Name of the Paper : Elements of Modern Physics
Name of the Course : B.Sc. (Hons.) Physics
Semester : IV (CBCS) Part – II
Duration : 3 hours
Maximum Marks : 75

2022

Attempt any four questions.
All questions carry equal marks.

Q. 1. Attempt any five parts :

(*a*) If $f = X^n$, show that f is an eigen function of the operator $x\left(\frac{d}{dx}\right)$. Also find the eigenvalue.

Sol.
$$\frac{xdf}{dx} = \frac{x\,d(x^n)}{dx}$$
$$= x \times nx^{n-1}$$
$$= n\,x^n$$
$$= nf$$

So, $f = x^n$ is an eigen function of sperator.

Eigen Value $= n$

(*b*) David Beckham takes a free-kick of a football of mass 400g. The curving ball mowith a velocity of 170km/hr while reaching the goalpost. Find the deBroglie wavelength associated with the ball at that time. Will this wavelength have any physical significance for the goalkeeper facing the free-kick?

Sol. $m = 400\text{g}$, Velocity = 170 km/hr

$m = \frac{400}{1000} = 0.4\ kg$ Velocity $= 170 \times \frac{5}{18}$ m/sec.

$= \frac{425}{9}$ m/sec.

deBroglie Wavelegth $= \lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{0.4 \times 425} \times 9$

$$= 0.35 \times 10^{-34}$$
$$= 3.5 \times 10^{-35}$$

The deBroglie wavelength is very small.

our of football. So, there is no physical significance for the goalkeeper.

(c) Can the following two functions be physically acceptable solution of the Schrodinger wave equation

(i) (A/2) tan (x)

(ii) (3/2C) sin (x)

where A and C are non-zero constants.

Sol. (i) $\frac{A}{2}\tan x$

$\tan x$ is infinite when $x = \frac{\pi}{2}$. So, this is not a physically acceptable solution of schrodinger wave equation.

(ii) $\frac{3}{2C}\sin x$

Sin x is definite, continuous and finite everywhere but it does'nt to zero when x ® ∞. So this is not physically acceptable.

(d) A 60 pm X-ray is incident on a calcite crystal. Find the wavelength of the X-rays scattered through an angle of 30°. What is the largest shift in wavelength that can be expected in this experiment?

Sol. Compton shift $= \Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$

$$\phi = 30$$

$$\Delta\lambda = \frac{6.6 \times 10^{-34}\ (1 - \cos 30)}{9.1 \times 10^{-31} \times 3 \times 10^{8}}$$

$$= \frac{6.6 \times 10^{-3} \times 10^{-8}\ (1 - \frac{\sqrt{3}}{2})}{9.1 \times 3}$$

$$= \frac{6.6 \times 0.134 \times 10^{-11}}{27.3}$$

$$\Delta\lambda = 0.32 \times 10^{-12}\text{m}$$

The largest shift is when $\cos\phi = 0$ or $\phi = 90^\circ$ and is given by

$$\Delta\lambda = \frac{h}{mc} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}}$$

$$\Delta\lambda = 2.4 \text{ pm}$$

(*e*) Find the combined kinetic energy of an electron and antineutrino, when free neutron decays into proton, electron and antineutrino. Given m_n = 1.008984*u*, m_p = 1.00759 *u*, m_e = 0.00055 *u*, *lu* = 1.673 × 10^{-27} kg.

Sol. $${}^{1}_{0}n \Rightarrow {}^{1}_{1}\text{P} + {}^{0}_{-1}e + \bar{\nu}$$

We will use the energy confreraction to do this since nutron is initially rest. So, its kinitic energy is zero

$$\text{E} = mc^2$$

$$m_n c^2 = m_p c^2 + k_p + m_e c^2 + ke + m_\nu c^2 + k\nu$$

$$k_e + k_\nu = \left[m_n c^2 - m_p c^2 - m_e c^2 - m_\nu c^2 + k_p\right]$$

We can ignore the last two terms, since mass of proton is large as compared to e^- and mass of antineutrino is negligible.

$$k_e + k_\nu = \left[m_n c^2 - m_p c^2 - m_e c^2\right]$$

$$= [1.008984 - 1.00759 - 0.00055] \times 1.67 \times 10^{-27} \times (3 \times 10^8)^2$$

$$= 0.00422844 \times 10^{-11}$$

$$k_e + k_\nu = 0.01268532 \times 10^{-11} \text{J}$$

(*f*) If ^{235}U loses 0.1% of its mass on undergoing fission, then how much energy is released when 1 Kg of ^{235}U undergoes fission?

Sol. Mass of uranium changed into energy

$$= 0.1\% \text{ of } 1 \text{ kg}$$

$$= 0.001 \text{ kg}$$

So, Energy corresponding to this energy as per einstein's relation is,

$$\text{E} = mc^2 = 0.001 \times (3 \times 108)^2$$

$$= 9 \times 10^{13} \text{ J}$$

(*g*) Why stimulated emission is necessary for laser action? (5 × 3)

Sol. See Q. 2. **[Page No. 116].**

Q. 2. (*a*) The work function of potassium in 230 eV. UV light of wavelength 30000A and intensity 2Wm^{-2} is incident on the potassium surface.

(*i*) Determine the maximum kinetic energy of the photo electrons.

(*ii*) If 40% of incident photons produce photo electrons, how many electrons are emitted per second if the area of the potassium surface is

Sol. (*i*) Work function = ϕ = 2.30eV

$$\lambda = 3000\text{A}$$

$$\text{Intensity} = 2\text{w/m}^2$$

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{3000 \times 10^{-10}}$$

$$E = 4.14\text{ev}$$

$$\text{Maximum K.E.} = \text{Kmax} = h\nu - \phi_0$$

$$= 4.14 - 2.3$$

$$= 1.84\text{eV}$$

(*ii*)

$$\lambda = 3000\text{A} = 3000 \times 10^{-10}\text{m}$$

$$\text{Intensity} = 2\text{wm}^{-2}$$

$$\text{Area} = 2\text{cm}^2 = 2 \times 10^{-4}\text{m}^2$$

n_p = no. of phatans rejected

$$n_p = \frac{\text{Total Energy}}{\text{Energy of phaton}}$$

$$= \frac{I \times A}{h\nu} = \frac{I \times A \times \lambda}{hc}$$

$$= \frac{2 \times 10^{-4} \times 2 \times 3000 \times 10^{-10}}{6.6 \times 10^{-34} \times 3 \times 10^{8}}$$

$$= \frac{12 \times 10^{-11} \times 10^{26}}{6.6 \times 3}$$

$$= 0.606 \times 10^{15}$$

$$\text{no of phatoelectrons} = \frac{40}{100} \times 0.606 \times 10^{15}$$

$$= 2.424 \times 10^{14}$$

(*b*) They energy of a free electron including its rest mass energy is 10 MeV. Calculate the group velocity and the phase velocity of the wave packet associated with the motion of this electron.

Sol. Engery = 10 meV

Kinetic energy = T.E – Rist mass energy

$$= 10 \times 10^{6} \times 1.6 \times 10^{-19} - m_0C^2$$

$$= 1.6 \times 10^{-12} - 9.1 \times 10^{-31} \times 9 \times 10^{16}$$

$$= [1.6 - 0.089] \times 10^{-12}$$
$$= 1.5104 \times 10^{-12} \text{J}$$

$$k = \frac{P^2}{2m}$$

$$P = \sqrt{2mk}$$

$$V_g = \frac{P}{m} = \sqrt{\frac{2mk}{m}} = \sqrt{\frac{2k}{m}}$$

$$V_g = \sqrt{\frac{2 \times 1.5 \times 10^{-12}}{9.1 \times 10^{-31}}}$$

$$V_g = \sqrt{0.32 \times 10^{19}} = \sqrt{3.2 \times 10^9}$$
$$V_g = 1.788 \times 10^9 \text{ m/s}$$

$$V_{ph} = \frac{V_g}{2} = \frac{1.788 \times 10^9}{2}$$

$$V_{ph} = 0.894 \times 10^9 \text{ ms}$$

(*c*) Deduce the Heisenberg's uncertainty principle for position and momentum from gamma ray microscope thought experiment.

(5 + 5 + 5)

Sol. See Q. 3 (*a*), **[Page No. 221].**

Q. 3. (*a*) Explain why it is plausible to define probability current density in quantum mechanics by the following expression

$$\mathbf{J = (-i\hbar / 2m)\ (\psi^* \text{ grad } \psi - \psi \text{ grad } \psi^*)}$$

The symbols have usual meaning.

Sol. The expression of prabability current density comes from the congeration of probability.

Let's take the time dirivative of $\langle \psi(t) | \psi(t) \rangle$

$$\frac{d}{dt}\langle \psi(t) | \psi(t) \rangle = \left(\frac{d}{dt} < \psi(t)|\right) | \psi(t) > + < \psi(t) | \left(\frac{d(\psi(t) >}{dt}\right) \quad ...(1)$$

$$\frac{d | \psi(t) >}{dt} = \frac{-i}{\hbar} \hat{H} | \psi(t) >$$

$$\frac{d}{dt} < \psi(t) | = \frac{i}{\hbar} < \psi(t) | \hat{H}^{+} = \frac{i}{\hbar} < \psi(t) | \hat{H}$$

Put these two in (1)

$$\frac{d}{dt} < \psi(t) | \psi(t) > = \left(\frac{i}{\hbar} - \frac{i}{\hbar}\right) < \psi(t) | \hat{H} | \psi(t) >$$

$$= 0 \qquad \text{...(2)}$$

So, probability does not with time

Now, time dependent schrodingers is

$$\frac{i\hbar\partial\psi}{dt} = \frac{-\hbar}{2m}\nabla^2\psi + v\psi \qquad \text{...(3)}$$

and its complex congugate

$$\frac{i\hbar\partial\psi}{dt} = \frac{-\hbar}{2m}\nabla^2\psi^* + v\psi^* \qquad \text{...(4)}$$

Multiply (3) by ψ^* and (4) by ψ and

Subtracting the two gives

$$\frac{i\hbar d}{dt}[\psi^*\psi] = \frac{\hbar^2}{2m}\left[\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right]$$

So, $\frac{\partial(\vec{r},t)}{dt} + \vec{\nabla}.\vec{J} = 0$

$$J = \psi^*(r, t)\, \psi(r,t) \quad \&$$

$$J = \frac{-i\hbar}{2m}[\psi^*\nabla\psi - \psi\nabla\psi^*]$$

(*b*) Name and explain an electron differaction experiment. Give the physical significance of this experiment in relation to the wave particle duality. (10 + 5)

Sol. See Q. 5 **[Page No. 67].**

Q. 4. (*a*) Explain nuclear binding energy and packing fraction. Discuss graphically the variation of average binding energy per nucleon with mass number. A and hence explain nuclear stability and phenomena of fusion and fission.

Sol. See Q. 1. **[Page No. 101].**

(*b*) Calculate the binding energy per nucleon of $_{26}Fe^{56}$ in MeV using semi-empirical mass formula. Given

$$a_1 = 1.41 \text{ MeV}, \; a_2 = 13.0 \text{ MeV}, \; a_3 = 0.595 \text{ MeV},$$

Sol. According to semi empirical

Mass formula

$$B(A, Z) = a_1 A - a_2 A^{2/3} - a_3 Z(Z-1)A^{-1/3}$$

$$- a_4 \frac{(A-2Z)^2}{A} \delta a_5 A^{-3/4}$$

$$A = 56, \quad Z = 26$$

$$a_1 = 1.41 \text{ MeV}, \; a_2 = 13.0 \text{ MeV}, \; a_3 = 0.595 \text{ MeV},$$

$$a_4 = 19.0 \text{ MeV}, \; a_5 = 33.5 \text{ MeV}, \quad \delta = 1$$

Put all the values and get answer.

Q. 5. A particle of mass m is confined in a field free region between impenetrable walls at $x = 0$ and $x = L$.

(a) Obtain an expression for energy of the particle.

(b) Obtain and draw the first three normalized wave functions.

(c) Find the minimum energy of the particle with mass 9.1×10^{-31} kg for L = 1 A. **(5 + 5 + 5)**

Sol. See Q. 1. **[Page No. 86].**

Q. 6. (a) Given the half life of 210P0 is 138 days, find

(i) the decay constant of Po.

(ii) the activity of 1 g of Po.

(iii) how many decays per second occur when the sample is one week old.

Sol. $\frac{\pi}{2} = 138$ days

(i) $$\frac{\pi}{2} = \frac{0.693}{\lambda}$$

λ = Decay constant

$$\lambda = \frac{0.693}{\frac{\pi}{2}}$$

$$\lambda = \frac{0.693}{138} = 0.00502 \,\text{day}^{-1}$$

(ii) $$A = \frac{-dN}{dt} = N\lambda \qquad \lambda = \frac{\ln 2}{138}$$

210 g will have 6.023×10^{23} atom

$$1 \text{ g will have } \frac{6.023 \times 10^{23}}{210} \text{ atm}$$

$$= 28.68 \times 10^{20}$$

$$A = \frac{\ln 2}{138 \times 24 \times 60 \times 60} \times 28.68 \times 10^{20}$$

$$A = 16.15 \times 10^{13}\, dpis$$

(*b*) What are the main difference among, alpha, beta and gamma decay?

Sol. See Q. 5. **[Page No. 106].**

(*c*) Name and explain which conservation laws seemed to be violated in beta decay. How did Pauli resolve these discrepancies? (5 + 5 + 5)

Sol. It was expected that the same consideration would hold for a parent nucleus breaking down to a daugther nucleus and a beta particle. Becuase only the electron and the recoiling daughter nucleus were observed in beta decay, the process was initially assumed to be a tow-body process, much like alpha decay. It would seem resonable to suppose that the beta particles aslo from a monoenergetic beam.

But the reality was diferent. The spectum of beta particles showed mutiple line on a diffuse background. Moreover, virtually all of the emitted beta particles have nergies below that predicted by energy conservation that appears to contradict the conservation of energy, under the then-current assumption that beta decay is the simple emission of an electron from a nucleus. When this was first observed, it threatened the survival of one of physics's most important conservation laws! To account for this energy release, Pauli proposed that there was emited in the decay process another particle, later named by Fermi the neutrino.

Q. 6. (*a*) What are the assumption made in liquid drop model of atomic nucleus? How do asymmetry and pairing of the nucleons affect the nuclear stability?

Sol. See Q. 2. **[Page No. 103].**