Free Study Material from All Lab Experiments

Classical Mechanics
for NET/Gate Physical Sciences
> Non-inertial frame of Reference <
> and Pseudo Forces, Part-2 <

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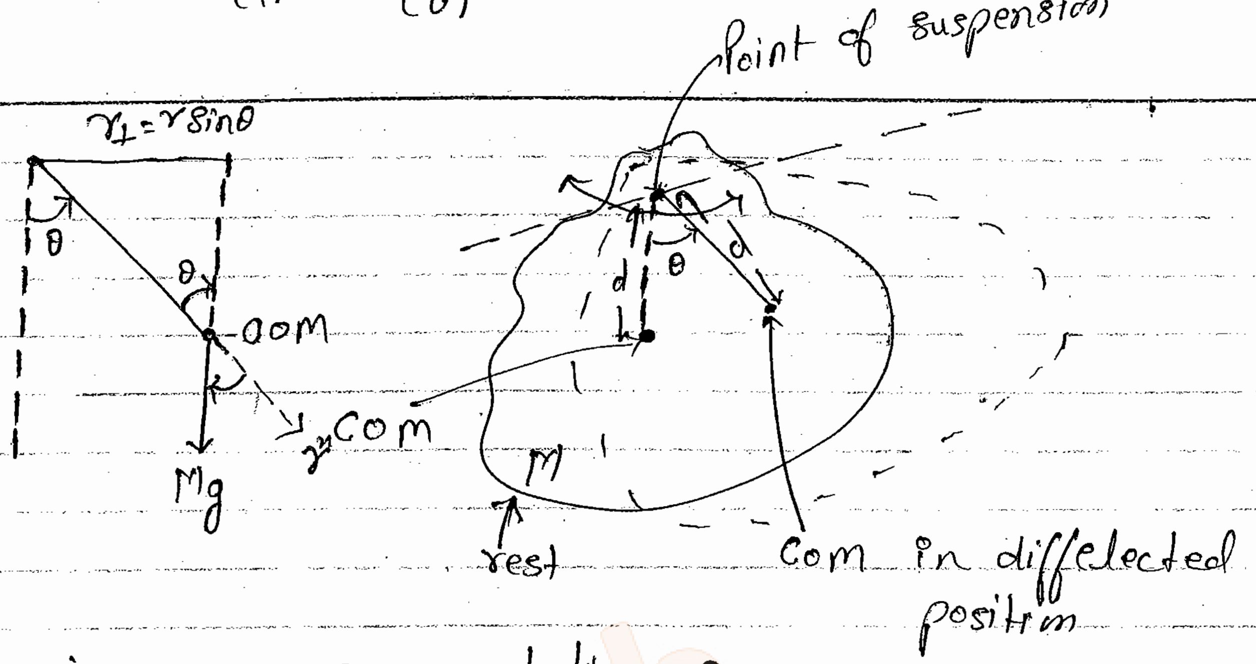
Direc^n of angular acc^n (α) = direc^n of Torque (τ).



It is a pure rotation case - So Equation of motion.

$$\boxed{\tau = I\alpha}$$

$$\boxed{Mg\,d\sin\theta = I\alpha}$$

I = M.I about horizontal axis through point of suspension.

If θ is small:-

$$\sin\theta \approx \theta$$

$$Mg\,d\,\theta = I\alpha$$

$$\boxed{\alpha = \frac{Mgd}{I}\theta}$$

Torque is opposite of θ acc to. direc^n

and $$\boxed{\vec{\alpha} = \frac{mgd}{I}(-\vec{\theta})}$$

This is the case of simple hormonic motion. Standard eq^n of S.H.M.

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$$\alpha = \omega^2 (-\vec{\theta})$$

↳ Angular frequency

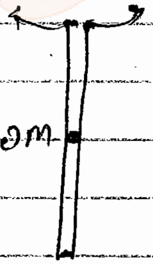
$$\omega^2 = \frac{mgd}{I}$$

$$\frac{2\pi}{T} = \sqrt{\frac{mgd}{I}}$$

$$\boxed{T = 2\pi\sqrt{\frac{I}{mgd}}}$$

ues A thin rod of mass m and length L is suspended from 1 m. what is its time period.

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$



$$I = \frac{ML^2}{3}$$

$$d = \frac{L}{2}$$

$$T = 2\pi\sqrt{\frac{ML^2/3}{MgL/2}}$$

$$\boxed{T = 2\pi\sqrt{\frac{2L}{3g}}}$$

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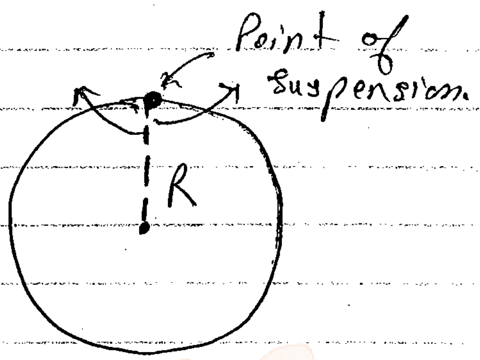
Q. A disc of radius R is suspended from a point of its periferi. If T_1 and T_2 be time period of oscillation parallel to and perpendicular to the plane of disc. What is the value of T_1 and T_2?

Sol^n



∵ $d = R$

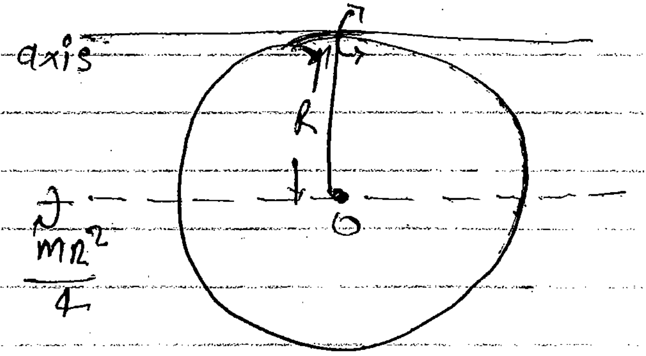
By parallel axis theorem –

$$I = \frac{MR^2}{2} + MR^2 = \frac{3}{2} MR^2$$

$$T_1 = 2\pi\sqrt{\frac{I}{Mgd}}$$

$$= 2\pi\sqrt{\frac{\frac{3}{2}MR^2}{M.g.R}}$$

$$\boxed{T_1 = 2\pi\sqrt{\frac{3R}{2g}}}$$



$$I = \frac{MR^2}{4} + MR^2$$

$$I = \frac{5}{4} MR^2$$

$$T_2 = 2\pi\sqrt{\frac{I}{Mgd}}$$

$$T_2 = 2\pi\sqrt{\frac{5MR^2/4}{Mg.R}}$$

$$\boxed{T_2 = 2\pi\sqrt{\frac{5R}{4g}}}$$

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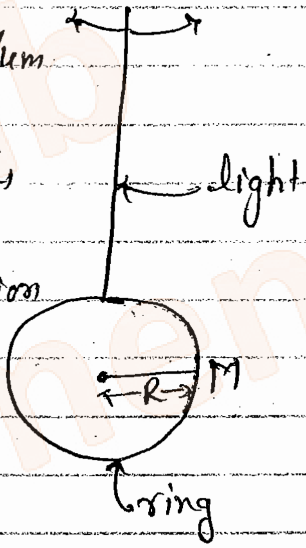
$$\frac{T_1}{T_2} = \sqrt{\frac{3}{2}\times\frac{4}{5}}$$

$$\boxed{\frac{T_1}{T_2} = \sqrt{\frac{6}{5}}}$$

+203

Q. System oscillates parallel ~~of~~ to plane of ring. What is time period.



It is not a simple pendulum becoz in simple pendulum ~~mass of bod is~~ boob is taken as a particle which not have radius and dimension but here Radius and dimension is $I = MR^2 + M(R+L)^2$ there.

$d = R+l$

$T = 2\pi\sqrt{\frac{\cancel{M}R^2+\cancel{M}(R+L)^2}{\cancel{M}g(R+l)}}$ It is compound ~~ple~~ pendulum.

$$= 2\pi\sqrt{\frac{\cancel{M}R^2+\cancel{M}R^2+l^2+2Rl}{(R+l)}}$$

$$\boxed{T = 2\pi\sqrt{\frac{2R^2+l^2+2Rl}{(R+l)}}}$$

Q. A thin rod of length ~~l~~ L is suspended from some point on its length what should be distance of point of

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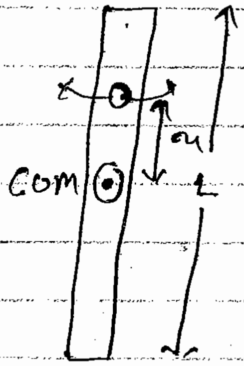
suspension from COM. So that time period should be minimum.

Sol^n

$d = x$

$$I = \frac{ML^2}{12} + Mx^2$$



$$T = 2\pi\sqrt{\frac{I}{Mgd}}$$

$$= 2\pi\sqrt{\frac{M\left(\frac{L^2}{12}+x^2\right)}{Mgx}}$$

$$T^2 = \frac{4\pi^2}{g}\left(\frac{L^2}{12x} + x\right)$$

If T is minimum then T^2 is also minimum.

$$\frac{dT^2}{dx} = 0$$

$$\frac{4\pi^2}{g}\left(\frac{-L^2}{12x^2} + 1\right) = 0$$

$$-\frac{L^2}{12x^2} + 1 = 0$$

$$x^2 = \frac{L^2}{12}$$

$$\boxed{x = \frac{L}{\sqrt{12}}}$$ Ans

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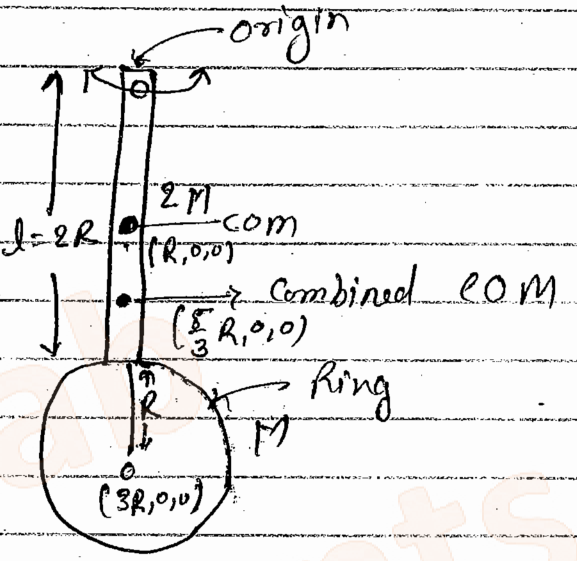
Suppose there is a rod and there is a ring, mass of Rod is 2M and mass of ring is M. What is T for oscillation.

$$T = 2\pi\sqrt{\frac{I_{total}}{M_{total}\, g d}}$$



d = distance of combined COM from point of suspension

$$I = I_c + M(dis)^2$$

$$I_{total} = MR^2 + M(R+l)^2 + \frac{(2M)l^2}{3}$$

Co-ordinate of center of mass :-

$$x_{com} = \frac{\sum m_i x_i}{\sum m_i}$$ } discreate case or for combination of object.

$$x_{com} = \frac{\int x\,dm}{\int dm}$$ } for a single continuous object.

In this case -

$$x_{com} = \frac{2M \times R + M \times 3R}{3M}$$

$$x_{com} = \frac{5}{3}R = d$$

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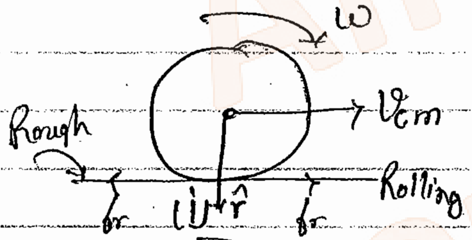
$$T = 2\pi\sqrt{\frac{\cancel{M}R^2+\cancel{M}(R+l)^2+(2\cancel{M})l^2/3}{\cancel{3}\cancel{M}\cdot g\cdot\frac{5}{\cancel{3}}R}}$$

$$= 2\pi\sqrt{\frac{R^2+R^2+l^2+2Rl+2l^2/3}{5gR}}$$

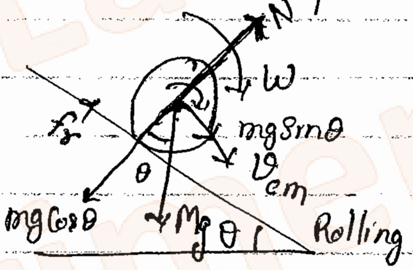
$$= 2\pi\sqrt{\frac{6R^2+3l^2+2Rl+2l^2}{15gR}}$$

$$\boxed{T = 2\pi\sqrt{\frac{6R^2+5l^2+2Rl}{15gR}}}$$ Ans

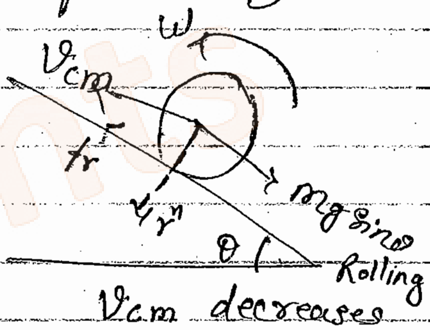
* Rolling Motion on Stationary Surface (horizontal or inclined) :-



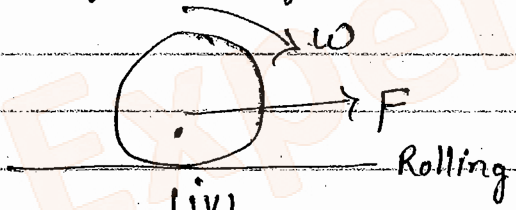
(i)



Vcm increases due to mg sinθ.
(ii)



Vcm decreases mg sinθ.
(iii)



(iv)

Condition for Rolling :-

$v_{cm} = \omega R$

$a_{cm} = \alpha R$

From fig (ii) :-

→ Here Vcm increases due to Mg sinθ

$\therefore v_{cm} = \omega R$

So due to this relation ω should also be increases but, Here torque is zero

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because Mg and N both are passing through center.
So Here ω is not increases so this Rolling is not possible.
So Rolling is not possible on smooth inclined plane.
For Rolling motion on incliened plane friction must be present.
When incliened plane is a rough surface then friction is works here.

i) from fig. (i) :-
In this case when friction works opposite to the motion then V_{cm} decreases but $r \times F$ and ω are in same direc^n then ω is increases so this can not satisfies the condition of Rolling.
In other hand when friction in the direc^n of motion then V_{cm} increases but $r \times F$ and ω is in opposite direction so ω is decreases so it is also not satisfies the condition of rolling.
So here for Rolling motion surface might be smooth then V_{cm} = constt. and ω = constt.

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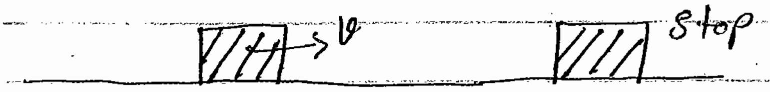
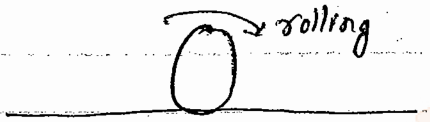
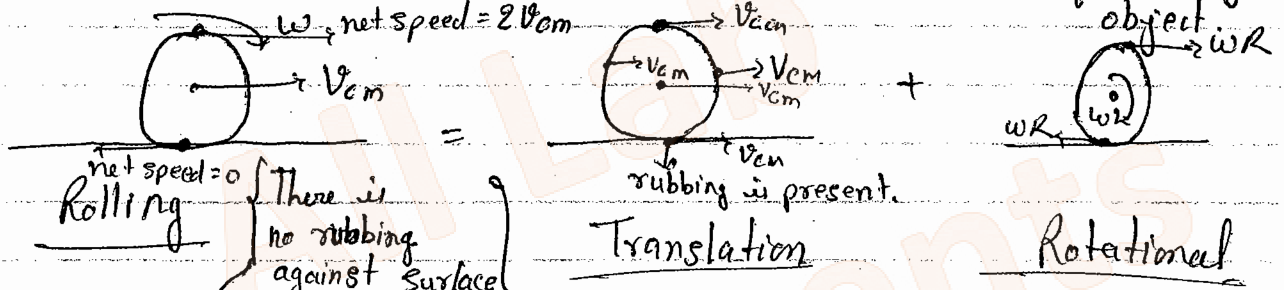
refectely Rigit that means we can't deform. Iron are not ~~pef~~ perfectily rigid.

* In rolling motion although surface might be rough there is no loss of energy against ~~further~~ friction.





never stop { True for perfect rigid object

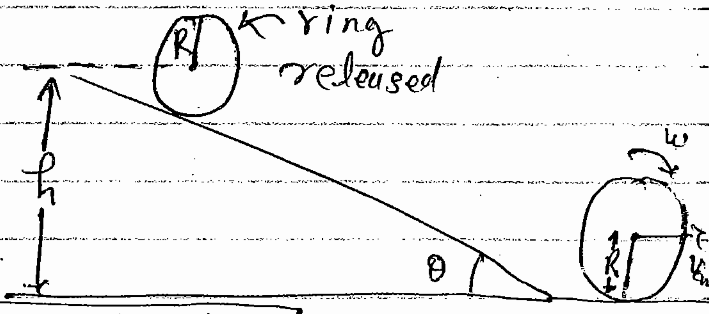


{ There is no rubbing against surface i.e. no loss of energy (So it not produce heat)

Net 2013 June.

Q. A ring is released from a inclined plane whose center is at h distance from ground. If ring rolls down the plane what is its angular speed when it reaches the bottom.

Sol^n



Here center decreases by (h-R) height.

$v_{cm} = \omega R$

Apply Conservation of energy:-

$\boxed{\text{Loss of P.E.} = \text{Gain of K.E.}}$

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$$Mg(h-R) = \frac{1}{2} I\omega^2 + \frac{1}{2} M v_{cm}^2$$

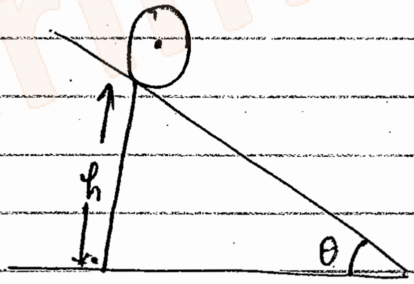
$$= \frac{1}{2} MR^2\omega^2 + \frac{1}{2} M\omega^2 R^2$$

$$\cancel{M}g(h-R) = \cancel{M}R^2\omega^2$$

$$\boxed{\omega = \sqrt{\frac{g(h-R)}{R^2}}}$$

Q. A ~~hollow~~ solid sphere and a disc rolls down from an inclined plane from same point what is ratio of velocity of sphere and velocity of disc when they reach at the bottom.

Sol^n



For Disc:-

By Conservation of energy :-

$$\boxed{\text{Loss in P.E.} = \text{Gain in K.E.}}$$

$$mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} m v_{cm}^2 \qquad \left\{ V_{cm} = \omega \cancel{r} \right.$$

$$= \frac{1}{2} \frac{m\cancel{r^2}}{2} \cdot \frac{V_{cm}^2}{\cancel{r^2}} + \frac{1}{2} m V_{cm}^2$$

$$\cancel{m}gh = \frac{3}{4} \cancel{m} V_{cm}^2$$

$$\boxed{(V_{cm})_{disc} = \sqrt{\frac{4}{3} gh}}$$

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$s = ut + \frac{1}{2}at^2$ when $a = 0$ then $s = ut$

and $\boxed{t = \frac{s}{u}}$ only When u = constant

$\text{time} = \frac{\text{distance}}{\text{Velocity}}$

For sphere:-

solid sphere

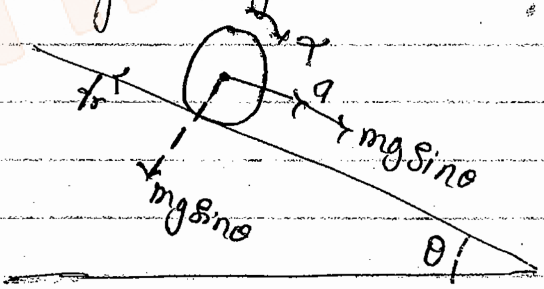
$$Mgh = \frac{1}{2}\left(\frac{2}{5}MR^2\right) \cdot \frac{V_{cm}^2}{R^2} + \frac{1}{2}MV_{cm}^2$$

$$\cancel{M}gh = \frac{7}{10}\cancel{M}V_{cm}^2$$

$$\boxed{(V_{cm})_{sphere} = \sqrt{\frac{10}{7}gh}}$$

$$\boxed{\frac{(V_{cm})_{sphere}}{(V_{cm})_{disc}} = \sqrt{\frac{15}{14}}}$$

* Acceleration of Rolling object on inclined plane:-



Equation of motion for Translation:-

$$mg\sin\theta - f_r = ma \quad \text{—} ①$$

For Rotation:-

$$\tau = I\alpha$$

$$f_r R = I\alpha$$

For Rolling:-

$$a = \alpha R$$

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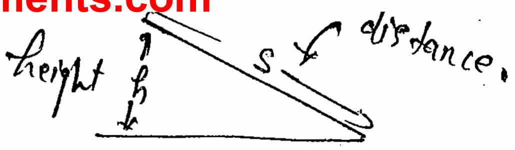


$$\boxed{\alpha = \frac{a}{R}}$$

$$f r R = \frac{Ia}{R}$$

$$\boxed{f r = \frac{Ia}{R}} \leftarrow \text{(ii)}$$

(i) + (ii) :-

$$mg \sin\theta = a\left[m + \frac{I}{R^2}\right]$$

$$g \sin\theta = a\left[1 + \frac{I}{mR^2}\right]$$

$$\boxed{a = \frac{g \sin\theta}{1 + \frac{I}{mR^2}}}$$

for friction put 'a' in eqⁿ (ii).

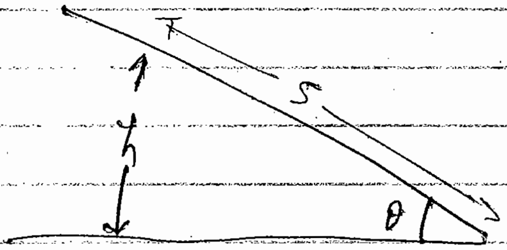
1-7
Q.5 A solid cylinder starts Rolling down an inclined plane of inclination θ. from height h. Time taken to reach the bottom is ?.

(a) $\sqrt{\frac{2h}{g}}$ (b) $\sqrt{\frac{3h}{g}}$ (c) $\frac{1}{\sin\theta}\sqrt{\frac{3h}{2g}}$

(d) $\frac{1}{\sin\theta}\sqrt{\frac{3h}{g}}$

Solⁿ

$$s = \frac{h}{\sin\theta}$$



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$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2}at^2$$

$$\boxed{t = \sqrt{\frac{2s}{a}}} \longrightarrow ⓐ$$

$$\left\{\because a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}\right\}$$

$$a = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2g\sin\theta}{3}$$

$\left\{\because\right.$ M.I. of ~~Sphere~~ cylinder $= \frac{1}{2}mR^2$

$$t = \sqrt{\cancel{2}\frac{h}{\sin\theta}\cdot\frac{3}{\cancel{2}g\sin\theta}}$$

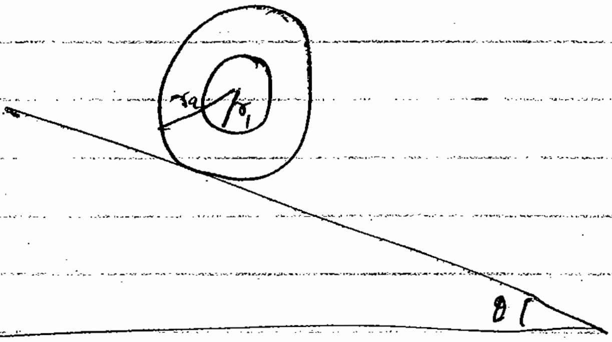
$$\boxed{t = \frac{1}{\sin\theta}\sqrt{\frac{3h}{g}}}$$

A-7

Q.15 A circular loop of inner and outer radii r_1 and r_2 rolls down an inclined plane of inclination θ. Angular acceleration of loop on the inclined plane is -?

[✓ (a)] $\frac{2r_2}{3r_2^2+r_1^2}g\sin\theta$ (b) $\frac{2r_1}{3r_2^2+r_2^2}g\sin\theta$ (c) $\frac{r_2}{r_2^2+r_1^2}g\sin\theta$

(d) $\frac{r_1}{r_2^2+r_1^2}g\sin\theta$.



Sol^n $a = \frac{g\sin\theta}{1+\frac{I}{mr_2^2}}$

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$$\boxed{a = \alpha r_2}$$

$$\boxed{a = \frac{g\sin\theta}{1+\frac{\frac{1}{2}m(r_1^2+r_2^2)}{m r_2^2}}}$$

$$\alpha = \frac{2\,\overset{\alpha}{r_2^2}\,g\sin\theta}{2r_2^2+(r_1^2+r_2^2)\times\cancel{r_2}}$$

$$\boxed{\alpha = \frac{2 r_2 g \sin\theta}{r_1^2+3r_2^2}}$$ Ans

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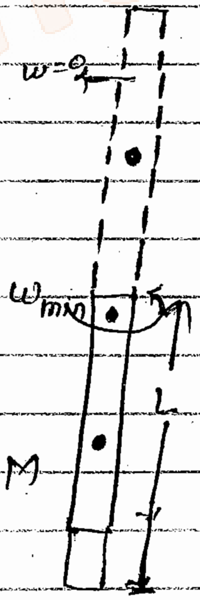

Since here one point is fixed so it is a case of pure rotation.

Loss in K.E. = Gain in P.E.

$$\frac{1}{2} I \omega_{min}^2 - 0 = mgL$$ { here L = height accended by center

$$\Rightarrow \frac{1}{2}\frac{\cancel{M}L^{\cancel{2}}}{3}\omega_{min}^2 = \cancel{m}g\cancel{L}$$

$$\boxed{\omega_{min} = \sqrt{\frac{6g}{L}}}$$

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$$w^2 = \frac{3g}{L}$$

$$\boxed{w = \sqrt{\frac{3g}{L}}}$$ Angular speed.

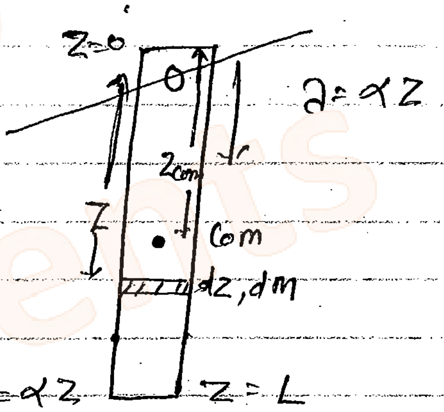
A-7
Q.10 A thin rod of length L is suspended from one end as shown in figure. The linear mass density of rod varies with distance from point of suspension as $\partial = \alpha s$. Then period of oscillation of the rod is ?

Sol^n

$$T = 2\pi\sqrt{\frac{I}{Mgd}}$$

I = MI about point of susp.

d = dist of (P.S.) from C.O.M.



$$I = \int dm\, r_\perp^2$$

$$\partial = \frac{dm}{dz} = \alpha z$$

$$dm = \alpha z dz$$

$$= \int_0^L \alpha z dz\, z^2$$

$$\boxed{M.I. = \frac{\alpha L^4}{4}}$$

Position of center of mass -

$$z_{com} = \frac{\int z dm}{\int dm} = \frac{\int z\, \alpha z dz}{\int \alpha z dz} = \frac{\int z^2 \alpha dz}{M}$$

$$= \frac{\alpha L^3}{3M} = d$$

So $$T = 2\pi\sqrt{\frac{\frac{\alpha L^4}{4}}{Mg\left(\frac{L^3\alpha}{3M}\right)}}$$

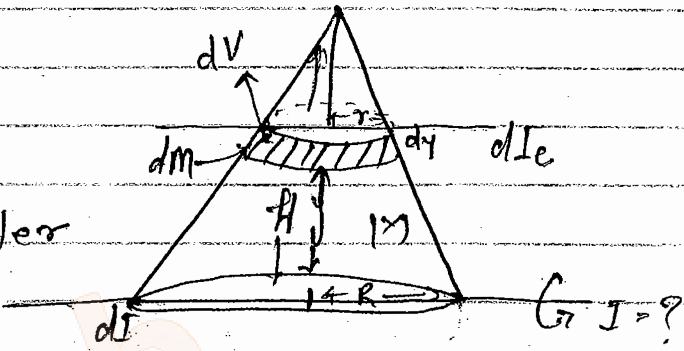
$$\boxed{T = 2\pi\sqrt{\frac{3L}{4g}}}$$ Ans

A-7

Q.19 A right circular cone has mass M, Radius R and height h. Moment of inertia of cone about a diameter of its base is?

Sol^n

$$I = \int dI$$



M.I of disc about the given axis (base diameter

$$dI = dI_c + dM y^2$$

$$= \frac{dM r^2}{4} + dM y^2$$

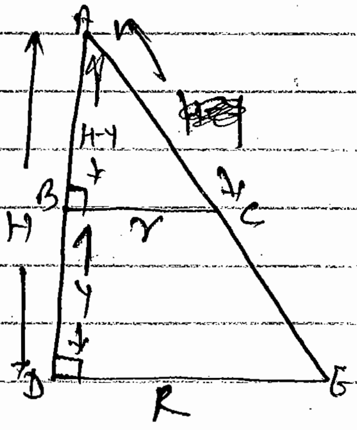
$$\boxed{dI = dM\left(\frac{r^2}{4} + y^2\right)} \quad — ①$$

$$\because \quad \rho = \frac{M}{\frac{1}{3}\pi R^2 H} = \frac{dM}{dV}$$

$$dm = \frac{M}{\frac{1}{3}\pi R^2 H} dV$$

$$\boxed{dM = \frac{3M}{\pi R^2 H} \pi r^2 dy} \quad — (ii)$$

∵ Here ΔABC and ΔADG is similar so -



$$\frac{r}{R} = \frac{H-y}{H}$$

$$\boxed{r = R\left(1 - \frac{y}{H}\right)} \quad — (iii)$$

$$\boxed{y = H\left(1 - \frac{r}{R}\right)}$$

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$$dr = \frac{-R}{H}\,dy$$

So $\boxed{dy = \frac{-H}{R}\,dr}$

So from ①

$$dI = \frac{3M}{\cancel{\pi}R^2\cancel{H}}\,\cancel{\pi}r^2\cdot\left(\frac{-\cancel{H}}{R}dr\right)\left(\frac{r^2}{4} + H^2\left(1 - \frac{r}{R}\right)^2\right)$$

$$\cancel{dI = \frac{3M}{R^3}\left[\frac{r^4}{4}dr + H r^2 dr - \frac{H}{R}r^3 dr\right]}$$

$$= \frac{3M}{R^3}\,r^2 dr\left[\frac{r^2}{4} + H^2\left(1 + \frac{r^2}{R^2} - 2\frac{r}{R}\right)\right]$$

$$\int dI = \frac{3M}{R^3}\left[\int_0^R \frac{r^4}{4}dr + H^2\left(\int_0^R r^2 dr + \int_0^R \frac{r^4}{R^2}dr - \frac{2}{R}\int^R r^3 dr\right)\right]$$

$$= \frac{3M}{R^3}\left[\frac{1}{4}\frac{R^5}{5} + H^2\left(\frac{R^3}{3} + \frac{1}{R^2}\left(\frac{R^5}{5}\right) - \frac{\cancel{2}}{\cancel{R}}\left(\frac{R^4}{\cancel{4}_2}\right)\right)\right]$$

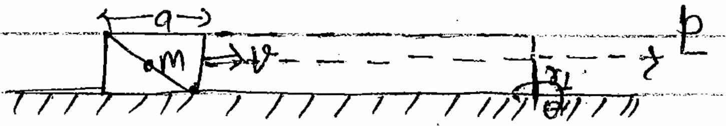
$$= \frac{3M}{R^3}\left[\frac{R^5}{20} + H^2R^3\left\{\frac{1}{3} + \frac{1}{5} - \frac{1}{2}\right\}\right]$$

$$= \frac{3MR^2}{20} + 3MH^2\left(\frac{10+6-15}{30}\right)$$

$$= \frac{3MR^2}{20} + \frac{\cancel{3}MH^2}{\cancel{30}\;10}$$

$$\boxed{I = \frac{3MR^2}{20} + \frac{MH^2}{10}}$$ Ans

22 A cubical block of side 'a' is moving with velocity 'v' on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is?

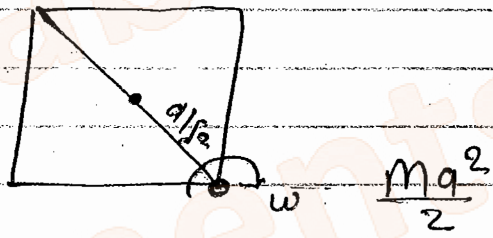


Sol^n So applying Conservation of Angular momentum -

$L_i = L_f$

$p r_\perp = I\omega$

$M v \cdot \frac{a}{2} = I\omega$



So $Mv.\frac{a}{2} = \left[\frac{Ma^2}{6} + \frac{Ma^2}{2}\right]\omega$

$\Rightarrow \cancel{M} v \cdot \frac{\cancel{a}}{2} = \cancel{M} a^{\cancel{2}} \omega \left[\frac{1}{6} + \frac{1}{2}\right]$

$\Rightarrow \frac{v}{2} = \omega a \left[\frac{1+3}{6}\right]$

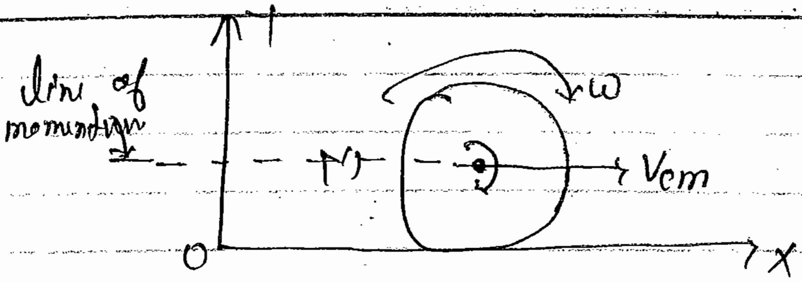
$\Rightarrow \frac{v}{\cancel{2}} = \frac{4\omega a}{\cancel{6}\,3}$

$\Rightarrow \boxed{\omega = \frac{3v}{4a}}$ So ans (a) is correct.

2.23 A disc of mass M and Radius R is rolling with angular speed ω on a horizontal plane as shown in figure. The magnitude of angular momentum of the disc about the origin O is?

Sol^n

All Lab Experiments



$$L = I\omega + M V_{cm} R$$

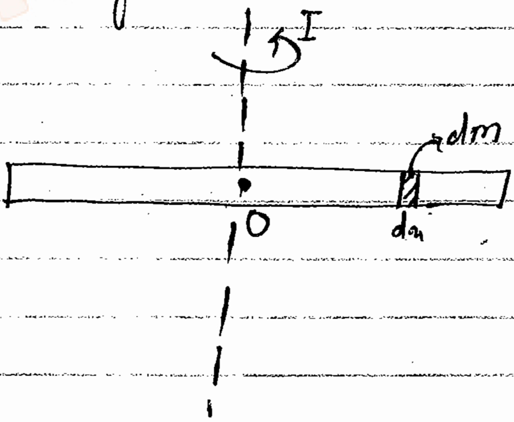
$$= \frac{MR^2}{2}\omega + M\omega R.R$$

$$= \frac{MR^2\omega}{2} + M\omega R^2$$

$$\boxed{L = \frac{3}{2} M\omega R^2}$$

A-6 Q. modified Calculate moment of Inertia about a perpendicular axis through its center of mass.

$\lambda = \lambda_0|x|$ x = dist. from center



$$\lambda = \frac{dm}{dx}$$

$$\lambda_0|x| = \frac{dm}{dx}$$

$$dm = \lambda_0|x|dx$$

$$I = \int dm\, r_{\perp}^2$$

$$= \int_{-l}^{+l} \lambda_0|x|dx.x^2$$

$$= \int_{-l}^{0} \lambda_0(-x)\, x^2 dx + \int_{0}^{l} \lambda_0(x)\, x^2 dx$$

All Lab Experiments

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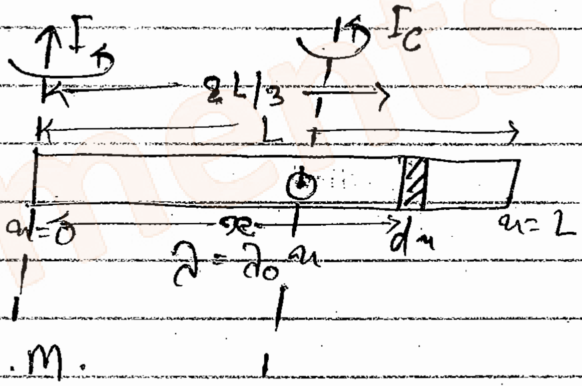
$$I = -\lambda_0\left[\frac{x^4}{4}\right]_{-\ell}^{0} + \lambda_0\left[\frac{x^4}{4}\right]_{0}^{\ell}$$

$$= +\lambda_0\frac{\ell^4}{4} + \lambda_0\frac{\ell^4}{4} = \frac{2\lambda_0\ell^4}{4}$$

$$\boxed{I = \frac{\lambda_0\ell^2}{2}}$$

$$x_{com} = \frac{\int x\,dm}{\int dm} = \frac{\int_{-L}^{+L} x\lambda_0|x|dx}{\int_{-L}^{+L}\lambda_0|x|dx} = 0$$

$$\boxed{x_{com} = 0}$$



Q2. When x is measured from one end. What is the M.I of Rod about a $\perp^{r}$ axis through its c.o.m.

Sol^n

$$\lambda = \frac{dm}{dx} = \lambda_0x$$

$$dm = \lambda_0x\,dx$$

$$I = \int dm\, r_\perp^2$$

$$\because r_\perp = x$$

$$\therefore I = \int_0^L \lambda_0x\,dx\,.\,x^2$$

$$\boxed{I = \frac{\lambda_0L^4}{4}}$$

$$X_{com} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^l x\cdot a_0 x\,dx}{\int_0^l a_0 x\,dx}$$

$$= \frac{A_0 \frac{L^3}{3}}{\boxed{a_0 \frac{L^2}{2}}} \rightarrow M$$

$$\boxed{X_{com} = \frac{2L}{3}}$$

So $\quad I = I_c + M\left(\frac{2L}{3}\right)^2$

$$\frac{A_0 L^4}{4} = I_c + \frac{a_0 L^{\cancel{2}}}{\cancel{2}}\cdot\frac{\overset{2}{\cancel{4}}L^2}{9}$$

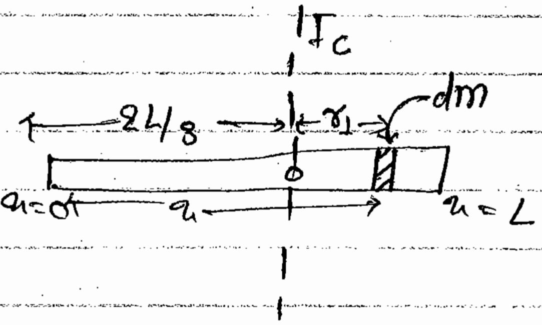
$$\frac{a_0 L^4}{4} = I_c + \frac{2a_0 L^4}{9}$$

$$I_c = a_0 L^4\left(\frac{1}{4} - \frac{2}{9}\right) = a_0 L^4\left(\frac{9-8}{36}\right)$$

$$\boxed{I_c = \frac{a_0 L^4}{36}}$$ Ans

Method-II :- Giving Ic directely :-

First calculate ~~c.o.~~ position of com.



$$I = \int dm\, r_1^2$$

$$= \int_0^L a_0 x\,dx\,\left(x - \frac{2L}{3}\right)^2$$

$$r_1 = x - \frac{2L}{3}$$

https://alllabexperiments.com

$$I = \lambda_0 \int_0^L x\left(x^2 + \frac{4}{9}L^2 - \frac{4}{3}Lx\right)dx$$

$$= \lambda_0\left[\int_0^L x^3\,dx + \frac{4L^2}{9}\int_0^L x\,dx - \frac{4L}{3}\int_0^L x^2\,dx\right]$$

$$= \lambda_0\left[\frac{L^4}{4} + \frac{\cancel{4}^{2}L^2}{9}\cdot\frac{L^2}{\cancel{2}} - \frac{4}{3}L\,\frac{L^3}{3}\right]$$

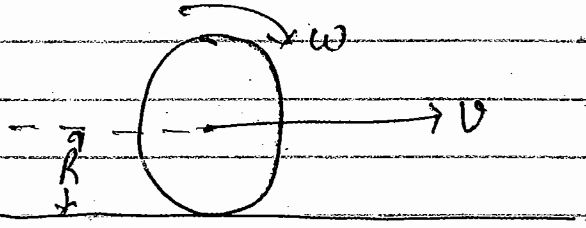
$$= \lambda_0\left[\frac{L^4}{4} + \frac{2L^4}{9} - \frac{4L^4}{9}\right]$$

$$= \lambda_0 L^4\left[\frac{1}{4} - \frac{2}{9}\right] = \lambda_0 L^4\left[\frac{9-8}{36}\right]$$

$$I = \frac{1}{36}\lambda_0 L^4$$

So $\boxed{I = \frac{\lambda_0 L^4}{36}}$ Ans

A circular disc of mass M and radius R is moving on a horizontal surface with speed v if angular momentum of the disc about the point on its path is 2MVR, what is K.E. of the disc?



$$L = I\omega + mvR$$

$$2mvR = I\omega + mvR$$

$$mvR = I\omega$$

$$\cancel{m}v\cancel{R} = \frac{\cancel{m}R^{\cancel{2}}}{2}\omega \Rightarrow \boxed{\omega = \frac{2v}{R}}$$

$$K.E. = \frac{1}{2} I \omega^2 + \frac{1}{2} mv^2$$

$$= \frac{1}{\cancel{2}} \frac{m\cancel{R^2}}{\cancel{2}} \cdot \frac{\cancel{4}v^2}{\cancel{R^2}} + \frac{1}{2} m v^2$$

$$\boxed{K.E. = \frac{3}{2} mv^2}$$

* Pure Rotational Motion of Rigid Body :-

At least one point of rigid body is fixed.

It has two types -

(I) Rotation about a fixed axis (Here point is also fixed)

(II) Rotation about a fixed point (axis is not fixed.)

Case I :-

$L_{axis} = I\omega$ → This may or may not be total angular momentum.

$$\boxed{K.E. = \frac{1}{2} I\omega^2}$$

Case II :-

$$\vec{L} = \overleftrightarrow{I} \cdot \vec{\omega}$$

$$\boxed{K.E = \frac{1}{2}\, \vec{\omega} \cdot \overleftrightarrow{I} \cdot \vec{\omega}}$$

All Lab Experiments

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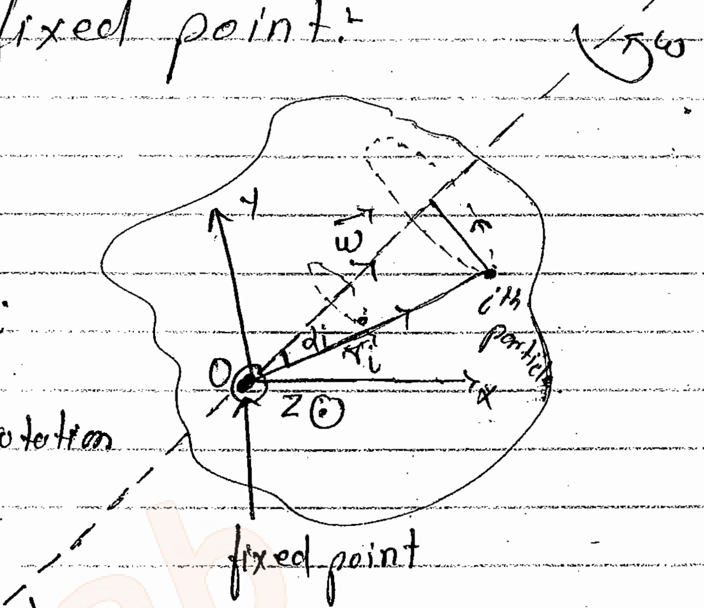
Rotation about a fixed point:-



Radius of circal in which ith particle moves is :- $r_i \sin\alpha_i$

tangential velocity of ith particle due to rotation

$$V_i = \omega \boxed{r} \rightarrow \text{Radius}$$

So speed of ith particle

Axis about which the rigid body is ~~esei~~ rotating at some instant of time.

$$v_i = \omega r_i \sin\alpha_i$$

$$\boxed{\vec{v_i} = \vec{\omega} \times \vec{r_i}}$$

{ We can't write $\vec{r_i} \times \vec{\omega}$ becoz it can't give correct sence of velocity.

Angular momentum of ith particle about point 'O'

$$\vec{L}_{i^{th}} = \vec{r_i} \times \vec{p_i}$$

$$= \vec{r_i} \times m_i \vec{v_i}$$

$$= m_i (\vec{r_i} \times \vec{v_i}) = m_i [\vec{r_i} \times (\vec{\omega} \times \vec{r_i})]$$

Angular momentum of rigid body about point O.

$$\vec{L} = \sum_{i=1}^{N} \vec{L_i}$$

$$\boxed{\vec{L} = \sum_{i=1}^{N} m_i [r_i \times (\vec{\omega} \times \vec{r_i})]}$$

All Lab Experiments

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$$\vec{r_i} = x_i\hat{x} + y_i\hat{y} + z_i\hat{z}$$

$$\vec{\omega} = \omega_x\hat{x} + \omega_y\hat{y} + \omega_z\hat{z}$$

$$\vec{L} = L_x\hat{x} + L_y\hat{y} + L_z\hat{z}$$

Put $\vec{r_i}$, $\vec{\omega_i}$ and $\vec{L}$ in above expression to get [equate Coefficients of $\hat{x}$, $\hat{y}$, and $\hat{z}$)

$$L_x = \underbrace{\sum m_i(y_i^2+z_i^2)}_{I_{xx}}\omega_x - \underbrace{\sum m_i x_i y_i}_{I_{xy}}\omega_y - \underbrace{\sum m_i x_i z_i}_{I_{xz}}\omega_z$$

$$L_y = \underbrace{-\sum m_i y_i x_i}_{I_{yx}}\omega_x + \underbrace{\sum m_i(x_i^2+z_i^2)}_{I_{yy}}\omega_y - \underbrace{\sum m_i y_i z_i}_{I_{yz}}\omega_z$$

$$L_z = \underbrace{-\sum m_i z_i x_i}_{I_{zx}}\omega_x - \underbrace{\sum m_i z_i y_i}_{I_{zy}}\omega_y - \underbrace{\sum m_i(x_i^2+y_i^2)}_{I_{zz}}\omega_z$$

So. $L_x = I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z$

$L_y = I_{yx}\omega_x + I_{yy}\omega_y + I_{yz}\omega_z$

$L_z = I_{zx}\omega_x + I_{zy}\omega_y + I_{zz}\omega_z$

$$\begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix} = \underbrace{\begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}}_{\text{Co-ordinate dependent}} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

values of elements will change if axis chosen are changed.

Here the axes is choosen in that manner that off diagonal elements will be zero.

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Here matrix (3×3) is II^{nd} rank Tensor.
It is symmetric and real.
So it is a Hermitian matrix.
its eigen values are real.
This matrix is known as Inertia Tensor.

So Symbolic representation:-

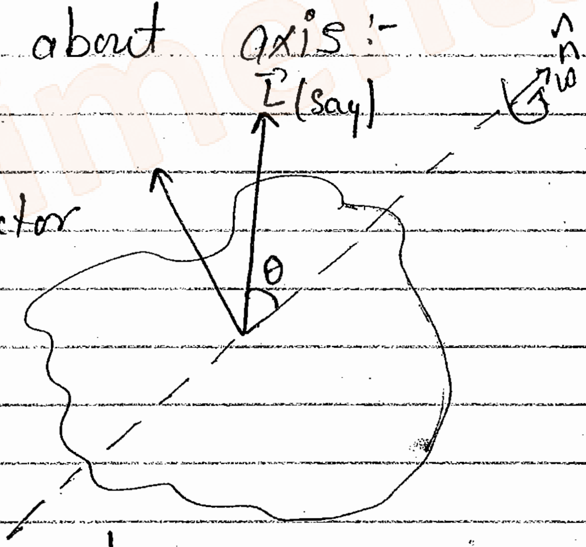
$$\boxed{\vec{L} = \overleftrightarrow{\underline{\underline{\mathbb{I}}}}\cdot\vec{\omega}}$$

Compact form of matrix relation.

Symbol for Inertia Tensor.

It is the general formula for Angular momentum.

Angular Momentum about axis:-



Let $\hat{n}$ is unit vector along the axis

$\vec{\omega} = \omega\hat{n}$

$L_{axis} = \vec{L}\cdot\hat{n}$

$= \hat{n}\cdot\vec{L}$

$= \hat{n}\cdot\overleftrightarrow{\mathbb{I}}\cdot\vec{\omega}$

$= (\hat{n}\cdot\overleftrightarrow{\mathbb{I}}\cdot\hat{n})\,\vec{\omega}$

$L_{axis} = L\cos\theta$

we know that -

$L_{axis} = I\omega$

Row Matrix

3x3 Matrix

Column matrix,

So $\boxed{\bar{I} = \hat{n}\cdot\overleftrightarrow{\mathbb{I}}\cdot\hat{n}}$

(Memorize)

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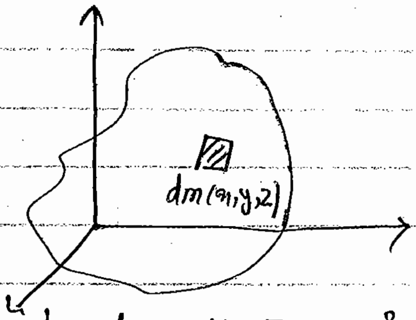
* Inertia Tensor :-

$$\mathbb{I} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$$



I_{xx} , I_{yy} , I_{zz} ⟶ M.I. about x, y, z axis

I_{ij} , $i \neq j$ ⟶ Product of inertia.

$$I_{xx} = \sum_{i=1}^{N} m_i (y_i^2 + z_i^2) \quad , \quad I_{xx} = \int dm (y^2 + z^2)$$

(x, y, z Co-ordinates of dm)

discreate — Continuous

$$I_{xy} = -\sum_{i=1}^{N} m_i x_i y_i \quad , \quad I_{xy} = -\int dm\, xy$$

* Principal Axes :-

The set of three axes for which product of inertia are zero, are called principal Axes [Mathematical Defination].

If x, y, z are principal axes -

$I_{ij} = 0 \quad i \neq j$, $I_{xy} = 0$, $I_{yx} = 0$ etc.

$$\mathbb{I} = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}$$

⟶ Principal Moment of Inertia.

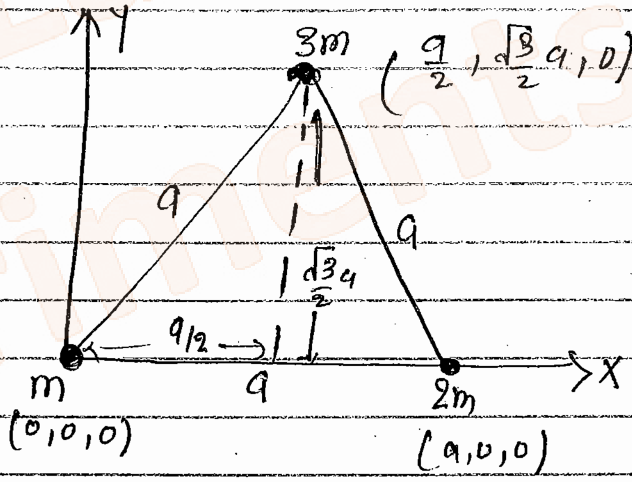
$$\boxed{\text{Principal M.I} = \text{Egen Value of Inertia Tensor}}$$

=> Direction of Principal Axes :-

Direction of principal Axes are given by eigen vectors of inertia tensor.

6/18 In the figure shown value of I_{xy} is ?

$$I_{xy} = -\sum_{i=1}^{3} m_i x_i y_i$$



$$= -\left(m\times 0 + 2m\times 0 + 3m \times \frac{a}{2} \times \frac{\sqrt{3}a}{2}\right)$$

$$\boxed{I_{xy} = -\frac{3\sqrt{3}ma^2}{4}}$$

$\because I_{xy} \neq 0$ (Product of Inertia $\neq 0$)

So X, Y, Z will ~~be~~ not be principal axes

* Draw the principal Axes about (0,0,0) :-

If We want to find orientation of principal axis then we need to write inertia tensor and find its eigen vectors.

All Lab Experiments

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$\therefore I_{yx} = I_{xy} = \frac{-3\sqrt{3}a}{4}$

$I_{yz} = I_{zy} = -\sum m_i y_i z_i = 0$

$I_{zx} = I_{xz} = 0$

$I_{xx} = \sum m_i (y_i^2 + z_i^2) = m \times 0 + 2m \times 0 + 3m\left(\frac{3a^2}{4} + 0^2\right)$

$I_{xx} = \frac{9a^2}{4}$

$I_{yy} = \sum m_i (x_i^2 + z_i^2) = m \times 0 + 2m(a^2 + 0) + 3m\left(\frac{a^2}{4} + 0\right)$

$= 2ma^2 + \frac{3ma^2}{4} = \frac{11ma^2}{4}$

$I_{zz} = \sum m_i (x_i^2 + y_i^2) = m \times 0 + 2m(a^2 + 0) + 3m\left(\frac{a^2}{4} + \frac{3a^2}{4}\right)$

$I_{zz} = 2ma^2 + 3ma^2 = 5ma^2$

$$\underline{\underline{I}} = ma^2 \begin{pmatrix} \frac{9}{4} & -\frac{3\sqrt{3}}{4} & 0 \\ -\frac{3\sqrt{3}}{4} & \frac{11}{4} & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

To find the direction of principal axis we will calculates eigen vector.

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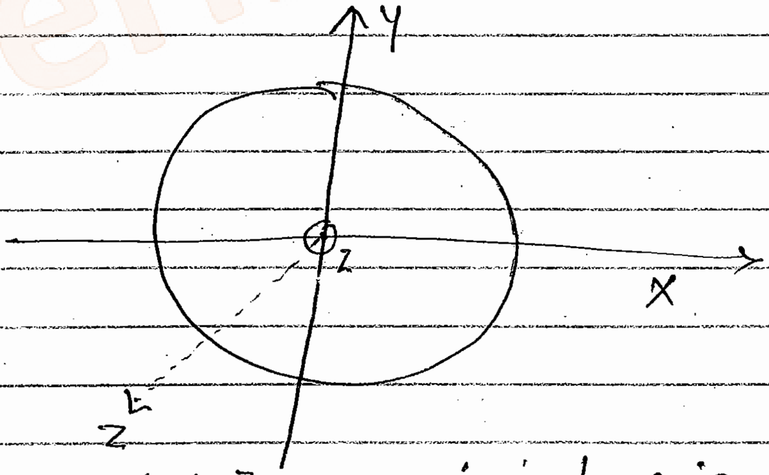
* Principal Axes :- {Defined through a point of rigid body}:-

Physical Defination :-
A symmetric axis is always a principal axis if the object has axial symmetry.

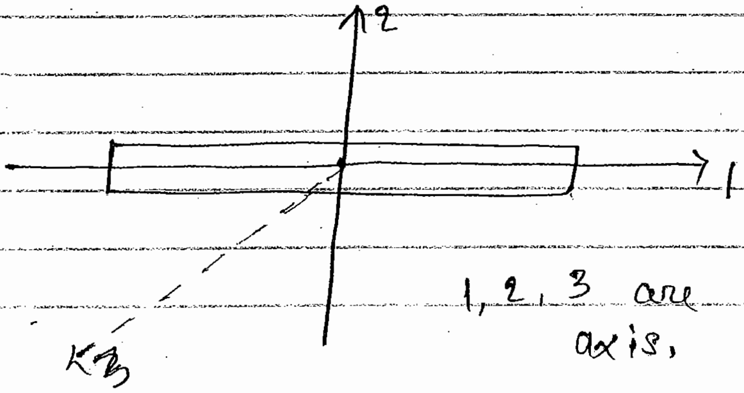
Ex -

① Sphere.



X, Y, Z are principal axis.

(2) Rod



1, 2, 3 are principal axis,

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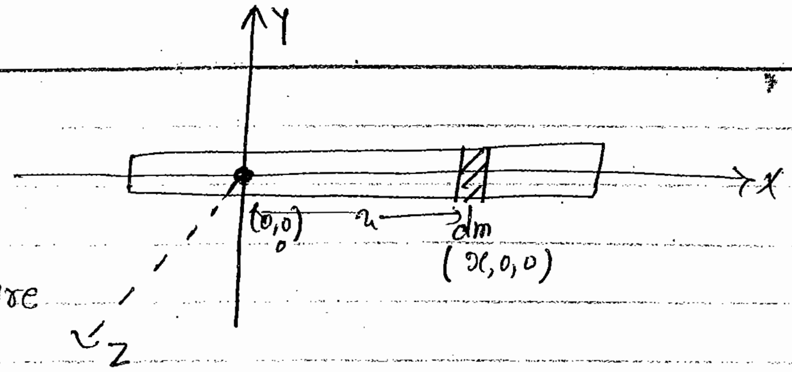


∵ $I_{ij} = 0$ $i \neq j$

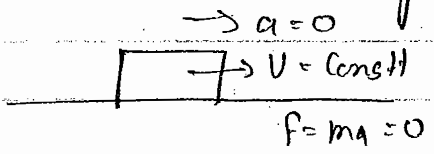
Product of inertia are zero ∴ x, y, z are principal axes.

* Principal axes may or may not be symmetric axes.
Every symmetric axes is a principal axes but every principal axes will not be symmetric.

* Actual Physical Defination of Principal Axes:-

"The set of ~~axe~~ axes about which uniform rotational motion can be maintained without application of a torque are called Principal Axes."

* The axes for which $\vec{L} \parallel \vec{\omega}$ are principal axes.



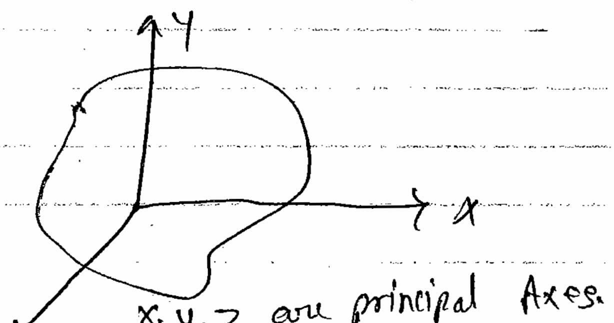
* Property of principal moment of inertia:-

x, y, z are principal axes
Then,

$I_{xx} = \sum m_i (y_i^2 + z_i^2)$

$I_{yy} = \sum m_i (x_i^2 + z_i^2)$

$I_{zz} = \sum m_i (x_i^2 + y_i^2)$



x, y, z are principal Axes.

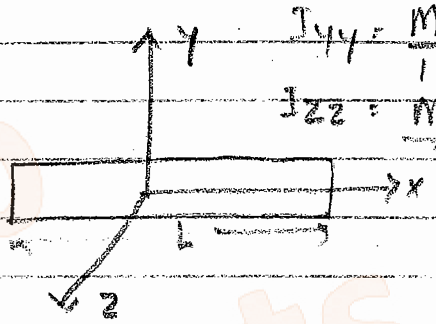
$$I_{xx} + I_{yy} = \sum m_i (x_i^2 + y_i^2 + 2z_i^2)$$

$$I_{xx} + I_{yy} = \cancel{\Sigma}\ I_{zz} + 2\sum m_i z_i^2$$

$$\boxed{I_{xx} + I_{yy} - I_{zz} = 2\sum m_i z_i^2 > 0}$$

$$I_{xx} + I_{yy} - I_{zz} \geq 0$$

$$\boxed{I_{xx} + I_{yy} \geq I_{zz}}$$



$I_{xx} = 0$

$I_{yy} = \frac{ML^2}{12}$

$I_{zz} = \frac{ML^2}{12}$

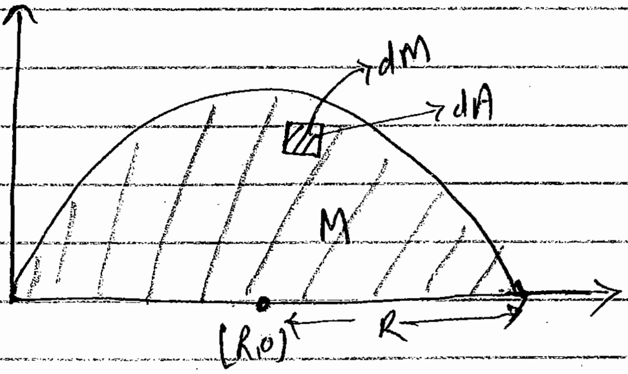
Similarly -

$$\boxed{I_{yy} + I_{zz} \geq I_{xx}} \quad \text{and} \quad \boxed{I_{xx} + I_{zz} \geq I_{yy}}$$

-6

.11 A semicircular disc of Radius R and mass M is kept in x-y plane as shown in figure. The product of inertia I_{xy} is?

ol^n



$$I_{xy} = -\int dm\, x.y$$

$$dm = \frac{M}{A} dA = \frac{M}{\frac{\pi R^2}{2}}$$

$$\frac{dm}{dA} = \frac{2M}{\pi R^2}$$

$$dm = \frac{2M}{\pi R^2} dx\, dy$$

$$I_{xy} = -\frac{2M}{\pi R^2} \iint x\, y\, dx dy$$

∵ Here ~~A~~ if we moving along the boundary then x and y both change. So limits are dependents.

So first we write equation of boundary -

$$(x-R)^2 + (y-0)^2 = R^2$$

$$\boxed{y = \sqrt{R^2-(x-R)^2}}$$

So $$I_{xy} = -\frac{2M}{\pi R^2}\int_0^{2R} x\,dx \int_0^{\sqrt{R^2-(x-R)^2}} y\,dy$$

$$\cancel{= -\frac{2M}{\pi R^2}\left[\frac{x^2}{2}\right]_0^{2R}\left[\frac{y^2}{2}\right]_0^{\sqrt{R^2-(x-R)^2}}}$$

$$\cancel{= -\frac{2M}{\pi \cancel{R^2}}\left[\frac{4\cancel{R^2}}{2}\right]\left[\frac{R^2-(x-R)^2}{2}\right]}$$

$$\cancel{= -\frac{2M}{\cancel{4}\pi}\left[\cancel{4}(\cancel{R^2}-x^2-\cancel{R^2}+2xR)\right]}$$

$$= -\frac{2M}{\pi R^2}\int_0^{2R}\left[\frac{y^2}{2}\right]_0^{\sqrt{R^2-(x-R)^2}} x\,dx$$

$$= -\frac{2M}{\pi R^2}\int_0^{2R}\left\{\frac{R^2-(x-R)^2}{2}\right\}\cdot x\,dx$$

$$= -\frac{\cancel{2}M}{\pi R^2}\int_0^{2R}\left[\frac{\cancel{R^2}-x^2-\cancel{R^2}+2xR}{\cancel{2}}\right]x\,dx$$

$$= -\frac{M}{\pi R^2}\int_0^{2R}(-x^3+2x^2R)\,dx$$

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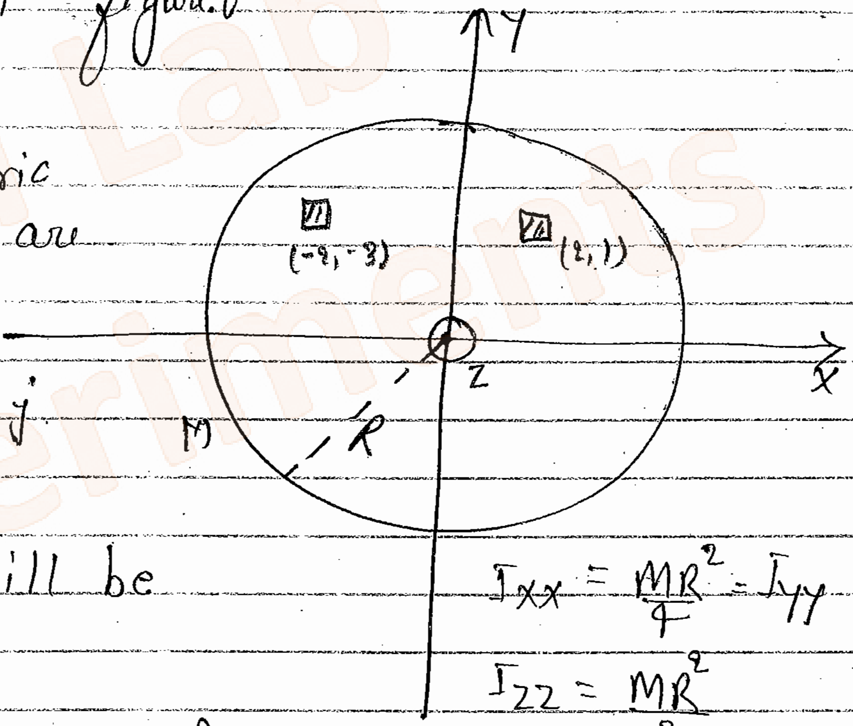
$$= \frac{-M}{\pi R^2}\left[-\left(\frac{x^4}{4}\right)_0^{2R} + 2R\left(\frac{x^3}{3}\right)_0^{2R}\right]$$

$$= \frac{-M}{\pi R^2}\left[\frac{-16R^4}{4} + \frac{2R\times 8R^3}{3}\right] = \frac{-M}{\pi R^2}\left[\frac{-16R^4}{4}+\frac{16R^4}{3}\right]$$

$$= +\frac{16R^{\cancel{4}}M}{\pi R^{\cancel{2}}}\left[\frac{1}{4}-\frac{1}{3}\right] = \frac{\overset{4}{\cancel{16}}MR^2}{\pi}\left[\frac{3-4}{\underset{3}{\cancel{12}}}\right]$$

$$I = -\frac{4MR^2}{3\pi} \quad \underline{\text{Ans}}$$

Write Inertia tensor of the disc about the axes shown in figure.



x, y, z are symmetric axes so these are principal axes.

$\therefore$ $I_{ij} = 0$ for $i \neq j$

$I_{xx} = \frac{MR^2}{4} = I_{yy}$

$I_{zz} = \frac{MR^2}{2}$

Inertia tensor will be written -

$$\mathbb{I} = \frac{MR^2}{4}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

matrix

Calculation of I from $\mathbb{I}$:-

10

Sol^n

$$\overleftrightarrow{I} = \begin{pmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix}$$

Magnitude of M.I about an axis.

$$\hat{n} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0 \right)$$

$$I = \underbrace{\hat{n}}_{\substack{\text{row} \\ \text{matrix} \\ 1\times 3}} \cdot \underbrace{\overleftrightarrow{I}}_{\substack{3\times3 \\ \text{matrix}}} \cdot \underbrace{\hat{n}}_{\substack{\text{coloumn} \\ \text{matrix} \\ 3\times 1}}$$

$$I = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0 \right) \begin{pmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ 0 \end{pmatrix}$$

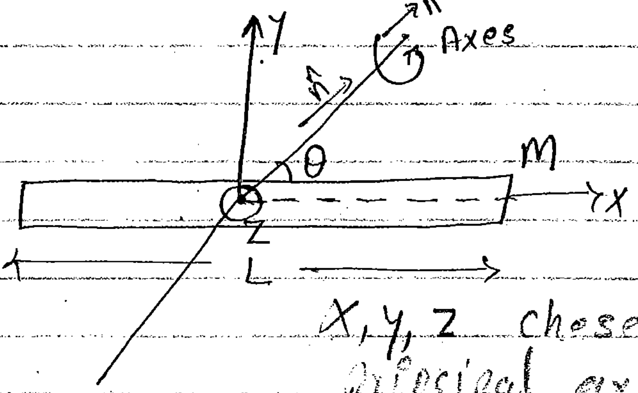
$$I = \left(4, 2\sqrt{3}, -2 \right) \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \\ 0 \end{pmatrix}$$

$$= 2 + 3 = 5$$

$\boxed{I = 5}$ Ans

Q. Find M.I. of thin rod about the axis shown in fig.

$$\overleftrightarrow{I} = \frac{ML^2}{12} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



X, Y, Z chosen as principal axis

All Lab Experiments

$$\hat{n} = (\cos\theta, \sin\theta, 0)$$

$$I = \hat{n}\cdot\overleftrightarrow{\mathbb{I}}\cdot\hat{n}$$

$$I = (\cos\theta, \sin\theta, 0)\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cos\theta\\ \sin\theta\\ 0\end{pmatrix}\frac{mL^2}{12}$$

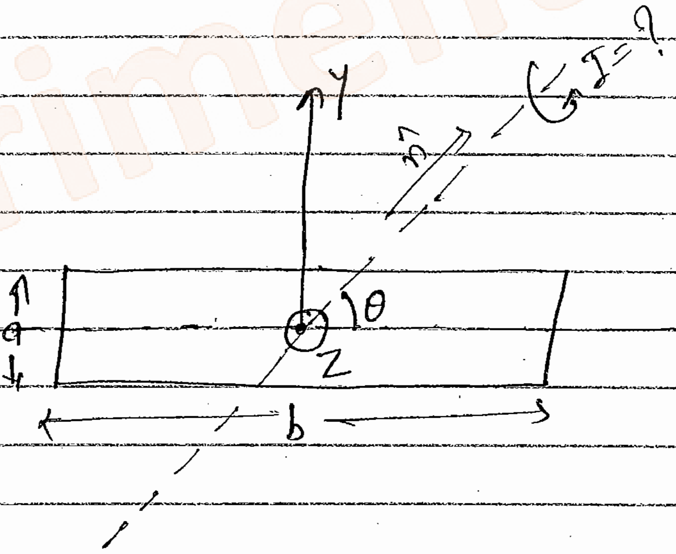
$$= (0, \sin\theta, 0)\begin{pmatrix}\cos\theta\\ \sin\theta\\ 0\end{pmatrix}\frac{ML^2}{12}$$

$$\boxed{I = \frac{ML^2}{12}\sin^2\theta}$$

Ans

6

21



$$I_{xx} = \frac{Ma^2}{12}$$

$$I_{yy} = \frac{Mb^2}{12}$$

$$I_{zz} = I_{xx} + I_{yy}$$

$$I_{zz} = \frac{M}{12}(a^2+b^2)$$

$$\overleftrightarrow{\mathbb{I}} = \frac{M}{12}\begin{pmatrix}a^2 & 0 & 0\\ 0 & b^2 & 0\\ 0 & 0 & (a^2+b^2)\end{pmatrix}$$

$$\hat{n} = (\cos\theta, \sin\theta, 0)$$

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So $I = \hat{n}\cdot\overleftrightarrow{I}\cdot\hat{n}$

$$= (\cos\theta, \sin\theta, 0)\frac{M}{12}\begin{pmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & (a^2+b^2) \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

$$= (a^2\cos\theta, b^2\sin\theta, 0)\begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}\frac{M}{12}$$

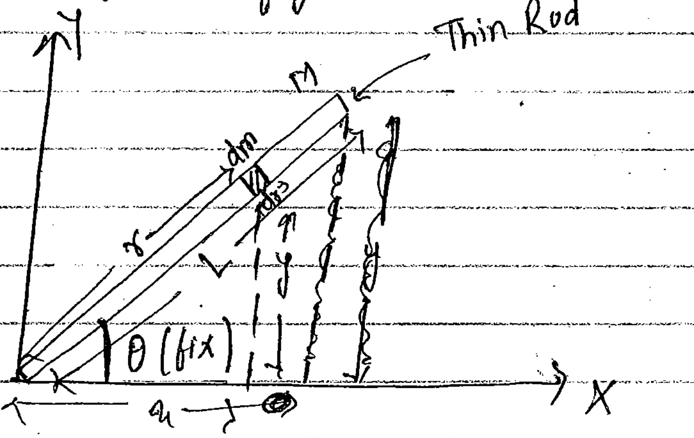
$$= (a^2\cos^2\theta + b^2\sin^2\theta)\frac{M}{12}$$

$$\boxed{I = \frac{Ma^2}{12}\cos^2\theta + \frac{Mb^2}{12}\sin^2\theta}$$

So option (d) is correct.

Q. Calculate $I_{xy} = ?$ of the given figure.

Sol^n



$I = -\int dm\, xy$

$dm = \frac{M}{L}dr$

So $I = \int \frac{M}{L}dr \cdot r\cos\theta \cdot r\sin\theta$ $\quad x = r\cos\theta$, $y = r\sin\theta$

$$= -\frac{M}{L}\cos\theta\sin\theta\int_0^L r^2 dr = -\frac{M}{L}\cos\theta\sin\theta\left[\frac{r^3}{3}\right]_0^L$$

$$\boxed{I = -\frac{ML^2}{3}\sin\theta\cos\theta}$$ Ans

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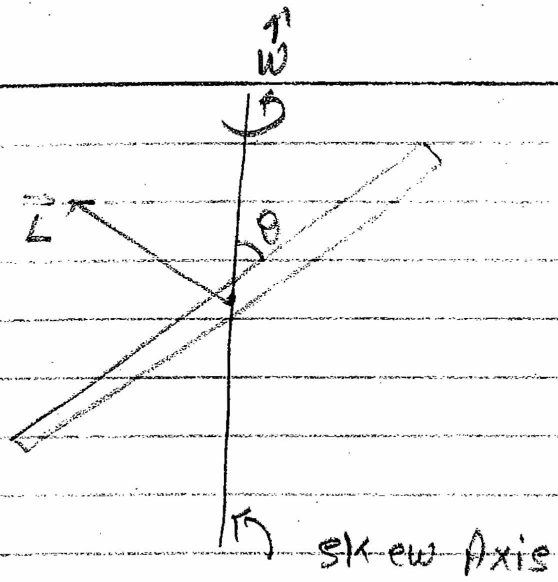
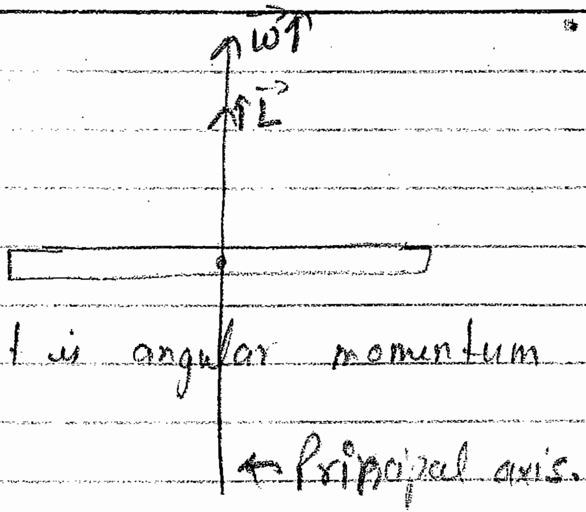
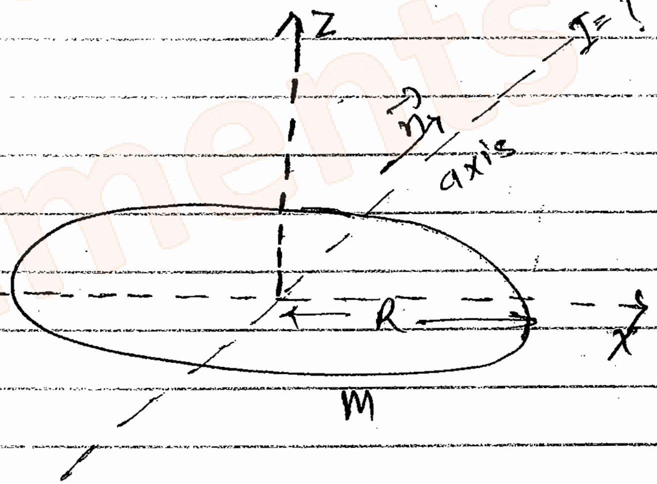




What is angular momentum

$$\mathbb{I} = \begin{pmatrix} \frac{MR^2}{4} & 0 & 0 \\ 0 & \frac{MR^2}{4} & 0 \\ 0 & 0 & \frac{MR^2}{2} \end{pmatrix}$$

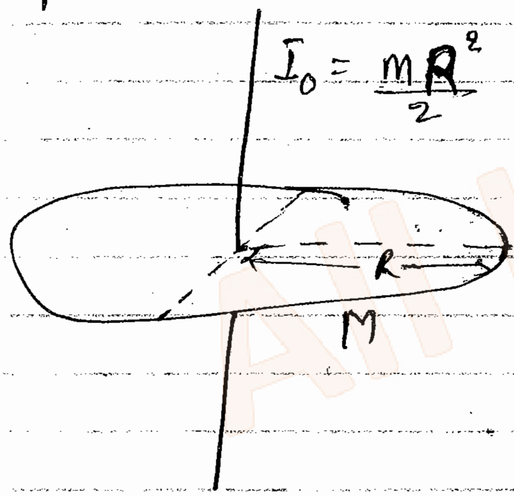
$$\hat{n} = \begin{pmatrix} \sin\theta \\ 0 \\ \cos\theta \end{pmatrix}$$



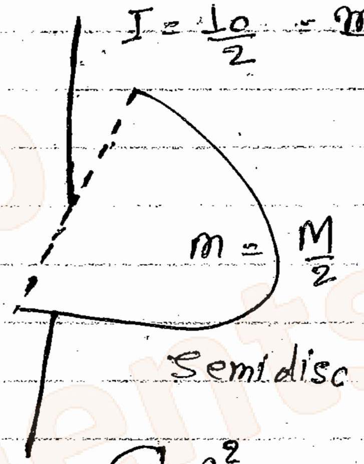
All Lab Experiments

Note : Two important points related to M.I.

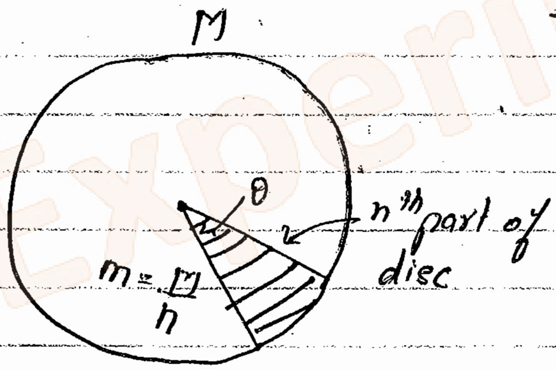
(i) If an object is cut symmetrically then form of M.I. of divided part is same as form of M.I of whole object when expressed in terms of mass of divided part.

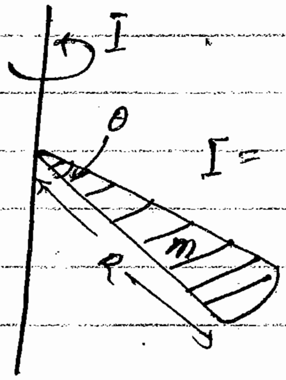


$I = \frac{I_0}{2} = \frac{mR^2}{2}$



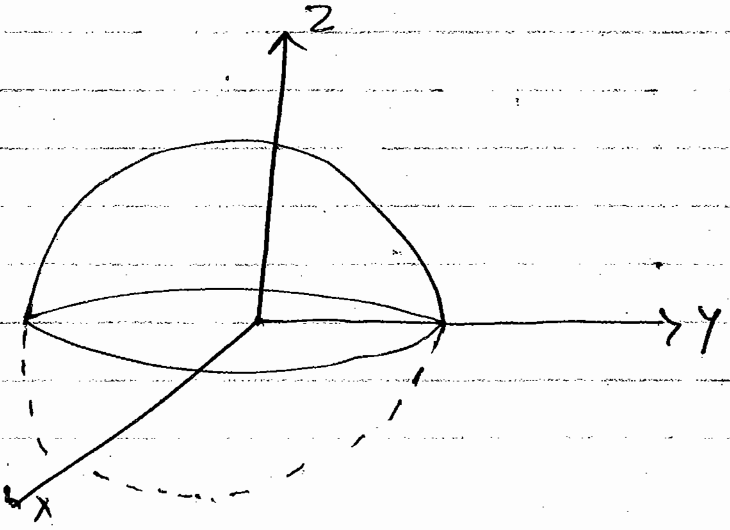
$I = \frac{I_0}{2} = \frac{M}{2 \times 2} R^2 = \frac{mR^2}{2}$





$I = \frac{mR^2}{2}$

A-6
Q10



Solⁿ I about x = $\frac{2}{3} MR^2$

I about z = $\frac{2}{3} MR^2$

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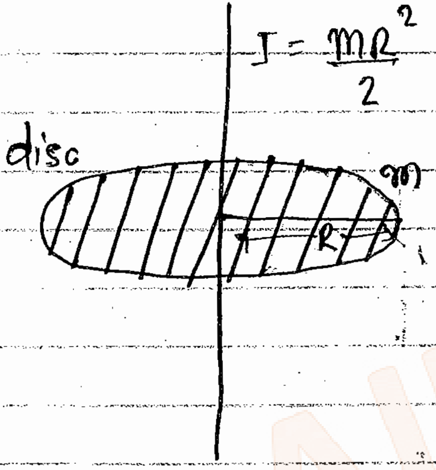
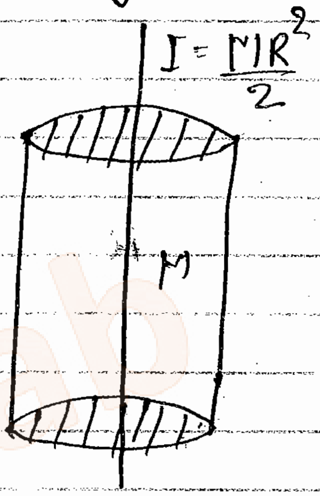
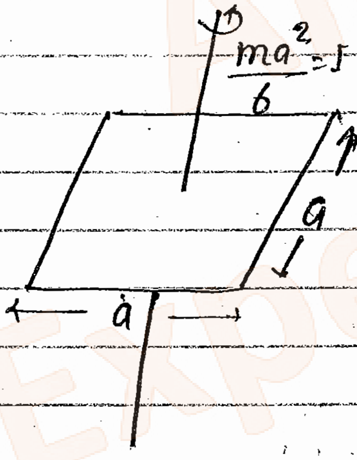
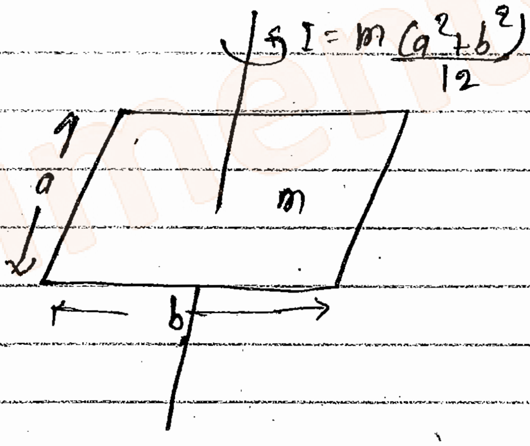
Assembling

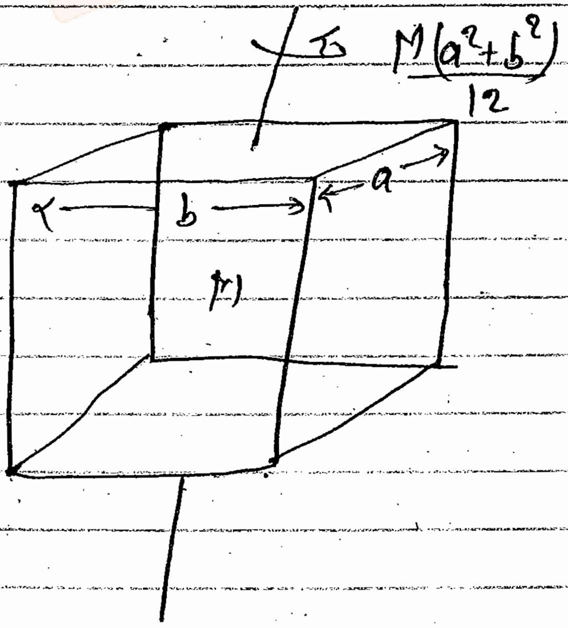
ii) If stacking of small ~~obejts~~ ~~is~~ objects is done to make a big object then form of M.I. of big objects is same as of M.I. of small objects.

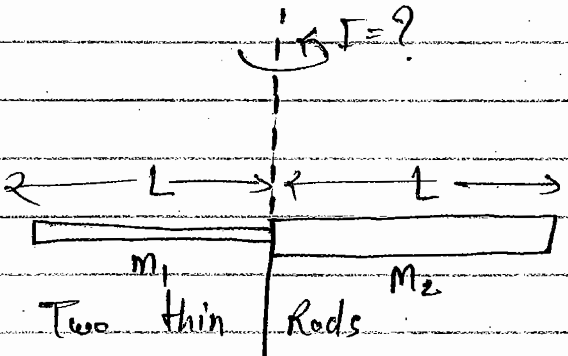












$$I = \frac{m_1L^2}{3} + \frac{M_2L^2}{3}$$

$$I = \frac{(m_1+m_2)L^2}{3}$$

All Lab Experiments

* Angular Momentum about Skew axis :-

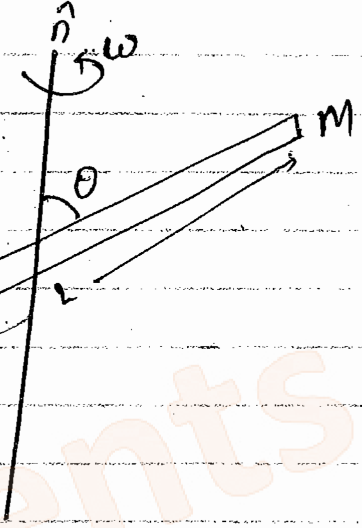
Here two questions can be asked.

* What is the angular momentum about axis.
* What is total angular momentum.

(i) $$\boxed{\vec{L}_{axis} = I_{axis}\,\omega}$$

(ii) $\vec{L}_{total} = \vec{L}_{axis}$ (if axis is principal axis)

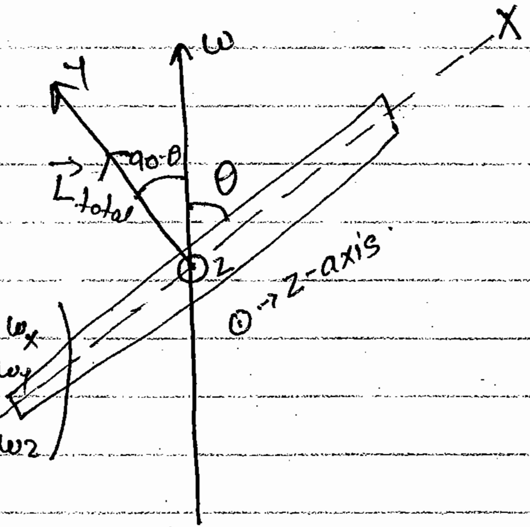
$\vec{L}_{total} = \overleftrightarrow{I}\cdot\vec{\omega}$ [if axis is not a principal axis]



(i) $\vec{L}_{axis} = I_{axis}\cdot\omega$

$$\boxed{\vec{L}_{axis} = \frac{ML^2}{12}\sin^2\theta\cdot\omega}$$

(ii) $\vec{L}_{total} = \overleftrightarrow{I}\cdot\vec{\omega}$



$$\begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{ML^2}{12} & 0 \\ 0 & 0 & \frac{ML^2}{12} \end{pmatrix}\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{ML^2}{12} & 0 \\ 0 & 0 & \frac{ML^2}{12} \end{pmatrix}\begin{pmatrix} \omega\cos\theta \\ \omega\sin\theta \\ 0 \end{pmatrix}$$

All Lab Experiments

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Comparing coefficient:-

$$\begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{mL^2}{12}\omega\sin\theta \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} L_x = 0 \\ L_y = \frac{mL^2}{12}\omega\sin\theta \\ L_z = 0 \end{matrix}$$

$$\vec{L}_{total} = \sqrt{L_x^2 + L_y^2 + L_z^2}$$

$$= \sqrt{0 + \left(\frac{mL^2}{12}\sin\theta\right)^2 + 0}$$

$$\boxed{\vec{L}_{total} = \frac{mL^2}{12}\omega\sin\theta}$$

Angular momentum about axis is just a component of total angular momentum.

$$L_{axis} = L_{total}\cos(90-\theta)$$

$$= L_{total}\sin\theta = \frac{mL^2}{12}\omega\sin\theta \cdot \sin\theta$$

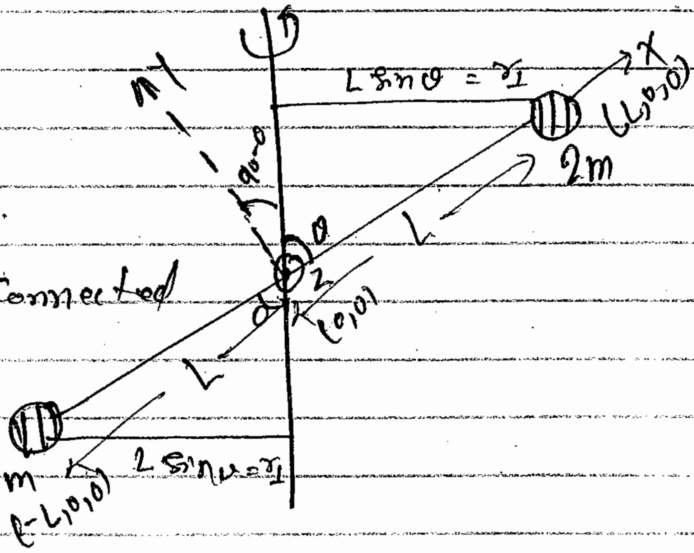
$$\boxed{L_{axis} = L_{total}\frac{mL^2}{12}\omega\sin^2\theta}$$

Ans

Find $L_{axis} = ?$

$L_{total} = ?$

Two small particle connected by a small rod.



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$$\vec{L}_{axis} = I_{axis} \cdot \omega$$

$$= \sum_{i=1}^{2} \left(m_i r_{1\,i}^{2} \right) \omega$$

$$= \left(m \times L^2 \sin^2\theta + 2m \times L^2 \sin^2\theta \right) \omega$$

$$\boxed{\vec{L}_{axis} = 3mL^2 \sin^2\theta \cdot \omega}$$

Total Angular momentum :-

$$I_{yy} = \sum m_i (x_i^2 + z_i^2)$$

$$= m(l^2 + 0) + 2m \times l^2$$

$$= 3ml^2$$

$$I_{zz} = \sum m_i (x_i^2 + y_i^2)$$

$$\text{Inertia Tensor } \underline{\underline{I}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3mL^2 & 0 \\ 0 & 0 & 3mL^2 \end{pmatrix}$$

$$\vec{\omega} = \begin{pmatrix} \omega\cos\theta \\ \omega\sin\theta \\ 0 \end{pmatrix}$$

$$\vec{L}_{total} = \underline{\underline{I}} \cdot \omega = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3mL^2 & 0 \\ 0 & 0 & 3mL^2 \end{pmatrix} \begin{pmatrix} \omega\cos\theta \\ \omega\sin\theta \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 3mL^2 \sin\theta\,\omega \\ 0 \end{pmatrix}$$

$$\boxed{\vec{L}_{total} = L_{2m} + L_m = 3mL^2 \omega \sin\theta.}$$

$$K.E. = \frac{1}{2} I_{axis} \omega^2 = \frac{1}{2} 3mL^2 \omega \sin^2\theta \cdot \omega^2$$

$$\boxed{K.E. = \frac{3}{2} mL^2 \omega^3 \sin\theta}$$

All Lab Experiments

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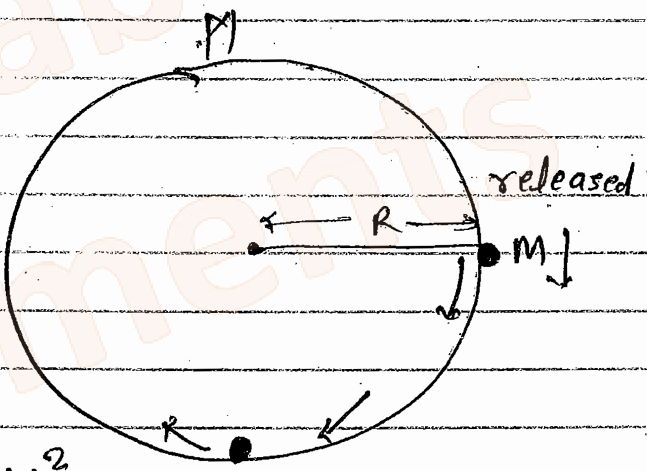
o A disc of mass M and Radius R is pivoted about a horizontal axis through its centre and a small body of the same mass M is attached to the rim of the disc. If the disc is released from rest with the small body at the end of a horizontal radius, the angular speed when the small body is at the bottom is ?

a) $\sqrt{\left(\frac{g}{4R}\right)}$ (b) $\sqrt{\left(\frac{g}{2R}\right)}$ (c) $\sqrt{\left(\frac{3g}{4R}\right)}$ ✓(d) $\sqrt{\left(\frac{4g}{3R}\right)}$

n Now apply Conservation of energy -

Loss in K.G. = Gain in K.E.



$$MgR = \frac{1}{2} I\omega^2$$

$$= \frac{1}{2}\left(\frac{MR^2}{2} + MR^2\right)\omega^2$$

$$\cancel{M}g\cancel{R} = \frac{3\cancel{M}R^{\cancel{2}}}{4} \cdot \omega^2$$

$$\boxed{\omega = \sqrt{\frac{4g}{3R}}}$$

-7

.21 An uniform bar of length 6a and mass 8m lies on a smooth horizontal table. Two point masses m and 2m moving in the same horizontal plane with speed 2v and v respectively strike the bar (see fig) and stick to the bar after collision.

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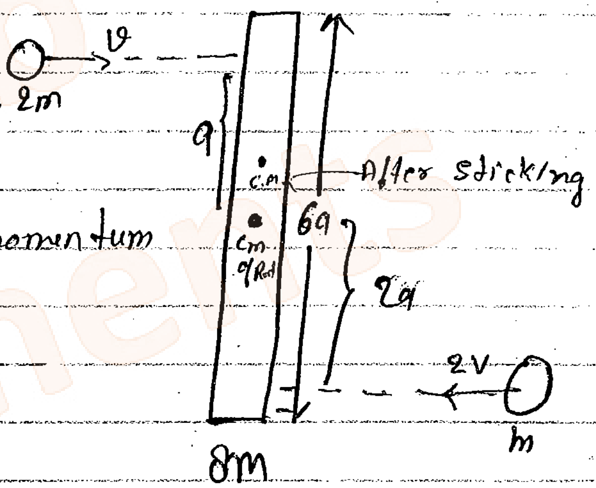
Denoting angular velocity (about the center of mass), total energy and centre of mass velocity by ω, E and v_c respectively, we have after collision.

(a) $v_c = 0$ (b) $\omega = \frac{3}{5}\left(\frac{v}{a}\right)$ (c) $\omega = \frac{v}{5a}$ (d) $E = \frac{3}{5}mv^2$

Sol^n The whole system lying on the smooth table therefore $\vec{F}_{ext} = 0$

$$\because \quad \vec{F}_{ext} = \frac{d\vec{p}}{dt}$$
$$\Downarrow$$
$$0 \Rightarrow \vec{p} = \text{Const.}$$



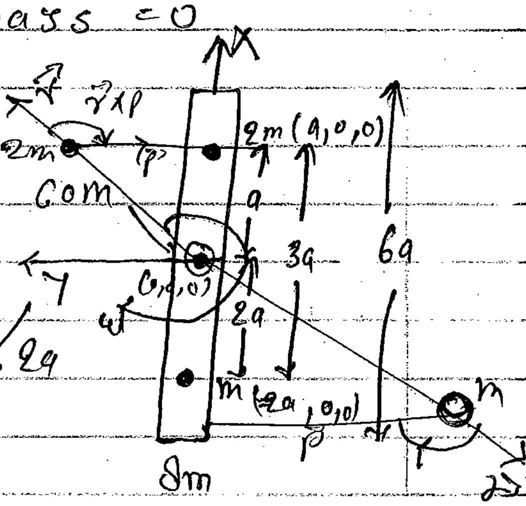
Now apply conservation of momentum

$$P_i = P_f$$

$$\cancel{2mv} - \cancel{m2v} = P_f$$

$$\boxed{P_f = 0} \Rightarrow \boxed{V_c = 0}$$

So Here initial and final momentum is = 0
So velocity of centre of mass = 0



$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$$= \frac{2\cancel{m}a + 8\cancel{m}\cdot 0 - \cancel{m}\cdot 2a}{11a}$$

$$= 0$$

So C.M. of system is mid of the rod.

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Here $F_{ext} = 0$ therefore there is no external force So there is no external torque.

$$\vec{\tau} = \frac{d\vec{L}}{dt} = 0$$

So $\vec{L}$ = Constant

So we apply Conservation of Angular momentum

$$L_i = L_f$$

$$V \times 2m \times a + m \times 2V \times 2a = I\omega$$

$$6maV = I\omega$$

$$6mVa = \left(\underset{\overset{\uparrow}{\text{Rod}}}{\frac{MR^2}{12}} + \underset{\overset{\uparrow}{\text{Particle}}}{m_1 r_1^2} + \underset{\overset{\uparrow}{\text{Particle}}}{m_2 r_2^2}\right)\omega$$

$$= \left[\frac{8m(6a)^2}{12} + 2ma^2 + m(2a)^2\right]\omega$$

$$6mva = \left(\frac{8m\cdot\overset{3}{\cancel{36}}a^2}{\cancel{12}} + 2ma^2 + 4a^2m\right)\omega$$

$$\cancel{6}\cancel{m}v\cancel{a} = \frac{\cancel{36}}{5}\cancel{m}a^2\omega$$

$$\boxed{\omega = \frac{V}{5a}}$$

Total Energy :- Pure Rotation:-

$$\text{Energy} = \frac{1}{2}I\omega^2$$

$$= \left(\frac{1}{\cancel{2}}\overset{3}{\overset{\cancel{6}}{\cancel{36}}}m\cancel{a^2} \times \frac{v^2}{\underset{5}{\cancel{25}}\cancel{a^2}}\right.$$

$$\boxed{E = \frac{3mv^2}{5}}$$

Ans