B.Sc. (Prog.) – Waves and Optics

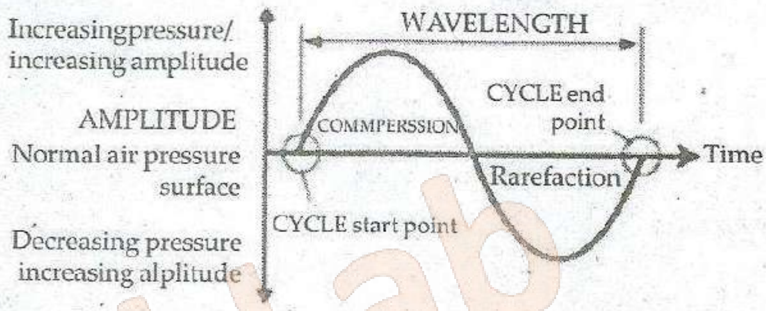
Solved Paper – 2016

Q.1 (a) What are sound waves? Describe their characteristic properties.

(b) Coherent light of wavelength 633nm from a He-Ne laser falls on a double slit with a slit separation of 0.103 mm. An interference pattern is produced on a screen 2.56 m from the slits. Calculate the separation on the screen of the two fourth-order bright fringes on either side of the central image.

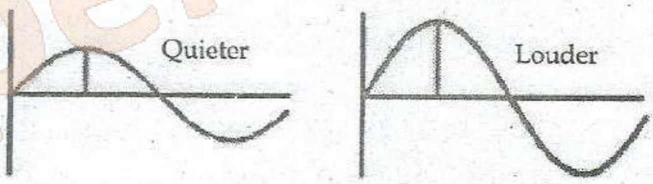
(c) The number of lines in a grating X is N_1 and the number of lines in a grating Y is N_2, N_1 is less than N_2 The total ruled width of the two gratings X and Y is same. Compare their resolving powers.

Ans. (a) Sound waves : These waves travel as pressure variation in the air i.e. are alternate regions of high pressure and low pressure.



Amplitude: It refers to the distance of the maximum vertical displacement of the wave from its mean position. Larger the amplitude, higher the energy. In sound, amplitude refers to the magnitude of compression and expansion experienced by the medium the sound wave is travelling through. This amplitude is perceived by our ears as loudness. High amplitude is equivalent to loud sounds.

Wavelength : A sound wave is made of areas of high pressure alternated by an area of low pressure. The high pressure areas are represented as the peaks of the graph. The low pressure areas

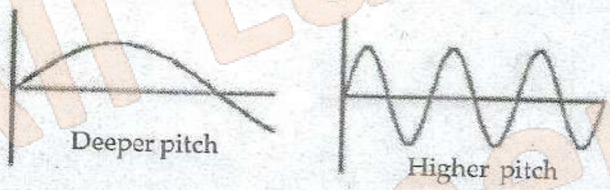


are depicted in the Valleys. The physical distance between two consecutive peaks or valleys in a sound wave is referred to as the Wavelength of the sound wave.

Frequency/ Pitch of the Sound Waves: It refers to the rate of the vibration of the sound travelling through the air. This parameter decides whether a sound is perceived as high pitched or low pitched. In sound, frequency is also known as Pitch. The frequency of the vibrating source of sound is calculated in cycles per second. The SI Unit for frequency being hertz and it is defined as $1/T$ where T refers to the time period of the wave. Wavelength and frequency of a sound wave are related mathematically as :

Velocity of Sound = Frequency × Wavelength

Two graphs showing the difference between sound waves with high and low frequencies and their corresponding pitches



Timbre: Timbre or quality is actually defined as; if two different sounds have the same frequency and amplitude, then by definition they have different timbres.

(b) The easiest way to handle this problem is to calculate the distance 1/4 of the fourth-order bright fringe on one side from the central image, and then double this value to obtain the distance between the two fourth-order images.

$$y_4 = \frac{4\lambda L}{d} = \frac{4(633 \times 10^{-9})(2.65)}{(0.10^3 \times 10^{-3})} = 6.29 \text{ cm.}$$

The distance between the two fourth-order fringes is therefore

$$2y_4 = 12.6 \text{ cm.}$$

The reflected light is weak in the red region of the spectrum and strong in the blue-violet region. The soap film will, therefore, possess a pronounced *blue* colour

(c) Resolving power of a grating is given by R.P. = $\lambda/d\lambda = n$N

According to grating equation (a + b) sin θ n λ or n (a + b) sin θ/λ

From equations (i) and (ii)

R.P. = N(a +b) sin θ /λ

But W = N (a + b), where W = width of the ruled surface

R.P = W sin θ /λ

From equation (iii), it is clear that the resolving power of both the gratings is same and it is independent of the number of lines for a given width of ruled surface.

Q. 2. (a) What is a wave front ? What did Huygen's principle say about formation of wave front?

(b) Differentiate in Young's double slit experiment and Llyod's mirror experiment.

(c) Describe Fresnel Biprism with its applications.

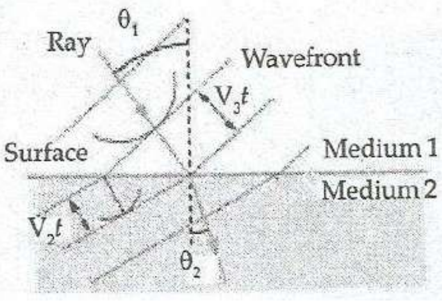
Ans. A wave front is the locus of points characterized by propagation of position of the same phase: a propagation of a line in 1-*D*, a curve in 2-*D* or a surface for a wave in 3-*D*.

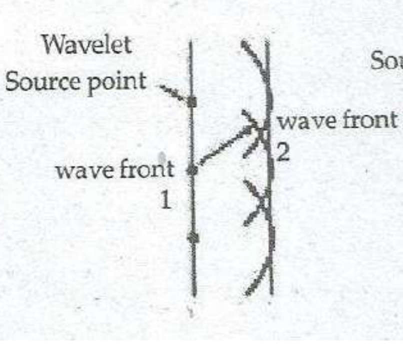
A wave front can also be described as a surface over which an optical wave has a constant phase such as wave front could be the surface over which the wave has a maximum (the crest of a water wave, for example) or a minimum (the trough of the same wave) value. The shape of a wave front is usually determined by the geometry of the source. A point source has wave fronts that are spheres whose centers are at the point source. A fluorescent tube would have wave fronts that are cylinders concentric with the tube itself. A very large sheet of material that is uniformly illuminated would generate wave fronts that are plane waves parallel to the sheet.

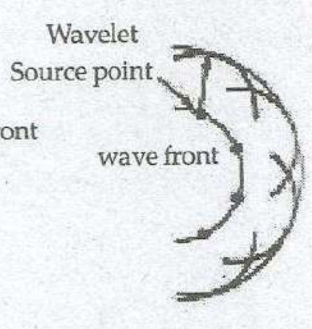
The direction of propagation of the wave is always perpendicular to the surface of the wave front at each point. Thus, the wave fronts of a point source are spheres and the wave propagates radially outward – the radius of a sphere is perpendicular to its circumference at each point. The same is true of the radius of the cylindrical wave fronts that would be generated by a fluorescent tube.

Although the wave fronts produced by a point source are always concentric spheres in principle, when the source is very far away the radii of the spheres are so large that they look like plane waves to an observer.

Huygens' principle (Christaan Huygens, 1629–1695, published about 1690) describes how a wave front moves in space. According to this principle, we imagine that each point on the wave front acts as a point source that emits spherical wavelets. These wavelets travel with the velocity of light in the medium. At any later time, the total wave front is the envelope that encloses all of these wavelets. That is, the tangent line that joins the front surface of each one of them. A simple example of how a plane wave front moves is shown below:







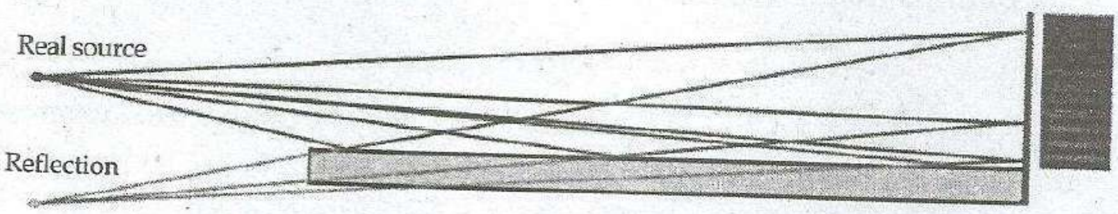
When a wave travels in a single medium at a constant speed, the Huygen's construction preserves the general form of the wave front. That is, spheres propagate and become larger spheres, cylinders become larger cylinders, etc.

If a portion of the wave front enters a different medium (enters glass from air, for example), then the wavelets generated by each portion of the wave front travel with the velocity that is appropriate for the medium that the wave front is in. That is, the wavelets in the medium where the speed of light is less will have smaller radii than the wavelets in the original medium.

Although Huygens' principle was initially stated without any proof, a slightly modified form of it was later (about 1815) derived by Fresnel from the mathematical theory of waves.

(b) Lloyd's Mirror is used to produce two-source interference patterns that have important differences from the interference patterns seen in Young's experiment.

In a modern implementation of Lloyd's mirror, a diverging laser beam strikes a front-surface mirror at a grazing angle, so that some of the light travels directly to the screen and some of the light reflects off the mirror to the screen. The reflected light forms a virtual second source that interferes with the direct light.

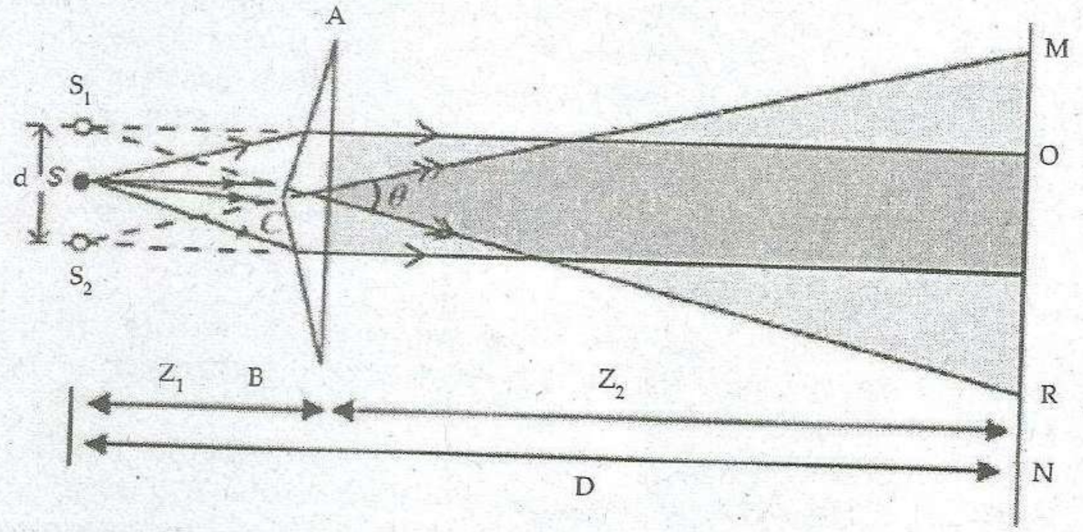


In Young's experiment, the individual slits display a diffraction pattern on top of which is overlaid interference fringes from the two slits. In contrast, the Lloyd's mirror experiment does not use slits and displays two-source interference without the complications of an overlaid single-slit diffraction pattern.

In Young's experiment, the central fringe representing equal path length is bright because of constructive interference. In contrast, in Lloyd's mirror, the fringe nearest to the mirror representing equal path length is dark rather than bright. This is because the light reflecting off the mirror undergoes a 180° phase shift, and so causes destructive interference when the path lengths are equal or when they differ by an integer number of wavelengths.

(c) A Fresnel Biprism is a thin double prism placed base to base and have very small refracting angle (0.5°). This is equivalent to a single prism with one of its angle nearly 179° and other two of 0.5° each.

The interference is observed by the division of wave front. Monochromatic light through a narrow slit S falls on biprism , which divides it into two components. One of these component is refracted from upper portion of biprism and appears to come from S_1 where the other one refracted through lower portion and appears to come from S_2. Thus S_1 and S_2 act as two virtual coherent sources formed from the original source. Light waves arising from S_1 and S_2 interfere in the shaded region and interference fringes are formed which can be observed on the screen



It can be used to determine the wavelength of a light source (monochromatic), thickness of a thin transparent sheet/thin film, refractive index of medium, etc.

Determination of wavelength of light : An expression for fringe width is

$$\beta = \frac{D\lambda}{d}$$

Biprism can be used to determine the wavelength of given monochromatic light using the expression

$$\lambda = \frac{d\beta}{D}$$

Determination of thickness of a thin film : To determine the thickness of transparent thin sheet (mica), the monochromatic source is replaced by white light source.

Measurement of fringe width: To get λ, fringes are first observed in the field of view of the microscope. The vertical wire of the eyepiece is made to coincide with one of the fringes and screw of micrometer is moved sideways and number of fringes is counted.

$$\beta = \text{Distance moved/number of fringes passed}$$

Measurement of D : This distance between source and eyepiece is directly measured on the optical bench scale.

A convex lens is placed between biprism and the eyepiece in such a way that for two positions of lens the image of virtual sources S_1 and S_2 are seen in eyepiece. Then

$$d = \sqrt{d_1 d_2}$$

where d_1 and d_2 are the distance between S_1 and S_2 for two position of lens.

Determination of thickness of a thin film : We know that the interference pattern shifts one side when a thin transparent film is put in the path of one ray. This shift is

$$\delta' = \beta \,/\, \lambda(\mu - 1)t$$

Thus, measuring fringe width β and shift δ′ using biprism the t can be calculated ifrefractive index μ and wavelength λ are known.

Q. 4 (a) Explain and derive the relation using double slit?

(b) If a yellow light with a wavelength of 540 nm shines on a double slit with the slits cut 0.0100 mm apart, determine what angle you should look away from the central fringe to see the second order fringe?

(c) For a single slit experiment apparatus like the one described above, determine how far from the central fringe the first order violet (λ = 350 nm) and red (λ = 700 nm) colours will appear if the screen is 10 m away and the slit is 0.050 cm wide.

Ans. Let A and B be two fine slits, a small distance 'd' apart. Let them be illuminated by a monochromatic light of wavelength l.

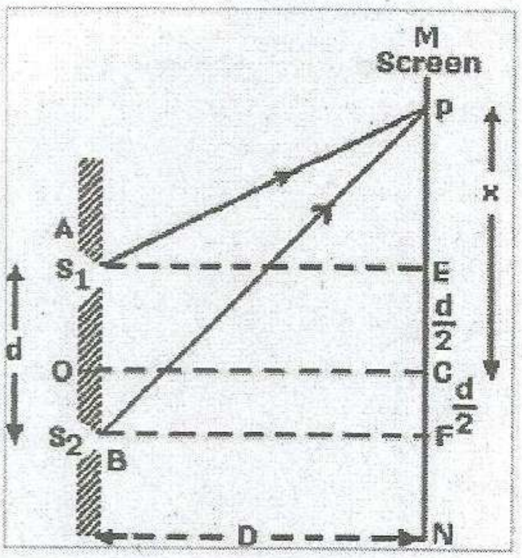
MN in the screen is at a distance D from the slits AB. The waves from A and B superimpose upon each other and an interference pattern is obtained on the screen. The point C is equidistant from A and B and therefore the path difference between the waves will be zero and so the point C will be of maximum intensity. It is called the central maximum.

For another point P at a distance 'x' from C, the path difference at $P = BP - AP$.

Now $AB = EF = d$, $AE = BF = D$ [Pythagoras theorem]

$$BP^2 = \left[BF^2 + PF^2\right]^{\frac{1}{2}} = \left[D^2 + \left(x \times \frac{d}{D}\right)^2\right]^{\frac{1}{2}}$$

$$= D\left[1 + \frac{\left(x + \frac{d}{2}\right)^2}{D^2}\right]^{\frac{1}{2}}$$



Similarly in Δ APE

$$AP = D\left[1 + \frac{\left(x - \frac{d}{2}\right)^2}{D^2}\right]^{\frac{1}{2}}$$

$$\therefore \quad BP - AP = \mathrm{D}\left[1 + \frac{1}{2}\frac{\left(x - \frac{d}{2}\right)^2}{\mathrm{D}^2}\right]^{\frac{1}{2}}$$

$$\therefore \quad BP - AP = D\left[1 + \frac{1}{2}\frac{\left(x - \frac{d}{2}\right)^2}{D^2}\right]^{\frac{1}{2}} - D\left[1 + \frac{1}{2}\frac{\left(x - \frac{d}{2}\right)}{D^2}\right]$$

(on expanding Binomially)

$$= \frac{1}{2D}\left[4 \times \frac{d}{2}\right] = \frac{xd}{D}$$

For bright fringes (constructive wavelength) the path difference is integral multiple of wavelength, i.e., path difference is *nl*.

$$\therefore \quad n\lambda = \frac{xd}{D}$$

$$x = \frac{n\lambda D}{d} \text{ where } n = 0, 1, 2, 3, 4 \ldots\ldots$$

(x therefore represents distance of n^{th} bright fringe from C)

Now,

$$\therefore \quad n = 0 \qquad x_0 = 0$$

$$n = 1 \qquad x_1 = \frac{\lambda D}{d}$$

$$\therefore \quad n = 2 \qquad x_2 = \frac{2\lambda D}{d}$$

$$n = 3 \qquad x_3 = \frac{3\lambda D}{d}$$

and so on.

$x_1 = \frac{\lambda D}{d}$ is the distance of the 1st bright fringe and therefore

$x_n = \frac{n\lambda D}{d}$ will be the distance of the nth bright fringe from C.

Therefore, separation between the centers of two consecutive bright fringes is the width of a dark fringe.

$$\therefore \qquad \beta_1 = x_n - x_{n-1} = \frac{x D}{d}$$

Similarly for dark fringes,

$$x_n = (2n-1)\frac{\lambda}{2}\frac{D}{d}$$

For $n = 1$ $\qquad x_1 = \frac{\lambda D}{2d} \rightarrow$ Position of 1st dark fringe

For $n = 2$ $\qquad x_2 = \frac{3\lambda D}{2d} \rightarrow$ Position of 2nd dark fringe

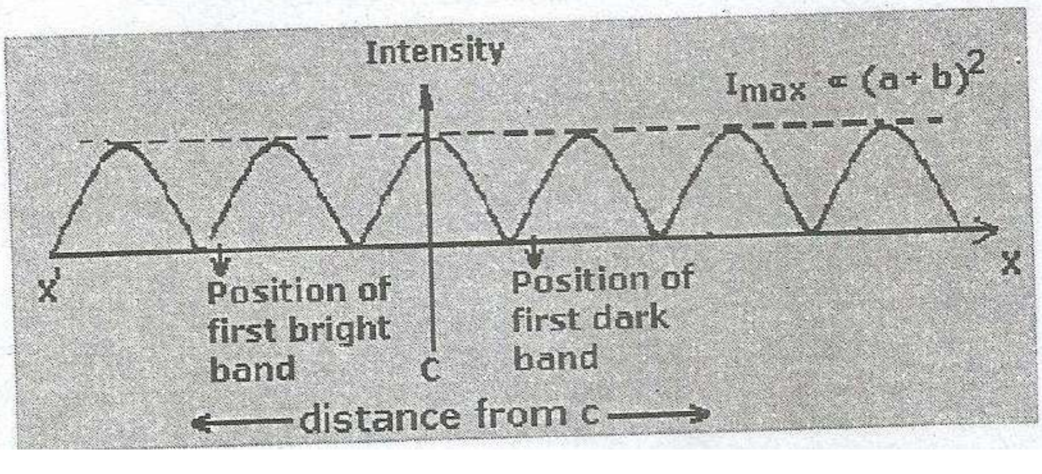
The separation between the centers of two consecutive dark interference fringes is the width of a bright fringe.

$$\beta_2 = x_n - x_{n-1} = \frac{\lambda D}{d}$$

The separation between the centers of two consecutive dark interference fringes is the width of a bright fringe.

All bright and dark fringes are of equal width as $b_1 = b_2$.

The intensity of all bright bands are the same. All dark bands also have same (zero) intensity. The intensity distribution Vs. distance is shown as :



(b) We have to change the wavelength into metres, the slit separation into metres.

"Second order" is a perfect number and has an infinite number of sig digs.

$$\lambda = \frac{d \sin 0}{n}$$

$$\sin 0 = \frac{\lambda n}{d}$$

$$\sin 0 = \frac{540e - 9x^2}{1.00e-5}$$

$$0 = 6.20°$$

(c) We need to solve the formula for "x", the distance from the central fringe.

$$\lambda = \frac{xd}{nL}$$

$$x = \frac{\lambda n L}{d}$$

For the violet light,

$$x = \frac{\lambda n L}{d}$$

$$x = \frac{350e - 9x1\ x\ 10}{0.00050}$$

$$x = 0.0070 \text{ m}$$

For the red light,

$$x = \frac{\lambda n L}{d}$$

$$x = \frac{700e - 9x^1 \times 10}{0.00050}$$

$$x = 0.0014 \text{ m}$$

Q. 5 (a) What are Newton's rings? How are they used to measure the wavelength of a monochromatic light.

(b) What are Fraunhofer Diffraction and Fresnel Diffraction? How are they different?

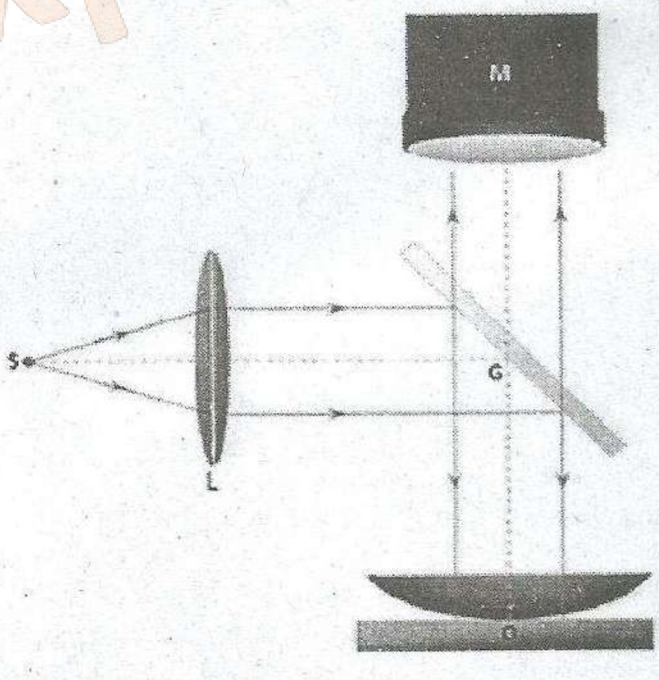
Ans. When a Plano convex lens of long focal length is placed in contact on a plane glass plate (as shown below), a thin air film is enclosed between the upper surface of the glass plate and the lower surface of the lens. The thickness of the air film is almost zero at the point of contact *O* and gradually increases as one proceeds towards the periphery of the lens. Thus, points where the thickness of air film is constant, will lie on a circle with *O* as center.

By means of a sheet of glass G, a parallel beam of monochromatic light is reflected towards the lens L. Consider a ray of monochromatic light that strikes the upper surface of the air film nearly along normal. The ray is partly reflected and partly refracted as shown in the figure. The ray refracted in the air film is also reflected partly at the lower surface of the film. The two reflected rays, i.e. produced at the upper and lower surface of the film, are coherent and interfere constructively or destructively. When the light reflected upwards is observed through microscope M which is focused on the glass plate, series of dark and bright rings are seen with center as O. These concentric rings are known as " Newton's Rings ".

At the point of contact of the lens and the glass plate, the thickness of the film is effectively zero but due to reflection at the lower surface of air film from denser medium, an additional path of $\lambda/2$ is introduced. Consequently, the center of Newton rings is dark due to destructive interference.

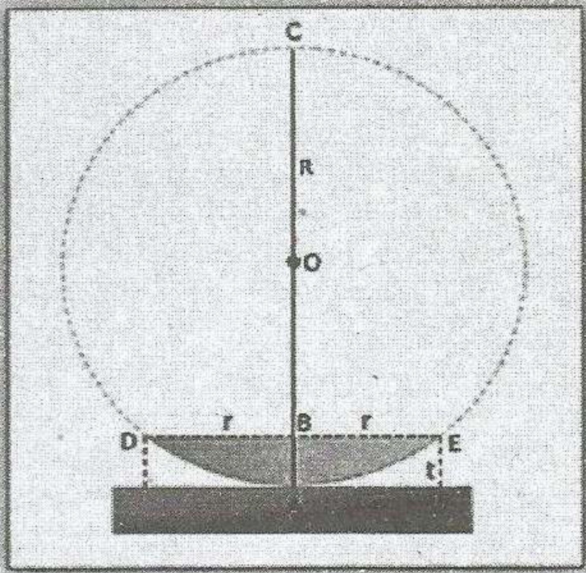
Let us consider a system of plano-convex lens of radius of curvature R placed on flat glass plate it is exposed to monochromatic light of wavelength λ normally.

The incident light is partially reflected from the upper surface of air film between lens and glass and light is partially refracted into the film which again reflects from lower surface with phase change of 180 degree due to higher index of glass plate. Therefore the two parts of light interfere constructively and destructively forming alternate dark and bright rings.



Experimental arrangement for observing Newton's rings

Now consider a ring of radius r due to thickness t of air film as shown in the figure given below:



According to geometrical theorem, the product of intercepts of intersecting chord is equal to the product of sections of diameter then,

$$DB \times BE = AB \times BC$$

$$r \times r = t\,(2R - t)$$

$$r^2 = 2Rt - t^2$$

As t is very small then t^2 will be so small which may be neglected, then,

$$r^2 = 2Rt$$

$$t = \frac{r^2}{2R} \qquad ...(1)$$

Radius for bright ring

The condition for constructive interference in thin film is,

$$2tn = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0, 1, 2, ...$$

From eq. (1) putting the value of t in the above equation we get,

$$2\left(\frac{r^2}{2R}\right)(1) = \left(m + \frac{1}{2}\right)\lambda$$

since $n = 1$ for air film

$$r^2 = \left(m + \frac{1}{2}\right)\lambda R$$

or

$$r = \sqrt{\left(m + \frac{1}{2}\right)\lambda R} \qquad ... (2)$$

For first bright ring, $m = 0$

$$r_1 = \sqrt{\frac{1}{2}\lambda R}$$

For second bright ring, $m = 1$

$$r_2 = \sqrt{\frac{3}{2}\lambda R}$$

For third bright ring, $m = 2$

$$r_2 = \sqrt{\frac{5}{2}\lambda R}$$

Similarly, for *Nth* bright ring,

$m = N - 1$

Radius for dark ring

The condition for destructive interference in thin film is,

$$2tn = m\lambda \qquad m = 0,1,2,....$$

By putting the value of t, we get,

$$2\left(\frac{r^2}{2R}\right)(1) = m\lambda$$

For air $n = 1$

$$r = \sqrt{m\lambda R}$$

For $m = 0 \Rightarrow r = 0$ *i.e.*, point of contact.

Now, if the radius of curvature of plano–convex lens is known and radius of particular dark and bright ring is experimentally measured then the wavelength of light used can easily be calculated using the above equations.

The wavelength of monochromatic light can be determined as,

$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}$$

Where, D_{m+p} is the diameter of the $(m + p)^{th}$ dark ring and D_m is the diameter of the m^{th} dark ring.

(b) Fraunhofer diffraction (named after Joseph von Fraunhofer), or far-field diffraction, is a form of wave diffraction that occurs when field waves are passed through an aperture or slit causing only the size of an observed aperture image to change due to the far-field location of observation and the increasingly planar nature of outgoing diffracted waves passing through the aperture.

It is observed at distances beyond the near-field distance of Fresnel diffraction, which affects both the size and shape of the observed aperture image, and occurs only when the Fresnel number $F << 1$, wherein the parallel rays approximation can be applied.

On the other hand, Fresnel diffraction or near-field diffraction is a process of diffraction that occurs when a wave passes through an aperture and diffracts in the near field, causing any diffraction pattern observed to differ in size and shape, depending on the distance between the aperture and the projection. It occurs due to the short distance in which the diffracted waves propagate, which results in a Fresnel number greater than 1 ($F > 1$). When the distance is increased, outgoing diffracted waves become planar and Fraunhofer diffraction occurs.

Characteristic	Fraunhofer Diffraction	Fresnel Diffraction
Wave fronts	Planar wave fronts	Cylindrical wave fronts
Observation distance	Observation distance is infinite. In practice, ofte at focal point of lens	Source of screen at finite distance from the obstacle
Movement of diffraction pattern	Fixed in position	Move in a way that directly corresponds with any shift in the object
Surface of calculation	Fraunhofer diffraction patterns on spherical surfaces	Fresnel diffraction patterns on flat surfaces

Q. 6. (a) Show that in a Fresnel zone plate there are more than one focus point.

(b) Describe the Spatial resolution of Fresnel zone plates and fabrication limitations.

Ans. The width dr_n of the rings of the circular grating decreases with increasing radius r_n of the rings. The ring radii can be calculated for monochromatic light of the wavelength λ emitted by a point source Q focused to a point p at a distance of $g + b = n$ with n a natural number, g the object distance and b the image distance. For each optical path from the source through the zone plate to the focus point, the light should interfere constructively. figure given on next page shows the optical path for light hitting the zone plate at a distance r_n from the optical axis.

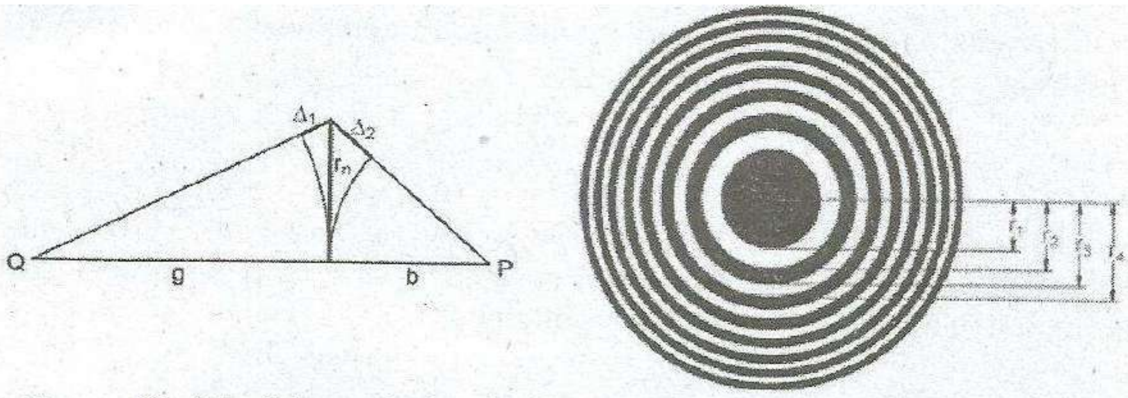


Figure : Sketch of the optical path difference between the source point Q and the focus point P at a distance from the optical axis

The optical path differences are

$$\Delta_1 = \sqrt{g^2 + r_n^2} - g$$

and

$$\Delta_2 = \sqrt{b^2 + r_n^2} - b$$

When the total optical path difference is

$$\Delta_{total} = \Delta_1 + \Delta_2 = n\lambda/2 = \sqrt{g^2 + r_n^2} - g + \sqrt{b^2 + r_n^2} - b$$

The light passing neighbour zones will interfere destructively. When now every second zone is absorbing the X-rays, the light passing the transmissive zones of a Fresnel zone plate will have an optical path difference of $n\lambda$ and hence will interfere constructively in the focus point P. Rearranging results in

$$\sqrt{g^2 + r_n^2} + \sqrt{b^2 + r_n^2} = g + b + n\lambda/2$$

or squared

$$g^2 + r_n^2 + b^2 + r_n^2 + \sqrt{\left(g^2 + r_n^2\right)\left(b^2 + r_n^2\right)} = (n\lambda/2 + g + b)^2$$

After rearranging, squaring and separating r_n the radius of the n^{th} Fresnel zone is

$$r_n^2 = \frac{n^4\lambda^4/16 + n^3\lambda^3(g+b)/2 + n^2\lambda^2(gb + (g+b)^2) + 4n\lambda gb\,(g+b)}{4(g+b)\,(n\lambda + (g+b)) + n^2\lambda^2}$$

As λ is small compared to b and g, neglecting all higher order terms in λ and using $n\lambda << g + b$ is a good approximation

$$r_n^2 \approx n\lambda \frac{gb}{g+b} = n\lambda f$$

So the focal length of a Fresnel zone plate depends on the wavelength. In the last step the equation for the focal length f of a thin lens was used

$$\frac{1}{f} = \frac{1}{g} + \frac{1}{b} \Rightarrow f = \frac{gb}{g+b}$$

Differentiating the approximated r_n with respect to n gives the width dr_n of the nth ring

$$dr_n = \frac{r_n}{2n}$$

The surface of each zone is constant over the zone plate, when the wavelength and the focal length are fixed. This means that each zone contributes to the focus intensity with an equal amount of transmitted light

$$F_{nthzone} = \pi\left(r_{n+1}^2 - r_n^2\right) = \pi\lambda f$$

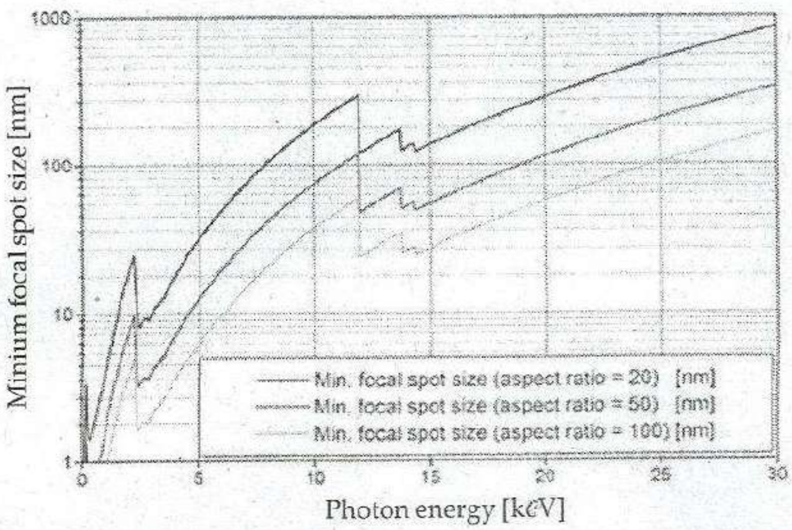
A grating mostly has more than one diffraction order. In a Fresnel zone plate this leads to more than one focus point. The negative diffraction orders even result in diverging beams, in other words: zone plates also behave like a dispersive lens with virtual focus points. When a zone plate is used as focusing lens, normally only the first diffraction order is used and all other orders have to be blocked by a suitable aperture.

(b) The spatial resolution δ_{ZP} of a Fresnel zone plate depends, according to the Abbe theory, on the numerical entrance aperture $NA_{E,\,ZP}$ of the zone plate, with the refractive index decrement n_{Material} (= real part of the refractive index n*) of the medium between the object and the zone plate, the entrance acceptance angle Θ_E of the zone plate and the diffraction order m used. The following approximations were made: normally the entrance acceptance angle Θ_E is small and the refractive index decrement nearly one.

Based on the Rayleigh-criterion (resolution of two neighbour points resolved with a lens with the numerical aperture NA) the largest theoretical possible spatial resolution δ_{ZP} can be calculated for incoherent, monochromatic illumination to be

$$\delta_{ZP} = 1.22\,\frac{\lambda}{2NA_{E,\,ZP}} = 1.22\,\frac{dr_n}{m}$$

In the fabrication process of zone plates the gold absorbers are mostly produced by electroplating a micro structured substrate. The aspect ratio of these micro structures is defined as the ratio of structures height to the smallest lateral structure size. The highest aspect ratio realized in 2009 was in the region of about 120 using the LIGA-process. If the permitted transmission of the absorbing gold structures is relatively high, e.g., 50%, and the aspect ratio achieved in the process is e. g., 100, zone plate for 10 keV light with a minimum spot size of about 40 nm are practicable. To achieve smaller spot sizes or the same spot size for larger photon energies either the required aspect ratio would have to be increased or the efficiency of the zone plate will decrease due to the increasing transmission of the absorber structures.



Q. 7(a) Describe half period zones of a plane wave and show that

(i) The amplitude due to a large wave front at a point is just half that due to first half period Fresnel zone.

(ii) The intensity will be one fourth that due to the first half period zone

(b) Describe three different types of polarisation.

Ans. Let a source S emits a plane wave front –*ABCD* travelling from left to right and has wavelength λ. To see the effect of wave front-point – *P* at a distance p from the wave front. Let's divide the wave front into Fresnel's zones with P as a centre and radii equal top+ n λ/2 ($n = 1, 2, 3 \ldots$).and drawing concentric spheres on the wave front as shown in figure. The area between two spheres is called zone. The secondary waves from any two consecutive zones reach the point *P* with a path difference of $A/2$ ($= T/2$) therefore, the name is half *O* period zones. Here *T* stands for period. The point *O* is called the pole of the wave front with respect to point *P*.

Radii of Half Period Zone : It will be

$$OM_1 = v\{(p + \lambda/2)^2 \text{-} P^2\} = v(p\lambda)$$
$$OM_2 = v\{(p + 2\lambda/2)^2 \text{-} p^2\} = v(2p\lambda)$$

Similarly,

$$OMn = v\{(p + n\lambda/2)^2 - p^2\}$$

Thus radii are proportional to the square roots of natural numbers.

Area of Half Period Zone : The area of nth zone will be

$$= p\ \{\ (p + n\lambda/2)^2 - p2\} - p\ \{\ ((p+(n-1)\lambda/2)^2 - p^2\}$$
$$= p\ \{\ p\lambda + \lambda^2(2n-1)/4\} = pp\lambda$$

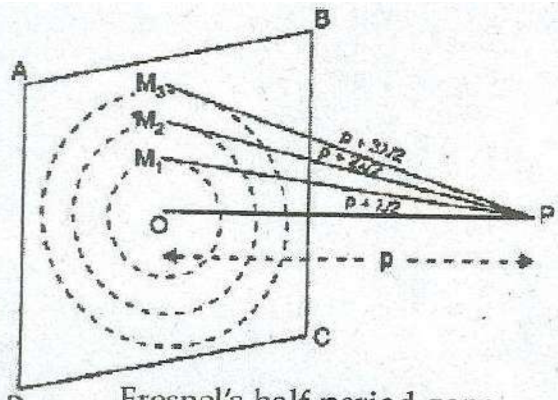
which says that area of each half period zone is nearly the same.

The distance of point P from half period zone : It is

$$= (p + n\lambda/2) + (p + (n - 1)\ \lambda/2)\ /\ 2 = p + (2n - 1)\ \lambda/\ 4)$$

Amplitude at point P due to one zone : It is given as

R_n = area of the zone/ distance of point P from zone x obliquity factor



Fresnel's half period zone

$$= p\lambda\,(1 + \cos \lambda n)$$

If n increases, cos λn decreases and hence Rn also decreases.

Resultant amplitude of point P due to whole wavefront : As the path difference between the two consecutive zones is λ/2 so they are reaching in opposite phase. If R_1, R_2, R_3 and so on are the amplitudes at point P from various zones then the resultant amplitude at point will be

$$R = R_1 - R_2 + R_3 - R_4 \ldots (-1)n - 1\ Rn$$

We can have

$$R_2 = R_1 + R_{3/2} \quad R_4 = R_3 + R_{5/2} \quad \text{and so on}$$

$$R = R_{1/2} + R\,n/2\text{, for n to be odd}$$

$$= R_{1/2} + R\,n/2 - Rn \text{ for } n \text{ to be even}$$

taking $\quad Rn - 1 = Rn \quad$ as n is very large, then

$$R = R_{1/2} + Rn/2 = R_{1/2}$$

Thus the amplitude due to a large wave front at a point is just half that due to first half period Fresnel zone. The intensity will be

$$I = R_2,$$

$$= R_{1/4}$$

i.e., one fourth that due to the first half period zone

(b) **Elliptical Polarization** : The tip of the electric field vector follows an elliptical trajectory in the xy plane; accordingly, the light is denoted as elliptically polarized. The orientation of the ellipse in the xy plane is constant in time; the polarization angle corresponds to the angle between the positive x axis and the semi-major axis of the ellipse (counted in counterclockwise direction).

Elliptical polarization is the most general state of polarization of an electromagnetic wave. Linear polarization occurs if the polarization ellipse degenerates into a line. Circular polarization corresponds to the – opposite – special case of the ellipse degenerating into a circle.

Linear Polarization : For the case 1 = 2, using here specifically 1 = 2 = 0 without loss of generality, we have the magnetic field perpendicular to the direction of travel and to the electric field. The amplitude of the magnetic field, B, is related to the amplitude of the electric field, E, like

$$B = E/c.$$

Polarization and Polarimetry

$$Ex(t) = Ex(0)\cos(\omega t)$$

$$Ey(t) = Ey(0)\cos(\omega t).$$

The orientation of E then depends only on the magnitudes of $Ex(0)$ and $Ey(0)$ and is independent of time; the angle is constant. The radiation is linearly polarized with polarization angle " [0, ω]. The orientation of the plane wherein the wave is located – the plane of polarization – as given by has an orientation but no direction; accordingly, the location of linear polarization in the xy plane is not a vector.

In physics and astronomy, the x and y components of linearly polarized light are commonly identified with horizontal (H) and vertical (V) polarizations, respectively.

Circular Polarization

In case of a relative phase shift 2 = 1 ± π/2, using here specifically 1 = 0 without loss of generality, and $Ex(0) =$

$Ey(0)$,

we have

$$Ex(t) = Ex(0)\cos(\omega t)$$

$$Ey(t) = \pm Ey(0)\sin(\omega t)$$

The tip of the electric field vector moves circularly in the xy plane with angular frequency : the radiation is circularly polarized. The sign of $Ey(t)$ – which derives from the relative phase – determines the sense of the motion of E. A positive sign, corresponding to counterclockwise motion, is commonly referred to as right-hand circular (RHC) polarization. Accordingly, a negative sign, corresponding to clockwise motion, is denoted as left-hand circular (LHC) polarization.