B.Sc. Physics (Waves and Optics)

Solved paper – 2018

1. Attempt any **FIVE** parts from the following :

(a) Calculate the minimum intensity of audibility in watt per sq.cm. for a note of 1000 c.p.s. If the amplitude of vibration is 10^{-9} cm. Assume the density of air = 0.0013 gm/c.c and velocity of sound = 340 m/sec.

(b) If the phase velocity is given by, $v_p = \left(\frac{2\pi S}{\rho\lambda}\right)^{1/2}$ (Here, S and ρ are constant), then find the expression of group velocity.

(c) Give three differences between travelling waves and stationary waves.

(d) What is meant by the term reverberation?

(e) What do you understand by wave front?

(f) Name one experiment each, which is based on division of wave front and division of amplitude.

(g) What is the highest order spectrum which may be seen with monochromatic light of wavelength 5000 A° by means of diffraction grating with 5000 lines/cm.

(h) Write two conditions for observing a sustained interference pattern. (5×3)

2. (a) What are Lissajous Figures? For the cases mentioned below, give the graphical or analytical representation of the Lissajous Figures (with direction) for the motion of a particle which is subjected to two perpendicular simple harmonic motions given by,

$$x = 3 \cos(\omega t)$$
$$y = 2 \cos(2\omega t + \alpha)$$

Case (i) $\alpha = 0$

Case (ii) $\alpha = \pi/2$

(b) Prove that the principle of superposition holds for linear homogenous differential equation of ruler two.

(10+5)

3. (a) Derive the expression for the differential equation of transverse vibrations of a uniform flexible stretched string fixed at the ends, $x = 0$ and $x = l$. Also find the expression for the velocity of transverse waves.

(b) Draw the shapes of first two modes of a stretched string. (11,4)

4. (a) Explain plane polarized, circularly polarized and elliptically polarized light? How can we analyze circularly polarized light?

(b) Derive an expression for the intensity of sound wave travelling in still air. (10,5)

5. (a) Describe briefly the construction of Michelson's interferometer. How it can be used to measure the (i) wavelength of a monochromatic light and (ii) refractive index of a thin transparent sheet.

(b) Show the formation of interference fringes due to Fresnel's biprism with the help of diagram. (12,3)

6. (a) Derive the expression for intensity distribution in case of Fraunhofer diffraction due to single slit.

(b) Find the positions of secondary minima and secondary maxima. (10,5)

7. What is zone plate and how is it made? Explain how a zone plate acts like a convergent lens having multiple foci. Derive an expression for its focal length. (15)

8. (a) Derive the expression for diameter of the Newton's ring pattern for reflected mode. How would you use Newton's rings to measure the wavelength of light?

(b) In Newton's ring experiment, the diameter of 10^{th} bright ring changes from 1.50 cm to1.25 cm when a liquid is introduced between the plate and the lens. Calculate the refractive index of the liquid. (12,3)

Q 1(a)

$f = 1000$ c.p.s

amplitude $= 10^{-9}$ cm

$\rho_{air} = 0.0013$ gm/cc.

$V_{sound} = 340$ m/sec.

$$I = 2\pi^2 \rho f^2 v \Delta s^2$$

$$= 2 \times (3.14)^2 \times 0.0013 \times 10^{-3} \times 10^{6} \times (1000)^2 \times 340 \times 10^{-22}$$

$$= 8.71 \times 10^{-13} \text{ watt}/m^2$$

$$\boxed{I = 8.71 \times 10^{-9} \text{ watt per sq. cm.}}$$

Que1(b)

Que: 1(b) $V_p = \left(\frac{2\pi S}{\rho\lambda}\right)^{1/2}$

S and ρ are constants

Now $V_g = V_p - \lambda \frac{dV_p}{d\lambda}$

$$= \left(\frac{2\pi S}{\rho\lambda}\right)^{1/2} - \lambda\left(\frac{2\pi S}{\rho}\right)^{1/2}\left(-\frac{1}{2}\frac{1}{\lambda^{3/2}}\right)$$

$$= \left(\frac{2\pi S}{\rho}\right)^{1/2}\left[\frac{1}{\lambda^{1/2}} + \frac{1}{2}\lambda^{1/2}\right]$$

$$\boxed{V_g = \frac{3}{2}\left(\frac{2\pi S}{\rho\lambda}\right)^{1/2}}$$

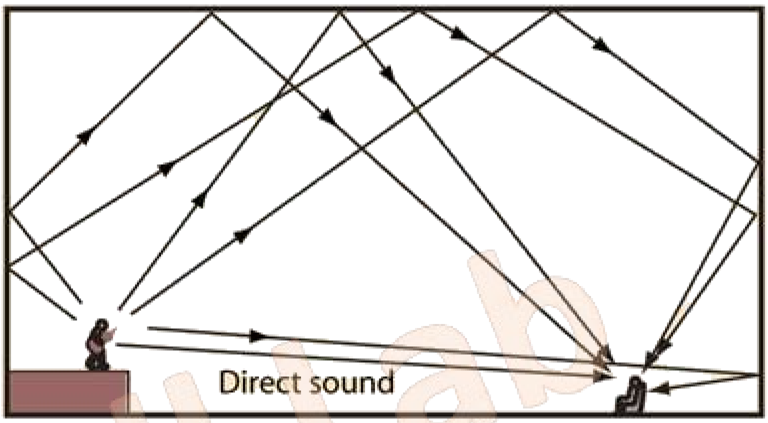
Que 1(c)

Travelling waves	**Standing waves**
Progressive waves or transverse waves	Stationary waves or longitudinal waves
a wave in which the medium moves in the direction of propagation	a vibration of a system in which some particular points remain fixed while others between them vibrate with the maximum amplitude

Move from place to place	Occur in confined space
Transport energy from a place to another.	Do not transport energy however there is energy associated with
Wave motion is perpendicular to the direction of the wave.	Wave motion is parallel to the direction of the waves
Wavelength is the distance from	Wavelength is distance from one

one crest to another crest.	compression to another compression
Can have any frequency	Frequency is quantized

Que 1(d) Reverberation Reverberation is the collection of reflected sounds from the surfaces in an enclosure like an auditorium. It is a desirable property of auditoriums to the extent that it helps to overcome the inverse square law dropoff of sound intensity in the enclosure. However, if it is excessive, it makes the sounds run together with loss of articulation - the sound becomes muddy, garbled. To quantitatively characterize the reverberation, the paramater called the reverberation time is used.



Que1(e)

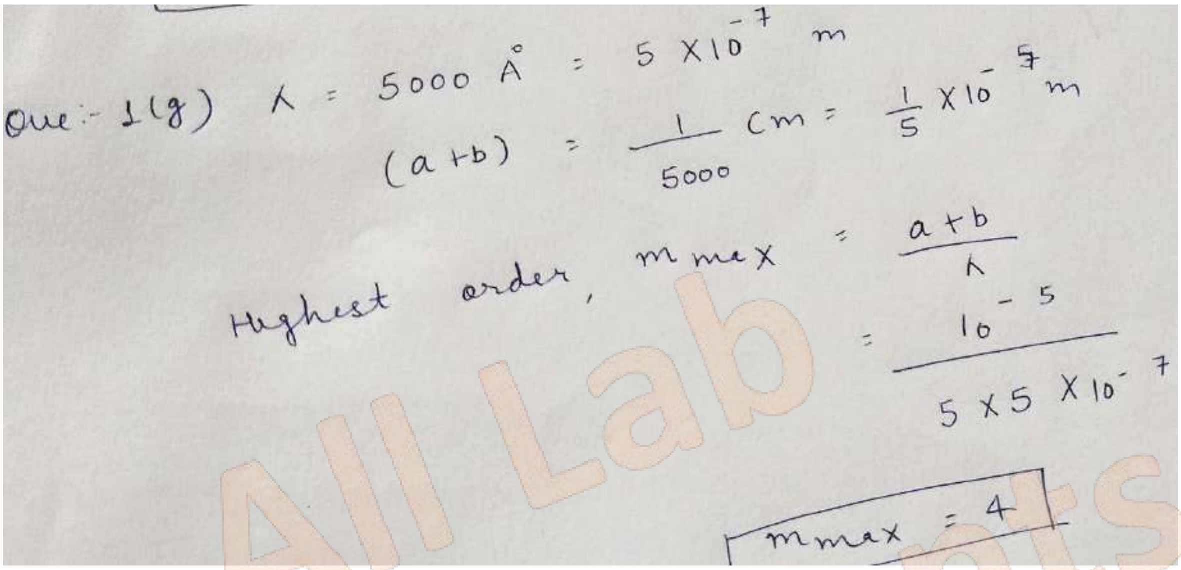
Ans: Wave front - A wave front is a surface over which an optical wave has a constant phase. For example, a wavefront could be the surface over which the wave has a maximum (the crest of a water wave, for example) or a minimum (the trough of the same wave) value. The shape of a wave front is usually determined by the geometry of the source. A point source has wave fronts that are spheres whose centers are at the point source. A fluorescent tube would have wave fronts that are cylinders concentric with the tube itself. A very large sheet of material that is uniformly illuminated would generate wave fronts that are plane waves parallel to the sheet.

The direction of propagation of the wave is always perpendicular to the surface of the wave front at each point. Thus, the wavefronts of a point source are spheres and the wave propagates radially outward – the radius of a sphere is perpendicular to its circumference at each point. The same thing is true of the radius of the cylindrical wavefronts that would be generated by a fluorescent tube.

Que 1(f) Ans: The phenomenon of interference may be grouped into two categories: **Division of Wave front**: Under this category, the coherent sources are obtained by dividing the wave front, originating from a common source, by employing mirrors, biprisms or lenses. This class of interference requires essentially a point source or a narrow- slit source. The instruments used to obtain interference by division of wave front are the Fresnel biprism, Fresnel mirrors, Lloyd's mirror, lasers, etc. **Division of Amplitude**: In this method, the amplitude of the incident beam is divided into two or more parts either by partial reflection or refraction. Thus, we have coherent beams produced by division of amplitude. These beams travel different paths and are finally brought together to produce interference.

The effects resulting from the superposition of two beams are referred to as two beam interference and those resulting from superposition of more than two beams are referred to as multiple beam interference. The interference in thin films, Newton's rings, and Michelson's interferometer are examples of two beam interference and Fabry-Perot's interferometer is an example of multiple beam interference.

Que 1(g)



Que:- 1(g) $\lambda = 5000\ \mathring{A} = 5 \times 10^{-7}$ m

$(a+b) = \frac{1}{5000}$ cm $= \frac{1}{5} \times 10^{-5}$ m

Highest order, $m_{max} = \frac{a+b}{\lambda}$

$= \frac{10^{-5}}{5 \times 5 \times 10^{-7}}$

$\boxed{m_{max} = 4}$

Que 1(h)

Ans: Conditions for sustained interference of light

To obtain well defined interference patterns, the intensity at points corresponding to destructive interference must be zero, while intensity at the point corresponding to constructive interference must be maximum. To accomplish this the following conditions must be satisfied.

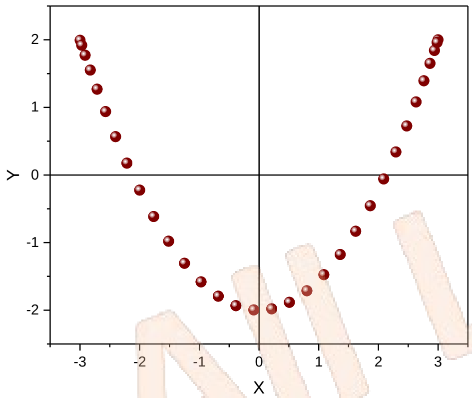
- The two interfering sources must be coherent, that is, they must keep a constant phase difference.
- The two interfering sources must emit the light of the same wavelength and time period. This condition can be achieved by using a monochromatic common original source, that is, the common source emits light of a single wavelength.
- The amplitudes or intensities of the interfering waves must be equal or very nearly equal so that the minimum intensity would be zero.

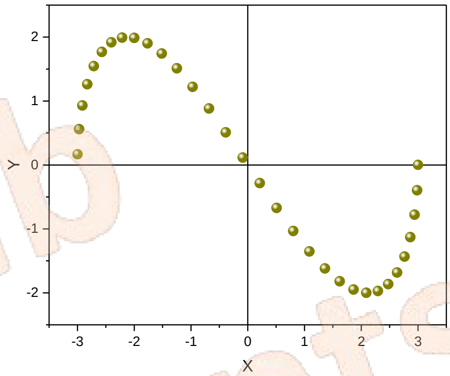
Que 2(a) :

Lissajous figure, also called BOWDITCH CURVE, pattern produced by the intersection of two sinusoidal curves the axes of which are at right angles to each other.

If the frequency and phase angle of the two curves are identical, the resultant is a straight line lying at 45° (and 225°) to the coordinate axes. If one of the curves is 180° out of phase with respect to the other, another straight line is produced lying 90° away from the line produced where the curves are in phase (i.e., at 135° and 315°).

Otherwise, with identical amplitude and frequency but a varying phase relation, ellipses are formed with varying angular positions, except that a phase difference of 90° (or 270°) produces a circle around the origin. If the curves are out of phase and differing in frequency, intricate meshing figures are formed.





Que 2(b):

Theorem (known as the Principle of Superposition): Consider the second-order, linear, homogeneous ordinary differential equation

$$p(x)y''(x) + q(x)y'(x) + r(x)y(x) = 0. \qquad (*)$$

If y_1 and y_2 are both solutions to $(*)$, then for any two constants c_1 and c_2,

$$y = c_1y_1 + c_2y_2$$

is also a solution to $(*)$.

Proof: The fact that y_1 and y_2 are solutions to $(*)$ imply that

$$p(x)y_1''(x) + q(x)y_1'(x) + r(x)y_1(x) = 0 \qquad \text{and} \qquad (1)$$
$$p(x)y_2''(x) + q(x)y_2'(x) + r(x)y_2(x) = 0. \qquad (2)$$

Since c_1 and c_2 are constants, we have

$$y'(x) = c_1y_1'(x) + c_2y_2'(x), \qquad \text{and}$$
$$y''(x) = c_1y_1''(x) + c_2y_2'(x).$$

Inserting these into $(*)$, we see that

$$p(x)y''(x) + q(x)y'(x) + r(x)y(x) = p(x)(\overbrace{c_1y_1''(x) + c_2y_2'(x)}^{y''(x)}) + q(x)(\overbrace{c_1y_1'(x) + c_2y_2'(x)}^{y'(x)})$$
$$+ r(x)(\underbrace{c_1y_1(x) + c_2y_2(x)}_{y(x)}) \qquad (3)$$

We now regroup the terms in (3) by those terms with c_1's and c_2's:

$$\begin{aligned} p(x)y''(x) + q(x)y'(x) + r(x)y(x) &= c_1p(x)y_1''(x) + c_2p(x)y_2'(x) + c_1q(x)y_1'(x) + c_2q(x)y_2'(x) \\ &\quad + c_1r(x)y_1(x) + c_2r(x)y_2(x) \\ &= c_1\left[p(x)y_1''(x) + q(x)y_1'(x) + r(x)y_1(x)\right] \\ &\quad + c_2\left[p(x)y_2''(x) + q(x)y_2'(x) + r(x)y_2(x)\right] \qquad (4) \end{aligned}$$

By equations (1) and (2), the right-hand side of (4) is zero. In other words,

$$p(x)y''(x) + q(x)y'(x) + r(x)y(x) = 0,$$

so that y is a solution to $(*)$.

Que 3(a):

Transverse vibration of a taut string: Referring to Figure 1, consider a taut string stretched between two fixed points at x = 0 and x = L. Let the cross-sectional area be S. If there is an initial stretching of L, the initial tension T must be

$$T = ES\frac{\Delta L}{L}$$

by Hooke's law, where E is Young's modulus.

Now study the lateral displacement of the string from the initial position. By the law of conservation of transverse momentum, the total lateral force on the string element must be balanced by its inertia. Let the lateral displacement be V (x, t) and consider a differential element between x and x+dx. The net transverse force due to the difference of tension at both ends of the element is

$$(T\sin\alpha)_{x+dx} - (T\sin\alpha)_x\,,$$

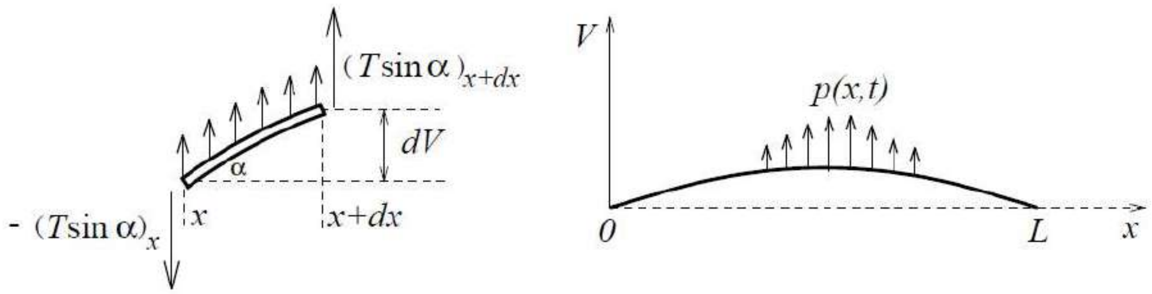


Figure 1: Deformation of a taut string

where

$$\sin\alpha = \frac{dV}{\sqrt{dx^2+dV^2}} = \frac{\frac{\partial V}{\partial x}}{\sqrt{1+(\frac{\partial V}{\partial x})^2}}.$$

We shall assume the displacement to be small everywhere so that the slope is also small: $\frac{\partial V}{\partial x} \ll 1$. The local value of $\sin\alpha$ can then be approximated by

$$\frac{\partial V}{\partial x} + O\left(\frac{\partial V}{\partial x}\right)^3,$$

where the expression $O(\delta)$ stands for *of the order of* δ. For any smooth function f, Taylor expansion gives

$$f(x+dx) - f(x) = \left(\frac{\partial f}{\partial x}\right) dx + O(dx)^2,$$

where the derivative is evaluated at x. Hence the net tension is

$$\frac{\partial}{\partial x}\left(T\frac{\partial V}{\partial x}\right) dx + O(dx)^2.$$

The instantaneous length $\ell(x,t)$ of the string from 0 to x is

$$\ell(x,t) = \int_0^x dx \left[1+\left(\frac{\partial V}{\partial x}\right)^2\right]^{1/2} = x\left[1+O\left(\frac{\partial V}{\partial x}\right)^2\right].$$

It follows that

$$\frac{\ell - x}{x} = O\left(\frac{\partial V}{\partial x}\right)^2 \quad \text{for all} \quad 0 < x < L,$$

which is of second-order smallness. The string length, hence the tension, is essentially unchanged with an error of $O\left(\partial V/\partial x\right)^2$, i.e., T can be taken as constant with a similarly small error. Thus the net tension in the string element is well represented by

$$T\frac{\partial^2 V}{\partial x^2}dx.$$

If the mass per unit length of the string is ρ, the inertia of the element is $\rho(\partial^2 V/\partial t^2)dx$. Let the applied load per unit length be $p(x,t)$. Momentum conservation requires that

$$\rho dx\frac{\partial^2 V}{\partial t^2} = T\frac{\partial^2 V}{\partial x^2}dx + pdx + O\left(dx\right)^2.$$

Eliminating dx and taking the limit of $dx \to 0$, we get

$$\frac{\rho}{T}\frac{\partial^2 V}{\partial t^2} - \frac{\partial^2 V}{\partial x^2} = \frac{p}{T}. \tag{1.1}$$

This equation, called the ***wave equation***, is a partial differential equation of the second order. It is linear in the unknown V and inhomogeneous because of the forcing term on the right-hand side.

Is the longitudinal displacement U important in this problem? Conservation of momentum in the x direction requires that

$$\rho dx\frac{\partial^2 U}{\partial t^2} = \left(T\cos\alpha\right)_{x+dx} - \left(T\cos\alpha\right)_x.$$

Since

$$\cos\alpha = \frac{dx}{\sqrt{(dx)^2 + (dV)^2}} = \frac{1}{\sqrt{1 + \left(\frac{\partial V}{\partial x}\right)^2}} \cong 1 + O\left(\frac{\partial V}{\partial x}\right)^2,$$

the acceleration is of second-order smallness

$$\frac{\rho}{T}\frac{\partial^2 U}{\partial t^2} = O\left(\frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right)^2\right) = O\left(\left(\frac{\partial V}{\partial x}\right)\frac{\rho}{T}\frac{\partial^2 V}{\partial t^2}\right).$$

Hence $U = O\left(\frac{\partial V}{\partial x}\right) V$ by twice integration with respect to t, and the longitudinal displacement can be ignored.

The differential equation (1.1) involves second-order derivatives with respect to both x and t. Two auxilliary conditions are needed for each variable. For example, at the initial instant, we may prescribe both the displacement and the velocity:

$$V(x,0) = f(x) \tag{1.2}$$

and

$$\frac{\partial V}{\partial t}(x,0) = g(x). \tag{1.3}$$

These statements are called the *initial conditions*. In addition we must also specify the *boundary conditions* at the ends. For a string stretched between two fixed ends, we require

$$V(0,t) = 0 \quad \text{and} \quad V(L,t) = 0. \tag{1.4}$$

Together with the partial differential equation, these auxilliary conditions define the initial-boundary-value problem. From the mathematical point of view, it is important to establish whether such a problem is well posed. This question involves the proof for the existence, uniqueness and stability of the solution.

As seen in this example, Taylor expansion is used at almost every step of the derivation. Indeed, it is indispensable not only in deriving governing equations, but also in obtaining approximate solutions of the equations, and in analyzing the physical content of the solution.

Note that the dimension of the coefficient T/ρ is

$$\left[\frac{T}{\rho}\right] = \frac{ML/t^2}{M/L} = \left(\frac{L}{t}\right)^2 = [\text{velocity}]^2 .$$

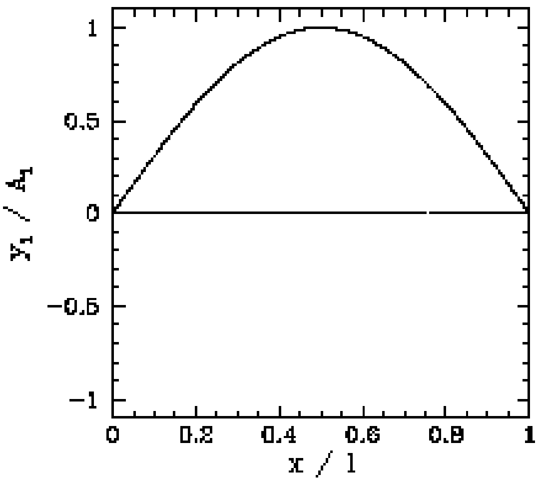
Now introduce the notation $c = \sqrt{T/\rho}$, which is a characteristic velocity of the physical problem. Equation (1.1) can then be written

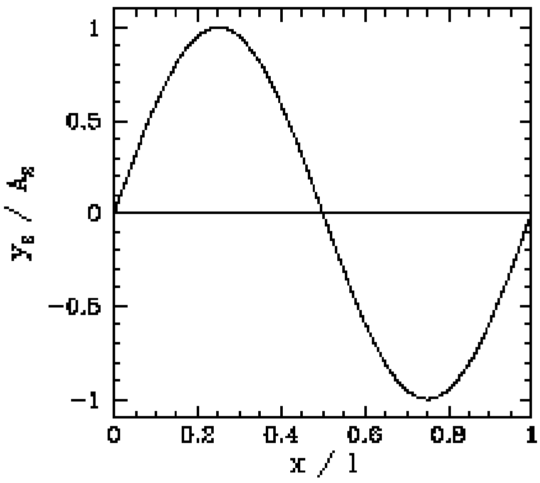
$$\frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} - \frac{\partial^2 V}{\partial x^2} = \frac{p}{T}, \tag{1.5}$$

which is called the *wave equation* arising in numerous contexts.

Que 3(b)

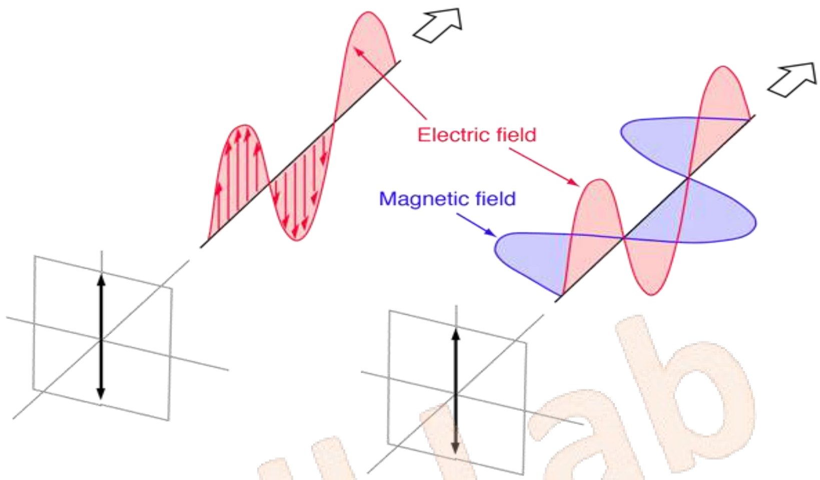
Ans: Figure below shows shapes of first two modes of a streched strings.





Que4(a):

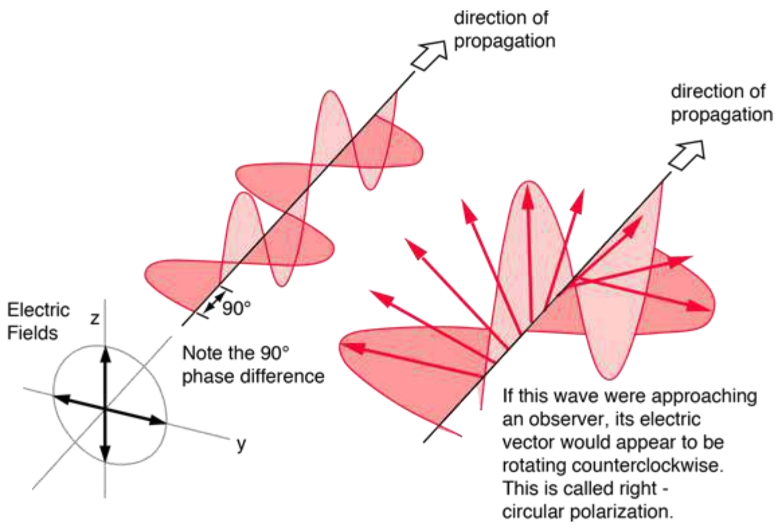
Linear Polarization : A plane electromagnetic wave is said to be linearly polarized. The transverse electric field wave is accompanied by a magnetic field wave as illustrated**.**



Circularly polarized light consists of two perpendicular electromagnetic plane waves of equal amplitude and 90° difference in phase. The light illustrated is right- circularly polarized.

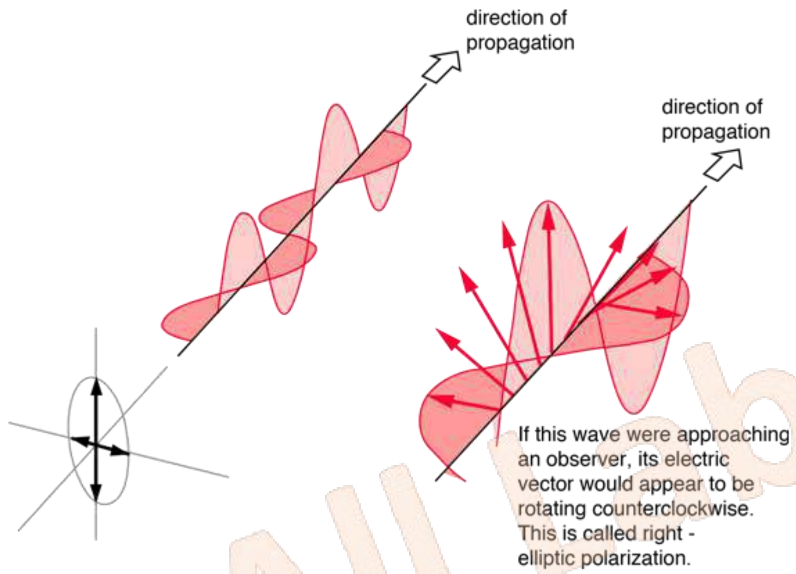
Circular Polarization

If light is composed of two plane waves of equal amplitude but differing in phase by 90°, then the light is said to be circularly polarized. If you could see the tip of the electric field vector, it would appear to be moving in a circle as it approached you. If while looking at the source, the electric vector of the light coming toward you appears to be rotating counterclockwise, the light is said to be right-circularly polarized. If clockwise, then left-circularly polarized light. The electric field vector makes one complete revolution as the light advances one wavelength toward you. Another way of saying it is that if the thumb of your right hand were pointing in the direction of propagation of the light, the electric vector would be rotating in the direction of your fingers.



Elliptical Polarization

Elliptically polarized light consists of two perpendicular waves of unequal amplitude which differ in phase by 90°. The illustration shows right- elliptically polarized light.



If the thumb of your right hand were pointing in the direction of propagation of the light, the electric vector would be rotating in the direction of your fingers.

Detection of circularly polarized light

The quarter wave plate doesn't destroy the polarization - it just changes its character. Circularly polarized light has an electrical vector that rotates without changing its magnitude; if you pass the light through a linear polarizing filter, and rotate the filter, the intensity of the observed light should not change.

Contrast this with the original light from the laser pointer: if you do the same experiment, you will see the intensity after the linear polarizer go from "almost zero", to "almost fully transmitted".

Que4(b):

For simple mechanical waves like sound, intensity is related to the density of the medium and the speed, frequency, and amplitude of the wave

Start with the definition of intensity. Replace power with energy (both kinetic and elastic) over time (one period, for convenience sake).

$$I = \frac{\langle P \rangle}{A}$$

$$I = \frac{\langle E \rangle / T}{A}$$

$$I = \frac{\langle K + U_s \rangle / T}{A}$$

Since kinetic and elastic energies are always positive we can split the time-averaged portion into two parts.

$$\langle P \rangle = \frac{\langle E \rangle}{T}$$

$$\langle P \rangle = \frac{\langle K + U_s \rangle}{T}$$

$$\langle P \rangle = \frac{\langle K \rangle}{T} + \frac{\langle U_s \rangle}{T}$$

Mechanical waves in a continuous medium can be thought of as an infinite collection of infinitesimal coupled harmonic oscillators. Little masses connected to other little masses with little springs as far as the eye can see. On average, half the energy in a simple harmonic oscillator is kinetic and half is elastic. The time-averaged total energy in then either twice the average kinetic energy or twice the average potential energy.

$$\langle P \rangle = \frac{2\langle K \rangle}{T} = \frac{2\langle U_s \rangle}{T}$$

Let's work on the kinetic energy and see where it takes us. It has two important parts — mass and velocity.

$$K = ½mv^2$$

The particles in a longitudinal wave are displaced from their equilibrium positions by a function that oscillates in time and space. Use the one dimensional wave equation for this.

$$\Delta s(x,t) = \Delta s \sin\left[2\pi\left(ft - \frac{x}{\lambda}\right)\right]$$

Where…

$\Delta s(x,t)$ = instantaneous displacement at any position (x) and time (t)

Δs = displacement amplitude

f = frequency

λ = wavelength

π = everyone's favorite mathematical constant

Take the time derivative to get the velocity of the particles in the medium (not the velocity of the wave through the medium).

$$v(x,t) = \frac{\partial}{\partial t}\Delta s(x,t)$$

$$v(x,t) = 2\pi f \Delta s \cos\left[2\pi\left(ft - \frac{x}{\lambda}\right)\right]$$

Then square it.

$$v^2(x,t) = 4\pi^2 f^2 \Delta s^2 \cos^2\left[2\pi\left(ft - \frac{x}{\lambda}\right)\right]$$

On to the mass. Density times volume is mass. The volume of material we're concerned with is a box whose area is the surface through which the wave is traveling and whose length is the distance the wave travels.

$$m = \varrho V = \varrho A \lambda$$

$$K = \int_0^\lambda dK(x,0)$$

$$K = \int_0^\lambda ½(\varrho A\, dx)\, v^2(x,0)$$

$$K = \int_0^\lambda ½(\varrho A)(4\pi^2 f^2 \Delta s^2)\cos^2\left[-2\pi\frac{x}{\lambda}\right] dx$$

Clean up the constants. $½(\varrho A)(4\pi^2 f^2 \Delta s^2) = 2\pi^2 \varrho A f^2 \Delta s^2$

$$\int_0^{\lambda} \cos^2\left[-2\pi \frac{x}{\lambda}\right] dx = ½\lambda$$

Put the constants together with the integral and divide by one period to get the time-averaged kinetic energy. (Remember that wavelength divided by period is wave speed.)

$$\frac{\langle K\rangle}{T} = \left\{(2\pi^2\varrho A f^2 \Delta s^2)(½\lambda)\right\}\frac{1}{T}$$

$$\frac{\langle K\rangle}{T} = \pi^2\varrho A f^2 v \Delta s^2$$

$$I = \frac{\langle P\rangle}{A} = \frac{2\langle K\rangle/T}{A}$$

$$I = \frac{2(\pi^2\varrho A f^2 v \Delta s^2)}{A}$$

One last bit of algebra and we're done. We now have an equation that relates intensity (I) to displacement amplitude (Δs).

$$I = 2\pi^2\varrho f^2 v \Delta s^2$$

Que5(a):

The Michelson interferometer is the best example of what is called an amplitude-splitting interferometer with an optical interferometer, one can measure distances directly in terms of wavelength of light used, by counting the interference fringes that move when one or the other of two mirrors are moved. In the Michelson interferometer, coherent beams are obtained by splitting a beam of light that originates from a single source with a partially reflecting mirror called a beam splitter. The resulting reflected and transmitted waves are then re-directed by ordinary mirrors to a screen where they superimpose to create fringes. This is known as interference by division of amplitude.

A simplified diagram of a Michelson interferometer is shown in the fig: 1.

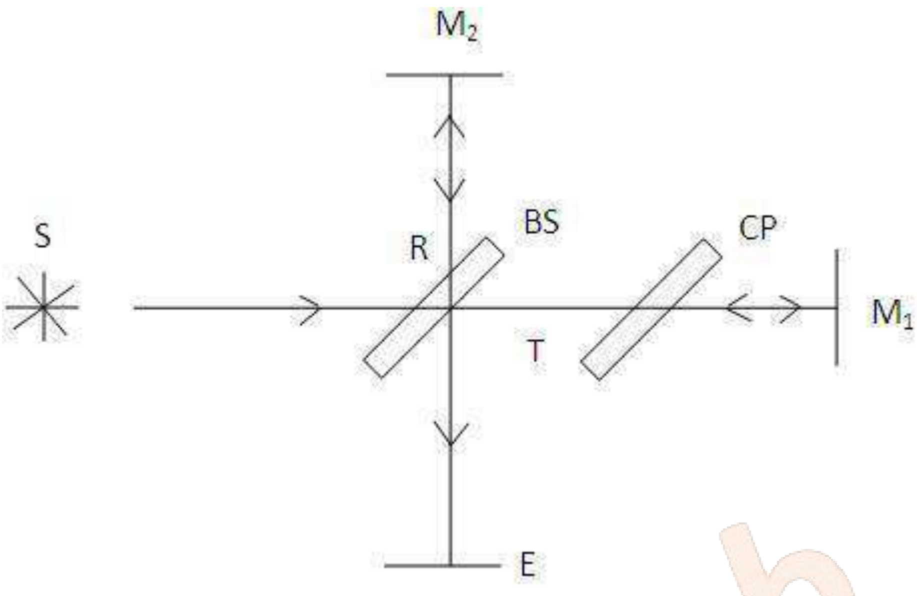


Fig.1


Fig. 2

Light from a monochromatic source S is divided by a beam splitter (BS), which is oriented at an angle 45° to the beam, producing two beams of equal intensity. The transmitted beam (T) travels to mirror M1 and it is reflected back to BS. 50% of the returning beam is then reflected by the beam splitter and strikes the screen, E. The reflected beam (R) travels to mirror M2, where it is reflected. 50% of this beam passes straight through beam splitter and reaches the screen.

Since the reflecting surface of the beam splitter BS is the surface on the lower right, the light ray starting from the source S and undergoing reflection at the mirror M2 passes through the beam splitter three times, while the ray reflected at M1 travels through BS only once. The optical path length through the glass plate depends on its index of refraction, which causes an optical path difference between the two beams. To compensate for this, a glass plate CP of the same thickness and index of refraction as that of BS is introduced between M1 and BS. The recombined beams interfere and produce fringes at the screen E. The relative phase of the two beams determines whether the interference will be constructive or destructive. By adjusting the inclination of M1 and M2, one can produce circular fringes, straight-line fringes, or curved fringes. This lab uses circular fringes, shown in Fig. 2.

From the screen, an observer sees M2 directly and the virtual image M1' of the mirror M1, formed by reflection in the beam splitter, as shown in Fig. 3. This means that one of the interfering beams comes from M2 and the other beam appears to come from the virtual image M1'. If the two arms of the interferometer are equal in length, M1' coincides with M2. If they do not coincide, let the distance between them be d, and consider a light ray from a point S. It will be reflected by both M1' and M2, and the observer will see two virtual images, S1 due to reflection at M1', and S2 due to reflection at M2. These virtual images will be separated by a distance 2d. If θ is the angle with which the observer looks into the system, the path difference between the two beams is $2d\cos\theta$. When the light that comes from M1 undergoes reflection at BS, a phase change of π occurs, which corresponds to a path difference of $\lambda/2$.

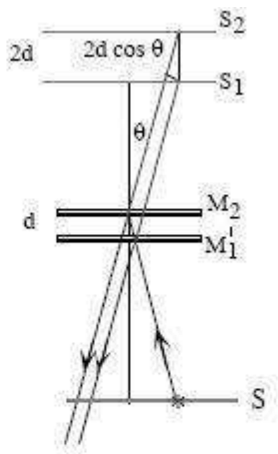


Fig. 3

Therefore, the total path difference between the two beams is,

$$\Delta = 2d\, Cos\,\theta + \frac{\lambda}{2}$$

The condition for constructive interference is then,

$$\Delta = 2d\, Cos\,\theta + \frac{\lambda}{2} = m\lambda, \quad m= 0, 1, 2... \qquad (1)$$

or a given mirror separation d, a given wavelength λ, and order m, the angle of inclination θ is a constant, and the fringes are circular. They are called fringes of equal inclination, or Haidinger fringes. If M1' coincides with M2, d = 0, and the path difference between the interfering beams will be λ/2. This corresponds to destructive interference, so the center of the field will be dark.

If one of the mirrors is moved through a distance λ/4, the path difference changes by λ/2 and a maximum is obtained. If the mirror is moved through another λ/4, a minimum is obtained; moving it by another λ/4, again a maximum is obtained and so on. Because d is multiplied by cosθ, as d increases, new rings appear in the center faster than the rings already present at the periphery disappear, and the field becomes more crowded with thinner rings toward the outside. If d decreases, the rings contract, become wider and more sparsely distributed, and disappear at the center.

For destructive interference, the total path difference must be an integer number of wavelengths plus a half wavelength,

$$\Delta_{destr} = 2d\, Cos\,\theta + \frac{\lambda}{2} = (m + \frac{1}{2})\lambda, \quad m= 0, 1, 2...$$

If the images S1 and S2 from the two mirrors are exactly the same distance away, d=0 and there is no dependance on θ. This means that only one fringe is visible, the zero order destructive interfrence fringe, where

$$\Delta_{destr} = \frac{\lambda}{2} = (m + \frac{1}{2})\lambda, \; m= 0$$

and the observer sees a single, large, central dark spot with no surrounding rings.

Measurement of wavelength:

Using the Michelson interferometer, the wavelength of light from a monochromatic source can be determined. If M1 is moved forward or backward, circular fringes appear or disappear at the centre. The mirror is moved through a known distance d and the number N of fringes appearing or disappearing at the centre is counted. For one fringe to appear or disappear, the mirror must be moved through a distance of λ/2. Knowing this, we can write,

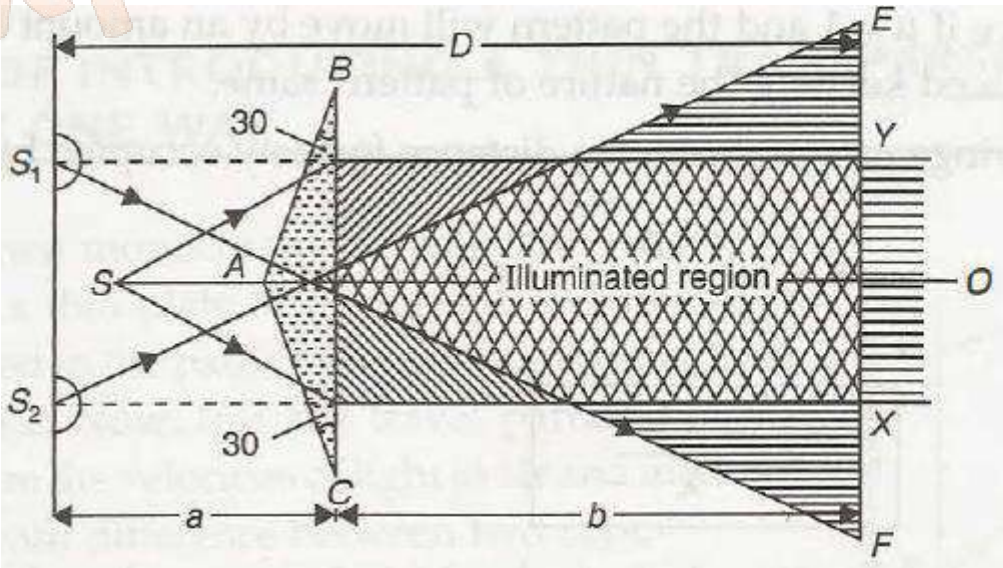
$$d = \frac{N\lambda}{2}$$

so that the wavelength is,

$$\lambda = \frac{2d}{N} \qquad (2)$$

Que 5(b)

The biprism consist of two active angled prisms with their bases in contact. Here two sources S1 and S2 are the virtual image of the fine slit S as shown in Figure below. The experimental arrangement consist of a slit S the biprism ABC and the microscope M. All are mounted on an optical bench. These are adjusted at the same height and can move and rotate as required. The light emerging from the slit fall on the biprism. The edge A of the biprism divides the incident wave front into two parts. One is through upper half AB of biprism and appears to coming from virtual source S1. Other is from lower half AC of biprism and appears to coming virtual S2 The interference fringes are seen in the overlapping region XY and can be seen by eyepiece.

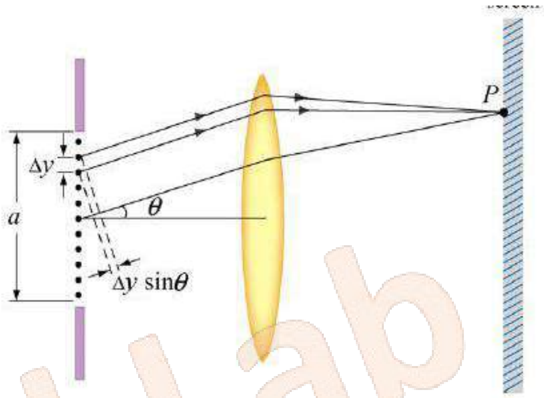


Que 6(a)

How do we determine the intensity distribution for the pattern produced by a single-slit diffraction? To calculate this, we must find the total electric field by adding the field contributions from each point.

Let's divide the single slit into N small zones each of width Δy = a / N , as shown in Figure. The convex lens is used to bring parallel light rays to a focal point P on the screen. We shall assume that Δy<< λ so that all the light from a given zone is in phase. Two adjacent zones have a relative path length δ = Δy sinθ . The relative phase shift Δβ is given by the ratio

$$\frac{\Delta\beta}{2\pi}=\frac{\delta}{\lambda}=\frac{\Delta y\sin\theta}{\lambda}, \quad \Rightarrow \quad \Delta\beta=\frac{2\pi}{\lambda}\Delta y\sin\theta$$



Suppose the wavefront from the first point (counting from the top) arrives at the point P on the screen with an electric field given by

$$E_1 = E_{10}\sin\omega t$$

The electric field from point 2 adjacent to point 1 will have a phase shift $\Delta\beta$, and the field is

$$E_2 = E_{10}\sin(\omega t+\Delta\beta)$$

Since each successive component has the same phase shift relative the previous one, the electric field from point N is

$$E_N = E_{10}\sin(\omega t+(N-1)\Delta\beta)$$

The total electric field is the sum of each individual contribution:

$$E = E_1+E_2+\cdots E_N = E_{10}\left[\sin\omega t+\sin(\omega t+\Delta\beta)+\cdots+\sin(\omega t+(N-1)\Delta\beta)\right]$$

Note that total phase shift between the point N and the point 1 is

$$\beta = N\Delta\beta=\frac{2\pi}{\lambda}N\Delta y\sin\theta=\frac{2\pi}{\lambda}a\sin\theta$$

where $N\Delta y = a$

The expression for the total field given can be simplified using some algebra and the trigonometric relation

Consider,

$$\cos(\alpha-\beta)-\cos(\alpha+\beta)=2\sin\alpha\sin\beta$$

$$\cos(\omega t-\Delta\beta/2)-\cos(\omega t+\Delta\beta/2)=2\sin\omega t\sin(\Delta\beta/2)$$
$$\cos(\omega t+\Delta\beta/2)-\cos(\omega t+3\Delta\beta/2)=2\sin(\omega t+\Delta\beta)\sin(\Delta\beta/2)$$
$$\cos(\omega t+3\Delta\beta/2)-\cos(\omega t+5\Delta\beta/2)=2\sin(\omega t+2\Delta\beta)\sin(\Delta\beta/2)$$
$$\vdots$$
$$\cos[\omega t+(N-1/2)\Delta\beta]-\cos[\omega t+(N-3/2)\Delta\beta]=2\sin[\omega t+(N-1)\Delta\beta]\sin(\Delta\beta/2)$$

Adding the terms and noting that all but two terms on the left cancel leads to

$$\begin{aligned}&\cos(\omega t-\Delta\beta/2)-\cos[\omega t-(N-1/2)\Delta\beta]\\&\quad=2\sin(\Delta\beta/2)\left[\sin\omega t+\sin(\omega t+\Delta\beta)+\cdots+\sin(\omega t+(N-1)\Delta\beta)\right]\end{aligned}$$

The two terms on the left combine to

$$\begin{aligned}&\cos(\omega t-\Delta\beta/2)-\cos[\omega t-(N-1/2)\Delta\beta]\\&\quad=2\sin(\omega t+(N-1)\Delta\beta/2)\sin(N\Delta\beta/2)\end{aligned}$$

with the result that

$$\begin{aligned}&\left[\sin\omega t+\sin(\omega t+\Delta\beta)+\cdots+\sin(\omega t+(N-1)\Delta\beta)\right]\\&\quad=\frac{\sin[\omega t+(N-1)\Delta\beta/2]\sin(\beta/2)}{\sin(\Delta\beta/2)}\end{aligned}$$

The total electric field then becomes

$$E=E_{10}\left[\frac{\sin(\beta/2)}{\sin(\Delta\beta/2)}\right]\sin(\omega t+(N-1)\Delta\beta/2)$$

The intensity I is proportional to the time average of E^2:

$$\langle E^2 \rangle = E_{10}^2 \left[\frac{\sin(\beta/2)}{\sin(\Delta\beta/2)} \right]^2 \langle \sin^2(\omega t + (N-1)\Delta\beta/2) \rangle = \frac{1}{2} E_{10}^2 \left[\frac{\sin(\beta/2)}{\sin(\Delta\beta/2)} \right]^2$$

and we express I as

$$I = \frac{I_0}{N^2} \left[\frac{\sin(\beta/2)}{\sin(\Delta\beta/2)} \right]^2$$

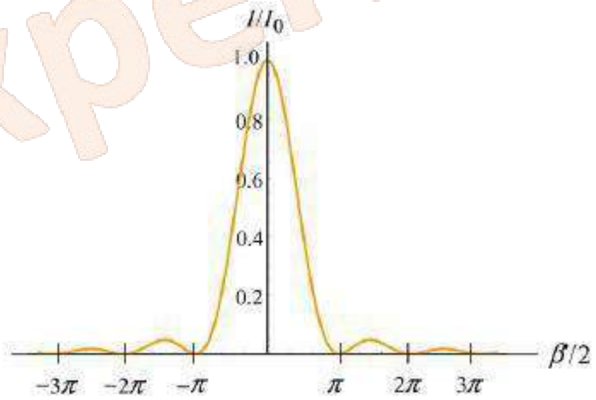
where the extra factor N^2 has been inserted to ensure that I_0 corresponds to the intensity at the central maximum $\beta = 0$ $(\theta = 0)$. In the limit where $\Delta\beta \to 0$,

$$N \sin(\Delta\beta/2) \approx N\Delta\beta/2 = \beta/2$$

and the intensity becomes

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2 = I_0 \left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda} \right]^2$$

we plot the ratio of the intensity I/I_0 as a function of $\beta/2$.



Que 6(b) From Eqn,

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2 = I_0 \left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda} \right]^2$$

We readily see that, condition for minimum intensity is

$$\frac{\pi}{\lambda} a \sin\theta = m\pi, \quad m = \pm 1, \pm 2, \pm 3, \dots$$

or

$$\boxed{\sin\theta = m\frac{\lambda}{a}, \quad m = \pm 1, \pm 2, \pm 3, \dots}$$

Similarly, condition for maximum intensity is,

$$\sin\theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{d}.$$

Que 7

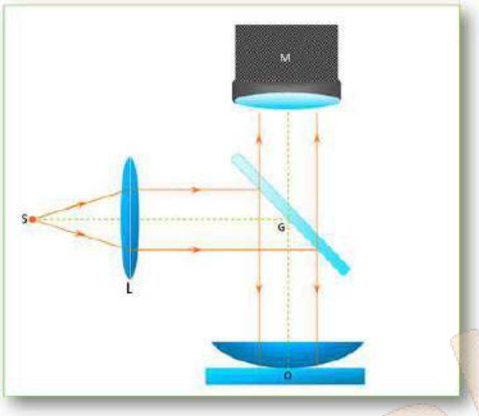
Ans: Please refer the textbook for this question

Que8 (a)

When a Plano convex lens of long focal length is placed in contact on a plane glass plate (Figure given below), a thin air film is enclosed between the upper surface of the glass plate and the lower surface of the lens. The thickness of the air film is almost zero at the point of contact O and gradually increases as one proceeds towards the periphery of the lens. Thus points where the thickness of air film is constant, will lie on a circle with O as center.

By means of a sheet of glass G, a parallel beam of monochromatic light is reflected towards the lens L. Consider a ray of monochromatic light that strikes the upper surface of the air film nearly along normal. The ray is partly reflected and partly refracted as shown in the figure. The ray refracted in the air film is also reflected partly at the lower surface of the film. The two reflected rays, i.e. produced at the upper and lower surface of the film, are coherent and interfere constructively or destructively. When the light reflected upwards is observed through microscope M which is focused on the glass plate, series of dark and bright rings are seen with center as O. These concentric rings are known as " Newton's Rings ".

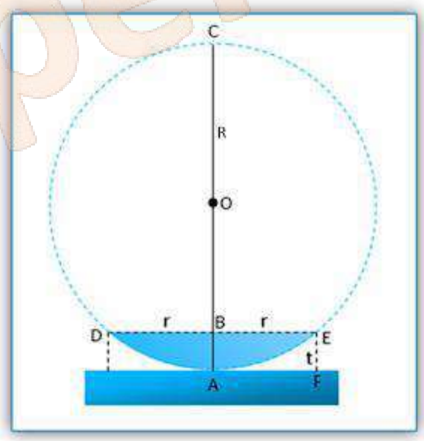
At the point of contact of the lens and the glass plate, the thickness of the film is effectively zero but due to reflection at the lower surface of air film from denser medium, an additional path of $\lambda/2$ is introduced. Consequently, the center of Newton rings is dark due to destructive interference.



Let us consider a system of plano-convex lens of radius of curvature R placed on flat glass plate it is exposed to monochromatic light of wavelength λ normally.

The incident light is partially reflected from the upper surface of air film between lens and glass and light is partially refracted into the film which again reflects from lower surface with phase change of 180 degree due to higher index of glass plate. Therefore the two parts of light interfere constructively and destructively forming alternate dark and bright rings.

Now consider a ring of radius r due to thickness t of air film as shown in the figure given below



According to geometrical theorem, the product of intercepts of intersecting chord is equal to the product of sections of diameter then

$$\overline{DB} \times \overline{BE} = \overline{AB} \times \overline{BC}$$

$$r \times r = t(2R - t)$$

$$r^2 = 2Rt - t^2$$

As **t** is very small then t^2 will be so small which may be neglected, then,

$$r^2 = 2Rt$$

$$t = \frac{r^2}{2R} \qquad \Rightarrow (1)$$

Radius for bright ring

The condition for constructive interference in thin film is,

$$2tn = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0,1,2,....$$

From equation (1) putting the value of t in the above equation we get,

$$2\left(\frac{r^2}{2R}\right)(1) = \left(m + \frac{1}{2}\right)\lambda$$

since **n = 1** for air film

$$r^2 = \left(m + \frac{1}{2}\right)\lambda R$$

Or

$$r = \sqrt{\left(m + \frac{1}{2}\right)\lambda R} \qquad \Rightarrow (2)$$

For first bright ring, m = 0

$$r_1 = \sqrt{\frac{1}{2}\lambda R}$$

For second bright ring, *m* = 1

$$r_2 = \sqrt{\frac{3}{2}\lambda R}$$

Radius for dark ring

The condition for destructive interference in thin film is,

$$\mathbf{2tn = m\lambda} \qquad m = 0,1,2,....$$

By putting the value of **t**, we get,

$$2\left(\frac{r^2}{2R}\right)(1) = m\lambda$$

For air **n = 1**

$$r = \sqrt{m\lambda R} \qquad \Rightarrow (4)$$

For $m = 0 \Rightarrow r = 0$ i.e. point of contact.

Now, if the radius of curvature of plano-convex lens is known and radius of particular dark and bright ring is experimentally measured then the wavelength of light used can be calculated from equation (3) and (4).

Que 8(b)

Que: 8(b) For bright rings, we have,

$$2tn = \left(m+\frac{1}{2}\right)\lambda \qquad t = \frac{r^2}{2R}$$

where, $n = 1$ for air, $r \rightarrow$ radius of Newton's ring

$$r^2 = \left(m+\frac{1}{2}\right)\lambda R \quad \text{for air,}$$

$$\left(m+\frac{1}{2}\right)\lambda R = \left(\frac{1.50}{2}\right)^2 \quad -(i)$$

For liquid, assuming refractive index to be μ,

$$\frac{\left(m+\frac{1}{2}\right)\lambda R}{\mu} = \left(\frac{1.25}{2}\right)^2 \quad -(ii)$$

Substituting (i) into (ii)

$$\frac{\left(\frac{1.50}{2}\right)^2}{\left(\frac{1.25}{2}\right)^2} = \mu$$

$$\boxed{\mu = 1.44}$$