B.Sc. (Prog.) Waves and Optics

Solved Paper - 2017

Q. 1. Attempt any five parts from the following :

(a) If two simple harmonic motions having angular frequencies 440 radians, sec and 396 radian/sec are superimposed . Calculate the time period of beats and the number of beats produced.

(b) Explain the physical characteristics that determine quality, pitch and loudness of a musical sound.

(c) Distinguish between Fresnel and Fraunhofer class of diffraction.

(d) Explain the reverberation time is larger for an empty hall than a crowded hall?

(e) Give the statements of Huygen's principle of propagation of wave front.

(f) Why do thin films appear coloured in white light?

(g) Why are Newtons rings circular?

(h) How is a zone plate different from a convex lens? **(5 × 3 = 15)**

Ans. (a) v_1=400 rads/sec, v_2 = 396 rads/sec

Therefore, $v_1 - v_2 = 400 - 396 = 4$ rads

$$T = 1/f = ¼ = 0.25 \text{ sec.}$$

(b) **Characteristics of musical sound :**

Pitch : The pitch is the characteristics of a musical sound which depends upon the frequency. The sound with low frequency is low pitchable sound and the sound with high frequency is high pitchable sound.

Loudness : The loudness of musical sound is related to the intensity of the sound the higher is the intensity, the higher will be the loudness. If I be the intensity of the sound, then the loudness is related to the I as,

Quality or Timber : It measure the complexity of sound. Quality of sound depends upon the number and intensity of harmonics, present in the sound. A pure sound produces comparatively less pleasing effect on ears then sound consisting of a number of harmonics. Usually a sounding body produce a complex sound of frequency. The f_0, $2f_0$, $3f_0$, etc. where is called fundamental frequency. The f_0, $2f_0$, $3f_0$...........etc., are called Ist, 2nd, 3rd .... harmonics. In the voice of different peoples different harmonics are present. Due to the different harmonics present in the voices, the characteristics of sound is called Quality or Timbre.

All Lab Experiments

(c) **Comparison chart**

Characteristic	Fraunhofer Diffraction	Fresnel Diffraction
Wave fronts	Planar wavefronts.	Cylindrical wave fronts.
Observation distance	Observation distance is infinite. In practice, often at focal point of lens.	Source of screen at finite distance from the obstacle.
Movement of diffraction pattern	Fixed in position.	Move in a way that directly corresponds with any shift in the object.
Surface of calculation	Fraunhofer diffraction patterns on spherical surfaces.	Fresnel diffraction patterns on flat surfaces.
Diffraction patterns	Shape and intensity of a Fraunhofer diffraction pattern stay constant.	Change as we propagate them further 'downstream' of the source of scattering.

(d) The audience is also absorber of sound. The total absorption in case of a crowded hall is more, therefore, as per Sabine formula, reverberation time of a crowded hall is smaller than that of an empty hall.

According to Sabine formula, the reverberation time is :

- directly proportional to the volume of the hall
- inversely proportional to the effective absorbing surface area of the walls and the materials inside the hall

$$\therefore \qquad T \alpha \frac{V}{\sum aA}$$

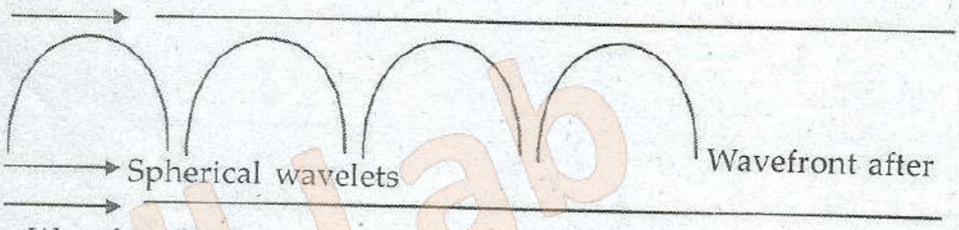
Where, V is the volume of the hall, a is the absorption coefficient of an area A. If the volume is measured in cubic feet and area in square feet, then the experimentally obtained value of the constant of proportionality, according to Sabine is 0.05.

(e) A wavefront is a surface over which an optical wave has a constant phase. For example, a wavefront could be the surface over which the wave has a maximum (the crest of a water wave, for example) or a minimum (the trough of the same wave) value. The shape of a wavefront is usually determined by the geometry of the source.

All Lab Experiments

- A point source has wavefronts that are spheres whose centers are at the point source.
- A fluorescent tube would have wavefronts that are cylinders concentric with the tube itself.

Huygens' principle describes how a wavefront moves in space. According to this principle, we imagine that each point on the wavefront acts as a point source that emits spherical wavelets. These wavelets travel with the velocity of light in the medium. At any later time, the total wavefront is the envelope that encloses all of these wavelets. That is, the tangent line that joins the front surface of each one of them as shown below.



The same construction is used for a wavefront of any other shape. When a wave travels in a single medium at a constant speed, the Huygen's construction preserves the general form of the wavefront. That is, spheres propagate and become larger spheres, cylinders become larger cylinders, etc.

If a portion of the wavefront enters a different medium (enters glass from air, for example), then the wavelets generated by each portion of the wavefront travel with the velocity that is appropriate for the medium that the wavefront is in. That is, the wavelets in the medium where the speed of light is less will have smaller radii than the wavelets in the original medium.

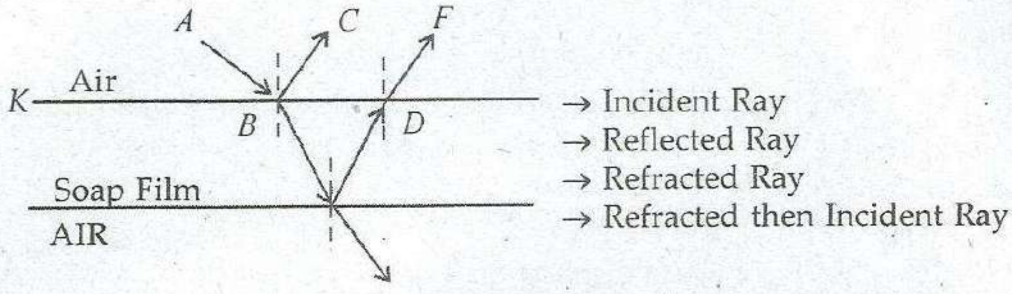
(f) A thin oil film spread on the surface of water appear beautifully coloured. The colours are due to interference between light waves reflected from the top and the bottom surfaces of thin films. When white light is incident on a thin film, the film appears coloured and the colour depends upon the thickness of the film and also the angle of incidence of the light on the surface.

Interference in thin films : Consider a transparent thin film of uniform thickness t and its refractive index μ bounded by two plane surfaces K and K' as shown in figure.

A ray of monochromatic light AB incident on the surface K of the film is partly reflected along BC and partly refracted into the film along BD. At the point D on the surface K_2, the ray of light is partly reflected along DE and partly transmitted out of the film along DG. The reflected light then emerges into air along EF which is parallel to BC. The ray EH after refraction at H, finally emerges along HJ.

BC and EF are reflected rays parallel to each other and DG and HJ are transmitted rays parallel to each other. Rays BC and EF interfere and similarly the rays DG and HJ interfere.

All Lab Experiments



(g) Newton's rings is a phenomenon in which interference pattern is created by the reflection of light between two surfaces; a spherical surface and an adjacent touching flat surface. Newton's rings appear as a series of concentric circles centered at the point of contact among the spherical and flat surfaces. When viewed with monochromatic light, Newton's rings emerge as alternating bright and dark rings; when viewed with white light, a concentric ring pattern of rainbow colours is observed.

The light rings are caused by constructive interference among the light rays reflected from both surfaces, while the dark rings are caused by destructive interference. The outer rings are spaced further closely than the inner ones because the slope of the curved lens surface increases outwards.

Moreover because the experiment is usually done with a flat mirror, and a spherical lens; neither of these changes the situation for the light on rotation, so the resulting light pattern cannot change. The only patterns that do not change if we rotate them have circular symmetry, so the rings are circular.

(h) A zone plate is a device which uses diffraction through alternate transparent and opaque rings to function as a lens.

- A zone plate has multiple foci but not a convex lens.
- The chromatic aberrations of a zone plate are much more severe than that of a convex lens.
- A zone plate acts both as concave and convex lens therefore forms real and virtual images simultaneously but not convex lens . It forms only real image at a given point.
- In zone plate the image formation is because of diffraction but in convex lens the image formation is primarily because of refraction.
- A zone plate can be effective for wide range of radiation of wavelength from microwaves to X-rays but convex lens are effective in visible range.

Q. 2. (a) Trace graphically or analytically the motion of a particle which is subjected to two perpendicular simple harmonic motions of equal frequencies, different amplitude and having a phase difference of

(i) $\alpha = 0$ **(ii)** $\alpha = \pi/2$

(b) Derive the expression for total energy contained in a simple harmonic motion. **(10 + 5 = 15)**

Ans. (a) Consider two SHM forces, F_1 and F_2, acting along the same straight line. Let the displacements be given by two equations,

All Lab Experiments

$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin \omega t + \varphi$$

The given two displacements reflect a convenient general case. Amplitudes are different. At any given instant, one of the two SHMs is 'ahead of' or 'lags behind' other, depending on the sign of phase constant 'φ'. As pointed out earlier, we keep the angular frequency 'ω' same for both SHMs.

Now, to find the net displacement of the particle at any given instant. It can be found using net displacement by evaluating vector relation,

$$r = r_1 + r_2$$

Since both SHMs are along the same straight line, one can drop the vector sign and can simply write this relation in the present context as :

$\Rightarrow$

$$x = x_1 + x_2$$

$$x = A_1 \sin \omega t + A_2 \sin \omega t + \varphi$$

Expanding trigonometric function,

$\Rightarrow$

$$x = A_1 \sin \omega t + A_2 \sin \omega t \cos \varphi + A_2 \cos \omega t \sin \varphi$$

Segregating sine and cosine functions, keeping in mind that "φ" is constant

$\Rightarrow$

$$x = A_1 + A_2 \cos \varphi \sin \omega t + A_2 \sin \varphi \cos \omega t$$

The expressions in the brackets are constant. Let,

$$C = A_1 + A_2 \cos \varphi \text{ and } D = A_2 \sin \varphi$$

Substituting in the expression of displacement, we have :

$$x = C \sin \omega t + D \cos \omega t$$

Following standard analytical method, let

$$C = A \cos\theta \text{ and } D = A \sin\theta$$

Substituting in the expression of displacement again, we have

$\Rightarrow$

$$x = A \cos\theta \sin \omega t + A \sin\theta \cos \omega t$$

$\Rightarrow$

$$x = A \sin \omega t + \theta$$

This is the final expression of the composition of two SHMs in the same straight line.

Clearly, the amplitude of resulting SHM is 'A'. Also, the resulting SHM differs in phase with respect to either of the two SHMs. In particular, the phase of resulting SHM differs by an angle 'θ' with respect of first SHM, whose displacement is given by '$A_1 \sin \omega t$' Its clear that frequency of the resulting SHM is same as either of two SHMs.

Phase constant : The phase constant of the resulting SHM is :

$\Rightarrow$

$$\tan \theta = DC = A_2 \sin \varphi A_1 + A_2 \cos \varphi$$

Amplitude : The amplitude of the resultant harmonic motion is obtained solving substitutions made in the derivation.

$$C = A \cos\theta \text{ and } D = A \sin\theta$$

All Lab Experiments

$$A = C_2 + D_2 = \{ A_1 + A_2 \cos \varphi_2 + A_2 \sin \varphi_2 \}$$

$\Rightarrow$ $$A = A_{1,2} + A_2 2 \cos 2\varphi + 2 A_1 A_2 \cos \varphi + A_2 2 \sin 2\varphi$$

$\Rightarrow$ $$A = A_{12} + A_{22} + 2 A_1 A_2 \cos \varphi$$

Important cases : Where, consider few interesting cases :

(*i*) Phase difference is zero : Two SHMs are in same phase. In this case, $\cos \varphi = \cos 0\ 0 = 1$.

$\Rightarrow$ $$A = A_{12} + A_{22} + 2 A_1 A_2 \cos \varphi = A_{12} + A_{22} + 2 A_1 A_2 = A_1 + A_{22}$$

$\Rightarrow$ $$A = A_1 + A_2$$

If additionally $A_1 = A_2$, then $A = 2 A_1 = 2 A_2$. Further, phase constant is given by :

$\Rightarrow$ $$\tan \theta = DC = A_2 \sin \varphi\ A_1 + A_2 \cos \varphi = A_2 \sin 0\ 0\ A_1 + A_2 \cos 0\ 0 = 0$$

$\Rightarrow$ $$\theta = 0$$

(ii) Phase difference is 'π' : Two SHMs are opposite in phase. In this case, $\cos \varphi = \cos \pi = -1$.

$\Rightarrow$ $$A = A_{12} + A_{22} + 2 A_1 A_2 \cos \varphi = A_{12} + A_{22} '' 2 A_1 A_2 = A_1 '' A_{22}$$

$\Rightarrow$ $$A = A_1 '' A_2$$

The amplitude is a non-negative number. In order to reflect this aspect, we write amplitude in modulus form :

$\Rightarrow$ $$A = | A_1 '' A_2 |$$

If additionally $A_1 = A_2$, then $A = 0$. In this case, the particle does not oscillate. Further, phase constant is given by

$\Rightarrow$ $$\tan \theta = DC = A_2 \sin \varphi\ A_1 + A_2 \cos \varphi = A_2 \sin \pi\ A_1 + A_2 \cos \pi = 0$$

$\Rightarrow$ $$\theta = 0$$

(b) Simple Harmonic motion is defined by the equation $F = -kx$. The work done by the force F during a displacement from x to x + dx is

$$dW = F dx = -kx dx$$

The work done in a displacement from $x = 0$ to x is

$$W = 0x - kx dx = -kx^2/2$$

as the change in potential energy corresponding to a force is negative of the work done by the force,

$$U(x) - U(0) = -W = \tfrac{1}{2} kx^2$$

Let us choose the origin to be zero of potential energy, then

$$U(0) = 0 \text{ and } U(x) = \tfrac{1}{2} kx^2$$

The kinetic energy at a time t is $K = \tfrac{1}{2} mv^2$

The total mechanical energy at time t is $E = U + K$

$$= \tfrac{1}{2} kx^2 + \tfrac{1}{2} mv^2 = \tfrac{1}{2} kA^2 \sin^2(\omega t + \phi) + \tfrac{1}{2} mA^2\omega^2 \cos^2(\omega t + \phi)$$

Putting $k = m\omega^2$

$$E = \tfrac{1}{2} m\omega^2 [A^2 \sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)] = \tfrac{1}{2} m\omega^2 A^2$$

All Lab Experiments

Thus, the total mechanical energy remains constant and is independent of time.

Q. 3 (a) Explain the formation of standing waves on a stretched string.

(b) A string 50 cm long is stretched by a load 25 kg and has a mass of 1.44 gm. Find the frequency of the second harmonic. **(10 + 5 = 15)**

Ans. When two progressive waves of same amplitude and wavelength travelling along a straight line in opposite directions superimpose on each other, stationary also called standing waves are formed.

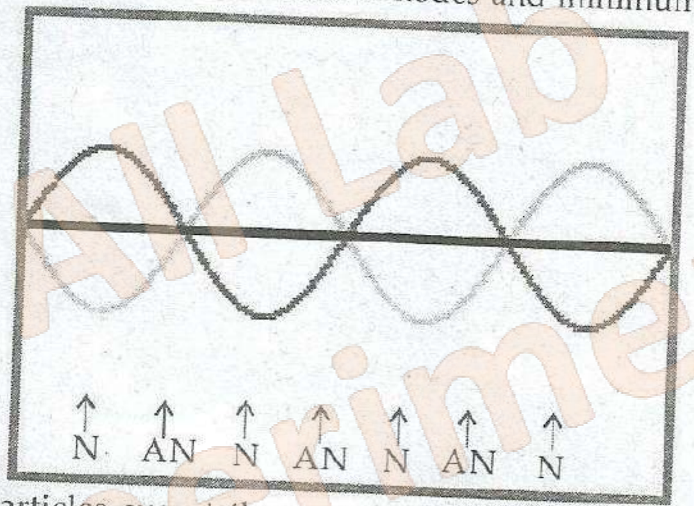
Characteristics of stationary waves :

— The waveform remains stationary.

— Nodes and antinodes are formed alternately.

— The points where displacement is zero are called nodes and the points where the displacement is maximum are called antinodes.

— Pressure changes are maximum at nodes and minimum at antinodes.



— All the particles except those at the nodes, execute simple harmonic motions of same period.

— Amplitude of each particle is not the same, it is maximum at antinodes decreases gradually and is zero at the nodes.

— The velocity of the particles at the nodes is zero. It increases gradually and is maximum at the antinodes.

— Distance between any two consecutive nodes or antinodes is equal to λ_2, whereas the distance between a node and its adjacent antinode is equal to $\lambda/4$.

— There is no transfer of energy. All the particles of the medium pass through their mean position simultaneously twice during each vibration.

— Particles in the same segment vibrate in the same phase and between the neighbouring segments, the particles vibrate in opposite phase.

— Standing waves in strings.

In musical instruments like sitar, violin, etc. sound is produced due to the ibrations of the stretched strings.

All Lab Experiments

When a string under tension is set into vibration, a transverse progressive wave moves towards the end of the wire and gets reflected. Thus, stationary waves are formed.

(b) Follow the relation $f = 1/2L[T/m]^{1/2}$

Substitute the values and find the frequency f.

Q. 4 (a) What do you understand by electromagnetic waves? Show that electromagnetic waves are transverse in nature.

(b) If the intensity is increased by factor 20, then how many decibel is the sound level increased? (10 + 5 = 15)

The electromagnetic waves are produced by an accelerated or decelerated charge or LC circuit. The frequency of e.m.f waves is

$$v = \frac{1}{2\pi\sqrt{LC}}$$

Characteristics of Electromagnetic Waves :

— The electromagnetic waves travel in free-space with the speed of light ($c = 3 \times 10^8\ ms^{-1}$); irrespective of their wavelength.

— Electromagnetic waves are neutral, so they are not deflected by electric and magnetic fields.

— The electromagnetic waves show properties of reflection, refraction, interference, diffraction and polarisation.

— In electromagnetic wave the electric and magnetic fields are always in the same phase.

— Ratio of magnitudes of electric and magnetic field vectors in free space is constant equal to c.

$$\frac{E}{B} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

— The speed of electromagnetic waves in a material medium is given by

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{n} \text{ where } n \text{ is the refractive index.}$$

— In an electromagnetic wave the energy is propagated by means of electric and magnetic field vectors in the direction of propagation of wave. In electromagnetic wave the average values of electric energy density and magnetic energy densities are equal $\left(\frac{1}{2}\varepsilon_0 E_0^2\right)_{av} = \left(\frac{B^2}{2\mu_0}\right)_{av}$

— The electric vector of electromagnetic wave is responsible for optical effects and is also called the light vector.

All Lab Experiments

Transverse Nature of Electromagnetic Waves : In an electromagnetic wave, the electric and magnetic field vectors oscillate perpendicular to the direction of propagation of electromagnetic wave. This shows that the electromagnetic waves are transverse in character. This may be proved as follows :

Let a plane electromagnetic wave propagate along positive X-axis. Then the propagating wavefront will be in $Y-Z$ plane. $ABCD$ is a portion of wavefront at any time t. The electric and magnetic field vectors at time t will be zero to the right of $ABCD$. To the left of $ABCD$, they will depend on x and t but not on Y and Z. Since we are considering a plane wave.

Considerring a closed surface $ABCDEFGH$. This surface does not enclose any charge, therefore by Gauss's theorem

$$\oint_{abcdefgh} \vec{E}.\vec{ds} = 0$$

or, $$\oint_{abcd} \vec{E}.\vec{ds} + \oint_{efgh} \vec{E}.\vec{ds} + \oint_{abef} \vec{E}.\vec{ds} + \oint_{dugh} \vec{E}.\vec{ds} + \oint_{aehd} \vec{E}.\vec{ds} + \oint_{begc} \vec{E}.\vec{ds} = 0 \qquad ...(i)$$

As electric field does not depend on Y and Z

$$\oint_{abef} \vec{E}.\vec{ds} = -\oint_{dcgh} \vec{E}.\vec{ds} \text{ and } \oint_{aehd} \vec{E}.\vec{ds} = -\oint_{begc} \vec{E}.\vec{ds}$$

in view of above equation (1) gives

$$\oint_{abcd} \vec{E}.\vec{ds} + \oint_{efgh} \vec{E}.\vec{ds} = 0$$

$$\Rightarrow \oint E_x \hat{i}.(dy\, dzi) + \oint E_x \hat{i}.(-dy\, dzi) = 0$$

or, $E_x dydz - Ex'\ dydz$

or, $E_x = E_x'$

i.e., component of electric field along the direction of propagation is constant. As a constant field can not produce a wave, this implies that $E_x = 0$. In a similar manner it may be shown that the component of magnetic field along the direction of propagation of wave is zero, i.e., $B_x = 0$. This shows that the electric and magnetic fields have no component along the direction of propagation. Thus, in an electromagnetic wave field vectors are perpendicular to the direction of propagation of wave, i.e., electromagnetic waves are transverse in nature.

(b) Let initial intensity be L and intensity level be β_1

When intensity is increased by 20 times, intensity level be β_2

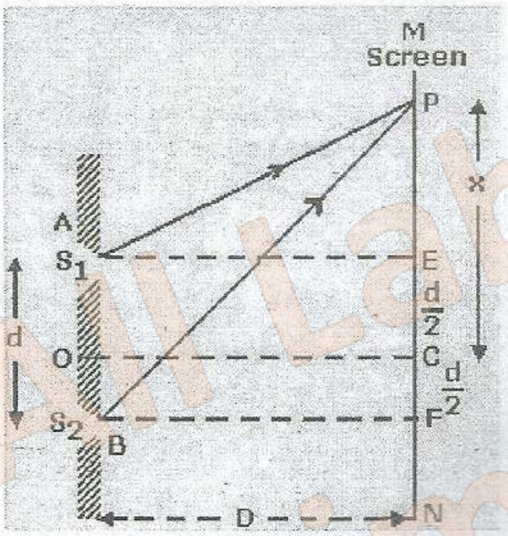
All Lab Experiments

$$\beta_1 = \log (I/I_0)$$
$$\beta_2 = \log (20I/I_0)$$
$$\beta_2 - \beta_1 = {}_{10}\log(20I/I_0)$$
$$= {}_{10}\log 20 = 13 \text{ dB}.$$

Q.5 (a) Show that in Young's double slit experiment, the fringe width is directly proportional to the wavelength of light.

(b) In case of Newton's ring experiment , calculate the diameter of ninth bright ring having radius of curvature of planoconvex lens 10 cm and wavelength of light λ= 40 nm. (10 + 5 = 15)

Ans. Young's Double Slit Experiment



Let *A* and *B* be two fine slits, a small distance '*d*' apart. Let them be illuminated by a monochromatic light of wavelength l.

MN in the screen is at a distance *D* from the slits *AB*. The waves from *A* and *D* superimpose upon each other and an interference pattern is obtained on the screen. The point C is equidistant from *A* and *B* and therefore the path difference between the waves will be zero and so the point C is of maximum intensity. It is called the central maximum.

For another point *P* at a distance '*x*' from *C*, the path difference at $P = BP - AP$.

Now $AB = EF = d$, $AE = BF = D$

$$BP^2 = \left[BF^2 + PF^2\right]^{\frac{1}{2}} = \left[D^2 + \left(x \times \frac{d}{d}\right)^2\right]^{\frac{1}{2}}$$

All Lab Experiments

$$= D\left[1 + \frac{\left(x + \frac{d}{2}\right)^2}{D^2}\right]^{\frac{1}{2}}$$

Similarly in Δ APE

$$AP = D\left[1 + \frac{\left(x - \frac{d}{2}\right)^2}{D^2}\right]^{\frac{1}{2}}$$

$$\therefore \qquad BP - AP = \mathrm{D}\left[1 + \frac{1}{2}\frac{\left(x - \frac{d}{2}\right)^2}{\mathrm{D}^2}\right]^{\frac{1}{2}}$$

$$\therefore \qquad BP - AP = D\left[1 + \frac{1}{2}\frac{\left(x - \frac{d}{2}\right)^2}{D^2}\right]^{\frac{1}{2}} - D\left[1 + \frac{1}{2}\frac{\left(x - \frac{d}{2}\right)}{D^2}\right]$$

(on expanding binomially)

$$= \frac{1}{2D}\left[4 \times \frac{d}{2}\right] = \frac{xd}{D}$$

For bright fringes (constructive wavelength) the path difference is integral multiple of wavelength, i.e., path difference is nl.

$$\therefore \qquad n\lambda = \frac{xd}{D}$$

All Lab Experiments

$$x = \frac{n\lambda D}{d} \text{ where } n = 0, 1, 2, 3, 4 \ldots\ldots$$

therefore represents distance of n^{th} bright fringe from C)

Now,

$\therefore \quad n = 0 \quad x_0 = 0$

$n = 1 \quad x_1 = \frac{\lambda D}{d}$

$\therefore \quad n = 2 \quad x_2 = \frac{2\lambda D}{d}$

$n = 3 \quad x_3 = \frac{3\lambda D}{d}$

and so on.

$x_1 = \frac{\lambda D}{d}$ is the distance of the 1^{st} bright fringe and therefore

$x_n = \frac{n\lambda D}{d}$ will be the distance of the n^{th} bright fringe from C.

Therefore separation between the centers of two consecutive bright fringe is equal to the width of a dark fringe.

$$\therefore \quad \beta_1 = x_n - x_{n-1} = \frac{x D}{d}$$

Similarly for dark fringes,

$$x_n = (2n - 1)\frac{\lambda}{2}\frac{D}{d}$$

For $n = 1$ $\quad x_1 = \frac{\lambda D}{2d} \rightarrow$ Position of 1st dark fringe

For $n = 2$ $\quad x_2 = \frac{3\lambda D}{2d} \rightarrow$ Position of 2nd dark fringe

The separation between the centers of two consecutive dark interference fringes is the width of a bright fringe.

$$\beta_2 = x_n - x_{n-1} = \frac{\lambda D}{d}$$

All Lab Experiments

(b) Radius for bright ring $r^2 = \left(m + \frac{1}{2}\right)\lambda R$

or, $$r = \sqrt{\left(m + \frac{1}{2}\right)\lambda R}$$

$$m = 9,$$

$$\lambda = 40 \text{ nm} = 40 \times 10^{-9}\text{m}$$

$$R = 10 \text{ cm} = 0.1\text{m}$$

$$\text{Diameter, } D = \sqrt{(19/2 \times 40 \times 10^{-9} \times 0.1)]} \times 2\text{m}$$

$$= 38 \times 10^{-9}\text{m} \times 2 = 76 \text{ nm}$$

Q. 6 (a) A zone plate has focal length of 50 cm at a wavelength of 6000 Å. What will be its focal length at a wavelength of 5000 Å?

(b) Explain with the help of diagram the intensity distribution due to Fresnel diffraction at a straight edge. **(4 + 11 = 15)**

Ans. (a) Given $\lambda_1 = 6000$ Å $\lambda_2 = 5000$ Å

$f_1 = 50$ cm $f_2 = ?$

With, $$f = r^2/n\lambda$$

$$f_2/f_1 = \lambda_1/\lambda_2$$

$$f_2 = f_1 \lambda_1/\lambda_2$$

$$= 50 \times 6000 \times 10^{-8} / 5000 \times 10^{-8} = 60 \; cm.$$

(b) Refer to the text on diffraction.

Q. 7 (a) Give the necessary theory to derive expression for the intensity distribution pattern in a plane transmission grating.

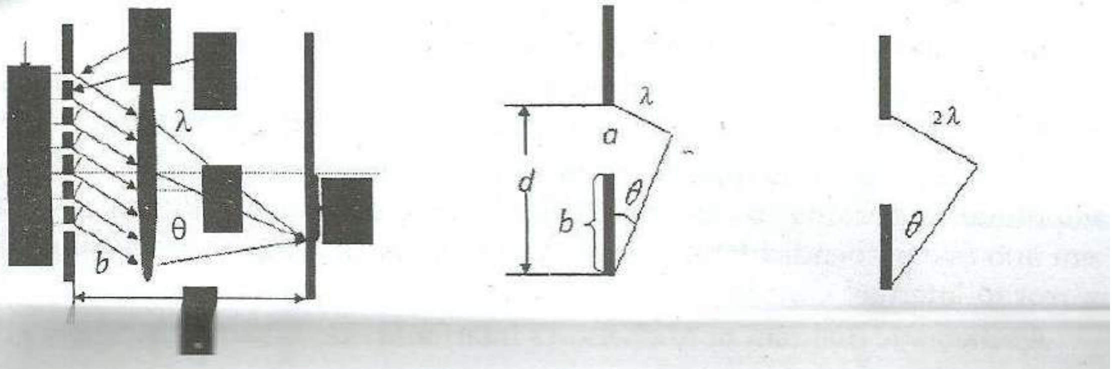
(b) A grating of width 2 inches is ruled with 15000 lines per inch. Find the smallest wavelength separation that can be resolved in second order at a mean wavelength of 5000 Å. **(12 + 3 = 15)**

Ans. Suppose that, instead of a single slit, or two slits side by side in Young's experiments, we have a very large number of parallel slits, all of the same width and spaced at regular intervals. Such an arrangement, known as diffraction grating, was first constructed by Fraunhofer.

$$d = a + b$$

d is called grating spacing or grating constant. Let's assume that the slits are so narrow that the diffracted beam from each spreads out over a sufficiently wide angle for it to interfere with all the other diffracted beams. Consider first the light proceeding from elements of infinitesimal width at the upper edge of each opening, and travelling in a direction making an angle θ with that of the incident beam. A lens at the right of the grating forms in its focal plane a diffraction pattern similar to that which would appear on a screen at infinity.

All Lab Experiments



First-order maximum when $AB = \lambda$, second order maximum when $AB = 2\lambda$.

It is easy to find that the diffraction grating equation (bright fringes) is

$$d \sin\theta = \pm m\lambda \qquad (m = 0, 1, 2, 3, \ldots)$$

The results are involved in interferences and diffractions. It is known that for the interference of the double-slit, the fringes are equally bright. But when we consider the diffraction by a slit, the final pattern actually observed is a combination of both effects. The interference pattern locates the position of each bright fringe, and the diffraction pattern from one slit modifies the intensity of each bright fringe. Diffraction modifies the interference pattern of a grating in just the same way to the double-slit. The interference pattern determines the position of each bright fringe, and the diffraction pattern determines its intensity. On the other hand, a grating produces much sharper bright fringes than a double slit. If a sharp bright fringe of interference occurs at the position where the first dark fringe of diffraction happens to be, the sharp bright fringe will be missing. This phenomenon is called order-missing phenomenon of a grating.

(b) Use the relation $(a + b) \sin\theta = n\lambda$ $(a + b) = 2.54/15000$ cm.

$N = (a + b)/\lambda$ [as $\theta = \pi/2$]

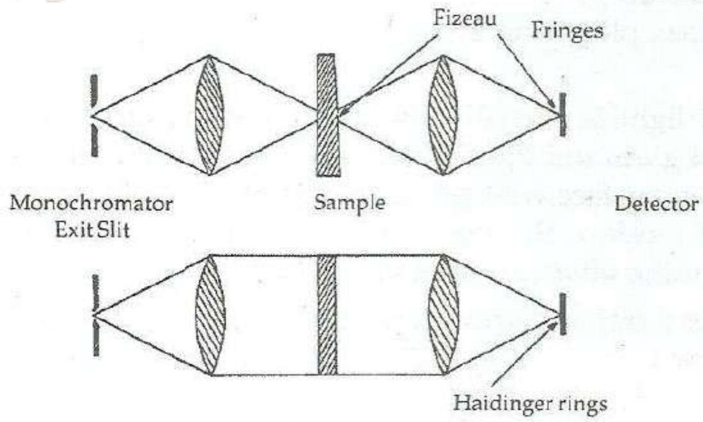
Q. 8 (a) Give the difference between Haidinger fringes and Fizeau fringes.

(b) Explain how Michelson's interferometer can be used to determine the wavelength of monochromatic light?

(c) Prove that the diameters of dark Newton rings are proportional to the square roots of natural numbers in reflected mode for normal incidence.

(3 + 6 + 6 = 15)

Ans. (a)

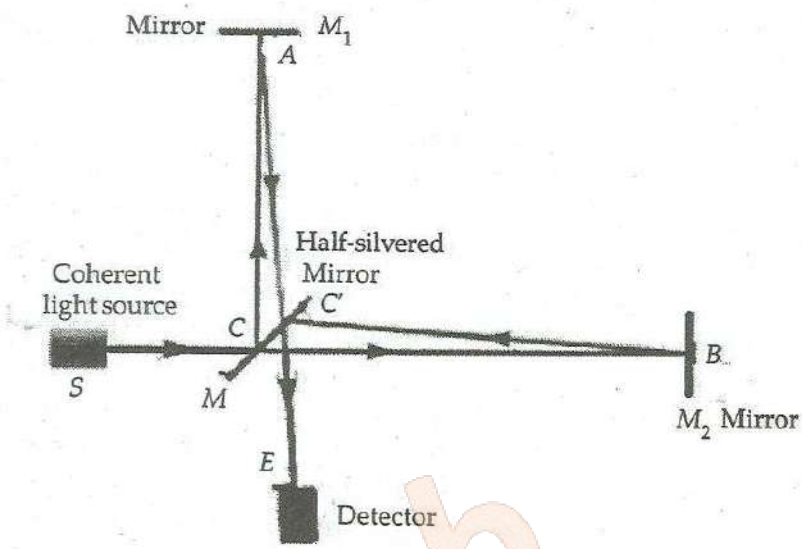


All Lab Experiments

(b) Michelson designed an instrument for the measurement of wavelength of sodium light, thickness of thin film and for many applications. The instrument is based on principle of interference of light known as Michelson's Interferometer.

It is based on principle of interference of light by the way of division of amplitude. According to this the incident beam is divided into two parts and sent into two perpendicular directions and brought back together by using plane mirror to interfere each other.

A schematic diagram of Michelson's Interferometer is shown in figure given below.



Measurement of wavelength : Using the Michelson interferometer, the wavelength of light from a monochromatic source can be determined. If M_1 is moved forward or backward, circular fringes appear or disappear at the centre. The mirror is moved through a known distance d and the number N of fringes appearing or disappearing at the centre is counted. For one fringe to appear or disappear, the mirror must be moved through a distance of $\lambda/2$. Knowing this,

we can write, $d = \frac{N\lambda}{2}$ so that the wavelength is, $\lambda = \frac{2d}{N}$

(c) Let us consider a system of plano-convex lens of radius of curvature R placed on flat glass plate it is exposed to monochromatic light of wavelength λ normally.

The incident light is partially reflected from the upper surface of air film between lens and glass and light is partially refracted into the film which again reflects from lower surface with phase change of 180 degree due to higher index of glass plate. Therefore the two parts of light interfere constructively and destructively forming alternate dark and bright rings.

Now consider a ring of radius r due to thickness t of air film as shown in the figure given below :

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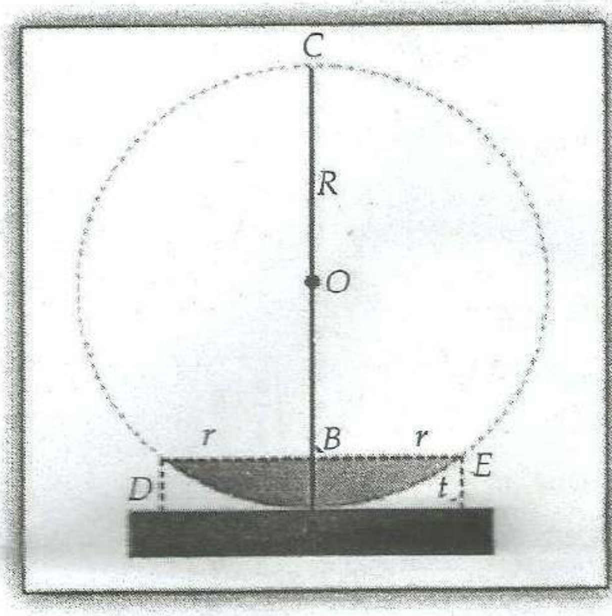

According to geometrical theorem, the product of intercepts of intersecting chord is equal to the product of sections of diameter then,

$$DB \times BE = AB \times BC$$

$$r \times r = t\,(2R - t)$$

$$r^2 = 2Rt - t^2$$

As t is very small then t^2 will be so small which may be neglected, then,

$$r^2 = 2Rt$$

$$t = \frac{r^2}{2R} \qquad \text{...(1)}$$

Radius for bright ring :

The condition for constructive interference in thin film is,

$$2tn = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0, 1, 2, ...$$

From Equation (1) putting the value of t in the above equation we get,

$$2\left(\frac{r^2}{2R}\right)(1) = \left(m + \frac{1}{2}\right)\lambda$$

since $n = 1$ for air film

$$r^2 = \left(m + \frac{1}{2}\right)\lambda R$$

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or $$r = \sqrt{\left(m + \frac{1}{2}\right)\lambda R} \quad \text{... (2)}$$

For first bright ring, $m = 0$

$$r_1 = \sqrt{\frac{1}{2}\lambda R}$$

For second bright ring, $m = 1$

$$r_2 = \sqrt{\frac{3}{2}\lambda R}$$

For third bright ring, $m = 2$

$$r_2 = \sqrt{\frac{5}{2}\lambda R}$$

Similarly, for Nth bright ring,

$$m = N - 1$$

Radius for dark ring :

The condition for destructive interference in thin film is,

$$2tn = m\lambda \qquad m = 0, 1, 2, ...$$

By putting the value of t, we get,

$$2\left(\frac{r^2}{2R}\right)(1) = m\lambda$$

For air $n = 1$

$$r = \sqrt{m\lambda R}$$

For $m = 0 \Rightarrow r = 0$ i.e., point of contact.

Now, if the radius of curvature of plano-convex lens is known and radius of particular dark and bright ring is experimentally measured then the wavelength of light used can be calculated from above equations.

The wavelength of monochromatic light can be determined as,

$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}$$

Where, D_{m+p} is the diameter of the $(m + p)^{th}$ dark ring and D_m is the diameter of the m^{th} dark ring.

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