Name of Paper	: **Thermal Physics**
Name of the Course	: **B.Sc. (Hons.) Physics (CBCS)**
Semester	: **III**
Duration	: **3 Hours**
Maximum Marks	: **75**

2021

Attempt any four questions.
All questions carry equal marks.

Q. 1. (a) Explain the concepts of temperature and thermal Equilibrium on the basis of Zeroth Law.

Ans. See back Q. 3. (b) **[Page No. 65]**

(b) Show that the heat transfered during an infinitesimal quasistatic process of an ideal gas can be written as

$$\delta Q = \frac{C_v}{nR}VdP + \frac{C_p}{nR}PdV$$

where n denotes the number of moles and R is the Gas Constant. Apply the above equation to an adiabatic process to show that PV^γ = Constant.

Ans. Consider,

$dQ = Q(V, P)$

$$dQ = \left(\frac{\partial Q}{\partial V}\right)_P dV + \left(\frac{\partial Q}{\partial P}\right)_V dP$$

$$dQ = \left(\frac{\partial Q}{\partial T}\right)_P \left(\frac{\partial T}{\partial V}\right)_P dV + \left(\frac{\partial Q}{\partial T}\right)_V \left(\frac{\partial T}{\partial P}\right)_V dP$$

$$dQ = C_P\left(\frac{\partial T}{\partial V}\right)_P dV + C_P\left(\frac{\partial T}{\partial P}\right)_V dP$$

$$PV = nRT$$

at constant Pressure

$$P\partial V = nR\partial T$$

$$\left(\frac{\partial T}{\partial V}\right)_P = \frac{P}{nR}$$

at constant Volume,

$$C\partial P = nR\partial T$$

$$\left(\frac{\partial T}{\partial P}\right)_V = \frac{V}{nR}$$

$$\therefore dQ = C_P \frac{1}{nR} dv + C_V \frac{V}{nR} dp$$

For adiabatic process,

$$\frac{C_P P}{nR} dV = -C_V \frac{V}{nR} dP$$

$$C_P P dV + C_V V dP = 0$$

Dividing by $C_V PV$ and putting $\dfrac{C_P}{C_V} = \eta$

$$\eta \frac{dV}{V} + \frac{dP}{P} = 0$$

Integrating, we get

$$\eta \ln V + \ln P = 0 \text{ const.}$$
$$\ln V^\eta + \ln P = 0 \text{ const.}$$
$$\ln (PV^\eta) = 0 \text{ const.}$$
$$PV^\eta = \text{const.}$$

(c) **The temperature of an ideal gas at initial pressure P_1 and volume V_1 is increased isochorically until the pressure is doubled. The gas is then expanded isothermally until the pressure drops to original value. Then it is compressed isobarically until volume returns to initial value. Sketch this process in the P-V and P-T plane. Calculate the total work done if $n = 2$ kilomoles, $P_1 = 10^5$ Pascal, $V_1 = 2m^3$.**

Ans. Since system has returned to its initial state $(P_1 V_1)$, the work done will be zero.
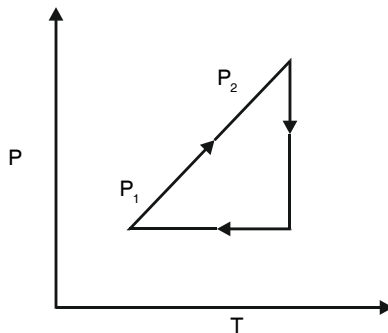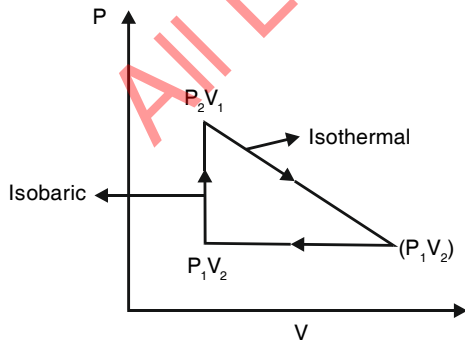
Q. 2. (a) Describe construction and working of Carnot's heat engine. Derive expression for its efficiency in terms of temperatures of its heat reservoirs.

Ans. See back Q. 4., **[Page No. 78]**

sink is 27°C. What is the temperature of the source? In order to raise its efficiency to 50%, what do you think, out of the following two, is more effective way.

> **(i) Decreasing the temperature of ths sink (while the temperature of the source remains the same).**

> **(ii) Increasing the temperature of the source (while the temperature of the sink remains the same)?**

Ans. efficiency of a Carnot's engine $\eta = 40\% = 0.4$

Temp. of sink = 27°C

= 300K

$$\eta = 1 - \frac{T_{sink}}{T_{source}}$$

$$0.4 = 1 - \frac{300}{T_{source}}$$

$$T_{source} = 500K$$

$$\eta = 1 - \frac{T_{sink}}{T_{source}}$$

In order to increase efficiency, decrease the temp. of sink while keeping stamp of source constant, as in formula $\frac{T_{sink}}{T_{source}}$ would get smaller increasing the efficiency.

(c) A heat engine employing Carnot's cycle with an efficiency $\eta = 20\%$ is used as a refrigerating machine, the thermal reservoirs being the same. Find the coefficient of performance of the machine.

Ans. $\eta = 20\% \rightarrow$ efficiency of heat engine.

Coeff. of performance $E = \dfrac{Q_2}{Q_1 - Q_2}$

$$E = \frac{Q_2 / Q_1}{1 - \dfrac{Q_2}{Q_1}} = \frac{1 - \eta}{\eta}$$

$$E = \frac{1}{\eta} - 1 = \frac{1}{0.2} - 1$$

$$E = 5 - 1 = 4$$

thermodynamics on the basis of the entropy.

Ans. See back Q. 1.**[Page No. 86]**

(*b*) **Consider that an ideal gas intially confined in a volume at any given temperature is allowed to expand and the whole arrangement is thermally insulated. Is it possible not to have any thermodynamic work even though there is expansion in its volume? Justify your answer and give an example. Find the entropy change of an ideal gas undergoing such a process.**

Ans. See back Q. 8., **[Page No. 92]**

Latent heat of ice = 3.35×10^3 J kg^{-1}

Latent heat of steam = 2.26×10^6 J kg^{-1}

Specific heat of steam = 2×10^3 J kg^{-1} K^{-1}

Ans. m = 3g = 0.003 kg

$C_w = 4180$ J kg^{-1} K^{-1} $T_1 = 0°C$

$C_s = 2 \times 10^3$ J kg^{-1} K^{-1} $T_2 = 100°C$

$L_i = 3.35 \times 10^5$ J Kg^{-1} $T_3 = 140°C$

$L_s = 2.26 \times 10^6$ J kg-1

Change in entropy δS

$$= \frac{mLi}{T_1} + mC_w 2.303 \log \frac{T_2}{T_1} + \frac{mL_s}{T_2} + mC_s 2.303 \log \frac{T_3}{T_2}$$

$$= m\left[\frac{3.35 \times 10^5}{273} + 4180 \times 2.303 \log\left(\frac{373}{273}\right) + \frac{2.26 \times 10^6}{373} + 2 \times 10^3 \right.$$

$$\left. \times 2.303 \log\left(\frac{413}{373}\right) \right]$$

$\delta S = 11.372$ J/K

Q. 4. (*a*) Describe how the process of Adiabatic demagnetisation leads to cooling in paramagnetic salt.

Ans. Magnetic cooling is one of the efficient methods of cooling objects. It capitalizes on the relationship between the applied magnetic field effects and the entropy of a material. Adiabatic demagnetization comes under magnetic cooling, exploiting paramagnetic properties to cool some materials down. It is based on the fact that the entropy of paramagnetic materials is lower in the magnetic field. The magnetic regions aligned in the paramagnetic field originate lower entropy. Thus, randomness is less in the presence of a magnetic field, and hence substance can reach a temperature below one Kelvin.

the partial derivatives $\left(\dfrac{\partial M}{\partial B}\right)_T$, $\left(\dfrac{\partial B}{\partial T}\right)_M$, $\left(\dfrac{\partial T}{\partial M}\right)_B$, **satisfy the cyclic relation.**

Ans. equation of state of paramagnetic material

$$M = \frac{kB}{T} \qquad \qquad ...(1)$$

$$\left(\frac{\partial B}{\partial T}\right)_T = \frac{k}{T} \; : \; \left(\frac{\partial T}{\partial M}\right)_B = \frac{-Bk}{M^2}$$

$$\left(\frac{\partial B}{\partial T}\right)_M = \frac{M}{k}$$

For cyclic relation,

$$\left(\frac{\partial M}{\partial B}\right)_T \left(\frac{\partial B}{\partial T}\right)_M \left(\frac{\partial T}{\partial M}\right)_B = -1$$

L.H.S.

$$\left(\frac{\partial M}{\partial B}\right)_T \left(\frac{\partial B}{\partial T}\right)_M \left(\frac{\partial T}{\partial M}\right)_B = \frac{k}{T} \times \frac{M}{K} \times \frac{-Bk}{M^2}$$

$$= \frac{-Bk}{T} \times \frac{1}{M}$$

$$= -M \times \frac{1}{M} \text{ (from equ. 1)}$$

$$= -1 = \text{R.H.S.}$$

(c) Show that

(i) $dH = C_p\, dT + V\,(1 - aT)\, dP$

Ans. Consider H = H (P.T)

$$dH = \left(\frac{\partial H}{\partial T}\right)_P dT + \left(\frac{\partial H}{\partial P}\right)_T dP$$

Also, $\qquad \qquad dH = T\, dS + V dP$

$$\left(\frac{\partial H}{\partial P}\right)_T = T\left(\frac{\partial S}{\partial P}\right)_T + V$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

Hence

$$\left(\frac{i\partial H}{\partial P}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_P$$

$$\therefore \qquad dH = C_P dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right]dP$$

$$d = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = dV$$

$$\therefore \qquad dH = C_p dT + [V - \alpha VT]dP$$

$$dH = C_p dT + V[1 - dT]dP$$

(ii) $dF = -(PaV + S)dT + P\beta V dP$

a **denotes volume expansivity and** β **denotes compressibility.**

Ans. $F = U - TS$

Consider $F = F(T, P)$

$$dF = \left(\frac{\partial F}{\partial T}\right)_P dT + \left(\frac{\partial F}{\partial P}\right)_T dP \qquad \text{...(1)}$$

Also,

$$dF = -SdT - PdV$$

$$\left(\frac{\partial F}{\partial T}\right)_P = -S - P\left(\frac{\partial V}{\partial T}\right)_F$$

Since

$$a = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \alpha V$$

So

$$\left(\frac{\partial F}{\partial T}\right)_P = -S - P\alpha V \qquad \text{...(2)}$$

Now,

$$\left(\frac{\partial F}{\partial P}\right)_T = 0 - P\left(\frac{\partial Y}{\partial P}\right)_T$$

Since
$$\beta = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \Rightarrow \left(\frac{\partial V}{\partial P}\right)_T = -V\beta$$

So,
$$\left(\frac{\partial F}{\partial P}\right)_T = PV\beta \qquad \qquad ...(3)$$

Substitute (2), (3) in (1)

$$dF = -(S + P\alpha V)\,dT + PVb\beta P$$

Q. 5. (a) Describe how the distribution of molecular velocities would be like for an ideal gas confined in a volume at a constant temperature and pressure. Consequently, derive the law which explains the molecular velocities on the basis of the kinetic theory of gases.

Ans. See Q. 7., **[Page No. 121]**

(b) Given that the molar mass of a gas is 44 g mol^{-1}, calculated the temperature at which most molecules in a given volume attain velocity equal to 355 m s^{-1}.

Ans. Molar Mass of gass = 44g mol^{-1}

$$= 0.044\text{kg mol}^{-1}$$

$$\text{Velocity} = \sqrt{\frac{3RT}{M}}$$

$$\text{Given velocity} = 355 \text{ ms}^{-1}$$

$$R = 8.31 \text{ S.I. unit}$$

$$(355)^2 = \left(\sqrt{\frac{3 \times 8.31 \times T}{0.044}}\right)^2$$

$$\frac{(355)^2 \times 0.044}{3 \times 8.31} = T$$

$$T = 222.4 \text{ K}$$

(c) Discuss what possible phenomena occurs when molecules of a gas at any temperature are moving with a range of velocities in a given volume under non equilibrium conditions. Discuss briefly about Brownian motion and its significance.

Ans. See Q. 8., **[Page No. 132]**

Q. 6. (a) Define critical temperature (T_C), critical pressure (P_C) and critical volume (V_C). Using van der Waa's equation of state, find their expressions in terms of van der Waal's constants 'a' and 'b'. Hence prove that for real gases,

$$\frac{RT_C}{\quad} \quad \frac{a}{\quad}$$

liquefied by increase of pressure alone.

Critical Pressure (Pc) and Critical Volume (Vc) : The pressure necessary to liquefy a gas at the critical temperature is called critical pressure (Pc) and the volume occupied by a unit mass of the gas at critical temperature is called critical volume.

Critical constants from Van der Waal's equation. Van der Waal's equation states

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

where P, V and T are the pressure, volume and absolute temperature of the gas respectively and a, b and R are constants. On expanding, we get

$$PV - Pb + \frac{a}{V} - \frac{ab}{V^2} = RT$$

Multiply by 1/2, we have

$$PV^3 - PbV^2 + aV - ab = V^2RT$$

Rearranging the terms, we have

$$PV^3 - V^2(Pb + RT) + aV - ab = 0$$

Dividing by P, we get

$$V^3 - V^2\left(b + \frac{RT}{P}\right) + V\frac{a}{p} - \frac{ab}{P} = 0 \qquad \qquad ...(i)$$

This is a cubic equation in V and has three roots for a given value or P and T. At the critical point all the three roots will be equal since a gas has only one value of volume, pressure and temperature corresponding to it.

Let x be the value of this critical volume, then

$$V = x \text{ or } V - x = 0$$

$$\therefore \qquad \qquad (V - x)^3 = 0$$

or $$V^3 - V^2.3x + V.3x^2 = 0 \qquad \qquad ...(ii)$$

Comparing the coefficients of $V3$, $V2$, V and the constant terms in (i) and (ii), we have

$$3x = b + \frac{RT}{P} \qquad \qquad ...(iii)$$

$$3x^2 = \frac{a}{p} \qquad \qquad ...(iv)$$

and
$$x^3 = \frac{ab}{p} \qquad \text{...(v)}$$

Dividing (v) by (iv), we get,

$$\frac{x}{3} = \frac{ab}{p} \times \frac{P}{a} = b$$

or
$$x = 3b \qquad \text{...(vi)}$$

$\therefore$ **Critical volume** $\qquad V_c = 3b$

Substituting the value of x in equation (iv), we have

$$3 \times 9b^2 = \frac{a}{P} \text{ or } P = \frac{a}{27b^2} \qquad \text{...(vii)}$$

$\therefore$ Critical pressure $\qquad P_c = \frac{a}{27b^2}$

Substituting the value of x and P in (iii), we have

$$3 \times 3b = b + \frac{RT}{a} \times 27b^2$$

$\therefore$
$$8a = 27b \, RT \text{ or } T = \frac{8a}{27Rb} \qquad \text{...(viii)}$$

$\therefore$ **Critical temperature** $\qquad T_c = \frac{8a}{27Rb}$

(b) Critical volume $\qquad V_c = 3b$

Critical pressure $\qquad P_c = \frac{a}{27b^2}$

Critical temperature $\qquad T_c = \frac{a}{27Rb}$

$\therefore$
$$\frac{P_c . V_c}{T_c} = 3b . \frac{a}{27b^2} . \frac{27Rb}{8a} = \frac{3}{8}R \qquad \text{...(ix)}$$

or
$$R = \frac{8}{3} \frac{P_c V_c}{T_c}$$

(b) The volume occupied by a gas at 300 K is 1.0 lit/mole. Compare the corresponding pressures considering the gas to be

(i) An ideal gas

Ans. V = 1.0 lit/mole.   T = 300K

Van der Waal's gas.

$$\left(P + \frac{a}{V^2}\right)(V-b) = RT$$

∴
$$P = \frac{RT}{(V-b)} - \frac{a}{V^2}$$

$$P = \frac{0.0821 \times 300}{1 - 0.312} - \frac{1.32}{1^2}$$

$$P = 34.95 \text{ atm}$$

Ideal gas

$$PV = RT$$

$$P = \frac{RT}{V}$$

$$= \frac{0.08206 \times 300}{1} \left\{ \begin{array}{l} R = 0.05203 \\ \text{lit atm/mol}^{-1} \end{array} \right\}$$

$$P = 24.61 \text{ atm}$$

(c) Express van der Waal's equation of state for 1 mole of a real gas in Virial form. Hence obtain the expression for the second Virial coefficient and Boyle's temperature of the gas.

Ans. See back Q. 6., **[Page No. 139]**, Q. 7., **[Page No. 141]**, and Q. 9., [P**age No. 146].**

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