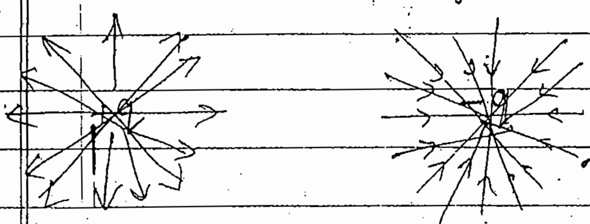
If source charge +ve → away
−ve → toward.

so $\boxed{\vec{F} = q\vec{E}}$

Electric Flux :- Flux = Electric field X area
$= \vec{E}\cdot\vec{A}$

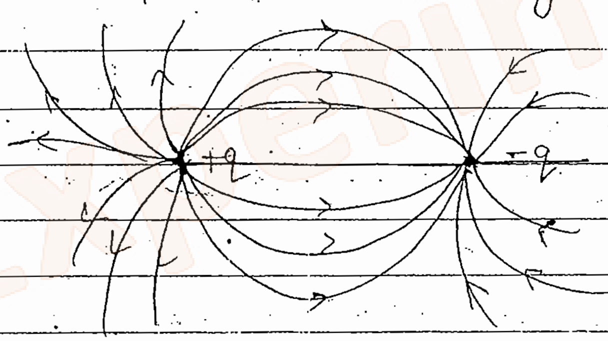
$\vec{E}\cdot\vec{A}$ = Total no. of electric field lines

Electric field lines :- It is a hypothetical concept. To find out the strength of a charge we use electric field lines concept.
from +ve charge, electric field lines diverges out, -ve charge, lines are inward

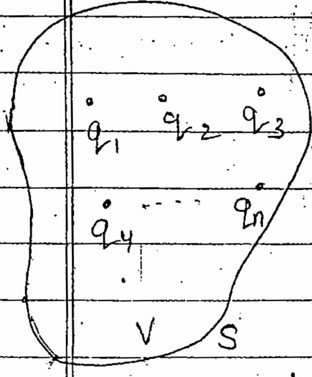


E. field lines show the dir^n of electric field.

E. field lines never crosses each other. If they cross, they show two dir^n.
E. field lines are open curves.
If two charges are placed together, then E. field lines originates at +ve charge & terminate at -ve charge.



Gauss Law :- Acc. to this law, Electric flux passing through a closed surface S (is equal to charge enclose in the surface divided by ϵ_0) is $1/\epsilon_0$ times charge enclose by the surface



$$\boxed{\oint \vec{E}\cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}}$$

q_{enc} → charge enclosed

This is Integral form of Gauss law.

Suppose we have no. of charges in the surface then flux passing through this surface. ~~This~~

$$\boxed{\oint \vec{E}\cdot d\vec{S} = \frac{\sum_{i=1}^{n} q_i}{\epsilon_0}}$$

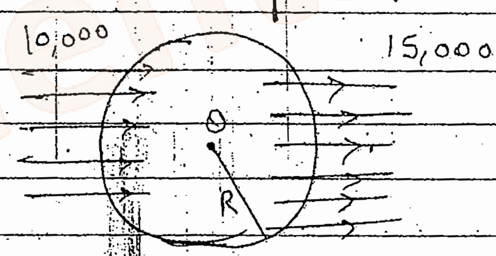
This flux is also known as ~~no.~~ net no. of electric field lines contained in the surface.

If we have some +ve charge in closed surface the electric field lines go out.

This law is valid only for closed surface

e.g. Suppose we have a sphere of radius R in this we have 10,000 electric field lines entering the surface & 15,000 electric field lines leaving the surface. Calculate the charge contained in the surface.

Net lines = 15,000 − 10,000
= 5,000



So there is a source inside the surface.

$$\oint \vec{E}\cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$$

$$q_{enc} = \epsilon_0 \times 5000$$
$$= 5000\,\epsilon_0 \quad \text{Ans}$$

* Gauss law is always valid but not useful, it is useful only if there is symmetry in the problem (it can be spherical symmetry, cylindrical symmetry or planer symmetry).

* Gauss law can not be used to calculate the electric field if monopole moment is zero.
(monopole mom → total charge)
Gauss law can be used only if "Inverse

square law" is valid.

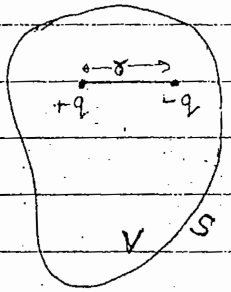
$$E \propto \frac{1}{r^2}$$

$$F = qE \quad \text{so} \quad F \propto \frac{1}{r^2}$$

If we have a closed surface bounded volume V & there place two equal & opposite charges seperated by a finite distance r so

charge enclosed is $= 0$

Monopole moment $= 0$



$$\oint \vec{E} \cdot d\vec{s} = 0$$

$$\boxed{E_{mono} = 0}$$

Here dipole term will be non-zero, only monopole mom is zero.

→ Electric field can be calculated by Gauss law only if monopole mom. is ~~zero~~ non-zero.

for dipole, $E \propto \frac{1}{r^3}$

So Inverse square law is not valid.

→ If flux is 0. It is not necessaryly mean that electric field is also 0.

→ <u>Differential form of Gauss Law:-</u>

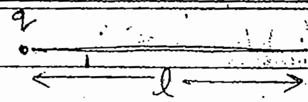
$$\oint_S \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow \int_V (\vec{\nabla} \cdot \vec{E})\, d\tau = \frac{q_{enc}}{\epsilon_0} \quad \text{——— (1)}$$

<u>Charge Densities :-</u>

(i) <u>Line charge density (λ)</u> — If a charge is concentrated at a point then it is a point charge. If charge is uniformely ~~ove~~

distributed over a line, then it is called line charge.

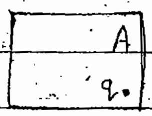


$$\lambda = \frac{q}{l}$$

$$\boxed{q = \int \lambda \, dl}$$

(ii) Surface charge density (σ) :- If charge is distributed over the surface then

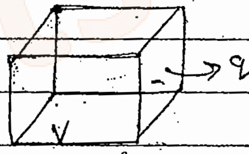
$$\sigma = \frac{q}{A} \text{ or } \frac{q}{S}$$



$$\boxed{q = \int \sigma \, ds}$$

(iii) Volume charge density (ρ) :- If charge is distributed over the volume (can be cube or sphere or ...)

$$\rho = \frac{q}{V}$$



$$\boxed{q = \int \rho \, dV}$$

These are the 3 charge density → line, surface & volume.

So finally, Gauss law in diff. form is

(1) ⇒
$$\int_V (\nabla \cdot \vec{E}) \, d\tau = \frac{1}{\epsilon_0} \int \rho \, d\tau$$

$$\int_V \left(\nabla \cdot E - \rho/\epsilon_0\right) d\tau = 0$$

$$\boxed{\nabla \cdot E = \frac{\rho}{\epsilon_0}}$$

Volume is arbitrary, it can not be zero.

So
$$\boxed{\nabla \cdot E = \frac{\rho}{\epsilon_0}}$$

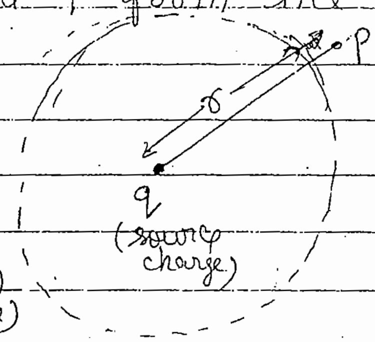
$\left[\begin{array}{l} \nabla \cdot D = \rho \\ \nabla \cdot E = \rho/\epsilon_0 \end{array} \right.$

later on it is known as Maxwell's eqn.

Problem - Electric field of a point charge.

Suppose we have a point charge, we have to find electric field at a point P from the point charge q.

i.e. At P, a unity test charge is placed.

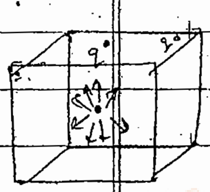


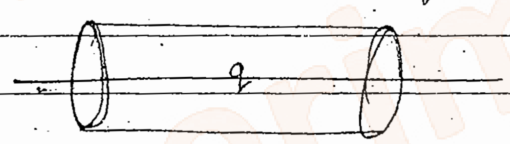
To find out flux, we need a closed surface (Gaussian surface)

→ We consider a sphere, becz each & every point of the spherical surface will be at same distace from the charge.

(for ex - If we take a cylender or cube than each point of cylender or cuble will not be at same distane from point charge).

→ [In case of line charge, we take cylender] i.e. cylendirical





Equation of the flux

$$\oint \vec{E}\cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

for spherical surface $d\vec{S}_r = r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$

$$\Rightarrow E r^2 \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi = \frac{q}{\epsilon_0}$$

$$\Rightarrow E r^2 \left[-\cos\theta\right]_0^{\pi} \left[2\pi\right] = \frac{q}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}}$$

Electric field is radially outward.

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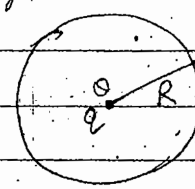
Note :- If there is no func^n of θ & φ then
$$d\tau = 4\pi r^2 dr$$

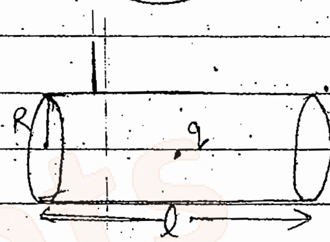
Problem Flux :-

If charge q is enclosed in a sphere of radius R, what is the flux passing through the surface.



Flux passing through the sphere $= \frac{q}{\epsilon_0}$

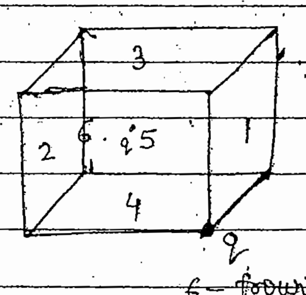
through cylinder $= \frac{q}{\epsilon_0}$



i.e. flux depends upon charge not on the surface.

→ If a charge q is placed in the cube, then total flux passing through the surface of cube $= \frac{q}{\epsilon_0}$ ~~each face~~

Flux passing through each face (6 faces are here)
$$= \frac{q}{6\epsilon_0}$$



(1) If charge placed at a corner.
then flux pass through 1 surface = 0
4 = 0
6 = 0

This charge placed at corner is not enclosed.
To cover this charge we need 8 cubes.
faces in one cube = 6
" 8 " = 48

Out of 48 faces from some face flux pass & from some face not pass.

The planes which are sharing the charge, from these planes no flux pass.

Surface 1, 4 & 6 share the charge so flux pass through these surface = 0

The flux is shared by 8 cubes i.e. $\frac{q}{\epsilon_0} \rightarrow 8$

So flux pass through the each single cube = $\frac{q}{8\epsilon_0}$

cube has 6 faces. Out of 6, ~~and~~ flux pass through only by 3 faces so

flux pass through this cube $= \frac{q}{8\epsilon_0} \times \frac{1}{3}$

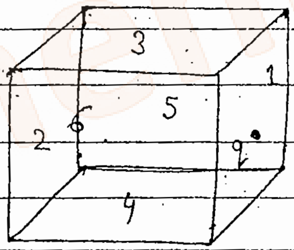
$= \frac{q}{24\epsilon_0}$

(2) Now,

charge is placed at face centred position.

To enclose the charge we need one more cube.

In two cubes → 12 faces



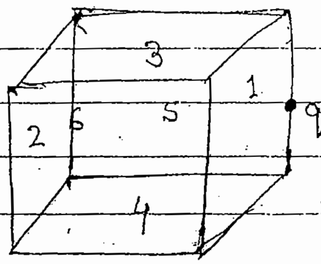
flux pass through this whole cube = $\frac{q}{2\epsilon_0}$

& Out of 6 faces, flux pass through 5 faces so flux pass through the cube $= \frac{q}{2\epsilon_0} \times \frac{1}{5}$

$= \frac{q}{10\epsilon_0}$

(3) To enclose charge we need 3 more cubes.

$= \frac{q}{4\epsilon_0}$

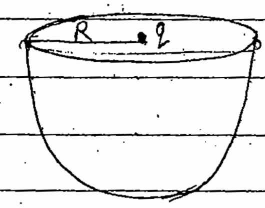


Out of 6 → flux pass through 4 ~~six~~ faces

So flux $= \frac{q}{4\epsilon_0} \times \frac{1}{4} = \frac{q}{16\epsilon_0}$

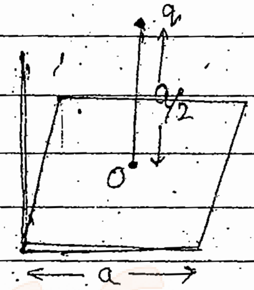
(4) How much flux pass through the Hemispherical surface.

→ To cover this charge, we need full sphere



$$\text{flux} = \frac{q}{2\epsilon_0}$$

(5) A charge is placed at $a/2$ distance from a surface.



To cover this charge we need to sketch a cube

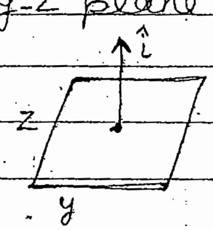
so $\text{flux} = \frac{q}{6\epsilon_0}$

Problem :- The Electric field in a region is given by

$$E = \frac{3}{5}E_0\,\hat{i} + \frac{4}{5}E_0\,\hat{j} + \frac{2}{3}E_0\,\hat{k}$$

where E_0 ~~as~~ $= 2 \times 10^3$ V/m

Find the flux of this field through a rectangular surface of area $0.2\ m^2$ parallel to y-z plane.



$$\text{flux } \phi_E = \vec{E}\cdot\vec{A}$$

$$= \frac{3}{5}E_0 \times 0.2$$

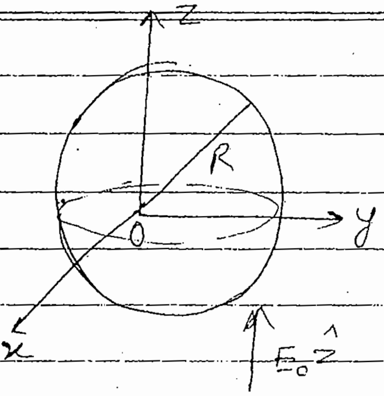
$$= \frac{3}{5} \times 2 \times 10^3 \times 0.2$$

$$= \frac{12 \times 10^2}{5} = \frac{1200}{5}$$

$$\boxed{\phi_E = 240 \text{ V-m}}$$

Mag. of electric field is not depending on space co-ordinate so field is uniform.

Prob 1- ~~Uniform~~ Electric field $E_0\hat{z}$ present in the space & a sphere of radius R is placed in that space. Find the flux passing through the sphere in space.

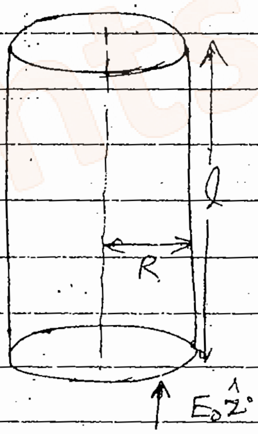


The flux entering inside the sphere is equal to the flux leaving the sphere so net flux is zero (passing through the sphere)

$$\boxed{\phi_E = \text{zero}}$$

Prob 1- flux passing through the cylender of length l & radius R. Ele. field is uniform.

Net flux $= 0$



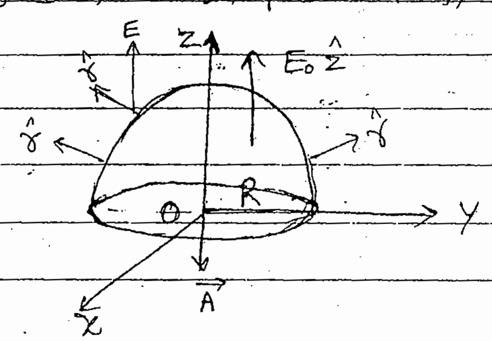
Note :- If Any close surface (sphere, cylender, cube) placed in the uniform electric field then net electric flux passing through the closed surface is zero always.

Net flux $= 0$

Prob 1- eg. - closed hemisphere of radius R is placed in uniform electric field.

Net flux $= 0$

$\phi_E = 0$



Note :- If we cut a hollow sphere then we'll get a open hemisphere & if we place a dist on it then it become close hemisphere.

Proof :- Area of cross-section $= \pi R^2$
Flux entering in cross-section $= -\pi R^2 E$ $\left\{ \pi R^2 E \cos 180^\circ = -\pi R^2 E \right.$

Let angle b/w $\vec{E_0}$ & $\hat{r}$ is θ.
bcz area vector is changing the dirn.

flux through curved surface $\phi_E = \int \vec{E} \cdot d\vec{S}$ $\boxed{\text{Entering flux :-}}$

$$= \int E \cdot R^2 \sin\theta \, d\theta \, d\phi$$

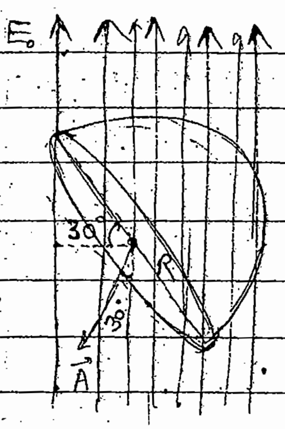
$$= ER^2 \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta \int_0^{2\pi} d\phi$$

$$= \pi R^2 E$$

Hence Net flux $= -\pi R^2 E + \pi R^2 E = 0$

Note :- Total flux passing through the (closed surface) of hemispherical surface $= 0$
& electric flux passing through the hemispherical surface $= \pi R^2 E$

Problem :- Calculate the total flux passing through the hemisphere →



Net $\phi_E = 0$

Electric flux passing through the hemispherical surface.

$$\phi_E = \int \vec{E} \cdot d\vec{s}$$

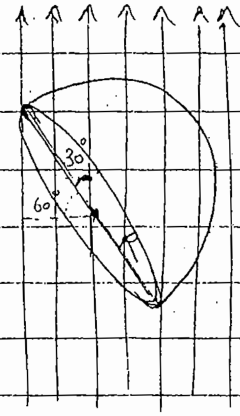
$\left\{ \begin{array}{l} ds = r\,d\theta\, d\phi \\ \int ds = \pi R^2 \end{array} \right.$

$$= E\, S \cos\theta$$

$$= E \pi R^2 \cos 30^\circ = \frac{\sqrt{3}}{2} \pi R^2 E$$

making the angle from horizontal

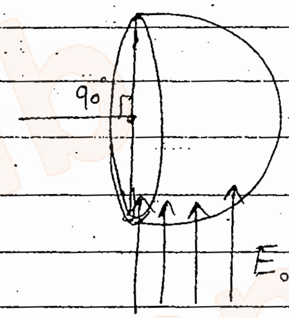
Electric Flux passing through the hemispherical surface



$$\phi_E = \pi R^2 E \cos 60^\circ$$

$$= \frac{\pi R^2 E}{2}$$

If hemisphere is tilted at 90°. then flux



$$\phi_E = 0.$$

* In Non uniform electric field, electric flux passing through a closed surface is not zero.

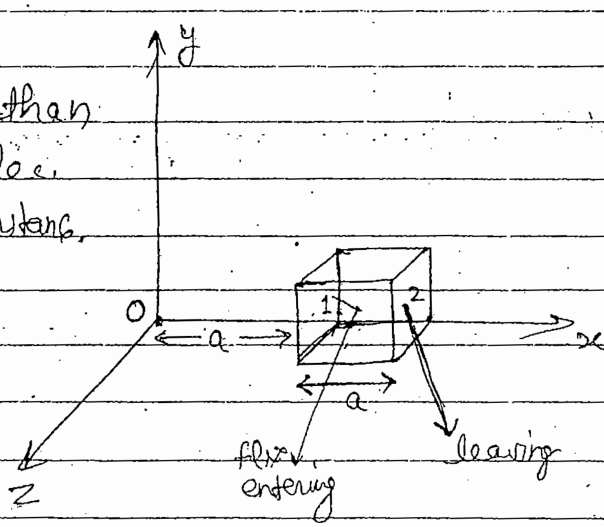
Prob 1:- Calculate the electric flux for a cube of side a. as shown in the figure where

$$E_x = b x^{1/2}, \quad E_y = E_z = 0$$

$$a = 10 \text{ cm}$$

$$b = 800 \text{ C/m}^2$$

leaving flux is more than entering flux bcz Elec. field is incresing with distanc.



$$\phi = EA\cos\theta$$

$$\phi_{net} = \phi_2 - \phi_1$$

$$= b(2a)^{1/2}a^2 - b(a)^{1/2}a^2$$

$(A = a^2)$

$$= b a^{5/2}[\sqrt{2} - 1]$$

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$\phi_{net} = (800)(10)^{5/2}[\sqrt{2}-1]$

Here, electric flux passing throug the cube is Non-zero becz electric field was non-uniform. becz in case of uniform electric field, flux is zero.

Net charge enclosed by the cube

$$\phi = \frac{q}{\epsilon_0}$$

$$q = \epsilon_0 b a^{5/2}(\sqrt{2}-1)$$

$$q = +ve$$

becz flux leaving is more.

→ If $E_x = \frac{b}{x^{1/2}}$

then flux entering is more than leaving

$$\phi_{net} = \phi_1 - \phi_2 = \frac{b}{a^{1/2}} \times a^2 - \frac{b}{(2a)^{1/2}} a^2$$

$$= b a^{3/2}\left[1 - \frac{1}{\sqrt{2}}\right]$$

By using this method we can calculate the charge contained in the surface

$$q = -\epsilon_0 b a^{3/2}\left[1 - \frac{1}{\sqrt{2}}\right]$$

{ flux entering is more so q is -ve }

Prob:- Electric field in some region is given by $\vec{E} = K r^3 \hat{r}$ where K is constant: find

(i) charge density ρ

(ii) Total charge contained in the sphere of radius R.

(i) $\vec{E} = K r^3 \hat{r}$

Diff form of Gauss law $\nabla \cdot E = \frac{\rho}{\epsilon_0}$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r) = \frac{\rho}{\epsilon_0}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k r^3\right) = \frac{\rho}{\epsilon_0}$$

$$\rho = \epsilon_0 \frac{1}{r^2} 5 k r^4$$

$$\boxed{\rho = 5\epsilon_0 k r^2}$$

(ii) Total charge Q

$$Q = \int_V \rho \, d\tau$$

$$= \int_V \rho \, r^2 \sin\theta \, d\theta \, d\phi \, dr$$

ρ is not depending upon θ & ϕ so

$$\int d\tau = 4\pi r^2 dr$$

$$Q = \int_V \rho \, 4\pi r^2 dr = \int_0^R 5\epsilon_0 k r^2 \, 4\pi r^2 dr$$

$$Q = \int 5\epsilon_0 k \times 4\pi \times \frac{R^5}{5}$$

$$\boxed{Q = 4\pi\epsilon_0 k R^5}$$

Prob:- Electric field in some region is given by

$$\vec{E} = \frac{(A\hat{r} + B\sin\theta\cos\phi\,\hat{\phi})}{r}$$

where A & B are constants, find

(i) ρ

(ii) Total charge contained in the sphere of radius R.

(i)
$$\vec{E} = \frac{A}{r}\hat{r} + \frac{B}{r}\sin\theta\cos\phi\,\hat{\phi}$$

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}E_\phi = \frac{\rho}{\epsilon_0}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{A}{r}\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left[\frac{B}{r}\sin\theta\cos\phi\right] = \frac{\rho}{\epsilon_0}$$

$$\frac{1}{r^2}A + \frac{1}{r\cancel{\sin\theta}}\frac{B}{r}\cancel{\sin\theta}\,(-\sin\phi) = \frac{\rho}{\epsilon_0}$$

$$\frac{A}{r^2} + \frac{B}{r^2}(-\sin\phi) = \frac{\rho}{\epsilon_0}$$

$$\rho = \frac{\epsilon_0}{r^2}\left[A - B\sin\phi\right]$$

(ii) $Q = \int_V \rho\, d\tau$

$$= \int_V \frac{\epsilon_0}{r^2}(A - B\sin\phi)\, r^2 \sin\theta\, d\theta\, d\phi\, dr$$

$$= \epsilon_0 \int_0^R\int_0^\pi\int_0^{2\pi} (A - B\sin\phi)\sin\theta\, d\theta\, d\phi\, dr$$

$$= \epsilon_0 R \int_0^\pi\int_0^{2\pi} (A\sin\theta\, d\theta\, d\phi - B\sin\phi\,\sin\theta\, d\theta\, d\phi)$$

$$= \epsilon_0 R\left[A\left(-\cos\theta\right)_0^\pi [2\pi] - B\left[-\cos\phi\right]_0^{2\pi}\left[-\cos\theta\right]_0^{2\pi}\right]$$

$$= \epsilon_0 R\left[-2\pi A(-1-1) - B(1-1)(-1-1)\right]$$

$$= \epsilon_0 R\,(4\pi A)$$

$$= 4\pi\epsilon_0 A R$$

Prob :- Electric field in some region is given by

$$\vec{E} = \frac{A\hat{r}}{r^2} + \frac{B\sin\theta\cos\phi}{r}\hat{\phi}$$

Find (i) ρ (ii) Q

(i) $\nabla\cdot E = \frac{\rho}{\epsilon_0}$

$\nabla\cdot\left(A\frac{\hat{r}}{r^2}\right)$ $\quad \underbrace{\frac{1}{\cancel{r^2}}\frac{\partial}{\partial r}\left(\cancel{r^2}\frac{A}{\cancel{r^2}}\right)} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{B}{r}\sin\theta\cos\phi\right) = \frac{\rho}{\epsilon_0}$

$A\nabla\cdot\left(\frac{\hat{r}}{r^2}\right)$ $\quad \underbrace{\cancel{\frac{1}{r^2}}(0)} + \frac{1}{r\cancel{\sin\theta}}\cdot\frac{B}{r}\cancel{\sin\theta}(-\sin\phi) = \frac{\rho}{\epsilon_0}$

$$A\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) - \frac{B}{r^2}\sin\phi = \frac{\rho}{\epsilon_0}$$

$$\rho = \epsilon_0 \left[A \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) - \frac{B}{r^2} \sin\phi \right]$$

$$\rho = \epsilon_0 \left[A \cdot 4\pi \delta^3(r) - \underbrace{\frac{B \sin\phi}{r^2}}_{\phi = 0} \right]$$

(ii) $\Phi = \int_V \rho \, d\tau$

when limit enclosing the surface

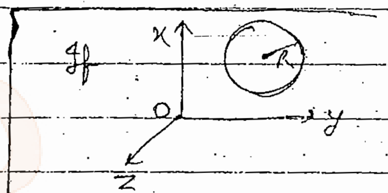
$$\int_V \delta^3(r)\, d\tau = 1$$

$$\Phi = \epsilon_0 4\pi A \int_0^R \delta^3(r)\, d\tau$$

$r \to 0$ to R

$$= \epsilon_0 4\pi A \int_0^R \delta^3(r)\, 4\pi r^2 dr$$

$$\boxed{\Phi = \epsilon_0 4\pi A}$$



$\int_V \delta^3(r)\, d\tau = 0$ bcz limit is not enclosing the sphere

Que :- $\vec{E} = \frac{A e^{-4r}}{r^2} \hat{r} + B \frac{\sin\theta \cos\phi}{r}$

$$\rho = \epsilon_0 \left[\nabla \cdot \left(\frac{A e^{-4r}}{r^2} \hat{r} \right) + \nabla \cdot \left(\frac{B \sin\theta \cos\phi}{r} \right) \right]$$

$$\nabla \cdot \left[A \overset{\text{II}}{e^{-4r}} \overset{\text{I}}{\left(\frac{\hat{r}}{r^2} \right)} \right] = A \left[\frac{\hat{r}}{r^2} \cdot \nabla \left(-4 e^{-4r} \right) + e^{-4r} \left(4\pi \delta^3(r) \right) \right]$$

$$\vec{\nabla} \cdot (\lambda \vec{A}) = \vec{A} \cdot \vec{\nabla} \lambda + \lambda \vec{\nabla} \cdot \vec{A}$$

$$= \frac{-4A}{r^2} e^{-4r} + A 4\pi \delta^3(r) e^{-4r}$$

$$= -\frac{4A}{r^2} e^{-4r} + A 4\pi \delta^3(r) \qquad [f(x)\delta(x) = f(0)\,\delta(x)]$$

$$\rho = \epsilon_0 \left[-\frac{4A}{r^2} e^{-4r} + A 4\pi \delta^3(r) - \frac{B \sin\phi}{r^2} \right]$$

$$\Phi = \int_V \rho \, d\tau = -4A\epsilon_0 \int_0^R \frac{e^{-4r}}{r^2} 4\pi r^2 dr + 4\pi A \epsilon_0 \int_0^R \delta^3(r)$$

$$= -16\pi A \epsilon_0 \left[\frac{e^{-4r}}{-4} \right]_0^R + 4\pi \epsilon_0 A$$

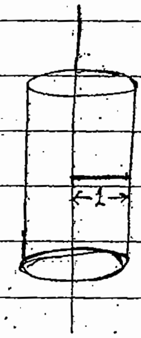
$$= 4\pi \epsilon_0 A \left[e^{-4R} - 1 \right] + 4\pi \epsilon_0 A$$

$$= 4\pi A \epsilon_0 e^{-4R}$$

Prob 1:- Calculate the charge density at $(1, \frac{\pi}{4}, 3)$ & the total charge enclosed by the cylender of radius 1 meter with $-2 \leq z \leq 2\,m$.

given that, electric field

$$\vec{E} = s z \cos^2\phi \, \hat{z} \quad N/C$$



$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\frac{\partial E_z}{\partial z} = \frac{\rho}{\epsilon_0}$$

$$\frac{\partial}{\partial z}(s z \cos^2\phi) = \frac{\rho}{\epsilon_0}$$

$$s\cos^2\phi = \frac{\rho}{\epsilon_0}$$

$$\rho = \epsilon_0 \, s\cos^2\phi$$

$$\rho = \epsilon_0 (1)\left(\cos\frac{\pi}{4}\right)^2$$

$$\rho = \frac{\epsilon_0}{2} \quad c/m^3 \quad Ans$$

$$\Phi = \int_V \rho \, d\tau = \int_0^1\int_0^{2\pi}\int_{-2}^{2} \rho \, s \, ds \, d\phi \, dz$$

$$= \int_0^1\int_0^{2\pi}\int_{-2}^{2} \epsilon_0 \, s\cos^2\phi \; s \, ds \, d\phi \, dz$$

$$= \epsilon_0 \left[\frac{s^3}{3}\right]_0^1 \int_0^{2\pi}\left(\frac{1+\cos 2\phi}{2}\right) d\phi \int_{-2}^{2} dz$$

$$= \epsilon_0 \frac{1}{3}\frac{1}{2}\left\{[\phi]_0^{2\pi} + \left[\frac{\sin 2\phi}{2}\right]_0^{2\pi}\right\} [z]_{-2}^{2}$$

$$= \epsilon_0 \frac{1}{3}\frac{1}{2}[2\pi + 0] (2+2)$$

$$= \frac{4}{3}\pi\epsilon_0 \quad c \quad Ans$$

$\int_0^R s(r)\,d\tau$

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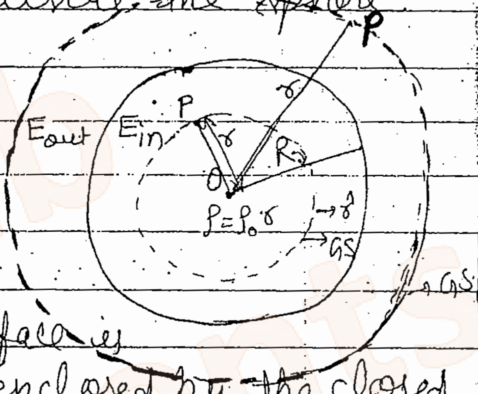
Note: In Problems based on symmetry, we can easily find out electric field using Gauss law but if symmetry is not present then we use Coulomb's law to calculate electric field.

Que(1): If sphere of radius R carries a vol. charge density $\rho = \rho_0 r$ where ρ_0 is the constant & r is the distance from the centre of the charged sphere. Calculate the electric field both inside & outside ~~the sphere~~

Sol:-

There is spherical symmetry.



$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$$

The flux passing through the surface is depend upon the total charge enclosed by the closed surface.

(i) charge enclosed by the sphere of radius r: $q_{enc} = ?$

$$q_{enc} = \int_V \rho\, d\tau = \int_0^r\int_0^{2\pi}\int_0^{\pi} \rho_0 r\, r^2 \sin\theta\, d\theta\, d\phi\, dr$$

$$= \int_0^r \rho_0 r\, 4\pi r^2 dr = \int_0^r \rho_0 4\pi r^3 dr$$

$$= \rho_0 4\pi \left[\frac{r^4}{4}\right]_0^r = \rho_0 4\pi \frac{r^4}{4}$$

$$= \pi \rho_0 r^4$$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$$

$$\vec{E} \cdot 4\pi r^2 = \frac{\pi \rho_0 r^4}{\epsilon_0}$$

$$\boxed{\vec{E}_{in} = \frac{\rho_0 r^2}{4\epsilon_0}\hat{r}}$$

(ii) Outside the sphere

$dS \rightarrow 4\pi r^2$ This r is the distance from O to P.

$$q_{enc} = \int_V \rho \, d\tau = \int_0^R \rho_0 r \, 4\pi r^2 dr$$

$$= \rho_0 4\pi \left[\frac{r^4}{4}\right]_0^R = \rho_0 \pi R^4$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\pi \rho_0 R^4}{\epsilon_0}$$

$$\boxed{\vec{E}_{out} = \frac{\rho_0 R^4}{4\epsilon_0 r^2} \hat{r}}$$

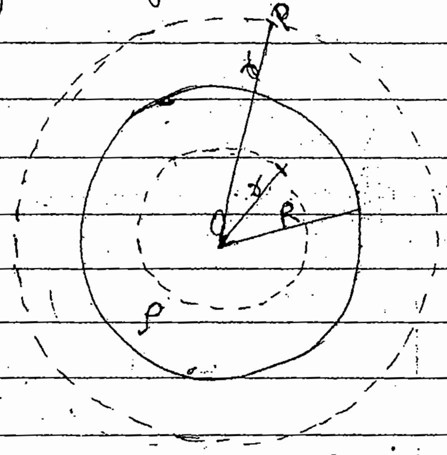
Hence $E_{in} \propto r^2$ & $E_{out} \propto \frac{1}{r^2}$

★ If there is spherical symmetry & ρ is given r^n ~~then~~ ($\rho \propto r^n$) then electric field inside will be

$$E_{in} \propto r^{n+1}$$

& outside will be $E_{out} \propto \frac{1}{r^2}$

Que ② Find the electric field inside & outside of a uniform charged sphere of radius R and total charge q.
[uniformly charged → ρ is constant with R]



We have to develope a relation b/w ρ & q

$$q = \int_V \rho \, d\tau = \int_0^R \rho \, 4\pi r^2 dr = 4\pi \rho \left(\frac{r^3}{3}\right)_0^R$$

$$q = \frac{4}{3}\pi R^3 \rho$$

ρ is uniform in the sphere of radius R so we can directly write $q = \frac{4\pi R^3 \rho}{3} \Rightarrow \rho = \frac{q}{\frac{4}{3}\pi R^3}$

(i) Electric field inside the sphere

~~q_{enc} in the sphere of radius r will be~~

$$\cancel{q_{enc} = \frac{4}{3}\pi r^3 \rho}$$

$$q_{enc} = \int_V \rho \, d\tau = \int_0^r \frac{q}{\frac{4}{3}\pi R^3} 4\pi r^2 dr$$

$$= \frac{3q}{R^3}\left[\frac{r^3}{3}\right]_0^r = \frac{q r^3}{R^3}$$

$$q_{enc} = \frac{q r^3}{R^3}$$

When $r \geq R$ then total charge enclosed within this sphere is

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} \frac{q r^3}{R^3}$$

$$E = \frac{q r}{4\pi R^3 \epsilon_0}$$

$$\boxed{\vec{E}_{in} = \frac{q r}{4\pi\epsilon_0 R^3}\hat{r}}$$

(ii) Electric field outside the sphere

$$q_{enc} = \int_V \rho \, d\tau = \int_0^R \frac{q}{\frac{4}{3}\pi R^3} 4\pi r^2 dr$$

$$= \frac{3q}{R^3}\left[\frac{r^3}{3}\right]_0^R = \frac{q}{R^3} R^3$$

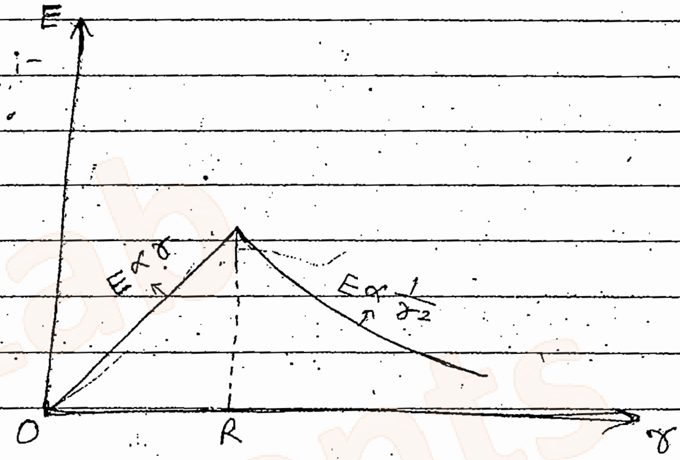
$$q_{enc} = q$$

$$\oint \vec{E}\cdot\vec{ds} = \frac{q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\boxed{\vec{E}_{out} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}}$$

Plot of Electric field :-



Q.3 :- Find the electric field for spherical charge distribution given by

$$\rho(r) = \begin{cases} \rho_0\left(1-\frac{r^2}{a^2}\right) & r \le a \\ 0 & r > a \end{cases}$$

where r is the distance from the centre of the sphere & a is the radius of the sphere. Also find the value of r for which electric field will be maximum.

(i) Inside the sphere,

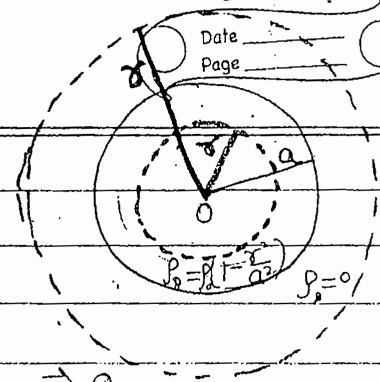
$$q_{enc} = \int_V \rho\, d\tau = \int_0^r \rho_0\left(1-\frac{r^2}{a^2}\right) 4\pi r^2 dr$$

$$= \rho_0 4\pi \left[\frac{r^3}{3} - \frac{1}{a^2}\frac{r^5}{5}\right]_0^r$$

$$= \rho_0 4\pi \left[\frac{r^3}{3} - \frac{r^5}{5a^2}\right]$$

$$\oint \vec{E}\cdot\vec{ds} = \frac{q_{enc}}{\epsilon_0}$$

$$E\cdot \cancel{4\pi} r^2 = \frac{1}{\epsilon_0}\rho_0 \cancel{4\pi}\left[\frac{r^3}{3} - \frac{r^5}{5a^2}\right]$$



$$\boxed{\vec{E}_{in} = \frac{\rho_0}{\epsilon_0}\left[\frac{r}{3} - \frac{r^3}{5a^2}\right]\hat{r}}$$

(ii) Outside the sphere,

$$q_{enc} = \int \rho\, d\tau = \int_0^{a} \cancel{\rho\, 4\pi r^2 dr} \cancel{= 0}$$

$$\cancel{E \neq 0}$$

$$q_{enc} = \int_0^a \rho_0\left(1 - \frac{r^2}{a^2}\right) 4\pi r^2 dr$$

$$= \rho_0 4\pi \left[\frac{a^3}{3} - \frac{a^3}{5}\right]$$

$$= 4\pi\rho_0 a^3 \times \frac{2}{15} = \frac{8}{15}\pi\rho_0 a^3$$

$$E \cdot \cancel{4\pi} r^2 = \frac{2}{\cancel{8}}{15}\cancel{\pi}\rho_0 a^3 \frac{1}{\epsilon_0}$$

$$\boxed{\vec{E}_{out} = \frac{2\rho_0 a^3}{15\epsilon_0 r^2}\hat{r}}$$

The value of ~~r~~ r at which field is maximum,

$$\frac{\partial E_{in}}{\partial r} = 0$$

$$\frac{\rho_0}{\epsilon_0}\left[\frac{1}{3} - \frac{1}{5a^2}3r^2\right] = 0$$

$$\frac{1}{5a^2}3r^2 = \frac{1}{3} \Rightarrow r^2 = \frac{5}{9}a^2$$

$$\boxed{r = \frac{\sqrt{5}}{3}a}$$

* Note :-
electric field can not be maximum, outside the sphere because outside electric field is decreasing as $E_{out} \propto \frac{1}{r^2}$

Note - We can use Gauss law to find elec field only if cylender is long

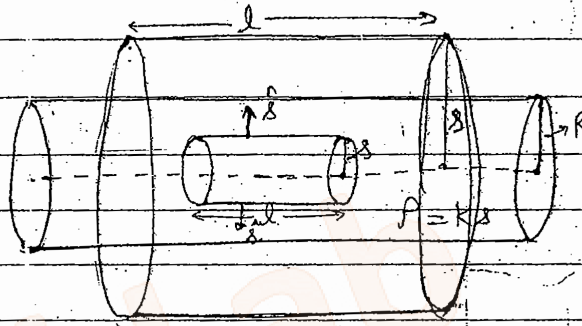
CYLENDER

Q.1 :- A long cylender of rad. R carries a charge density i.e. proportional to the distance from the axis

$\rho = k s$

where k is constant.

Find the electric field inside & outside the cylend



(i) Total charge enclosed by the Gaussian surface

$$q_{enc} = \int_V \rho \, d\tau$$

$$= \int_0^s \int_0^{2\pi} \int_0^l k s \, s \, ds \, d\phi \, dz$$

$$= k\left(\frac{s^3}{3}\right)_0^s (\phi)_0^{2\pi} (z)_0^l = \frac{k}{3} s^3 \cdot 2\pi \cdot l$$

$$q_{enc} = \frac{2\pi k s^3 l}{3}$$

Electric field $\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$

flux only pass through the curve surface of cylender bcz dir^n of $\vec{E}$ is always normal to the axi

Along curved surface $\vec{E}$ is normal to axis

So $\cancel{E \cdot 2\pi s l} = \oint E \cdot s \, d\phi \, dz = \frac{q_{enc}}{\epsilon_0}$

$$E \cdot 2\pi s l = \frac{2\pi k s^3 l}{3\epsilon_0}$$

$$\boxed{\vec{E}_{in} = \frac{k s^2}{3\epsilon_0} \cdot \hat{s}}$$

(ii) Outside the cylender,

$$q_{enc} = \int_V \rho \, d\tau = \int_0^R \int_0^{2\pi} \int_0^l k s \, s \, ds \, d\phi \, dz$$

$$= \frac{2\pi k R^3 l}{3}$$

$$\oint E \cdot ds = \frac{q_{enc}}{\epsilon_0}$$

$$E \cdot 2\pi s l = \frac{1}{\epsilon_0} \frac{2\pi k R^3 l}{3}$$

$$\boxed{\vec{E}_{out} = \frac{k R^3}{3\epsilon_0 s} \hat{s}}$$

Note :- If $\rho \propto s^n$

then $\boxed{E_{in} \propto s^{n+1} \quad E_{out} \propto \frac{1}{s}}$

e.g. If given $\rho \propto s^3$

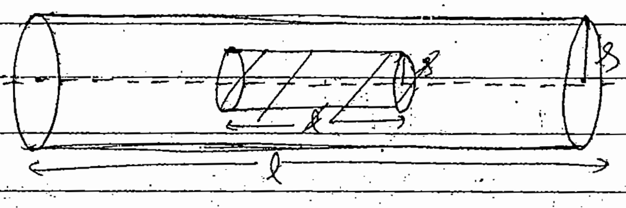
then $n = 3$

but $\rho = \rho_0 \left(1 - \frac{s^3}{a^2}\right)$ then there is no clear dependence of r. We can' not use this method.

*. To find out $\vec{E}$ outside the cylender Gaussian surface should be infinitely long including cross-section also for Gauss law. (complicated).

We use a short cylender which include curved surface only.

Qu:2 :- Find the $\vec{E}$ at a distance s from an infinitely long straight wire carrying a uniform line charge density λ.



$$q_{enc} = \int \lambda \, dl$$ (suppose length → l)
$$= \lambda l$$

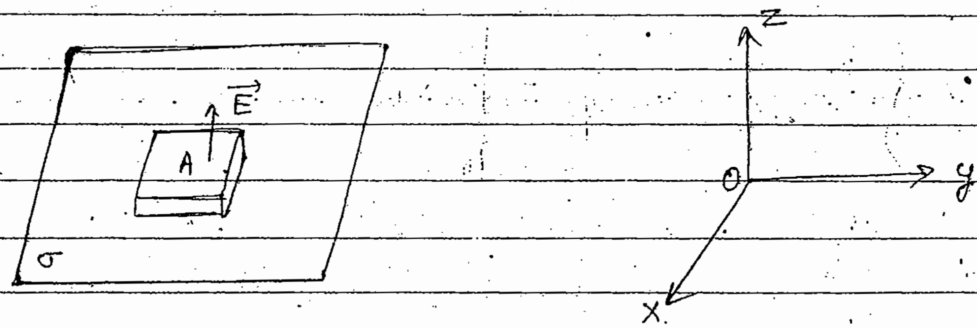
$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

$$E \cdot 2\pi s l = \frac{\lambda l}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}}$$

Plane Surface

Q.1 :- An infinite plane carries a uniform surface charge density σ. find its electric field.



dir^n of $\vec{E}$ can be $+\hat{z}$ or $-\hat{z}$
(Gaussian surface → Gaussian pill box) half part is upper & half part is lower to the plane.

$\vec{E}$ will pass from area A only. so flux will pass through area A.

charge enclosed $q_{enc} = \sigma A$

$$\oint E \cdot ds = \frac{q}{\epsilon_0}$$

$$[EA + EA] = \frac{\sigma A}{\epsilon_0}$$

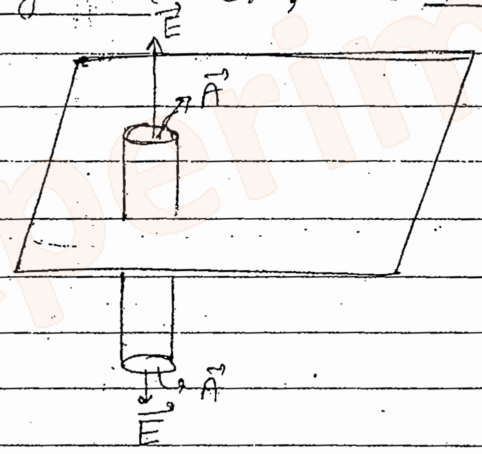
flux = 2 EA
from −z to z.

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}}$$

$\hat{n} \rightarrow$ dirⁿ of $\vec{E}$ normal to the surface.

* In place of Gaussian pill box we can also use a Gaussian cylender also, as a Gaussian surface.

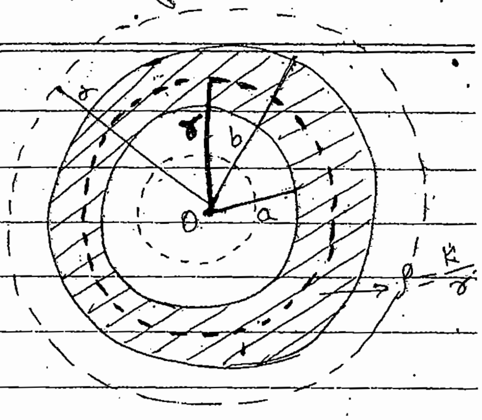


Gaussian surface must be symmetrical from the point so it should be half up & half down.

Que :- A hollow spherical shell carries a charge density $\rho = \frac{K}{r^2}$, $a \leq r \leq b$. Find the electric field in 3 regions :-

(i) $r < a$
(ii) $a < r < b$
(iii) $r > b$

Thick hollow sphere → have 2 radius
Thin " " → have 1 radius (inner & outer radius same)



(i) $q_{enc} = \int_V \rho \, d\tau$

$= \int_0^{r} k(0) 4\pi r^2 dr$

$= \cancel{4\pi k a}\ 0$

$\oint \vec{E} \cdot ds = \frac{q_{enc}}{\epsilon_0} \Rightarrow \boxed{E = 0}$

(ii) $q_{enc} = \int_V \rho \, d\tau = \int_a^r \frac{k}{\cancel{r^2}} 4\pi \cancel{r^2} dr$

$= \int_0^a k 4\pi dr + \int_a^r k \cdot 4\pi dr$
$\Downarrow$
0

$= 4\pi k (r-a)$

$\oint E \cdot ds = \frac{q_{enc}}{\epsilon}$

$E \cdot 4\pi r^2 = \frac{4\pi k (r-a)}{\epsilon_0}$

$$\boxed{\vec{E} = \frac{k}{\epsilon_0 r^2}(r-a)\hat{r}}$$

(iii) $q_{enc} = \int_V \rho \, d\tau = \int_a^b \frac{k}{r^2} 4\pi r^2 dr$

$= 4\pi k (b-a)$

$\oint E \cdot ds = \frac{q_{enc}}{\epsilon_0}$

$E \cdot 4\pi r^2 = \frac{4\pi k (b-a)}{\epsilon_0}$

$$\boxed{\vec{E} = \frac{k}{\epsilon_0 r^2}(b-a)\hat{r}}$$

$\vec{E}$ is Maximum for $\frac{\partial \vec{E}_{(ii)}}{\partial r} = 0$

$$\frac{\partial}{\partial r}\left[\frac{K}{\epsilon_0 r^2}(r-a)\right] = 0$$

$$\frac{K}{\epsilon_0}\left[\frac{-1}{r^2} + \frac{2a}{r^3}\right] = 0$$

$$\frac{K}{\epsilon_0 r^2}\left(-1 + \frac{2a}{r}\right) = 0$$

$$\frac{2a}{r} = +1$$

$$\boxed{r = +2a}$$

$$E_{max} = \frac{K}{\epsilon_0 (2a)^2}(2a-a)$$

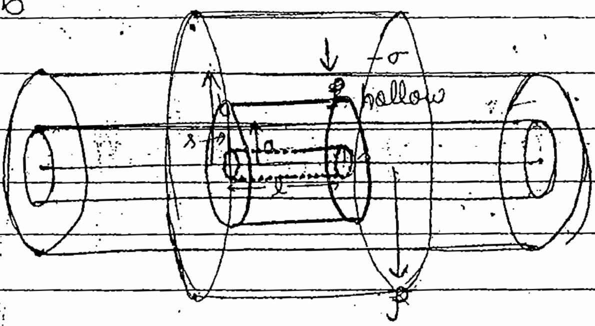
$$\boxed{E_{max} = \frac{K}{4a\epsilon_0}}$$

Ques:- A long co-axial cable carries a uniform volume charge density ρ on inner cylinder (radius a) & a uniform surface charge density $(-\sigma)$ on the outer cylender (radius b) surface charge is such that cable as a whole is electrically neutral. Find the electric field in 3 regions -

(i) $s < a$

(ii) $a < s < b$

(iii) $s > b$



(i) $s < a$

$$q_{enc} = \int_v \rho\, d\tau = \int_0^{s}\int_0^{2\pi}\int_0^{l} \rho\, s\, ds\, d\phi\, dz$$

$$= \rho \frac{s^2}{2} (z)_0^l 2\pi$$

$$q_{enc} = \rho \pi s^2 l$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

$$E \cdot 2\pi s l = \frac{\rho \pi s^2 l}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho s}{2\epsilon_0} \hat{s}}$$

(ii) $a < s < b$

$$q_{enc} = \int_0^a \int_0^{2\pi} \int_0^l \rho\, s\, d\phi\, dz\, ds$$

$$= \rho \pi a^2 l$$

$$E \cdot 2\pi s l = \frac{\rho \pi a^2 l}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho a^2}{2\epsilon_0 s} \hat{s}}$$

(iii) $s > b$

$$q_{enc} = 0$$

So $\boxed{E = 0}$

Que: ① The electric field due to an unknown charge distribution is given by $\vec{E} = \frac{q}{r^2} e^{-4r} \hat{r}$.

Find charge density ρ & total charge in the sphere of radius R centred at the origin.

Que ② A charged ball of radius R carries a charge density $\rho = 2\epsilon_0 \left(3a + \frac{b}{r}\right)$

(i) Total charge in the ball.
(ii) Electric field inside & outside the ball
(iii) Net no. of electric field lines passing through the ball.

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(1) $\vec{E} = \frac{q}{r^2} e^{-4r} \hat{r}$

ρ & $Q = ?$

$\nabla \cdot E = \frac{\rho}{\epsilon_0}$

$\nabla \cdot \left(\frac{q}{r^2} e^{-4r} \hat{r} \right) = \frac{\rho}{\epsilon_0}$

$q \left[\frac{\hat{r}}{r^2} \nabla \cdot (e^{-4r}) + e^{-4r} \nabla \cdot \left(\frac{\hat{r}}{r^2} \right) \right] = \frac{\rho}{\epsilon_0}$

$q \left[\frac{\hat{r}}{r^2} (-4e^{-4r}) + e^{-4r} (4\pi \delta^3(r)) \right] = \frac{\rho}{\epsilon_0}$

$\frac{-4q}{r^2} e^{-4r} + q\, 4\pi \delta^3(r) e^{-4r} = \frac{\rho}{\epsilon_0}$

$\boxed{\rho = \epsilon_0 q \left[4\pi \delta^3(r) - \frac{4}{r^2} e^{-4r} \right]}$ $\{ f(x) \delta(x) = f(0) \delta(x) \}$

Now $Q = \int_V \rho\, d\tau = -4q\epsilon_0 \int_0^R \frac{e^{-4r}}{r^2} 4\pi r^2 dr +$
$4\pi q \epsilon_0 \int_0^R \delta^3(r) dr$

$= -16 q \epsilon_0 \pi \left[\frac{e^{-4r}}{-4} \right]_0^R + 4\pi q \epsilon_0$

$= +4 q \epsilon_0 \pi \left[e^{-4R} - 1 \right] + 4\pi q \epsilon_0$

$= 4\pi \epsilon_0 q \left[e^{-4R} - 1 + 1 \right]$

$\boxed{Q = 4\pi \epsilon_0 q e^{-4R}}$

(2) $\rho = 2\epsilon_0 \left(3a + \frac{b}{r} \right)$

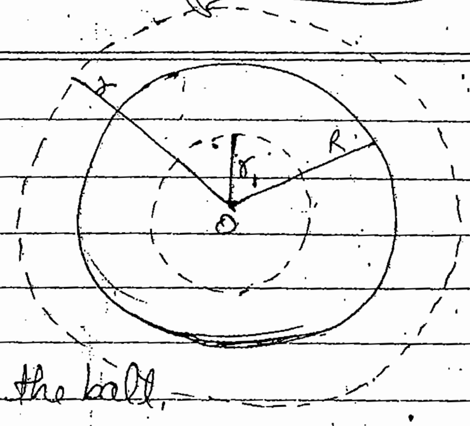
(i) Total charge in the ball

$q = \int_V \rho\, d\tau \Rightarrow q = \int_0^R 2\epsilon_0 \left(3a + \frac{b}{r} \right) 4\pi r^2 dr$

$q = 4\pi\, 2\epsilon_0 \left[3a \frac{r^3}{3} + b \frac{r^2}{2} \right]_0^R$

$$q = 8\pi\epsilon_0\left[aR^3 + \frac{b}{2}R^2\right]$$

$$\boxed{q = 8\pi\epsilon_0 R^2\left[aR + \frac{b}{2}\right]}$$



(ii) Electric field inside & outside the ball.

Inside :-

$$q = \int_V \rho\, d\tau = \int_0^r 2\epsilon_0\left(3a + \frac{b}{r}\right)4\pi r^2 dr$$

$$= 8\pi\epsilon_0\left[ar^3 + \frac{b}{2}r^2\right]$$

$$\oint E\cdot ds = \frac{q}{\epsilon_0}$$

$$E\cdot 4\pi r^2 = 8\pi\left[ar^3 + \frac{b}{2}r^2\right]$$

$$\boxed{\vec{E} = 2\left(ar + \frac{b}{2}\right)\hat{r}}$$

Outside :-

$$q = 8\pi\epsilon_0 R^2\left[aR + \frac{b}{2}\right]$$

$$\oint E\cdot ds = \frac{q}{\epsilon_0}$$

$$E\cdot 4\pi r^2 = 8\pi R^2\left[aR + \frac{b}{2}\right]$$

$$\boxed{E = \frac{2R^2}{r^2}\left(aR + \frac{b}{2}\right)\hat{r}}$$

(iii) Net no. of electric field lines passing through the ball.

Net no. of elec. field lines = flux

$$\text{flux} = \oint E\cdot ds = \frac{q}{\epsilon_0}$$

$$= \frac{8\pi\epsilon_0 R^2\left[aR + \frac{b}{2}\right]}{\epsilon_0}$$

$$= 8\pi R^2\left[aR + \frac{b}{2}\right]$$

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Neutral Points :- Neutral points are those points at which electric field is zero.

$\vec{E} = 0$

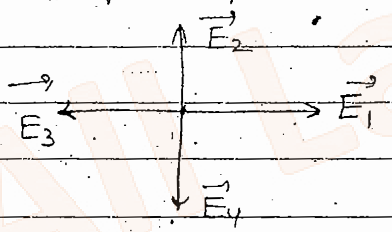
If we place a charge at the neutral point, force on the charge will be zero

$\vec{F} = q\vec{E}$

$F = 0$

By Superposition principle :-
Coulomb force (or electric field) follow the superposition principle.

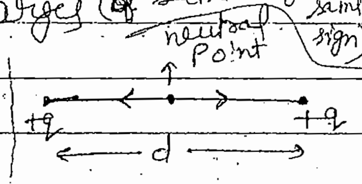
If



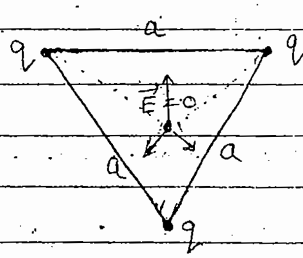
then electric field can be vectorially sum

$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \vec{E_4} + \cdots$$

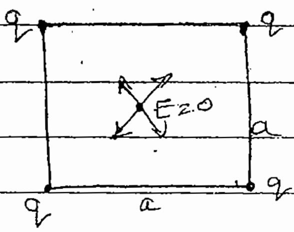
Suppose we have two point charges (of same mag. & same sign)
Electric field due to L.H.S. charge →
R.H.S. ←
So, Electric field is zero



Similarly for 3 charges,
At the centre $\vec{E} = 0$, This is based on symmetry.



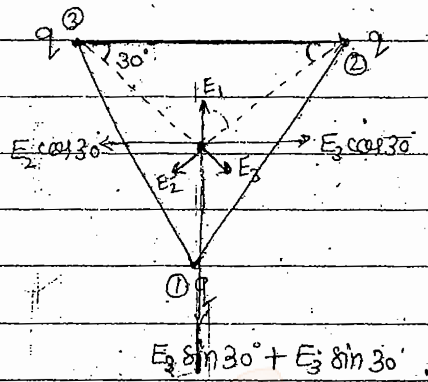
If there are 4 charges
At centre $\vec{E} = 0$
Similarly, If we have, regular pentagon then at centre $\vec{E} = 0$



*** Digonal of a square of sides a will be a√

Triangle is the polygon of 3 sides &
Square " " " 4 "

* If we have any regular polygon then at its centre $\vec{E} = 0$.



→ The side angle are of 60°.
→ Magnitude of $E_1 = E_2 = E_3$ are same.
→ The resultant field at the centre is 0.

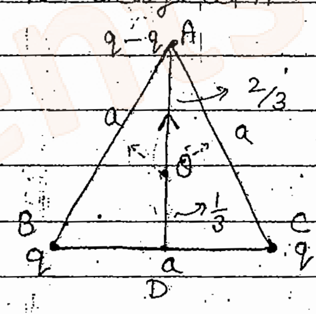
$$\vec{E} = 0$$

* Suppose we have 2 charges at B & C, & No charge at A.

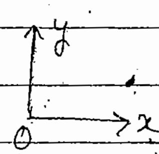
$$AD = \frac{\sqrt{3}}{2}a$$

$$AO = \frac{2}{3} \times \frac{\sqrt{3}}{2}a$$

$$AO = \frac{a}{\sqrt{3}}$$



At A, No charge i.e. $+q-q$
The dir^n of $\vec{E}$ will be perpendicular to the charge at B & C so
dir^n of $\vec{E} \longrightarrow \hat{y}$



$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{3q}{a^2}\hat{y}}$$

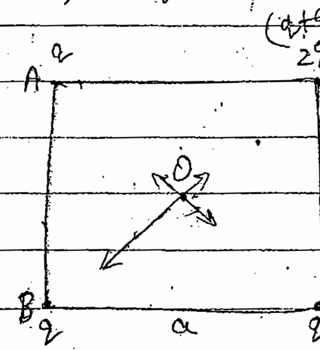
A → +q, −q
+q charge at A, B, [C] makes $\vec{E} = 0$ at O,
Now $\vec{E}$ will be only due to (−q) charge at A.

*

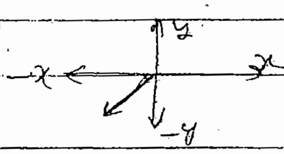
$$2q = q + q$$

$\vec{E}$ due to +q at D,

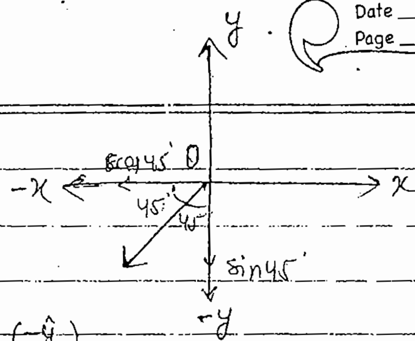
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2q}{a^2}\left(\frac{-(\hat{x}+\hat{y})}{\sqrt{2}}\right)$$



Dir^n



(***) $DO = \frac{a}{\sqrt{2}}$



Magnitude of this vector

$r = a/\sqrt{2}$

& vector –

$\frac{a}{\sqrt{2}}\cos 45(-\hat{x}) + \frac{a}{\sqrt{2}}\sin 45°(-\hat{y})$

$\frac{a}{\sqrt{2}}\frac{1}{\sqrt{2}}(-\hat{x}) + \frac{a}{\sqrt{2}}\frac{1}{\sqrt{2}}(-\hat{y})$

$\vec{r} = -\frac{a}{2}\hat{x} - \frac{a}{2}\hat{y}$

Now $\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r} = \frac{q}{4\pi\epsilon_0}\frac{\vec{r}}{r^3}$

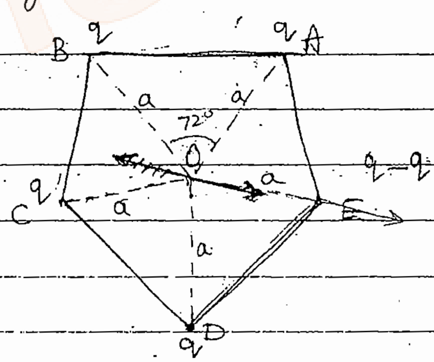
$= \frac{q}{4\pi\epsilon_0\left(\frac{a^3}{2\sqrt{2}}\right)}\left[-\frac{a}{2}(\hat{x}+\hat{y})\right]$

$$\boxed{\vec{E} = \frac{q\sqrt{2}}{4\pi\epsilon_0 a^2}\left[-(\hat{x}+\hat{y})\right]}$$

* Find the electric field at O. No charge at E.

Suppose at pt E, charge is $q-q$.

or 5 → q, $\vec{E}=0$ & $\vec{E}$ at O will be due to $-q$ at E pt.



$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{a^2}(\hat{OE})$

Dir^n of $\vec{E} \rightarrow (+\hat{OE})$

Prob:- Find the electric field $\vec{E}$ at O.

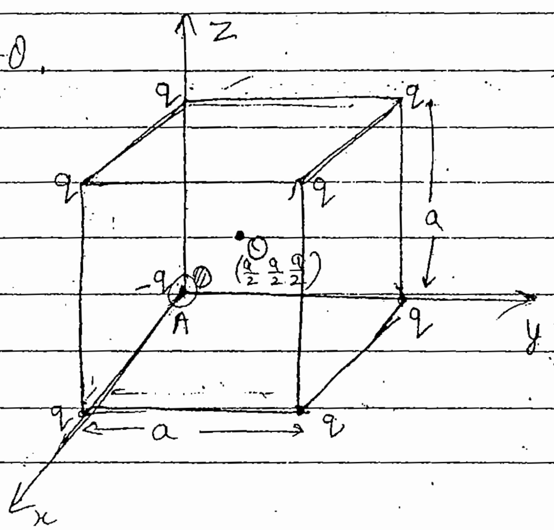
At A, charge $= -q$

$= -2q + q$

Field will be due to $-2q$.

$A = (0, 0, 0)$

$O = \left(\frac{a}{2}, \frac{a}{2}, \frac{a}{2}\right)$



$\vec{r} = \vec{OA} = -\frac{a}{2}\hat{x} - \frac{a}{2}\hat{y} - \frac{a}{2}\hat{z}$

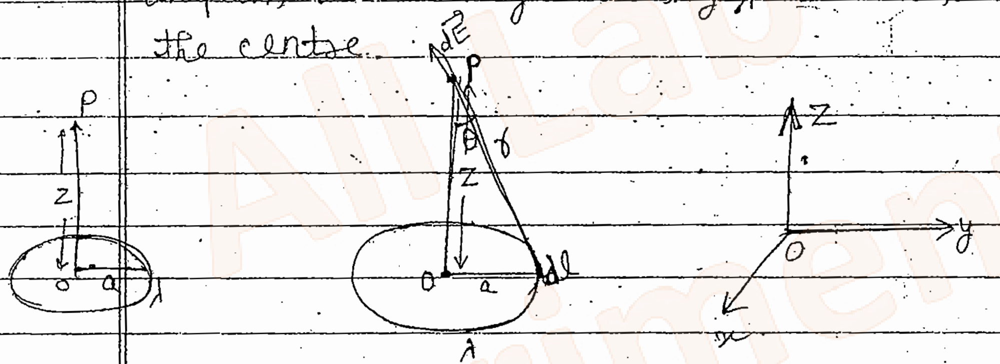
$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^3}\vec{r}$

$$|\vec{r}| = \frac{\sqrt{3}\,a}{2}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\,\frac{2q}{\left(\frac{3\sqrt{3}\,a^3}{2^3}\right)}\left[-\frac{a}{2}(\hat{x}+\hat{y}+\hat{z})\right]$$

$$\boxed{\vec{E} = \frac{8q}{4\pi\epsilon_0(3\sqrt{3}\,a^2)}\left[-(\hat{x}+\hat{y}+\hat{z})\right]}$$

<u>Que 1-</u> Electric field of a circular Ring of radius a having uniform line charge density λ at a distance z above the centre.



Circular ring is not a closed surface so can not apply Gauss' law.

Let a small element on the ring dl. charge on $dl \rightarrow$ $dq = \lambda dl$

This is a point charge so electric field will be pointed away from the charge.

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

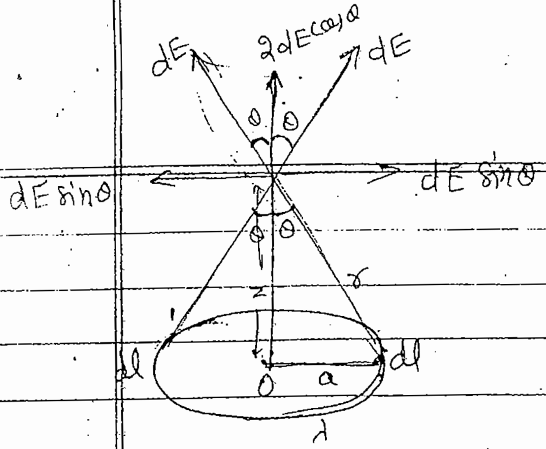
$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda dl}{(a^2+z^2)}$$

Now,

Take a similar ~~angle~~ element on opposite side.

Sine comp. will cancle out

$$\cos\theta = \frac{z}{r} = \frac{z}{(a^2+z^2)^{1/2}}$$



Total electric field,

$$E = \int 2\,dE\cos\theta$$

$$E = \int \frac{1}{4\pi\epsilon_0}\,\frac{2\lambda\,dl\;z}{(z^2+a^2)^{3/2}}$$

$$E = \frac{2(\pi a)\lambda z}{4\pi\epsilon_0(z^2+a^2)^{3/2}}$$

$\left\{\int dl = \pi a\right\}$

$$\boxed{E = \frac{\lambda a z}{2\epsilon_0(z^2+a^2)^{3/2}}}$$

{ Note :- If we take $E = \int dE\cos\theta$ then $\int dl = 2\pi a$ }

* If Total charge in the ring is q then

$$q = \lambda \cdot 2\pi a$$

then $\boxed{E = \dfrac{qz}{4\pi\epsilon_0(z^2+a^2)^{3/2}}}$

* Now, find the value of z at which $\vec{E}$ is maximum

$$\frac{dE}{dz} = 0$$

$$\frac{\lambda a}{2\epsilon_0}\,\frac{d}{dz}\,\frac{z}{(z^2+a^2)^{3/2}} = 0$$

$$\frac{\lambda a}{2\epsilon_0}\left[\frac{1}{(z^2+a^2)^{3/2}} + z\left(\frac{-3}{\cancel{2}}\right)(z^2+a^2)^{-5/2}(\cancel{2}z)\right] = 0$$

$$\frac{1}{(z^2+a^2)^{3/2}} - \frac{3z^2}{(z^2+a^2)^{5/2}} = 0$$

$$\frac{3z^2}{(z^2+a^2)^{5/2}} = \frac{1}{(z^2+a^2)^{3/2}}$$

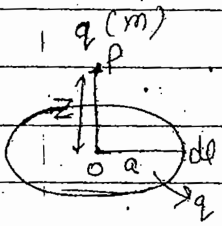
$$3z^2 = z^2+a^2$$

$$2z^2 = a^2$$

$$\boxed{z = \pm\frac{a}{\sqrt{2}}}$$

* Now if the distance z is small & put a charge q at the small distance z from the centre of ring.

If total charge = q
then force on it



$$F = \frac{q^2 z}{4\pi\epsilon_0 (z^2+a^2)^{3/2}}$$

$F \propto z$ $\quad F = \frac{q^2 z}{4\pi\epsilon_0 a^3}$ (as $z << a$)

So its motion will be simple harmonic.

* Find its freq. of oscillation & time period of oscillation

$$F = kz$$

$$\text{So } k = \frac{q^2}{4\pi\epsilon_0 a^3}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$\left\{ \omega = \sqrt{\frac{k}{m}} \quad T = 2\pi/\omega \right.$

$$T = 2\pi\sqrt{\frac{m \cdot 4\pi\epsilon_0 a^3}{q^2}}$$

$$T = \frac{4\pi a}{q}\sqrt{\pi\epsilon_0 m a}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{q^2}{4\pi\epsilon_0 a^3 m}} = \frac{q}{2a}\sqrt{\frac{1}{\pi\epsilon_0 a m}}$$

* Find $\vec{E}$ at the centre of the ring. (If we put charge at centre)

at centre $z = 0$

so $\vec{E} = 0$

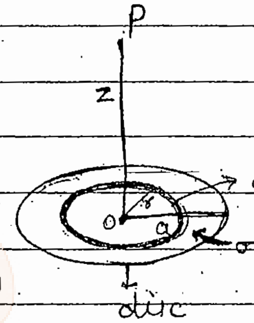
* At far distance from the ring, $z >> a$ then neglect a.

$$\vec{E} = \frac{q}{4\pi\epsilon_0 z^2}$$

At very far distance ring will behave like a point charge.

Electric field of a Disc :-

If we have a disc of radius a & surface charge density σ. charge is distributed over surface.
We need to find electric field at point P.



If we put cocentric rings (first put ring of radius o & then increasing radius) then it will from a disc.

Suppose a ring of radius r inside the disc,

$$dq = 2\pi r \, dr \, \sigma \qquad (\text{area} = 2\pi r \, dr)$$

Find the field due to this ring,

$$dE = \frac{2\pi r \sigma \, dr \cdot z}{4\pi\epsilon_0 (z^2+r^2)^{3/2}} \qquad \left\{ dE = \frac{qz}{4\pi\epsilon_0 (z^2+a^2)^{3/2}} \right.$$

Total electric field due to a disc

$$E = \frac{z\sigma}{2\epsilon_0} \int_0^a \frac{r \, dr}{(z^2+r^2)^{3/2}}$$

$$= \frac{z\sigma}{2\epsilon_0} \left[-(z^2+r^2)^{-1/2} \right]_0^a$$

$$= \frac{z\sigma}{2\epsilon_0} \left[\left\{ \frac{-1}{\sqrt{z^2+a^2}} \right\} + \frac{1}{z} \right]$$

$$\boxed{E = \frac{z\sigma}{2\epsilon_0} \left[\frac{1}{z} - \frac{1}{(z^2+a^2)^{1/2}} \right]}$$

Electric field of a finite disc.

let $z^2 + r^2 = t$

$2r \, dr = dt$

$r \, dr = dt/2$

$$\int \frac{r \, dr}{(z^2+r^2)^{3/2}} = \int \frac{dt}{2t^{3/2}}$$

$$= \frac{1}{2} \frac{t^{-1/2}}{(-1/2)}$$

$$= -t^{-1/2}$$

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(i) Electric field of a infinite disc,

$$a \to \infty$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

This is the same expression obtained by Gauss law.

Note :- If disc is a finite disc then we can not find $\vec{E}$ by Gauss law.

(ii) If z is very small & disc is finite

i.e. $z << a$

$E = ?$

$$(z^2+a^2)^{-1/2} = \frac{1}{a}\left[1+\frac{z^2}{a^2}\right]^{-1/2}$$

$$= \frac{1}{a}\left[1-\frac{1}{2}\frac{z^2}{a^2}\right]$$

{ $(1+x)^n$, cond^n of Binomal expansion is $x << 1$ }

$$E = \frac{z\sigma}{2\epsilon_0}\left[\frac{1}{z} - \frac{1}{(z^2+a^2)^{1/2}}\right]$$

Since a is large $\quad \propto$ neglect

So $\quad \boxed{E = \frac{\sigma}{2\epsilon_0}}$

(iii) Disc will behave as a point charge

$$z >> a$$

$$(z^2+a^2)^{-1/2} = \frac{1}{z}\left[1+\frac{a^2}{z^2}\right]^{-1/2}$$

$$= \frac{1}{z} - \frac{a^2}{2z^3}$$

$$E = \frac{z\sigma}{2\epsilon_0}\left[\frac{1}{z} - \frac{1}{z} + \frac{a^2}{2z^3}\right]$$

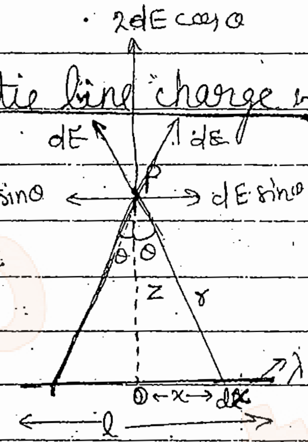
$$\boxed{E = \frac{\sigma a^2}{4\epsilon_0 z^2}}$$

(iv) If Total charge in the disc is q then

$$q = \sigma \cdot \pi a^2$$

So. $$\boxed{E = \frac{q}{4\pi\epsilon_0 z^2}}$$

Electric field of a finite wire or fintie line charge :-



At a distance z from centre O.

(We can't use Gauss law)

Take a small element dx at a distance x from O.

$$dq = \lambda\, dx$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$dE = \frac{\lambda\, dx}{4\pi\epsilon_0 (z^2+x^2)}$$

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2+x^2)^{1/2}}$$

Total electric field

$$E = \int_0^{l/2} 2dE\cos\theta$$

{If we take $2dE$ then integration will be from 0 to $l/2$.
" " dE " " " " $-l/2$ to $l/2$}

$$E = \int_0^{l/2} 2\frac{\lambda\, dx \cdot z}{4\pi\epsilon_0 (z^2+x^2)^{3/2}}$$

$$= \frac{\lambda z}{2\pi\epsilon_0}\int_0^{l/2}\frac{dx}{(z^2+x^2)^{3/2}}$$

let $x = z\tan\theta$

$dx = z\sec^2\theta\, d\theta$

$$\int \frac{dx}{(z^2+x^2)^{3/2}} = \int \frac{z\sec^2\theta\, d\theta}{z^3\sec^3\theta}$$

$$= \frac{1}{z^2}\int \cos\theta\, d\theta$$

$$= \frac{1}{z^2}\sin\theta$$

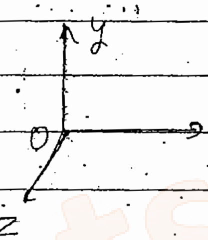
$\tan\theta = \frac{x}{z}$, $\sin\theta = \frac{x}{r} = \frac{x}{(z^2+x^2)^{1/2}}$

$$\int_0^{l/2} \frac{dx}{(z^2+x^2)^{3/2}} = \left[\frac{1}{z^2}\frac{x}{\sqrt{x^2+z^2}}\right]_0^{l/2}$$

$$= \frac{1}{z^2}\left[\frac{l/2}{\sqrt{\frac{l^2}{4}+z^2}} - 0\right]$$

$$\boxed{\vec{E} = \frac{\lambda z}{2\pi\epsilon_0}\frac{1}{z^2}\frac{l/2}{\sqrt{\frac{l^2}{4}+z^2}}\hat{y}}$$

dir^n of $\vec{E}$ is $+\hat{y}$



Cases:-

(i) If $l \to \infty$

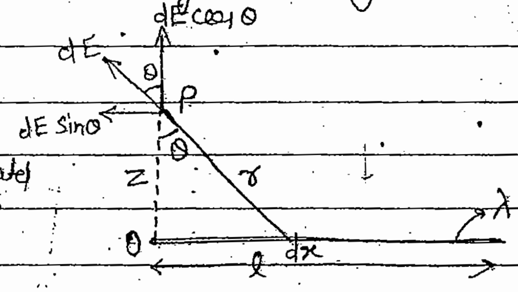
$$E = \frac{\lambda z}{2\pi\epsilon_0}\frac{1}{z^2}\frac{l/2}{l/2\sqrt{1+\frac{4z^2}{l^2}}}$$

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 z}}$$

(ii) If $z >> l$

$$E = \frac{\lambda l}{4\pi\epsilon_0 z^2} \Rightarrow \boxed{E = \frac{q}{4\pi\epsilon_0 z^2}}$$

Prob:- Finite wire of length l & charge density λ. Calculate the electric field at P.



$E = \int dE\cos\theta$ i.e.

E along $\hat{y} \Rightarrow E_y$ we have calculated above.

Now,

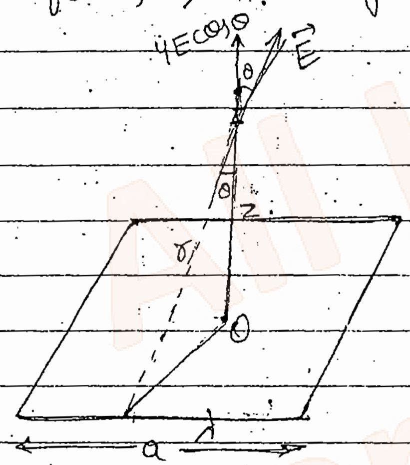
$$E_x = \int dE\sin\theta$$

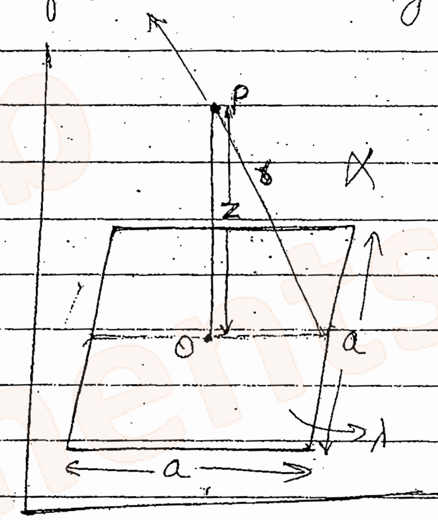
$$= \int_0^l \frac{\lambda\, dx}{4\pi\epsilon_0(z^2+x^2)^{3/2}}\, x$$

$$E_x = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{-1}{(z^2+a^2)^{1/2}}\right]_0^l$$

$$\boxed{E_x = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{z} - \frac{1}{(z^2+l^2)^{1/2}}\right]}$$

Q4 Find the electric field at a distance z above at the centre of a square loop of side a carrying uniform line charge λ.





Sin θ - Component will cancel out.
Electric field of a finite wire

$$E = \frac{\lambda(l/2)}{2\pi\epsilon_0 z\left(z^2+\frac{l^2}{4}\right)^{1/2}}$$

so for a square loop,

$$E = \frac{\lambda(a/2)}{2\pi\epsilon_0 r\sqrt{r^2+\frac{a^2}{4}}}$$

$$r = \sqrt{z^2+\frac{a^2}{4}}$$

$$E = \frac{\lambda a}{4\pi\epsilon_0\sqrt{z^2+\frac{a^2}{4}}\sqrt{z^2+\frac{a^2}{2}}}$$

Electric field due to 1 wire along z is

$$E_z = E\cos\theta$$

$$\cos\theta = \frac{z}{\sqrt{z^2+a^2/4}}$$

$$E_z = \frac{\lambda a z}{4\pi\epsilon_0\left(z^2+\frac{a^2}{4}\right)\sqrt{\left(z^2+\frac{a^2}{2}\right)}}$$

Total electric field due to 4 wires i.e. for square loop

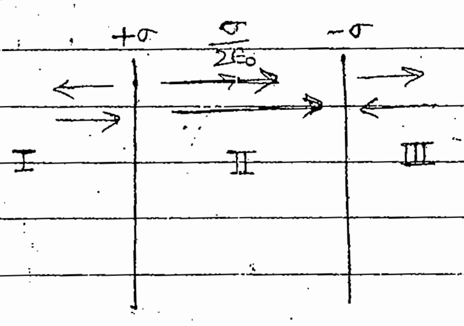
$$E_z = 4\,\frac{\lambda z a}{4\pi\epsilon_0\left(z^2+\frac{a^2}{4}\right)\sqrt{z^2+\frac{a^2}{2}}}$$

$$\boxed{E_z = \frac{\lambda a z}{\pi\epsilon_0\left(z^2+\frac{a^2}{4}\right)\sqrt{z^2+\frac{a^2}{2}}}}$$

Ans

Electric field $= \frac{\sigma}{2\epsilon_0}$

Que:-① If there are two plane sheets having charge density σ & $-\sigma$ respectively then find out $\vec{E}$ in I, II & III region.



Arrow shows the dirn of electric field
I. Draw the arrow for $+\sigma$ sheet
II. for $-\sigma$ sheet & so on

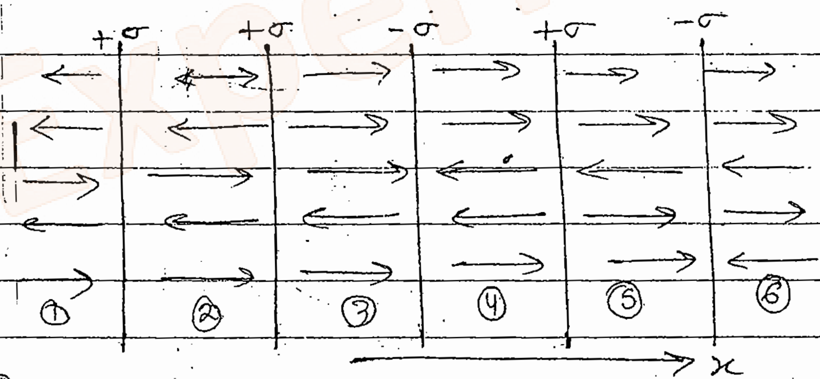
∴ Magnitude of electric field $= \frac{\sigma}{2\epsilon_0}$

$\vec{E}$ in region I $\rightarrow$ 0
II $\rightarrow$ $\frac{\sigma}{\epsilon_0}$ $\left\{ \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} \right.$
III $\rightarrow$ 0

Dirn of $\vec{E}$ is $+\hat{x}$

* Dirn of $\vec{E}$ is always from + to −.

Que:-②



Region ① $\rightarrow E = \frac{\sigma}{2\epsilon_0}$ $(-\hat{x})$

(2) $\rightarrow E = \frac{\sigma}{2\epsilon_0}$ $(+\hat{x})$

(3) $\rightarrow E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{3\sigma}{2\epsilon_0}$ $(+\hat{x})$

(4) $\rightarrow E = \frac{\sigma}{2\epsilon_0}$ $(+\hat{x})$

(5) $\rightarrow E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{3\sigma}{2\epsilon_0}$ $(+\hat{x})$

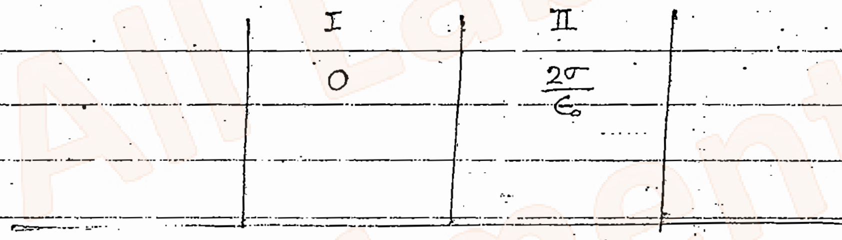
(6) $\rightarrow E = \frac{\sigma}{2\epsilon_0}$ $(+\hat{x})$

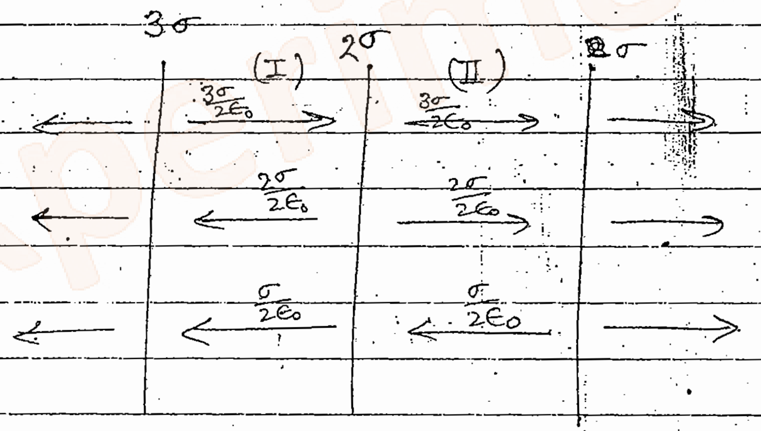
Note :- Dir^n of Electric field

for $+q \rightarrow$ away

$-q \rightarrow$ towards

Q.2 - Three infinite non-conducting sheets with uniform surface charge densities, σ, 2σ & 3σ are arranged to be parallel. What is there order from left to right if electric field $\vec{E}$ produced by the arrangement has the magnitude, $E = 0$ in one region & $E = \frac{2\sigma}{\epsilon_0}$ in the another region.





In I region, $\vec{E} = \frac{3\sigma}{2\epsilon_0}\hat{x} - \frac{2\sigma}{2\epsilon_0}\hat{x} - \frac{\sigma}{2\epsilon_0}\hat{x}$

$$\vec{E} = \frac{3\sigma}{2\epsilon_0}\hat{x} - \frac{3\sigma}{2\epsilon_0}\hat{x} \Rightarrow \boxed{\vec{E} = 0}$$

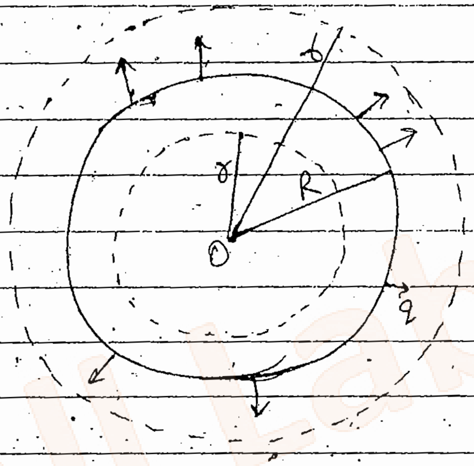
In II Region, $\vec{E} = \frac{3\sigma}{2\epsilon_0}\hat{x} + \frac{2\sigma}{2\epsilon_0}\hat{x} - \frac{\sigma}{2\epsilon_0}\hat{x}$

$$= \frac{5\sigma}{2\epsilon_0}\hat{x} - \frac{\sigma}{2\epsilon_0}\hat{x} = \frac{4\sigma}{2\epsilon_0}\hat{x}$$

$$\boxed{\vec{E} = \frac{2\sigma}{\epsilon_0}\hat{x}}$$

Conducting Surfaces :-

Q.1 :- Find the electric field inside & outside a conducting sphere of radius R and carrying total charge q.



Inside :-

charge enclosed

$q_{enc} = 0$

$\vec{E} = 0$

Electric field inside the conducting material is zero. No static charge can remain inside the conducting material.

Volume charge density for conducting material is zero.

Properties of conducting sphere,

(i) $\vec{E} = 0$
(ii) $\rho = 0$

Outside :- charge enclosed = q

$E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$

$$\boxed{\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}}$$

(iii) Electric field $\vec{E}$ is normal to the conducting surface.

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✓(iv) Conductors are equipotential surface i.e. at every point of the conductor, potential is same.

for reference, $\boxed{\text{Pot}^n \text{ of earth} = 0}$ bcoz

if we apply some charge, pot^n of earth does not change. It remain constant. ~~Pot~~ That's why for reference, we consider pot^n of earth zero. Since Capacitance of earth is very larger so its pot^n does not change. If we connect sphere to earth then it will take the pot^n of earth.

* Only free charge goes into the ground. Induced charge never goes in the ground.

→ Similarly, ~~for~~
If we have a conducting cylender then

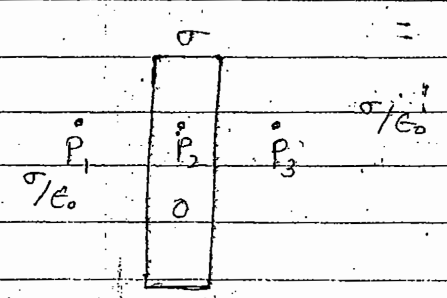
Electric field inside the cylender $\vec{E} = 0$
" " outside " " $\vec{E}$ normal to the surface.

$$E_{out} = \frac{KR^3}{3\epsilon_0 s}\hat{s}$$

Conducting Plates :-

We have a plate made up of conducting ~~sphere~~ surface having charge denside σ.

(i) at P_2, $\vec{E}_{in} = 0$
(ii) at P_1 & P_2, $\vec{E} = \frac{\sigma}{\epsilon_0}$



Conducting plate have two surface ~~wh~~ even if it is very thin.

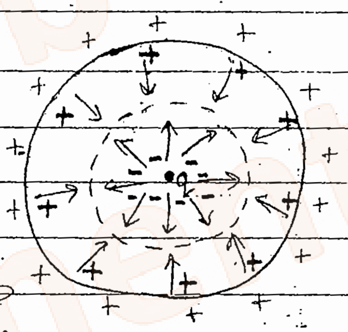
→ Why $\vec{E}$ is zero inside the conding material.

* In conductors, there are free e^-.
In Insulator, there are not present free e^-.
This is the difference b/w conductor & insulator.

If we take a neutral conducting sphere then total $q_{enc} = 0$ so $E_{in} = 0$ bcoz no. of free e^- = +ve charges created when e^- become free.

If we take a conducting sphere and charge q is placed at centre.

then still $q_{enc} = 0$



bcoz dir^n of $\vec{E}$ created by q at centre is away from charge. Due to +q, inward force, $F = -eE$. Due to free

q charge outward force $F = qE$

force apply on free e^-, they become free & +ve charge create over the surface. Dir^n of $\vec{E}$ ~~this~~ by +ve charge is ~~towards q (inside) (from + to −)~~

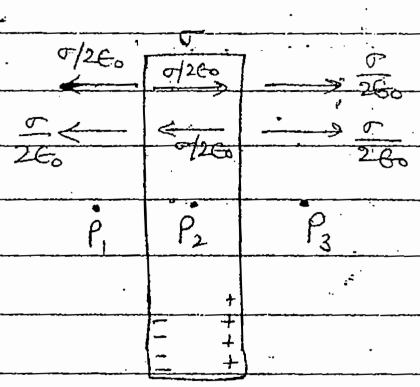
So Total $q_{enc} = 0$

So $E_{in} = 0$ (due to charge seperation)

but outside E_{out} = Non zero.

⇒ field due to I surface

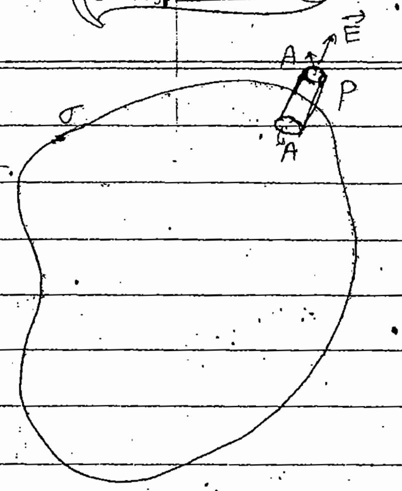
push ~~create~~ −ve charge on I & +ve charge on ~~+ve~~ II surface.



Again field due to II surface push −ve charges on I surface. So Net $\vec{E}$ at P_2 is 0. $\boxed{\vec{E} = 0}$

Arbitrary Surfaces :-

If we take a arbitrary surface of charge density σ.



- Find out $\vec{E}$ at point P which is just outside the conductor. What is the total flux inside the conductor.

If area is A. $\boxed{\text{charge} = \sigma A}$.

The flux passing through the curved part is zero ($\vec{E}$ is ⊥ to the surface) The flux passing through the lower cross section which is inside the surface is also zero & flux passing through upper cross section is Non zero.

Electric field inside the conductor $E = 0$

" " outside " " is

$$\oint E \cdot dS = \frac{q}{\epsilon_0} \Rightarrow EA = \frac{\sigma A}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

$$\boxed{\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}}$$

- Electrostatic field is conservative field i.e. work done by the field is path independent

Electrostatic field is

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\boxed{\vec{\nabla} \times \vec{E} = 0} \rightarrow \text{differential form}$$

$$\boxed{\oint_c E \cdot dl = 0} \rightarrow \text{Integral form}$$

$$\nabla \times E = 0 \Rightarrow \int (\vec{\nabla} \times \vec{E}) \cdot d\vec{S} = 0 \Rightarrow \oint_c \vec{E} \cdot d\vec{l} = 0$$

{ for conservative field, curl of field should be ...

$E \cdot dl \rightarrow$ work done for a unit test charge.

$\left\{ E = \frac{F}{q} \right\}$ & $\{ W = F \cdot dl \}$ $\left\{ \text{for } q = 1, \; E = F, \; E = \frac{W}{dl}, \; E \cdot dl = W \right\}$

Que 1- $\vec{E} = k(xy\,\hat{x} + 2yz\,\hat{y} + 3xz\,\hat{z})$

$\vec{E} = k(y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\,\hat{z})$

Find wheather these two electric fields are possible electrostatic fields.

OR

Which one of the following is impossible electrostatic field.

(i) $\vec{E} = k(xy\,\hat{x} + 2yz\,\hat{y} + 3xz\,\hat{z})$

$$\vec{\nabla} \times \vec{E} = k \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix}$$

$$= k\left[\hat{x}(0 - 2y) - \hat{y}(3z - 0) + \hat{z}(0 - x)\right]$$

$$= k\left[-2y\,\hat{x} - 3z\,\hat{y} - x\,\hat{z}\right]$$

$\nabla \times E \neq 0$

(ii) $\vec{E} = k\left[y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\,\hat{z}\right]$

$$\nabla \times \vec{E} = k \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & 2xy + z^2 & 2yz \end{vmatrix}$$

$$= k\left[\hat{x}(2z - 2z) - \hat{y}(0 - 0) + \hat{z}(2y - 2y)\right]$$

$$= k\left[0\,\hat{x} + 0\,\hat{y} + 0\,\hat{z}\right] = 0$$

(i) Impossible Electric field.

(ii) Possible Electric field

$\nabla \times E = 0$ (only in Electrostatic)
$\nabla \times E \neq 0$ (Non electrostatic)

Que :- If $\vec{\nabla} \times \vec{E} = 0$

$\vec{E} = -\vec{\nabla} V$

where

$V \rightarrow$ Scalar Pot^n

It is electrostatic when it does not depend on t.
If V depends on t then $\rightarrow$ Non electrostatic.

We can find V by $E \Rightarrow$

$$\boxed{V = -\int_{O}^{r} E \cdot dr}$$

$O \rightarrow$ reference point

We know that

$$\text{Pot}^n \; V = \frac{W}{q}$$

Pot^n is work done per unit charge.

Pot^n at ∞ (ref. point) is zero. Pot^n of any finite point V w.r. to ∞ is pot^n. At ∞, Pot^n is 0.
Pot^n of any finite point w.r. to a finite point is pot^n difference.

V_a V_b

$V = 0 \longrightarrow$ Pot^n
$V_a - V_b \neq 0 \rightarrow$ Pot^n diff.

In Pot^n Energy we take a ~~finite~~ unity test charge; for unity test charge, pot^n is the pot^n energy itself.

$$U = qV$$

Work done on a system is stored in terms of energy. If work is done on unity test charge then pot^n enrgy is = pot^n.
& if work is done on finite test charge then (i.e. work done to move a finite charge from ∞ to a point) is stored as pot^n energy.

We generally take reference point from $0 \rightarrow \infty$

In all cases we take ∞ as a reference point except where charge itself extended to ∞, In this case we take a finite ref. point.

e.g. line charge
| it extended to itself to ∞. So we can't take ∞ as ref. point.

For point charge, $V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$

for line charge $V = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dl}{r}$

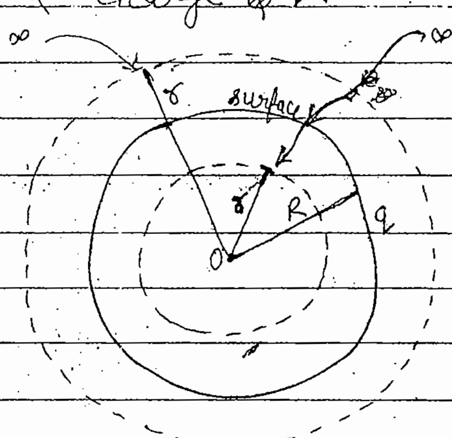
Surface charge $V = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma da}{r}$

Volume charge $V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho d\tau}{r}$

→ Where symmetry is present, I find electric field & then pot^n.

→ In Non symmetry cases → I find pot^n & then electric field.

Que - Find the pot^n inside & outside a conducting sphere of radius R & charge q.



Pot^n $\quad V_{out} = -\int \vec{E} \cdot d\vec{r}$

Outside electric field $\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$

charge is not extended to ∞ itself so we'll take ∞ as reference point.

Work done required to take a unity test charge from ∞ to r is pot^n.

$$V_{out} = -\int_{\infty}^{r} E \cdot dr$$

$$= -\int_{\infty}^{r} \frac{q}{4\pi\epsilon_0 r^2} \cdot dr$$

$$= -\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r}$$

$$\boxed{V_{out} = \frac{q}{4\pi\epsilon_0 r}}$$

To find V_{in} → Work done to take a unity test charge from ∞ to surface & surface to r.

$$V_{in} = -\int_{\infty}^{r} E \cdot dr$$

$$= -\int_{\infty}^{R} \frac{q}{4\pi\epsilon_0 r^2} dr - \int_{R}^{r} 0 \, dr$$

$$\boxed{V_{in} = \frac{q}{4\pi\epsilon_0 R}}$$

$W = F \cdot dr$

for unity test charge $E = \frac{F}{1}$ bcoz $E = \frac{F}{q}$

then work done to take the unity test charge from surface to r is 0.

$$\boxed{V_{centre} = \frac{q}{4\pi\epsilon_0 R}}$$

→ Pot^n inside the conductor is equal to the pot^n at centre.

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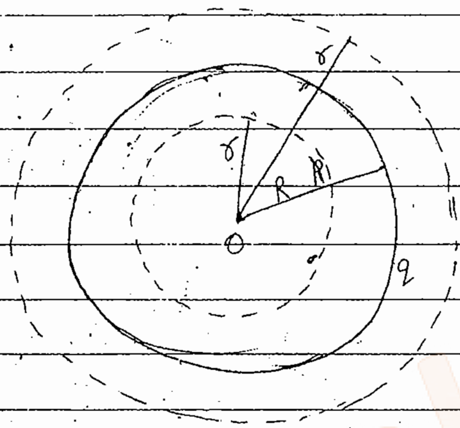
Qn :- Find the pot^n inside & outside of a uniformly charged sphere having radius R & total charge q.



Uniformly charged means ρ is constant throughout.

$$\vec{E}_{out} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{E}_{in} = \frac{qr}{4\pi\epsilon_0 R^3}\hat{r}$$

$$V_{out} = -\int_{\infty}^{r}\vec{E}\cdot d\vec{r} = -\int_{\infty}^{r}\frac{q}{4\pi\epsilon_0 r^2}dr = \frac{-q}{4\pi\epsilon_0}\left[\frac{-1}{r}\right]_{\infty}^{r}$$

$$\boxed{V_{out} = \frac{q}{4\pi\epsilon_0 r}}$$

$$V_{in} = -\int_{\infty}^{R}\frac{q}{4\pi\epsilon_0 r^2}dr - \int_{R}^{r}\frac{qr}{4\pi\epsilon_0 R^3}dr$$

$$= -\frac{q}{4\pi\epsilon_0}\left[\frac{-1}{r}\right]_{\infty}^{R} - \frac{q}{4\pi\epsilon_0 R^3}\left[\frac{r^2}{2}\right]_{R}^{r}$$

$$= \frac{q}{4\pi\epsilon_0 R} - \frac{q}{4\pi\epsilon_0 R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$= \frac{q}{4\pi\epsilon_0 R}\left[1 - \frac{r^2}{2R^2} + \frac{1}{2}\right]$$

$$= \frac{q}{4\pi\epsilon_0 R}\left[\frac{2R^2 - r^2 + R^2}{2R^2}\right]$$

$$\boxed{V_{in} = \frac{q}{8\pi\epsilon_0 R^3}(3R^2 - r^2)}$$

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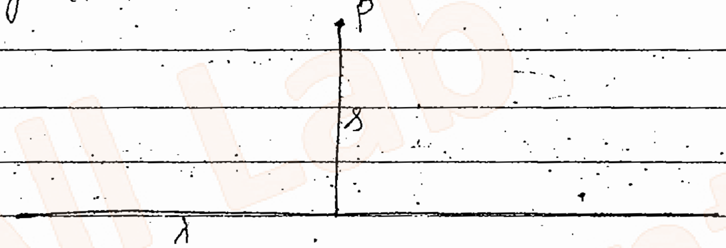
$$\boxed{V_{centre} = \frac{3q}{8\pi\epsilon_0 R}} \quad (\text{at centre } r = 0)$$

$$\boxed{V_{centre}\left(\begin{matrix}\text{uniformly}\\ \text{charged}\end{matrix}\right) = \frac{3}{2} V_{centre}\,(\text{conducting})}$$

Que :- Find the pot^n at a distance s from an infinitely long straight wire that carries uniform line charge λ.



$\{\cancel{q = \lambda s}\}$

at P, $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}$

$$V = -\int_a^s E \cdot ds$$

{a → finite point at which we know the pot^n}

$$= -\int_a^s \frac{\lambda}{2\pi\epsilon_0 s} ds$$

$$= \frac{-\lambda}{2\pi\epsilon_0}(\ln s)_a^s = \frac{-\lambda}{2\pi\epsilon_0}\left(\ln \frac{s}{a}\right)$$

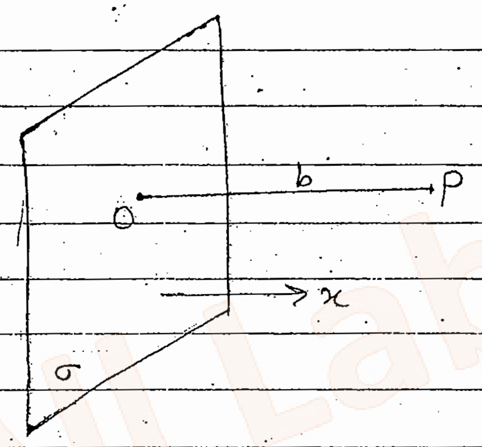
$$\boxed{V = \frac{\lambda}{2\pi\epsilon_0}\ln(a/s)}$$

a → reference point

s → at which we want to find pot^n.

Que:- find the potⁿ at a distance b from an infinite sheet of charge σ

→ Sheet is extended to itself ∞ so ref. point can'not be ∞.



at P, $\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}$

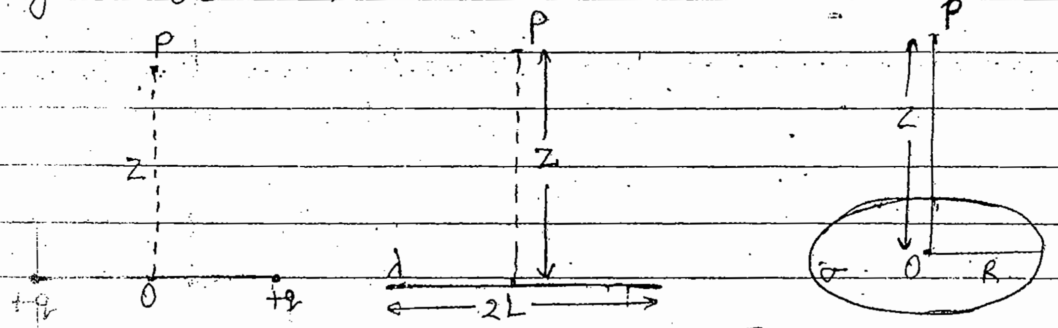
$$V = -\int_a^b \frac{\sigma}{2\epsilon_0}\,dx$$

a → reference point

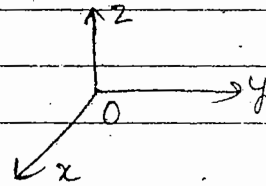
$$= -\frac{\sigma}{2\epsilon_0}[x]_a^b = -\frac{\sigma}{2\epsilon_0}[b-a]$$

$$\boxed{V = \frac{\sigma}{2\epsilon_0}(a-b)}$$

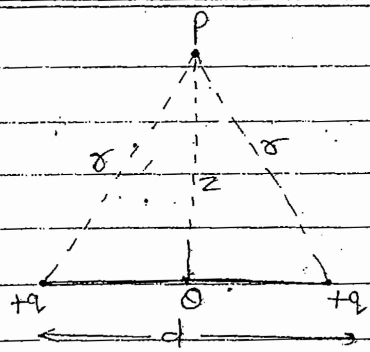
Q.1:- Calculate the potⁿ of 3 charge configurations given below:



Co-ordinate system:-



(i)



$$V_2 = V_1 = \frac{q}{4\pi\epsilon_0 r}$$

$$= \frac{q}{4\pi\epsilon_0\sqrt{z^2+\frac{d^2}{4}}}$$

Potⁿ is a scalar quantity. Total potⁿ at P will be

$$V = V_1 + V_2$$

$$V = \frac{2q}{4\pi\epsilon_0\sqrt{z^2+\frac{d^2}{4}}} \Rightarrow \boxed{V = \frac{q}{2\pi\epsilon_0\sqrt{z^2+\frac{d^2}{4}}}}$$

Electric field at point P will be

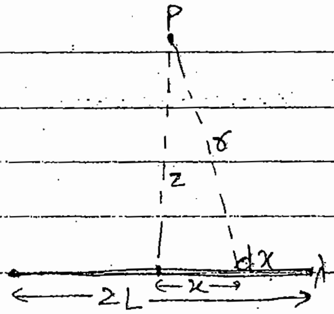
$$\vec{E} = -\vec{\nabla}V$$

$$= -\frac{\partial V}{\partial z}\hat{z}$$

$$= -\frac{q}{2\pi\epsilon_0} \cdot -\frac{1}{2}\left(z^2+\frac{d^2}{4}\right)^{-3/2}(2z)$$

$$\boxed{\vec{E} = \frac{qz}{2\pi\epsilon_0\left(z^2+\frac{d^2}{4}\right)^{3/2}}\hat{z}}$$

(ii)



$$r = \sqrt{z^2+x^2} \quad , \quad V = \frac{1}{4\pi\epsilon_0}\int\frac{\lambda\, dl}{r}$$

$$V = \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{\lambda\, dx}{\sqrt{z^2+x^2}}$$

$$V = \frac{\lambda}{2\pi\epsilon_0}\ln\left[x+\sqrt{z^2+x^2}\right]_0^L$$

$$\left\{\text{Using } \int\frac{dx}{(a^2+x^2)^{1/2}} = \ln\left[x+\sqrt{a^2+x^2}\right]\right\}$$

$$V = \frac{\lambda}{2\pi\epsilon_0}\ln\left[\frac{L+\sqrt{z^2+L^2}}{z}\right]$$

Electric field $\vec{E} = -\frac{\partial V}{\partial z}\hat{z}$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{\frac{L+\sqrt{L^2+z^2}}{z}}\left\{\frac{z\frac{1}{2}(z^2+L^2)^{-1/2} - (\sqrt{z^2+L^2}+L)}{z^2}\right\}\hat{z}$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{L+\sqrt{L^2+z^2}}\left[\frac{1}{2\sqrt{z^2+L^2}} - \frac{(\sqrt{z^2+L^2}+L)}{z^2}\right]\hat{z}$$

$$= \frac{\lambda}{2\pi\epsilon_0}\frac{1}{L+\sqrt{L^2+z^2}}\left[\frac{z^2-2z^2-2L^2}{2\sqrt{z^2+L^2}\;z^2}\right]\hat{z}$$

$$= \frac{\lambda}{2\pi\epsilon_0(L+\sqrt{L^2+z^2})}\frac{z^2-2L^2}{2\sqrt{z^2+L^2}\,z^2}\hat{z}$$

$$= \frac{\lambda}{4\pi\epsilon_0}$$

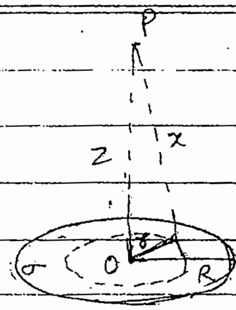
$$\vec{E} = -\frac{\partial}{\partial z}\left[\frac{\lambda}{2\pi\epsilon_0}\left(\ln\left\{L+\sqrt{z^2+L^2}\right\} - \ln z\right)\right]$$

$$= -\frac{\lambda}{2\pi\epsilon_0}\left[\frac{2z}{(L+\sqrt{L^2+z^2})\,2\sqrt{z^2+L^2}} - \frac{1}{z}\right]\hat{z}$$

$$= -\frac{\lambda}{2\pi\epsilon_0}\left[\frac{z}{(L+\sqrt{z^2+L^2})\sqrt{z^2+L^2}} - \frac{1}{z}\right]\hat{z}$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0}\left[\frac{1}{z} - \frac{z}{(L\sqrt{z^2+L^2}+z^2+L^2)}\right]\hat{z}$$

(iii)



$$dq = 2\pi r\, dr\, \sigma$$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{z^2+r^2}}$$

$$x = \sqrt{z^2+r^2}$$

$$V = \frac{\sigma}{4\pi\epsilon_0}\int_0^R \frac{2\pi r\, dr}{(z^2+r^2)^{1/2}}$$

$$z^2+r^2 = t$$

$$2r\, dr = dt$$

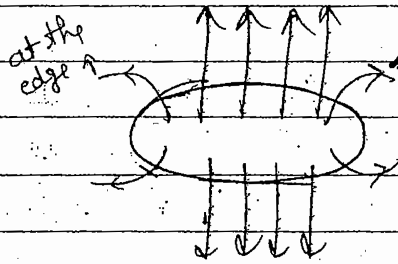
$$V = \frac{\sigma . 2\pi}{4\pi\epsilon_0}\int \frac{dt}{2(t)^{1/2}}$$

$$V = \frac{\sigma}{4\epsilon_0}\left[\frac{t^{1/2}}{1/2}\right]$$

$$= \frac{\sigma}{2\epsilon_0}\left[(z^2+r^2)^{1/2}\right]_0^R \Rightarrow \boxed{V = \frac{\sigma}{2\epsilon_0}\left[(z^2+R^2)^{1/2} - z\right]}$$

$$\boxed{V_{centre} = \frac{\sigma R}{2\epsilon_0}} \quad \{\text{at Centre} \rightarrow z=0\}$$

Potential at the edge,



$$\boxed{V_{edge} = \frac{\sigma R}{\pi\epsilon_0}}$$

$\therefore$ $\boxed{V_{centre} > V_{edge}}$ Due to Edge Effect.

Electric field lines at the edges are not entirely normal to the surface.

Hence * V_{edge} is different from the V_{centre}

Qu 1 - Find whether the Electric field $\vec{E} = (12x^2 - y^2)\hat{x} - 2xy\hat{y}$ is conservative or not. If conservative find the potential as a func^n of position.

$$\vec{E} = (12x^2 - y^2)\hat{x} - 2xy\,\hat{y}$$

$\vec{E}$ will be conservative if $\vec{\nabla} \times \vec{E} = 0$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (12x^2 - y^2) & -2xy & 0 \end{vmatrix}$$

$$= \hat{x}[0] - \hat{y}[0] + \hat{z}[-2y + 2y]$$

$$\vec{\nabla} \times \vec{E} = 0$$

So $\vec{E}$ is conservative

Now find $V(r) = ?$

$$\vec{E} = -\vec{\nabla}V \Rightarrow V = -\int E \cdot dr$$

$$= -\left[\int (12x^2 - y^2)\,dx - \int 2xy\,dy\right]$$

$$\cancel{= -\left[12\frac{x^3}{3} + xy^2 - 2x\frac{y^2}{2}\right]}$$

$$\cancel{V = -4x^3 + 2xy^2}$$

$$V = -\int d(4x^3 - xy^2)$$

$$\boxed{V = xy^2 - 4x^3 + C} \quad \text{Ans}$$

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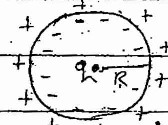
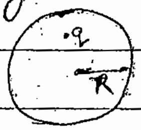
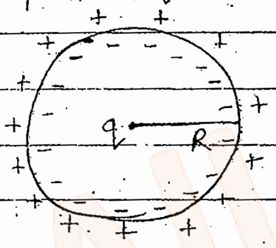
If we have a conductor & +q is place at centre of the conductor. So at centre
$\boxed{q_{inside} = 0}$ & $\rho = 0$



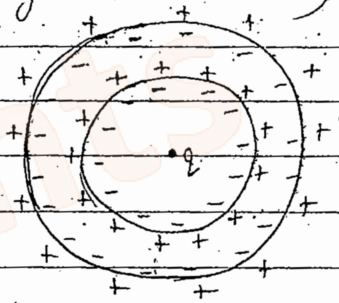
If instead of centre, we place charge q anywhere else then still, $\rho_{inside} = 0$



If we have a spherical shell or hollow conducting sphere of radius R then
On inner surface charge will be $-q$ & on outer surface $+q$. (+ve charge is uniformly distributed)

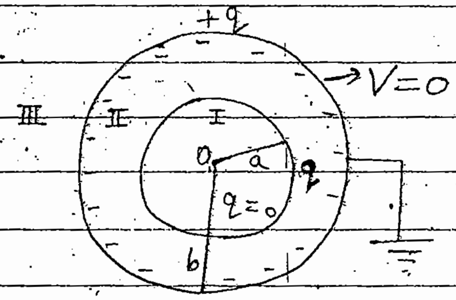


If we have further place the conducting shell.



Que: A spherical conductor of radius 'a' has a charge +q placed on it. This is surrounded by thin spherical conducting shell of radius b. It is connected to the earth. Find
(a) charges on outer & inner surface of the shell
(b) potⁿ in region I, II & III.
(c) The electric field in region I, II & III.

Conductors are equipotⁿ surface.
It is not necessary that — if potⁿ = 0 then q = 0
So only satisfy the potⁿ condition.



for conducting sphere of radius a, inner charge $\boxed{q=0}$

On Inner surface of conducting sphere of radius b the -ve charge is uniformly distributed & on outer surface $q=?$

It is grounded so $V=0$

At Outer surface

$$V=\frac{1}{4\pi\epsilon_0}\frac{q}{b}-\frac{1}{4\pi\epsilon_0}\frac{q}{b}+\frac{1}{4\pi\epsilon_0}\frac{q}{b}=0$$

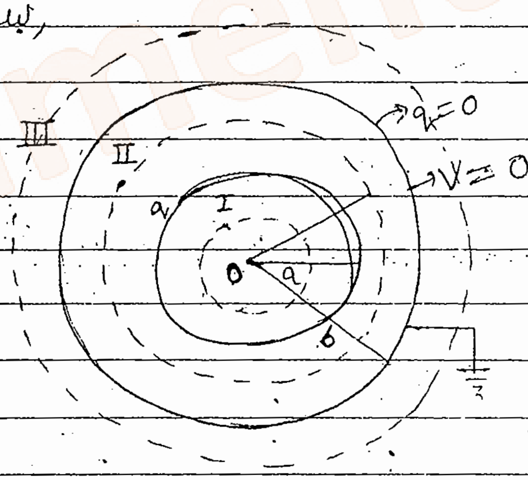
$$V=\frac{1}{4\pi\epsilon_0}\frac{q}{b}=0$$

$$\Rightarrow \boxed{q=0}$$

So charge is zero at outer surface.

(b) Now find $\vec{E}$ using Gauss law,

Region I, $\boxed{E_I=0}$



Region II,

$$\boxed{E_{II}=\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$$

Region III,

$$\boxed{E_{III}=0}$$

(c) Potential :- $V_{III}=-\int_{\infty}^{r}E_{III}\cdot dr=0$

$$\boxed{V_{III}=0}$$

$$V_{II}=-\int_{\infty}^{b}E_{III}\,dr-\int_{b}^{r}\frac{q}{4\pi\epsilon_0}\frac{dr}{r^2}$$

$$=0+\frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_b^r$$

$$\boxed{V_{II}=\frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}-\frac{1}{b}\right]}$$

Conducting sphere shield the inner charge from outer charge but
Not " " outer " " inner "

$$V_I = -\int_{\infty}^{r} \vec{E}\cdot d\vec{r}$$

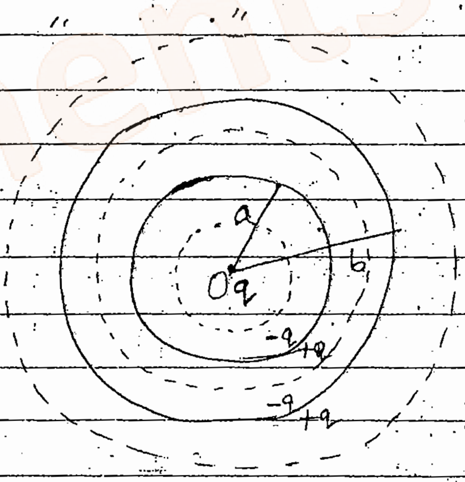
$$= -\int_{\infty}^{b} E_{III}\cdot dr - \int_{b}^{a} \frac{q}{4\pi\epsilon_0 r^2}dr - \int_{a} E_I \cdot dr$$

$$= 0 + \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_b^a - 0$$

$$\boxed{V_I = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]}$$

Que: If we have 2 conducting spherical shells of radius a & b. find
(i) distribution of charge for both surface
(ii) pot^n V for $r<a$, $a<r<b$, $r>b$
(iii) $\vec{E}$ " " "

(i) inner $\to -q$
outer $\to +q$



(iii) Net charge $\to +q$
In all III regions,
behaviour of $\vec{E}$ is same
for $r<a$, $a<r<b$, $r>b$

$$\boxed{E = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}}$$

(ii) for $r>b$

$$V = -\int_{\infty}^{r} E\cdot dr \Rightarrow V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

for $a<r<b$,

$$V = -\int_{\infty}^{b} E\cdot dr - \int_{b}^{r} E\cdot dr = +\frac{1}{4\pi\epsilon_0}\frac{q}{b} + \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r}-\frac{1}{b}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

$r<a$ = $V = -\int_{\infty}^{r} E\cdot dr = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

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Note :- If all the charges are inside then ~~cal~~ take sum of all the charges & distance where we calculate the potⁿ i.e.

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

[for previous figure]

If some charge are inside & some outside then for inside charges → takes distance where we calculate potⁿ
for outside " → " " where ~~these~~ these charges are placed.

If outer charge is +q & −q then

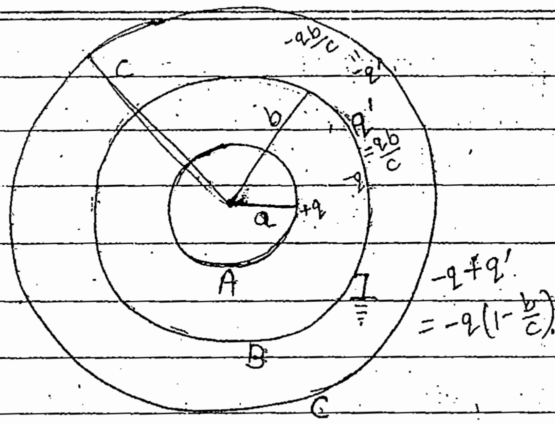
$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{b} - \frac{1}{4\pi\epsilon_0}\frac{q}{b} = 0$$

Note :- " To find the potⁿ in spherical symmetric cases place all the inside charges at the distance where we are calculating the potⁿ & all outside charges must be placed at the distance where they are."

This is the direct method.

Que :- 3 cocentric spherical shells A, B & C of radii a, b & c respectively. The shell A is given a charge +q & shell C, −q. Shell B is grounded. Find the charges & potentials on the surfaces of A, B & C.

charge on B can not be exactely grounded due to shell C. & Also can not be exactely equal to q. Let ~~ch~~ induced charge on shell B is q'.
i.e. At B shell, potⁿ = 0
but charge may or may not be zero.



Potⁿ At B,

$$V_B = \frac{q'}{4\pi\epsilon_0 b} + \frac{q}{4\pi\epsilon_0 b} - \frac{q}{4\pi\epsilon_0 b} - \frac{q}{4\pi\epsilon_0 c} - \frac{q'}{4\pi\epsilon_0 c} + \frac{q'}{4\pi\epsilon_0 c}$$

$$V_B = \frac{q'}{4\pi\epsilon_0 b} - \frac{q}{4\pi\epsilon_0 c} = 0 \qquad \} \text{ At B, } V=0$$

$$\frac{q'}{b} - \frac{q}{c} = 0$$

$$\Rightarrow \boxed{q' = \frac{qb}{c}}$$

We know that $\boxed{V_B = 0}$

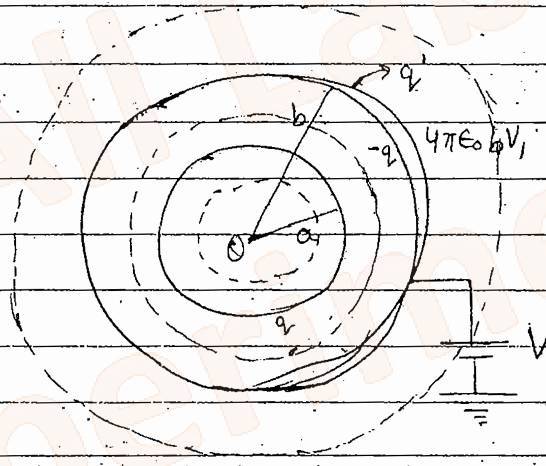
$$V_A = \frac{q}{4\pi\epsilon_0 a} - \frac{q}{4\pi\epsilon_0 b} + \frac{q\cancel{b}}{4\pi\epsilon_0 \cancel{b}c} - \frac{qb}{4\pi\epsilon_0 c \cdot c} - \frac{q}{4\pi\epsilon_0 c} + \frac{qb}{c\,4\pi\epsilon_0 c}$$

$$\boxed{V_A = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]}$$

$$V_C = \frac{q}{4\pi\epsilon_0 c} - \frac{q}{4\pi\epsilon_0 c} + \frac{qb}{c\,4\pi\epsilon_0 c} - \frac{qb}{c\,4\pi\epsilon_0 c} - \frac{q}{4\pi\epsilon_0 c} + \frac{qb}{c\,4\pi\epsilon_0 c}$$

$$\boxed{V_c = \frac{-q}{4\pi\epsilon_0 c} + \frac{qb}{4\pi\epsilon_0 c^2}}$$

Ques :- A spherical conductor of radius 'a' has a charge q placed on it. This is surrounded by a thin spherical shell of radius b & connected to the earth through a battery of potⁿ V_1.
(a) Find the charges on the outer & inner surface of shell.
(b) Find the potⁿ & Electric fields at a distance r from the centre of the sphere where
(i) $r < a$
(ii) $a < r < b$
(iii) $r > b$



Let charge on outer surface is q'.
On outer surface potⁿ is V_1

$$V_1 = \cancel{\frac{q}{4\pi\epsilon_0 b}} - \cancel{\frac{q}{4\pi\epsilon_0 b}} + \frac{q'}{4\pi\epsilon_0 b}$$

$$\Rightarrow \boxed{q' = 4\pi\epsilon_0 b V_1}$$

On inner surface → $-q$

(b) Potⁿ :-
(i) $r > b$

$$V = \frac{q}{4\pi\epsilon_0 r} - \frac{q}{4\pi\epsilon_0 r} + \frac{4\pi\epsilon_0 b V_1}{4\pi\epsilon_0 r}$$

$$\boxed{V = \frac{bV_1}{r}}$$

(ii) $a<r<b$

$$V = \frac{q}{4\pi\epsilon_0 r} - \frac{q}{4\pi\epsilon_0 b} + \frac{4\pi\epsilon_0 b V_1}{4\pi\epsilon_0 b}$$

$$\boxed{V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{b}\right] + V_1}$$

(iii) $r<a$

$$V = \frac{q}{4\pi\epsilon_0 a} - \frac{q}{4\pi\epsilon_0 b} + \frac{4\pi\epsilon_0 b V_1}{4\pi\epsilon_0 b}$$

$$\boxed{V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] + V_1}$$

Electric field

i) $r>b$,

either $\vec{E} = -\vec{\nabla} V$ or by Gauss law.

$$\cancel{V = -\int_\infty^r \vec{E}\cdot d\vec{r}} \quad \cancel{= -\int_\infty^r \frac{bV_1}{r}\, dr}$$

$$\cancel{V = -bV_1 [\log r]_\infty^r}$$

$$\oint E\cdot ds = \frac{q}{\epsilon_0}$$

$$E\cdot 4\pi r^2 = \frac{4\pi\epsilon_0 b V_1}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{bV_1}{r^2}\hat{r}}$$

$$\left\{ \text{by gradient} \quad E = -\vec{\nabla} V = -\vec{\nabla}\left(\frac{bV_1}{r}\right) = \frac{bV_1 \vec{r}}{r^3} = \frac{bV_1}{r^2}\hat{r} \right.$$

$$\because r = \sqrt{x^2+y^2+z^2}$$

(ii) $a<r<b$,

$$\oint E\cdot ds = \frac{q}{\epsilon_0}$$

$$E\cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}}$$

(iii) $r < a$,

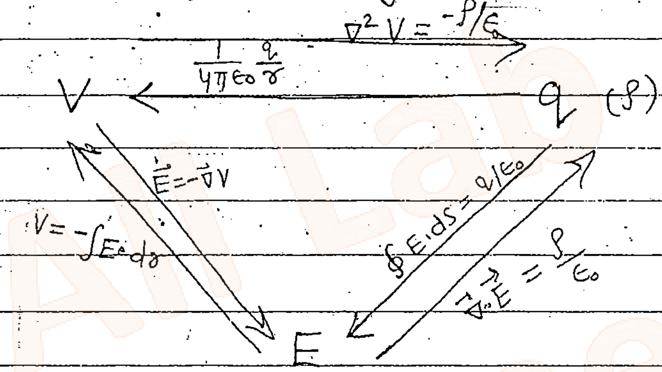
$q_{enc} = 0$

$$\oint E \cdot ds = \frac{q}{\epsilon_0}$$

$$\boxed{E = 0}$$

In Electrostatics

3 Main things:, charge, electric field, pot^n.



Ques :- The pot^n of a charge distribution is given by

$$V(r, \theta, \phi) = \left(\frac{V_0}{2}\right)\left(3 - \frac{r^2}{R^2}\right) \quad , r < R$$

$$= \frac{V_0 R}{r} \quad , r > R$$

where V_0 & R are constants.

(a) Obtain the corresponding Electric field distribution.

(b) Obtain the charge distribution ρ.

(c) If R is the radius of sphere which is centred at the origin. Calculate the total charge enclosed by this sphere.

(a) $$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial r}\hat{r}$$

$r < R$, $$\vec{E} = -\vec{\nabla}\left[\frac{V_0}{2}\left(3 - \frac{r^2}{R^2}\right)\right]$$

$$= -\frac{V_0}{2}\left(\cancel{3} - \frac{2r}{R^2}\right)\hat{r}$$

$$\boxed{\vec{E} = \frac{V_0 r}{R^2}\hat{r}}$$

$r > R$, $\vec{E} = -\frac{\partial V}{\partial r}\hat{r} = -\frac{\partial}{\partial r}\left(\frac{V_0 R}{r}\right)\hat{r}$

$$\vec{E} = -V_0 R\left(\frac{-1}{r^2}\right)\hat{r}$$

$$\boxed{\vec{E} = \frac{V_0 R}{r^2}\hat{r}}$$

(b) $\rho = ?$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \Rightarrow \rho = \epsilon_0(\vec{\nabla}\cdot\vec{E})$$

$r < R$, $\rho = \epsilon_0\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r)\right]$

$$= \epsilon_0\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{V_0 r}{R^2}\right)\right]$$

$$= \epsilon_0\left[\frac{1}{r^2}\frac{V_0}{R^2}3r^2\right]$$

$$\boxed{\rho = \frac{3V_0\epsilon_0}{R^2}}$$

$r > R$, $\rho = \epsilon_0\left[\cancel{\frac{1}{r}}\,\vec{\nabla}\cdot\left(V_0 R\frac{\hat{r}}{r^2}\right)\right]$

$$= \epsilon_0 V_0 R\, 4\pi\, \delta^3(r)$$

for $r > R$, is not enclosing R. so $\delta^3(r) = 0$

$$\therefore \boxed{\rho = 0}.$$

(c) $q = \int \rho\, d\tau$

$\{r < R\}$ $q = \int_0^R \frac{3V_0\epsilon_0}{R^2} 4\pi r^2 dr$

$$= \frac{3V_0\epsilon_0}{R^2} 4\pi \frac{R^3}{3}$$

$$\boxed{q = 4\pi\epsilon_0 R V_0}$$

This is the total charge enclosed by this sphere.

Ques 1- Electric potⁿ in some configuration is given by

$$V(r) = A\,\frac{e^{-\lambda r}}{r} \qquad A \;\&\; \lambda \text{ are constants.}$$

find (i) ρ

(ii) $\vec{E}$

(iii) Total q in the sphere of radius R centred at the origin.

(i) $$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial r}\,\hat{r}$$

$$\vec{E} = -\frac{\partial}{\partial r}\left[A\,\frac{e^{-\lambda r}}{r}\right]\hat{r}$$

$$= -A\left[\frac{1}{r}(-\lambda)e^{-\lambda r} + e^{-\lambda r}\left(\frac{-1}{r^2}\right)\right]\hat{r}$$

$$= A\left[\frac{\lambda e^{-\lambda r}}{r} + \frac{e^{-\lambda r}}{r^2}\right]\hat{r}$$

$$\vec{E} = A\left[\frac{\lambda}{r} + \frac{1}{r^2}\right]e^{-\lambda r}\,\hat{r} = A e^{-\lambda r}\left[\lambda r + 1\right]\frac{\hat{r}}{r^2}$$

(ii) $$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\rho = \epsilon_0\left[\vec{\nabla}\cdot\vec{E}\right] = \epsilon_0\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A\left\{\frac{\lambda}{r} - \frac{1}{r^2}\right\}e^{-\lambda r}\right)\right]$$

$$= A\epsilon_0\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left[(\lambda r - 1)e^{-\lambda r}\right]\right]$$

$$= A\epsilon_0\left[\frac{1}{r^2}\left\{\lambda e^{-\lambda r} + (\lambda r - 1)e^{-\lambda r}(-\lambda)\right\}\right]$$

$$= A\epsilon_0\left[\frac{1}{r^2}\left\{\lambda e^{-\lambda r} \mp \lambda^2 r e^{-\lambda r} \pm \lambda e^{-\lambda r}\right\}\right]$$

$$= A\epsilon_0\left[\frac{1}{r^2}\left(2\lambda e^{-\lambda r} - \lambda^2 r e^{-\lambda r}\right)\right]$$

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$$\rho = \frac{A\epsilon_0}{r^2} \lambda e^{-\lambda r}(2 - \lambda r)$$

$$\rho = \epsilon_0 [\vec{\nabla}\cdot\vec{E}] = \epsilon_0\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A\frac{\lambda}{r}e^{-\lambda r}\right)\right] + \epsilon_0\left[A\,\vec{\nabla}\cdot\left(e^{-\lambda r}\frac{\hat{r}}{r^2}\right)\right]$$

$$= \epsilon_0\left[\frac{1}{r^2}A\lambda\left(r(-\lambda)e^{-\lambda r} + e^{-\lambda r}\right)\right] + \epsilon_0 A\left[\frac{\hat{r}}{r^2}(-\lambda)e^{-\lambda r}\cdot\hat{r} + e^{-\lambda r}4\pi\delta^3(r)\right]$$

$$\{\vec{\nabla}\cdot(\lambda\vec{A}) = \vec{A}\cdot\vec{\nabla}\lambda + \lambda\vec{\nabla}\cdot\vec{A}\}$$

$$= \epsilon_0\left[\frac{A\lambda}{r^2}\{(-\lambda r + 1)e^{-\lambda r}\}\right] + \epsilon_0 A\left[\frac{-\lambda e^{-\lambda r}}{r^2} + 4\pi\delta^3(r)\right]$$

$$= \frac{\epsilon_0 A\lambda}{r^2}e^{-\lambda r} - \frac{\epsilon_0 A\lambda^2}{r}e^{-\lambda r} - \frac{\epsilon_0 A\lambda e^{-\lambda r}}{r^2} + \epsilon_0 A\, 4\pi\delta^3(r)$$

$$\rho = -\frac{\epsilon_0 A\lambda^2}{r}e^{-\lambda r} + 4\pi\delta^3(r)\,\epsilon_0 A$$

$$\rho = \epsilon_0 A\left[\frac{-\lambda^2 e^{-\lambda r}}{r} + 4\pi\delta^3(r)\right]$$

OR

$$\rho = \epsilon_0\left[\vec{\nabla}\cdot\{\underbrace{A e^{-\lambda r}(\lambda r + 1)}_{\mathrm{I}}\,\underbrace{\frac{\hat{r}}{r^2}}_{\mathrm{II}}\}\right]$$

$$= \epsilon_0 A\left[\frac{\hat{r}}{r^2}\cdot\vec{\nabla}\left(e^{-\lambda r}(1+\lambda r)\right) + e^{-\lambda r}(1+\lambda r)\,\vec{\nabla}\cdot\frac{\hat{r}}{r^2}\right]$$

$$= \epsilon_0 A\left[\frac{\hat{r}}{r^2}\left(-\lambda e^{-\lambda r} + \lambda e^{-\lambda r} - \lambda^2 r e^{-\lambda r}\right)\cdot\hat{r} + e^{-\lambda r}\left(4\pi\delta^3(r)(1+\lambda r)\right)\right]$$

$$\rho = \epsilon_0 A\left[\frac{-\lambda^2}{r}e^{-\lambda r} + 4\pi\delta^3(r)\right]$$

(Using $f(r)\,4\pi\delta^3(r) = f(0)\,4\pi\delta^3(r)$)

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$q = \int \rho \, d\tau \neq +\cancel{q_p}$

$= -\epsilon_0 A \lambda^2 \int_0^R \frac{e^{-\lambda r}}{r} 4\pi r^2 dr + 4\pi\epsilon_0 A \int_V \underset{1}{\underset{\parallel}{\delta^3(r)}}\, dr$

$= -4\pi\epsilon_0 A \lambda^2 \int_0^R e^{-\lambda r} \cdot r\, dr + 4\pi\epsilon_0 A$

$= -4\pi\epsilon_0 A \lambda^2 \left[r \cdot \frac{e^{-\lambda r}}{-\lambda} - \frac{e^{-\lambda r}}{+\lambda^2} \right]_0^R + 4\pi\epsilon_0 A$

$= -4\pi\epsilon_0 A \lambda^2 \left[\frac{r e^{-\lambda r}}{-\lambda} - \frac{e^{-\lambda r}}{\lambda^2} \right]_0^R + 4\pi\epsilon_0 A$

$= -4\pi\epsilon_0 A \lambda^2 \left[\frac{R e^{-\lambda R}}{-\lambda} - \frac{e^{-\lambda R}}{\lambda^2} + \cancel{\frac{1}{\lambda^2}} \right] + \cancel{4\pi\epsilon_0 A}$

$= 4\pi\epsilon_0 A \left[e^{-\lambda R} + R\lambda e^{-\lambda R} \right]$

$\boxed{q = 4\pi\epsilon_0 A [1 + R\lambda] e^{-\lambda R}}$

$\Rightarrow \nabla^2 V = -\frac{\rho}{\epsilon_0}$

$\rho = ?\quad , q = ?$

$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right)$

$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\left(A \frac{e^{-\lambda r}}{r} \right) \right]$

$= \frac{A}{r^2}\frac{\partial}{\partial r}\left[r^2 \left(\frac{-\lambda e^{-\lambda r}}{r} - \frac{e^{-\lambda r}}{r^2} \right) \right]$

$= -\frac{A}{r^2}\frac{\partial}{\partial r}\left[(\lambda r + 1) e^{-\lambda r} \right]$

$= -\frac{A}{r^2}\left[(\lambda r + 1)(-\lambda) e^{-\lambda r} + e^{-\lambda r}(\lambda) \right]$

$= \frac{A}{r^2}\left[\lambda^2 r e^{-\lambda r} - \cancel{\lambda e^{-\lambda r}} + \cancel{\lambda e^{-\lambda r}} \right]$

$= \frac{A}{r} \lambda^2 e^{-\lambda r}$

$\rho = -\epsilon_0 \frac{A}{r} \lambda^2 e^{-\lambda r}$

$$q = \int \rho \, d\tau = \int_0^R -\frac{\epsilon_0 A \lambda^2 e^{-\lambda r}}{r} 4\pi r^2 dr$$

$$= -\epsilon_0 A \lambda^2 4\pi \int_0^R r e^{-\lambda r} dr$$

$$= -4\pi \epsilon_0 A \lambda^2 \left[-r \frac{e^{-\lambda r}}{\lambda} + \frac{e^{-\lambda r}}{-\lambda} \right]_0^R$$

$$= -4\pi \epsilon_0 A \lambda^2 \left[-\lambda R e^{-\lambda R} - e^{-\lambda R} + 0 + 1 \right]$$

$$= -4\pi \epsilon_0 A \lambda^2 \left[-e^{-\lambda R} - \lambda R e^{-\lambda R} + 1 \right]$$

$$= 4\pi \epsilon_0 A \lambda^2 \left[1 - e^{-\lambda r}(1 - \lambda R) \right]$$

$$= 4\pi \epsilon_0 A \lambda \left[r e^{-\lambda r} + e^{-\lambda r} \right]_0^R$$

$$= 4\pi \epsilon_0 A \lambda \left[R e^{-\lambda R} + e^{-\lambda R} - 1 \right]$$

$$q = 4\pi \epsilon_0 A \lambda \left[e^{-\lambda R}(R+1) - 1 \right]$$

Qu:- Yukawa Pot^n is given by $V = -\frac{q}{r} e^{-\alpha t}\left(1 + \frac{\alpha r}{2}\right)$

Find (i) $\vec{E}$ (ii) ρ (iii) Total q in a sphere of radius R.

(i) $\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial r} \hat{r}$

$$\vec{E} = -\frac{\partial}{\partial r}\left[\frac{-q}{r} e^{-\alpha t}\left(1 + \frac{\alpha r}{2}\right) \right] \hat{r} = q e^{-\alpha t} \frac{\partial}{\partial r}\left[\frac{1}{r}\left(1 + \frac{\alpha r}{2}\right) \right]$$

$$= q e^{-\alpha t} \frac{\partial}{\partial r}\left[\frac{1}{r} + \frac{\alpha}{2} \right] \hat{r}$$

$$= q e^{-\alpha t}\left[\frac{-1}{r^2} \right] \hat{r} = -\frac{q e^{-\alpha t}}{r^2} \hat{r}$$

$$\boxed{\vec{E} = -\frac{q e^{-\alpha t}}{r^2} \hat{r}}$$

(ii) $\rho = ?$ $\quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

$$\rho = \epsilon_0 \left[-q e^{-\alpha t} \left[\vec{\nabla} \cdot \frac{\hat{r}}{r^2} \right] \right] = -q \epsilon_0 e^{-\alpha t} 4\pi \delta^3(r)$$

$$\boxed{\rho = -4\pi \epsilon_0 q e^{-\alpha t} \delta^3(r)}$$

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$$Q = \int \rho \, d\tau = \int_0^R \cancel{-4\pi\epsilon_0 q e^{-\alpha t}\, \delta^3(r)\, 4\pi r^2 dr}$$

$$= \cancel{(4\pi)^2} \epsilon_0$$

$$= -4\pi\epsilon_0 q e^{-\alpha t} \int_V \delta^3(r)\, dr$$

{sphere is centred at origin so $\int_V \delta^3(r)\, d\tau = 1$}

$$\boxed{Q = -4\pi\epsilon_0 q e^{-\alpha t}}$$

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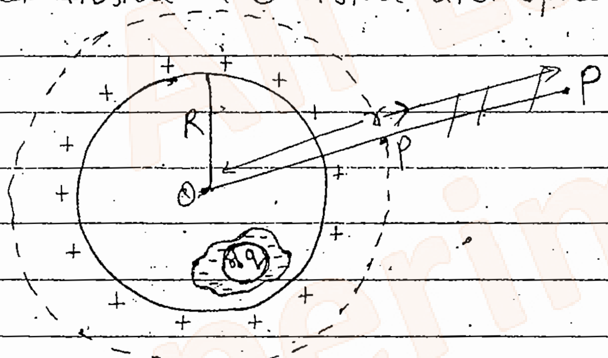
Q.1 :- An uncharged spherical conductor centred at the origin has a cavity of some arbitrary shape. Somewhere within the cavity charge is q. Then find electric field inside & outside the sphere.



Empty space inside the sphere is cavity.

Electric field inside the sphere

$$\boxed{\vec{E}_{in} = 0}$$

Electric field outside the sphere

$$\boxed{\vec{E}_{out} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}}$$

Electric field inside & outside independent on shape on the cavity but dependent on charge of the cavity.

Electric field within the cavity, (it is not 0 bcoz it encloses the charge q).

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}}$$

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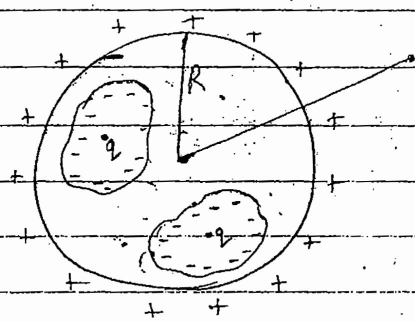
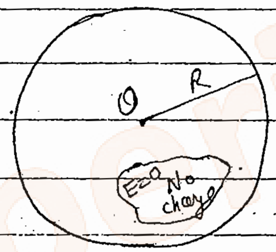
⇒ If we have more than one cavity.



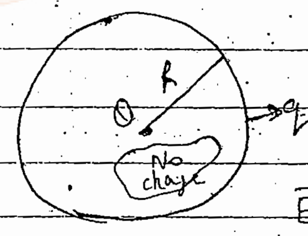
Note:- Charges inside the different cavities are independent of each other.

⇒ If we have a conducting sphere of radius R. We have a cavity which is empty itself, Electric field inside the cavity is ZERO. (whether the conductor is charged or not)



$\vec{E} = 0$

If conductor is charged.



still $\vec{E} = 0$.

Q.2- A metal sphere of radius R carrying a small charge q is surrounded by a thick cocentric metal shell inner radius 'a' & outer radius 'b'. The shell carries no net charge.

(a) Find the charge densities at a, b & R

(b) Find the potⁿ at centre using ∞ as a reference point.

(c) Find the electric field for

(i) $r < R$

(ii) $R < r < a$

(iii) $a < r < b$

(iv) $r > b$

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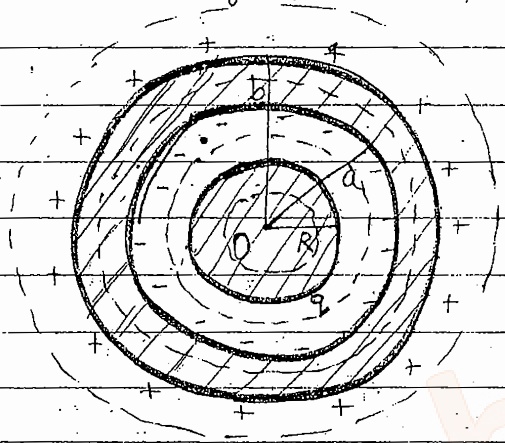
(d) Now if the outer surface is touched to the ground how the answers of part (a), (b), (c) will be change?



(a) charge density

$$\sigma = \frac{\text{charge}}{\text{Area}}$$

$q = \sigma A$

$\therefore \sigma = \frac{q}{A}$

$$\sigma_R = \frac{q}{4\pi R^2}$$

$$\sigma_a = \frac{-q}{4\pi a^2}$$

$$\sigma_b = \frac{q}{4\pi b^2}$$

(b)
$$V_{centre} = \frac{q}{4\pi\epsilon_0 R} - \frac{q}{4\pi\epsilon_0 a} + \frac{q}{4\pi\epsilon_0 b}$$

$$V_{centre} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{a} + \frac{1}{b}\right]$$

(c) $\vec{E} = ?$

for $r < R$, $\boxed{E = 0}$ (No charge)

for $R < r < a$ $\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$

$r < b$ $\boxed{\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}}$

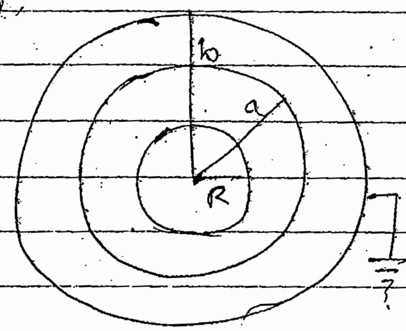
$a < r < b$ $\boxed{\vec{E} = 0}$

(d) Now outer surface is grounded.

On σ_A & σ_R → No effect

$\sigma_b = 0$

$$V_{centre} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{a}\right]$$



$\vec{E}$ in region $x > b$

$$\vec{E} = 0$$

In others region → No effect.

Q.3:- Two spherical cavities of radie a & b are hollowed out from the interior of a neutral conducting sphere of radius R at the centre of each cavity point charge q_a & q_b, ~~having radie a & b respectively~~ are placed. ~~q_a charge~~ Find

(a) The surface charge densities σ_a, σ_b & σ_R.

(b) Electric field inside & outside the conductor

(c) " " " the cavities.

(d) force b/w ~~on~~ charges q_a & q_b.

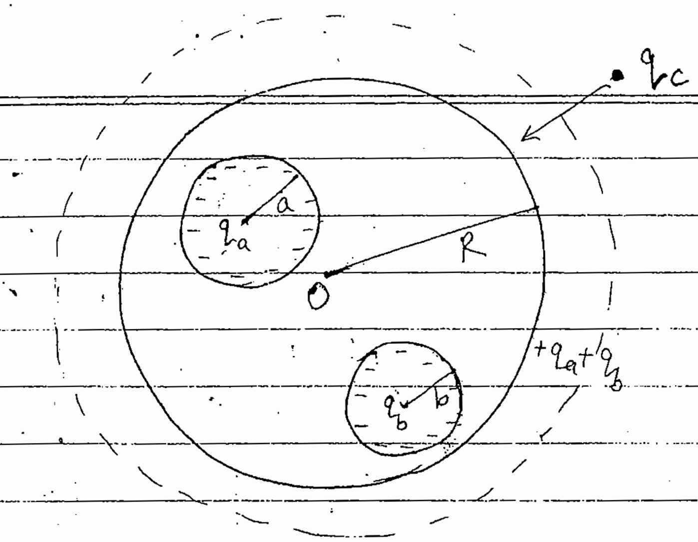
(e) Which of the answers (a), (b), (c) & (d) will change if a III charge q_c were brought near the conductor.

(a) $$\sigma_a = \frac{-q_a}{4\pi a^2}$$

$$\sigma_b = \frac{-q_b}{4\pi b^2}$$

$$\sigma_R = \frac{q_a + q_b}{4\pi R^2}$$

(b) $$\boxed{\vec{E}_{in} = 0}$$



$$\vec{E}_{out} = \frac{q_a+q_b}{4\pi\epsilon_0 r^2}\hat{r}$$

(c) Electric field inside the cavities

$$\frac{q_a}{4\pi\epsilon_0 r^2}\hat{r} \quad , \quad \frac{q_b}{4\pi\epsilon_0 r^2}\hat{r}$$

(d) $\boxed{F=0}$

$[F_a = 0 \quad , F_b = 0]$

(e) $\sigma_a, \sigma_b \rightarrow$ No change

σ_R will change. Now charge will be Non-uniform

$E_{in} = 0$

$E_{out} = \frac{q_a+q_b}{4\pi\epsilon_0 r^2}\hat{r}$ will change

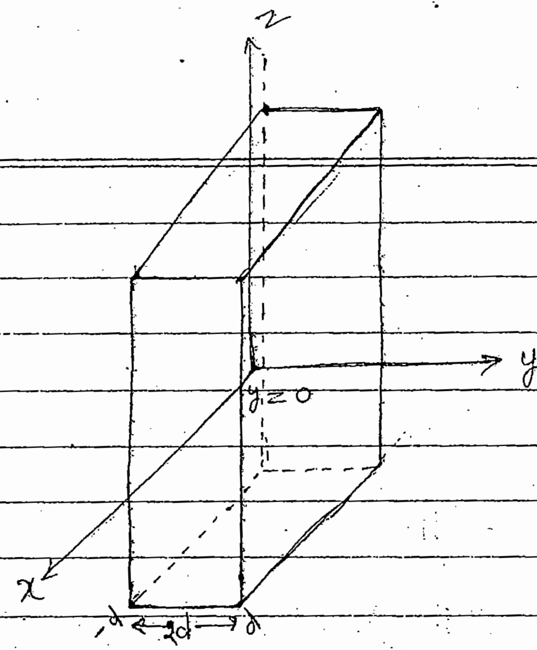
$\vec{E}$ inside the cavity $\rightarrow$ No change

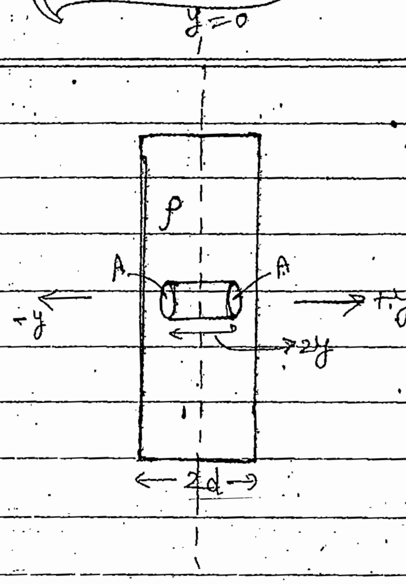
force " " $\rightarrow$ No change.

Q.4 :- An infinite plane slab of thickness 2d carries a uniform charge density ρ. Find the electric field ~~in~~ as a funcⁿ of y where y = 0 at the centre. Plot $\vec{E}$ v/s y. ~~Only~~ $\vec{E}$ calling E +ve when it points in +y dirⁿ & -ve when it points in -y dirⁿ.

conducting → $E = \frac{\sigma}{\epsilon_0}$
Non- " → $E = \frac{\sigma}{2\epsilon_0}$

$y = -d$ $y = +d$





dir^n of $\vec{E}$ will be in $+y$ or $-y$ dir^n. We can also take a Gaussian pillbox instead of cylinder as a Gaussian surface.

charge enclosed in the cylinder

$q_{enc} = \rho \cdot A \cdot 2y$

$\left\{ \begin{array}{l} vol = \pi R^2 l \\ \quad\;\; = A \cdot l \\ \rho = q/vol \end{array} \right.$

The flux passing through both cross section surface is $\phi = 2E \cdot A$

$$2EA = \frac{q}{\epsilon_0}$$

$$2EA = \frac{\rho A \, 2y}{\epsilon_0}$$

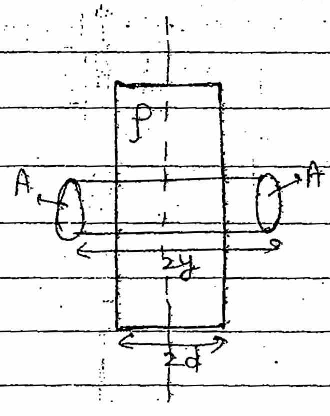
$$\boxed{\begin{array}{ll} \vec{E}_{in} = \frac{\rho y}{\epsilon_0} \hat{y} & 0 < y < d \\ \vec{E}_{in} = \frac{\rho y}{\epsilon_0} (-\hat{y}) & -d < y < 0 \end{array}}$$

For Outside :-

$q_{enc} = \rho \cdot A \cdot 2d$

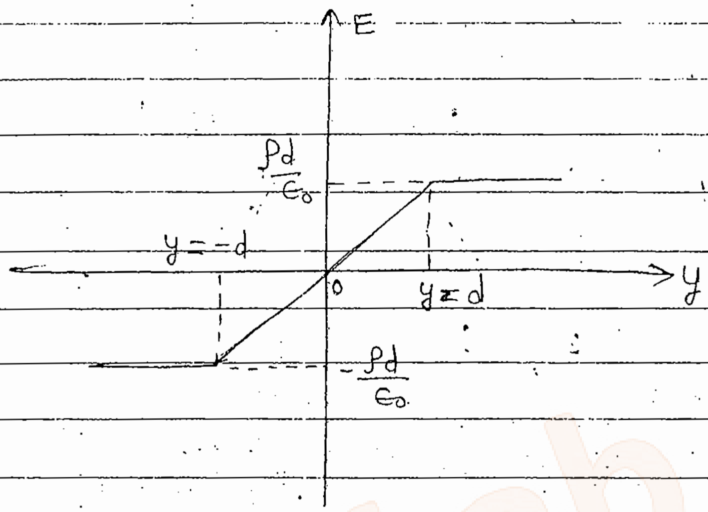
flux $= 2E \cdot A$

$2E \cdot A = \frac{\rho A 2d}{\epsilon_0}$



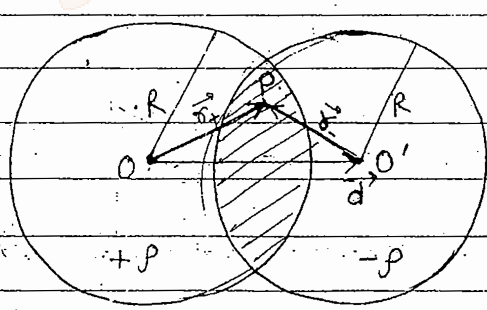
$$\boxed{\begin{array}{ll} \vec{E}_{out} = \frac{\rho d}{\epsilon_0} \hat{y} & y > d \\ \vec{E}_{out} = \frac{\rho d}{\epsilon_0} (-\hat{y}) & y < -d \end{array}}$$

Behaviour of $\vec{E}$:- Plot $\vec{E}$ v/s y.



⇒ Outside of a thin conducting sheet $\vec{E}$ is constant.

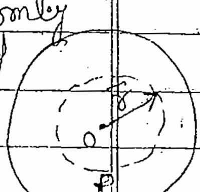
Q.51 - Two spheres each of radius R carrying the uniform charge densities $+\rho$ & $-\rho$ respectively are placed so that they partially overlap. Take a vector from the +ve centre to -ve centre, $\vec{d}$. Show that the field in the region of overlap is constant & find its value.



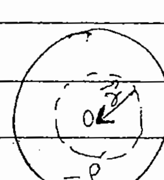
$\vec{OO'} = \vec{d}$
$\vec{OP} = \vec{r}_+$
$\vec{O'P} = \vec{r}_-$

$\vec{E}$ follow the superposition principle.

uniformly charged sphere



$$\vec{E} = \frac{\rho r}{3\epsilon_0}\hat{r} = \frac{\rho \vec{r}}{3\epsilon_0}$$



$$\vec{E} = \frac{\rho r}{3\epsilon_0}(-\hat{r})$$

$$q_{enc} = \int \rho \, d\tau = \int \rho \, 4\pi r^2 \, dr = \rho \, 4\pi \frac{r^3}{3}$$

$$E \cdot 4\pi r^2 = \frac{q}{\epsilon} = \frac{\rho \, 4\pi r^3}{3\epsilon}$$

$$\boxed{E = \frac{\rho r}{3\epsilon}\hat{r}}$$

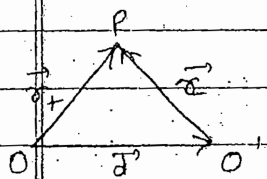
Electric field due to the $+\rho$ & $-\rho$ will be

$$\vec{E}_+ = \frac{\rho \vec{r}_+}{3\epsilon_0} \quad \text{——— (1)}$$

$$\vec{E}_- = \frac{\rho}{3\epsilon_0}(-\vec{r}_-) \quad \text{——— (2)}$$

Total Electric field $\vec{E} = \vec{E}_+ + \vec{E}_-$

$$\vec{E} = \frac{\rho}{3\epsilon_0}(\vec{r}_+ - \vec{r}_-)$$



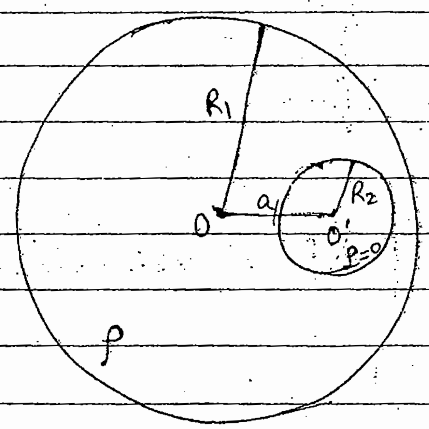
$$\boxed{\vec{E} = \frac{\rho \vec{d}}{3\epsilon_0}}$$

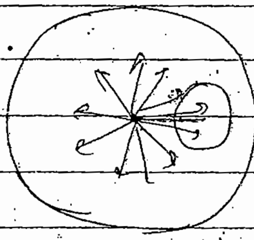
$$\left\{ \begin{aligned} \vec{r}_+ &= \vec{d} + \vec{r}_- \\ \vec{r}_+ - \vec{r}_- &= \vec{d} \end{aligned} \right\}$$

$d \rightarrow$ constant

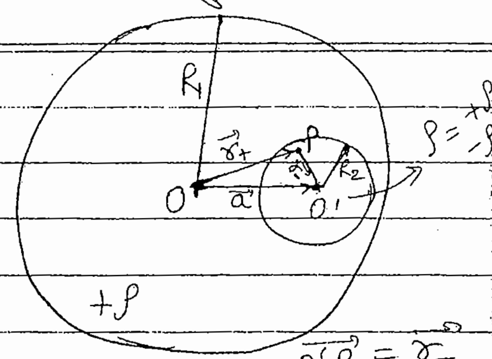
so $\vec{E}$ is constant

Q.6 :- A sphere of radius R_1 has a charge density ρ uniform within its volume except for a small spherical hollow region of radius R_2 located at a distance a from the centre. Find the electric field inside the hollow sphere.





In case of conducting sphere $q = 0$ inside the sphere so $\vec{E} = 0$ i.e. no electric field lines. But here uniformly charged sphere having charge uniformly distributed so field lines radially outward & will pass through the cavity also. so In cavity $\vec{E} \rightarrow$ Non. zero.



In Cavity $\rho = 0$

let $\rho = \rho_+ + \rho_-$

$\rho = +\rho - \rho$

Electric field inside the cavity = ?

$\vec{O'P} = \vec{r}_-$

$\vec{OP} = \vec{r}_+$

$\vec{OO'} = \vec{a}$

$$\vec{E}_+ = \frac{\rho \vec{r}_+}{3\epsilon_0} \quad \text{---(1)}$$

$$\vec{E}_- = \frac{\rho(-\vec{r}_-)}{3\epsilon_0} \quad \text{---(2)}$$

Total $\vec{E} = \vec{E}_+ + \vec{E}_-$

$$\vec{E} = \frac{\rho}{3\epsilon_0}(\vec{r}_+ - \vec{r}_-)$$

$$\boxed{\vec{E} = \frac{\rho \vec{a}}{3\epsilon_0}}$$ depends on the distance b/w centre of the sphere & centre of cavity

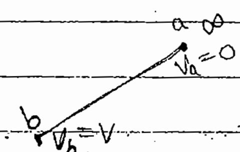
$\vec{a} \longrightarrow$ Constant. So Electric field inside the cavity is constant.

<u>Note</u> :- If centre of sphere & centre of cavity Coincide then $\vec{E} = 0$

• <u>Energy</u> :- <u>Work done to move a point charge</u>

Work done $= qV$

$$\boxed{W = qV} \text{ (finite)}$$



If we want to take a point charge from ∞ to a point ~~will~~ then work done to move a charge will store in terms of '<u>potential Energy</u>'.

If we want to move a charge from point a to b then

$$\boxed{W = q[V_b - V_a]}$$