

This question paper contains 4 printed pages.

Your Roll No.

Sl. No. of Ques. Paper : 102 I
Unique Paper Code : 32221101
Name of Paper : Mathematical Physics - I
Name of Course : B.Sc. (Hons.) Physics
Semester : I
Duration : 3 hours
Maximum Marks : 75

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

*Attempt five questions in all.
Question No. 1 is compulsory.*

1. Do any five questions :

(a) Solve: $\frac{dy}{dx} = (1+x^2)(1+y^2)$.

(b) By calculating the Wronskian of the functions e^x , e^{-x} , and e^{-2x} check whether the functions are linearly dependent or independent.

(c) Find the area of the triangle with vertices P(2, 3, 5), Q(4, 2-1), and R(3, 6, 4).

(d) Find the unit vector normal to the surface $x^2 + y^2 + z^2 = 4$ at the point $(1, \sqrt{2}, -1)$.

(e) Show that :

$$\oiint_S (\vec{\nabla} r^2) \cdot d\vec{S} = 6V$$

where S is the closed surface enclosing the volume V.

P.T.O.

(f) Evaluate :

$$\iint_R \sqrt{x^2 + y^2} dx dy$$

(g) Verify that :

$$\int_{-\infty}^{+\infty} \delta(a-x)\delta(x-b) dx = \delta(a-b)$$

(h) Form a differential equation whose solutions are e^{2x} and e^{3x} . 5×3=15

2. (a) Solve the inexact equation :

$$y(1+xy) dx + x(1+xy+x^2y^2) dy = 0. \quad 5$$

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x. \quad 4$$

(c) Using method of undetermined coefficients, solve the differential equation :

$$\frac{d^2y}{dx^2} + 4y = 2\sin 2x. \quad 6$$

3. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = x^2 + 5. \quad 9$$

(b) Solve the differential equation using method of variation of parameter

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 e^{2x}. \quad 6$$

4. (a) Show that

$$\begin{aligned} & [(\vec{A} \times \vec{B}) \times \vec{C}] \times \vec{D} + [(\vec{B} \times \vec{A}) \times \vec{D}] \times \vec{C} + \\ & [(\vec{C} + \vec{D}) \times \vec{A}] \times \vec{B} + [(\vec{D} \times \vec{C}) \times \vec{B}] \times \vec{A} = 0. \end{aligned} \quad 6$$

(b) Show that :

$$\vec{F} = (y^2 \cos x + z^3) \hat{i} + (2y \sin x - 4) \hat{j} + (3xz^2 + 2) \hat{k}$$

is a conservative force field and then evaluate

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is any path from $(0, 1, -1)$ to $(\frac{\pi}{2}, -1, 2)$. 9

5. (a) If \vec{a} is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then prove that:

$$\text{curl} \left(\frac{\vec{a} \times \vec{r}}{r^3} \right) = \frac{3(\vec{a} \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{a}}{r^3}. \quad 7$$

(b) Evaluate :

$$\iint_S \vec{A} \cdot \hat{n} \, dS$$

where $\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ located in the first octant. 8

6. (a) Evaluate :

$$\oint_C (y - \sin x) \, dx + \cos x \, dy$$

(i) directly

(ii) using Green's theorem in the plane, where C is the boundary of a triangle enclosed by the lines $y = 0$,

$$x = \frac{\pi}{2}, \text{ and } y = \frac{2}{\pi}x. \quad 10$$

(b) Verify that :

$$\nabla^2 r^n = n(n+1)r^{n-2}. \quad 5$$

7. (a) Verify divergence theorem for

$$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$$

taken over the rectangular parallelepiped $0 \leq x \leq a$,
 $0 \leq y \leq b$, $0 \leq z \leq c$. 10

(b) Express the position and velocity of a particle in cylindrical coordinates. 5

8. (a) Derive an expression for the divergence of a vector field in orthogonal curvilinear coordinate system. 10

(b) Evaluate Jacobian $J\left(\frac{x, y, z}{u_1, u_2, u_3}\right)$ for the transformation from rectangular coordinate system to spherical coordinate system. 5