Name of Paper : **Mathematical Physics–III**
Name of the Course : **B.Sc. (Hons.) Physics**
Semester : **IV**
Duration : **3 Hours**
Maximum Marks : **75**

2022

Attempt five questions in all.
Question No. 1 compulsory.

Q. 1. Attempt any five parts : (5 × 3 = 15)

(a) Find the cube root of $z = 1 + i$ and locate them in the plane.

(b) Show that $u(x, y) = x^2 - y^2$ is a harmonic function in the whole complex plane, find its harmonic conjugate, $v(x, y)$.

(c) "If a complex function $f(z)$ is analytic in a domain D and $|f(z)|$ = Const. K in D, then show that $f(x)$ is also constant in D.

(d) Show that the Laplace Transform of Dirac delta function is 1, *i.e.*, $L\{\delta(t)\} = 1$.

(e) If Laplace Transform of a function $L\{f(t)\} = F(s)$ show that

$$L\{t^n f(t)\} = (-1)^n F^{(n)}(s),$$

where $F^{(n)}$ represent n-th derivative of $F(s)$.

(f) If Fourier Transform of $f(x)$ is $F(\omega)$, find Fourier Transform of $f(x) \cos ax$, where $a > 0$.

(g) Evaluate the following integrals

(*i*) $\int_0^\infty e^{3t}\delta(t-4)dt$

(*ii*) $\int_0^\infty \sin 2t\, \delta(t-\pi/4)dt$

Q. 2. (a) Find all values of $\sin^{-1} 2$. (5)

(b) Expand $f(z) - e^{z(z-2)}$ in the Laurent series about $z = 2$ and determine the region of convergence of this series. Also classify the singularity. (6)

(c) Evaluate

$$\oint_c \frac{z}{z^2+9}dz, \quad \text{where} \quad C : |z - 2i| = 4$$

Q. 3. Using Contour Integeration, solve any two of the followings :

$\int_0 \quad x^4 + 1$

(*b*) $\int_0^{\pi} \frac{a}{a^2 + \sin^2 \theta} d\theta \qquad a > 0$

(*c*) $\int_0^{x} \frac{\cos x \, dx}{(x^2 + a^2)(x^2 + b^2)} \qquad a \,\&\, b > 0$

(*d*) $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$

Q. 4. (*a*) Obtain Fourier Integral representationn of the function

$$f(x) = \begin{cases} 0 & x < 0 \\ a & 0 \le x \le 3 \\ 0 & x > 3 \end{cases}$$

(*b*) Find the Fourier Transform of the function

$$f(x) = \frac{x}{x^2 + 1}$$

(*c*) Find Fourier sine transform of *e–mx*, $m > 0$ and hence evaluate the integeral

$$\int_0^{\infty} \frac{\omega \sin \omega x}{a^2 + \omega^2} d\omega$$

Q. 5. (*a*) For a periodic function $f(t)$ having periodicity T, such that $f(t + T) = f(t)$, show that the Laplace Transform is given by

$$L\{f(t)\} = \frac{\int_0^{T} e^{-st} f(t) \, dt}{1 - e^{sT}}.$$

(*b*) If a function is piece-wise continuous on $0 < t \le T$ and is of exponential order for $t > T$ then show that

$$\lim_{s \to \infty} L\{f(t)\} = \lim_{s \to \infty} F(s) = 0,$$

and hence further show that

where F(s) represents the Laplace Transform of $f(t)$.

Q. 6. (*a*) Plot the given function

$$f(x) = \begin{cases} 1 & |x| < 2 \\ 0 & |x| > 2 \end{cases}$$

Finding its Fourier Transform, F(s), plot it.

(*b*) Solve the differential equation $y''(t) + 4y(t) = 9t$ with initial condition $y(0) = 0$ and $y'(0) = 7$.

Q. 7. (*a*) Show that the Dirac delta function can be expressed as the derivative of Heaviside's unit step function.

(*b*) For the Dirac delta function $\delta(x)$, prove that

$$\delta(x^2 - a^2) = \frac{1}{2|a|}\left[\delta(x + |a|) + \delta(x - |a|)\right].$$

(*c*) If a continuous function $f(t)$ is an even function, then show that its Fourier Transform F(ω) will also be an even function.

Q. 8. (*a*) Find the Fourier Transform of the function

$$f(x) = e^{-a|x|}, \quad a > 0$$

and hence show that

$$\int_{-\infty}^{\infty} \frac{e^{-ikx}}{(a^2 + k^2)} dk = \frac{\pi}{\alpha} e^{-a|x|}.$$

(*b*) For a function

$$h(t) = \begin{cases} e^{-xt} g(t) & t>0 \\ 0 & t<0 \end{cases}$$

show that F{$h(t)$} = L{$g(t)$).

(*c*) For the equation $z^4 - 3z^2 + 1 = 0$, find the sum of its roots.