

Name of Paper : DSE : Solid State Physics

Name of the Course : B.Sc. (Prog.) DSE

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

2022

Answer five questions in all.

Question No. 1 is compulsory.

All questions carry equal marks.

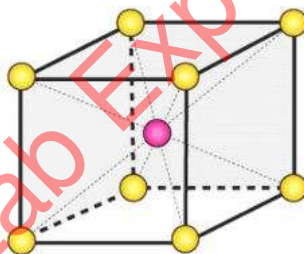
Non-programmable scientific calculator is allowed.

Q. 1. Attempt any Five :

(a) Determine the volume of the primitive unit cell of a bcc lattice.

Sol. Let a be the lattice constant r be the radius of the atom. For BCC structure
 $a\sqrt{3} = 4r$

As the volume of a cube is $a^3 = \left(\frac{4}{\sqrt{3}}r\right)^3$



(b) Calculate the lattice constant of a simple cubic lattice whose interplanar spacing between (100) planes is 3 \AA .

Sol. The interplanar spacing is given by

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$3 = \frac{a}{\sqrt{1}}$$

$$a = 3 \text{ \AA}$$

(c) Show that the number of normal modes of vibration in a linear monoatomic lattice of finite length is equal to number of unit cells in the lattice.

Sol. The assumption that the last atom is joined to the first atom makes the

apply periodic boundary condition.

Therefore,

$$u_n(x=0) = u_{N+n}(x=L)$$

where $L = Na$ is the length of ring of atoms. substituting eqn 3 in eqn 9

$$e^{ikL} = 1$$

$$KL = 2n\pi$$

$$k = \frac{2n\pi}{L} = n \frac{2\pi}{L}$$

where $n = 0, 1, 2, \dots$. Only discrete values of k are allowed which are integral multiples of $\frac{2\pi}{L}$. For large L the points form a quasi-continuous mesh along k axis. Thus the total number of points inside the first zone will be given by

$$\frac{2\pi/a}{2\pi/L} = \frac{L}{a} = N$$

where N is the total number of atoms or unit cells in the lattice and is equal to the allowed values of k inside the first Brillouin zone. This is expected because values of k inside the zone uniquely describes the vibrational modes of the lattice. Therefore, the number of modes must be equal to the number of degrees of freedom in the lattice, which is N .

(d) State Dulong and Petit's law of specific heat for solids?

Sol. Refer theory part of this book for this question. **Page No. 36 in Q. No. 4.**

(e) Calculate the Hall Coefficient when number of electrons in a semiconductor is 10^{20} m^{-3} . Given that $e = 1.6 \times 10^{-19} \text{ coulomb}$.

Sol. The Hall coefficient is given by

$$\begin{aligned} R_H &= 1/ne \\ &= 1/(1.6 \times 10^{-19} \times 10^{20}) \\ &= 0.0625 \end{aligned}$$

(f) The Debye temperature for silver is 25 K. Calculate the highest possible frequency for lattice vibrations in silver and its molar heat capacity at 25 K.

Sol. The maximum frequency of vibration of the atoms of solid bodies is given by

$$f_v = \frac{k_B T_D}{h} = \frac{1.39 \times 10^{-23} \text{ J/K} \cdot 25 \text{ K}}{6.62 \times 10^{-34} \text{ J}\cdot\text{s}} = 5.22 \times 10^{11} \text{ Hz}$$

where $k_B = 1.38 \times 10^{-23} \text{ J/K}$ is the Boltzmann constant; $h = 6.62 \times 10^{-34} \text{ J}\cdot\text{s}$ is the Planck constant; T_D is the Debye temperature.

$$C_v(T) = 9NK_B \left(\frac{T}{T_v} \right)^3 \int_0^{T_D/T} \frac{\xi^4 e^\xi}{(e^\xi - 1)^2} d\xi$$

$$C_v(T = 500) = 9 \left(\frac{25}{25} \right)^3 6.02 \times 10^{23} \times 1.32 \times 10^{-23} \text{ J/K} \left(\int_0^{225/500} \frac{\xi^4 e^\xi}{(e^\xi - 1)^2} d\xi \right) = J/K$$

Hence $f_D = \frac{K_B T_D}{h} \times 5.22 \cdot 10^{11} \text{ Hz}$; $C_v(T = 25 \text{ K}) = J/K$

where N is the number of atoms in a solid body

(g) Distinguish between normal and anomalous dispersion.

Sol.

- Ordinary dielectric transparent media will have a higher index for blue than for red. There are always some absorption bands in the infrared and in the ultra-violet.
- In between two absorption bands; the index always *increases* toward shorter wavelengths. **This is normal dispersion.**
- In the absorption band itself, the index will *decrease* toward shorter wavelengths. This latter phenomenon is called "**anomalous dispersion**". (or sometimes "resonant dispersion".)
- There is a close relationship between the absorption versus wavelength and the index versus wavelength. Mathematically, it is known as the Kramers-Kronig relationship, which means any two media with identical absorption bands will have identical dispersion relationships.

(h) For a given specimen of a superconductor, the critical fields are $1.4 \times 10^5 \text{ A/m}$ and $4.2 \times 10^7 \text{ A/m}$ respectively for 14K and 13K. What will be the critical field at 4.2 K?

Sol. Her Given

$$H_c(T_1) = 1.4 \times 10^5 \text{ A/m} \quad \text{at } T_1 = 14 \text{ K}$$

$$H_c(T_2) = 4.2 \times 10^7 \text{ A/m} \quad \text{at } T_2 = 13 \text{ K}$$

The critical field can be calculated by using equation

$$H_c(T_1) = H_c(0) [1 - (T_1/T_c)^2] \quad \dots(1)$$

$$H_c(T_2) = H_c(0) [1 - (T_2/T_c)^2] \quad \dots(2)$$

Hence using equation (1-2) and dividing eq. 1 by eq. 2, we will get

$$1.4 \times 10^5 \text{ A/m} = H_c(0) [1 - (14/T_c)^2] / 4.2 \times 10^7 \text{ A/m} = H_c(0) [1 - (13/T_c)^2]$$

$$1.4/420 = 1 - (14/T_c)^2 / 1 - (13/T_c)^2$$

$$0.003 - 0.507/T_c^2 = 1 - 196/T_c^2$$

$$1 - 0.003 = 196/T_c^2 - 0.507/T_c^2$$

$$0.997 = 195.493/T_c^2$$

$$T_c^2 = 196.081$$

$$T_c = 14.002 \text{ K}$$

Hence

$$H_c(0) = H_c(T_1)/[1 - (T_1/T_c)^2] \quad \dots(3)$$

$$H_c(0) = 1.4 \times 10^5 \text{ A/m} [1 - (14/14.002)^2]$$

$$H_c(4.2 \text{ K}) = 1.4 \times 10^8 \text{ A/m} \times [1 - (4.2/14.002)^2]$$

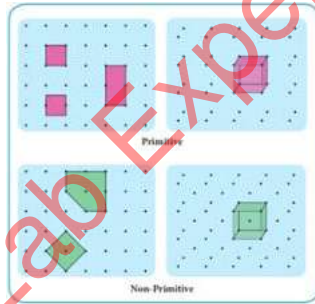
$$= 1.4 \times 10^8 \text{ A/m} \times 1.911$$

$$= 1.28 \times 10^8 \text{ A/m}$$

Q. 2. (a) Distinguish between a primitive and non-primitive unit cells with diagram.

Ans. There are two types of unit cells; primitive and non-primitive. A unit cell that contains only one lattice point is called a primitive unit cell, which is made up from the lattice points at each of the corners.

In case of non-primitive unit cells, there are additional lattice points, either on a face of the unit cell or with in the unit cell.



(b) Draw the primitive unit cell of a fcc lattice and obtain its reciprocal lattice vectors. Determine the volumes of unit cells in direct lattice and reciprocal lattice.

Sol. Refer the book for the solution to this question Page N. 18, Q. 15 (iii)

(c) In crystal, a lattice plane makes an intercepts at a , $2b/3$ and $c/2$ along the three primitive axes a , b and c of the unit cell. Determine the Miller Indices of the given plane.

Sol. Given,

$$\text{Intercepts of plane} = \left(a, \frac{2b}{3}, \frac{c}{2} \right)$$

$$\text{Intercept} = \left(1, \frac{2}{3}, \frac{1}{2} \right)$$

Reciprocal of intercept = $1, \frac{1}{2}, 2$

L.C.M. = 2

Miller indicates of the plane = L.C.M. \times Reciprocal of intercept

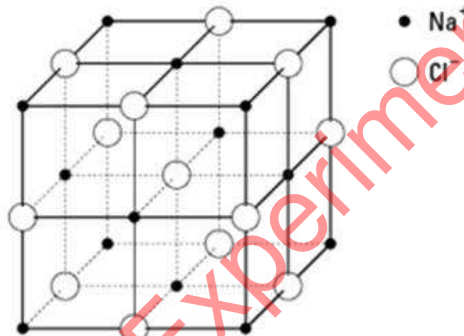
\therefore Miller indicates of the plane = (2, 3, 4)

Q. 3. (a) Explain why X-rays are used in the study of crystals? Derive Bragg's law of X-ray diffraction.

Sol. Refer book for the solution to this question Page No. 6, Q. 4. and Q. 6.

(b) Draw the nearest planes containing exclusively Na^+ and Cl^- ions in a sodium chloride lattice of lattice constant 5.63 \AA and determine the spacing between them.

Sol. The crystal structure of NaCl is as follows :



The nearest plane containing all Na atoms is considered as origin and the chlorine atoms at the center of these edges are closest to this. Its intercepts are $\frac{1}{2}$, $\frac{1}{2}$ and the hkl value of miller indices will be 2, 2, 2.

The inter planar spacing will be $d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{5.63}{\sqrt{12}} = 1.63 \text{ \AA}$

Q. 4. (a) Describe qualitatively and Einstein's theory of specific heat of solids. Discuss its low and high temperatures behaviour. Describe its shortcomings.

Sol. Refer book for the solution to this question Page No. 35, Q. 4.

(b) The Debye temperature for diamond is 2230° K . Calculate the highest possible vibrational frequency in diamond, ($h = 6.6 \times 10^{-34} \text{ Js}$, $k_B = 1.35 \times 10^{-23} \text{ J/K}$).

Sol. The maximum frequency of vibration of the atoms of solid bodies is given by

$$f_v = \frac{k_B T_D}{h} = \frac{1.37 \cdot 10^{-23} \text{ J/K} \cdot 2230 \text{ K}}{6.62 \cdot 10^{-34} \text{ J}\cdot\text{s}} = 4.65 \times 10^{13} \text{ Hz}$$

Planck constant; T_D is the Debye temperature.

Q. 5. (a) Explain Hall Effect. Derive the expression for Hall coefficient in semiconductor.

Sol. Hall Effect : It was suggested by E.H. Hall in 1897 that when a mag. field is applied \perp to a current carrying conductor, a voltage is developed in the specimen its a direction \perp to both current and the magnetic field. This phenomenon is called 'Hall Effect' and generated voltage is called Hall Voltage.

Let's consider a thin slab of conductor subjected to electric field E . This produces a current I and causes a force of magnitude eE to act on the charge carriers. When this conductor is placed in a magnetic field, a magnetic force is proportional to the strength of the mag. field (B) and charge (e) and velocity act on charge carriers. This force is at right angle to the direction of B & V . Therefore each charge is deflected towards one side of the conductor. When charge reaches the surface of conductors, an electrical charge is built up there, which in turn gives rise to transverse electric field E_H called Hall field. It causes a compensating drift such that force due to hall field exactly balances the Lorentz force.

At equilibrium

$$eE_H = eV_d B_Z$$
$$E_H = V_d B_Z$$
$$V_d = \frac{E_H}{B_Z} \quad \dots(1)$$

The electric current in slab

$$I = nAeV_d = n.Wt.ev_d$$

$$V_d = \frac{I}{nWte} \quad \dots(2)$$

Comparing eq. (1) & (2)

$$\frac{E_H}{B_Z} = \frac{I}{nWte}$$

Since

$$E_H = \frac{V_H}{W}$$

$$\frac{V_H}{WB} = \frac{I}{(Wt)ne}$$

$$V_H = \left(\frac{t}{ne}\right) \left(\frac{BI}{t}\right)$$

$$R_H = \frac{1}{ne}$$

Hall coefficient = $-ne$ for n type

$$R_H = \frac{1}{ne}$$

= $+ne$ for p type

(b) Hall coefficient of a material is $3.66 \times 10^{-4} \text{ m}^3 \text{ C}^{-1}$ and the resistivity of that material is $8.93 \times 10^{-3} \text{ ohm-m}$. Find the mobility and density of the charge carriers. Given that $e = 1.6 \times 10^{-19} \text{ coulomb}$.

Sol. Hall coefficient $R_H = 3.66 \times 10^{-4} \text{ m}^3 \text{ C}^{-1}$

Resistivity $\rho = 8.93 \times 10^{-3} \Omega \text{ m}$

(i) Density of holes

$$n_h = \frac{1}{R_H e}$$

$$= \frac{1}{3.66 \times 10^{-4} \times 1.6 \times 10^{-19}}$$

$$n_h = 1.7076 \times 10^{22} \text{ m}^{-3}$$

(ii) Mobility of holes $\mu_n = \frac{1}{\rho n e}$

$$= \frac{1}{8.93 \times 10^{-3} \times 1.7076 \times 10^{22} \times 1.6 \times 10^{-19}}$$

$$\mu_n = 0.041 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

Q. 6. (a) Assuming the existence of Weiss molecular field, explain the phenomenon of spontaneous magnetization and obtain Currie-Weiss law for ferromagnetism.

Sol. Refer to Page No. 50. Q. 15.

(b) Explain the significance of critical temperature in a ferromagnetic material.

Sol. The critical temperature above which ferromagnetic materials lose their magnetic property is known as Curie temperature.

The magnetic susceptibility decreases with an increase in temperature. So, the ferromagnetism decreases with rising temperature. It is maximum at absolute zero temperature and becomes zero at Curie temperature. Above this temperature, the ferromagnetic material behaves as a paramagnetic substance.

Q. 7. (a) Derive an expression for electronic polarizability of a material in a varying electric field.

Sol. Refer to book for its solution Page No. 61, Q. 2.

K respectively. Calculate the critical current density for a wire of Gallium of diameter 1 mm at 1.0° K.

Sol. The critical current density J_c is given by equation

$$\begin{aligned} J_c &= \frac{B_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right] \times 2\pi r}{A} \\ &= \frac{(5 \times 10^{-3} \text{ T}) \left[1 - \left(\frac{1.0 \text{ K}}{1.1 \text{ K}} \right)^2 \right] \times 2\pi r}{\pi r^2} \\ &= \frac{(5 \times 10^{-3} \text{ T}) \times 0.1738 \times 2}{0.5 \times 10^{-3}} \\ &= 3.476 \text{ Am}^{-2} \end{aligned}$$

Q. 8. (a) Derive an expression for dispersion relation for the vibration modes of diatomic linear lattice having masses m and M .

Sol. Refer to Page No. 29, Q. 2.

(b) Obtain expressions for two dispersion curves. Draw the dispersion curves.

Sol. Refer to Page No. 29, Q. 2.

(c) Prove that a diatomic lattice acts as a band pass filter.

Sol. Refer to Page No. 29, Q. 2.

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