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All Lab Experiments

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B.Sc. => Solid-State Physics
Chapter - 7
Superconductivity

7

Superconductivity

Syllabus: Experimental Results. Critical Temperature. Critical magnetic field. Meissoner effect. Type I and II Superconductors, London's Equation and Penetration Depth. Isotope effect. Idea of *BCS* theory (No derivation)

Q.1. What are Isotope Effect, Critical Magnetic Field and Meissoner's Effect?

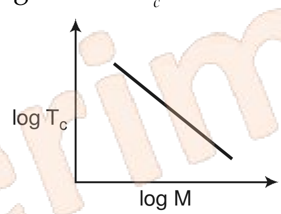
Ans. Isotope Effect: It was established that the critical temperature (T_c) of superconductors varies with their isotopic cases (M). This was independently found by Maxwell and Serin. This variation is given by the relation

$$T_c M^{1/2} = \text{constant} \quad ...(1)$$

or $$M^{1/2} \alpha \frac{1}{T_c} \text{ i.e. } T_c \text{ decrease as}$$

isotopic mass M increase

The variation of log T_c with log M different isotopes of mercury is shown in Fig. It is clear from the figure that T_c decreases as M increases.



According to the relation 3.1, if M increases, T_c decreases, e.g. T_c is found to be 4.185 K and 4.146 K for two isotopes of mercury with mass 199.5 and 203.4 amu respectively. The it is also known that Debye temperature θ_D of the phonon (quantum of energy in elastic waves) spectrum is also proportional to $M^{-1/2}$ and therefore,

$$\theta_D M^{-1/2} = \text{constant} \quad ...(2)$$

From (1) and (2), we have

$$\frac{T_c}{\theta_D} = \text{constant}$$

This can be expressed as

$$T_c \alpha\, \theta_D \alpha\, M^{-1/2} \qquad ...(3)$$

The relations (2) and (3) suggest that lattice vibrations might be related to super conducting state of the material. This led to the belief that electorn phonon interaction is responsible for super conductivity.

This was found true when Frohlich showed that two electrons can attract each other with the help of a phonon (lattice vibration).

In 1957, Bardeen, Cooper and Schrieffer proposed a detailed microscopic theory, known as BCS theory for superconductivity in which these interactions are included.

Critical Magnetic Field (H_c): It is observed that if the superconductor is placed in a sufficiently strong magnetic field, the superconductor becomes a normal conductor i.e., resistance again comes into picture. The field at which suter conductivity is destroyed is called the critical field H_c. This critical field is a function of temperature. A graph for H_c versus temperature is shown in Fig. This type of graph is known as magnetic phase diagram. It is found that superconducting state is stable only for certain ranges of temperature and field. The relation between critical field and temperature is given as

$$H_c(T) = H_c(0)\left[1 - \frac{T^2}{T_c^{\,2}}\right] \qquad ...(3.4)$$

Here H_c (0) is the critical magnetic field at $0K$ and $H_c(T)$ be its value at temperature T. This relation shows that the graph will be a parabolic curve as shown in Fig.

According to this relation at $0K$ i.e. at T = 0

$$H_c(T) = H_c(0)$$

and at $T = T_c$

$$H_c(T_c) = 0$$

The curve defines the boundary below which superconductivity is present and outside, the conductor behaves as normal as shown in Fig.

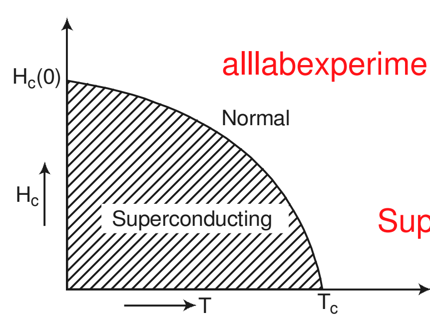


Fig: Variation of critical field with temperature.

Silsbee effect: It is found that when a strong current is passed through a superconductor, its superconductivity gets destroyed. It occurs only when the current is above the certain critical value, called critical current. It was thought that by passing a large current through a superconductor, a strong permanent, a magnet could be produced but it was found by Silsbee that the magnet produced in this way also destroys teh superconductivity of the material. Hence the reason of the destruction of superconductivity is magnetic field associated with the current and not the current itself. This effect poses a limit to the current to be passed through a superconductor.

Meissoner Effect (Perfect Diamagnetism).

Meissoner and Ochsenfeld found that a superconductor expels all the magnetic flux as soon as transition temperature T_c occurs while the sample is lying in an external magnetic field. This means that superconductors behave as perfect diamagnet. This situation occurs even if T_c is reached first and then applying magnetic field. In this case again no magnetic flux enters the sample. This phenomenon of expelling magnetic flux by a superconductor is known as Meissoner effect. This is illustrated on Fig.

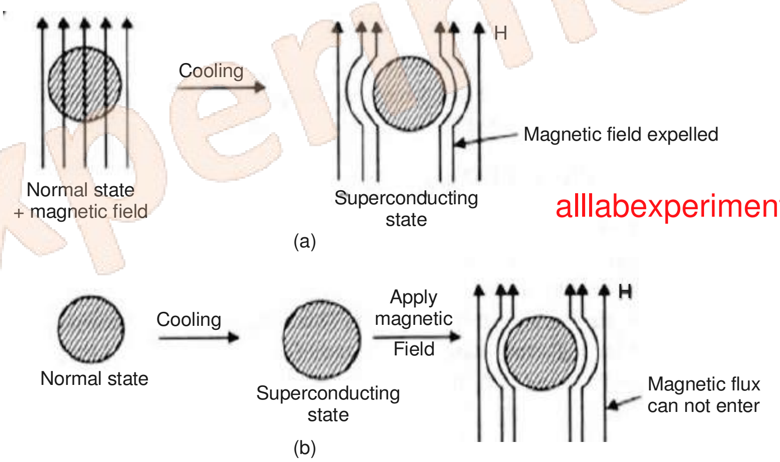


Fig. (a) Field applied before T_c is reach (*b*) Filed applied after *Tc* is reached (*H* is less than the critical field H_c)

According to this kleffect, magnetic flux density (B) does not exist in the superconductor. Therefore,

$$B = \mu_0(H + M) = 0, M \text{ is the magnetisation}$$

i.e. $$H = -M$$

As we know teh susceptibility χ is given as

$$\chi = \frac{M}{H}$$

Hence, here $$\chi = -1$$

This is the case with perfect ideal diamagnetic substance. Hence the superconductor becomes a perfect diamagnet.

Contradiction to Maxwell's equation.

The perfect diamagnetism of the superconductor cannot be explained simply on the basis of zero resistivity. From Ohm's law.

$$\vec{J} = \sigma\vec{E}, (\sigma, \text{conductivity})$$

$\therefore$ $\vec{E} = \frac{1}{\sigma}\vec{J}$ or $\vec{E} = \rho\,\vec{J}, (\rho, \text{resistivity})$, we see if $\rho = 0$; then

$\vec{E} = 0$, Also the Maxwell's equation

$$\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt} \text{ becomes}$$

$$\frac{d\vec{B}}{dt} = 0$$

i.e. $\vec{B} = \text{constant}$ ...(1)

The meaning of this is obvious that Magnetic flux through the perfect conductor does not change even if it is cooled to its transition temperature.

Hence, we find that zero resistivity leads us to a situation which says that magnetic flux through the conductor remains constant and is not expelled. Thus Meissner effect contradicts the Maxwell's equation.

Therefore, we conclude that the two properties viz. state of zero resistivity and perfect diamagnetism are mutually independent to each other. Thus, for a superconductor, we have

$$E = 0 \text{ and } B = 0$$

Q.2. Derive London equation for superconductor and obtain expression for penetration depth?

Ans. To explain the phenomenon of flux through thin films of superconductors *F*. London and *H*. London proposed a modification of electrodynamic equations which can explain Meissner effect as well as flux penetration in thin films. These equations are known as London equations. These equations are based on a postulate that in the superconducting state the current density is directly proportion to the vector potential $\vec{A}$ of the local magnetic field, where

$$\vec{B} = \text{Curl } \vec{A}.$$

$\therefore$ $\vec{J}_s \propto \vec{A}$

or $$\vec{J}_s = -\frac{1}{\mu_0\lambda_L^2}\vec{A} \quad ...(2)$$

where $\frac{1}{\mu_0 \lambda_L^2}$ is constant of proportionally and λ_L is called penetration depth.

This is the London equation.

Taking the curl of both sides of equation (36).

$$\therefore \quad \text{Curl}\,\vec{J} = -\frac{1}{\mu_0 \lambda_L^2}\vec{\nabla}\times\vec{A} = -\frac{1}{\mu_0 \lambda_L^2}\vec{B} \quad ...(3)$$

Equation (3) is also a London equation, which holds true, independent of geometry of superconductor (ring, cylindrical etc).

Now we know that for conditions

$$\vec{B} = \mu_0 \vec{J}_s \quad ...(4)$$

Taking curl of both sides

$$\therefore \quad \text{curl}\ \vec{B} = \mu_0\ \text{curl}\ J_s$$

$$\text{or grad div} \quad (\vec{B}) - \nabla^2\vec{B} = \mu_0\ \text{curl}\ \vec{J}_s$$

$$\text{or} \quad -\nabla^2\vec{B} = \mu_0\ \text{curl}\ \vec{J}_s \quad (\because \text{div}\ \vec{B} = 0)$$

Use equation (3)

$$\therefore \quad -\nabla^2\vec{B} = \mu_0\left(-\frac{1}{\mu_0 \lambda_l^2}\vec{B}\right)$$

$$\text{or} \quad \nabla^2\vec{B} = \frac{\vec{B}}{\lambda_L^2} \quad ...(5)$$

This equation does not have a solution uniform in space.

Hence a uniform Magnetic field can not exist in a superconductor. This explains the Meissner effect.

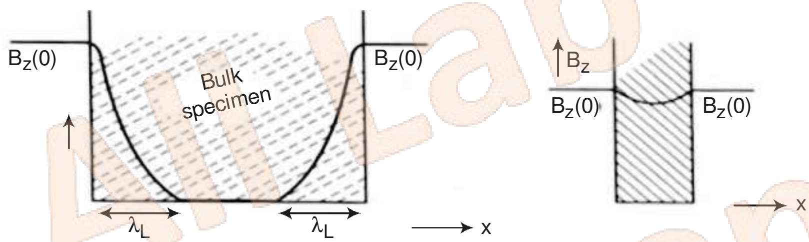
If the magnetic field is applied along Z-direction on the semi-infinite slab extending in the x-direction, the equation

$$\text{(5) becomes} \quad \frac{d^2 B_z}{dx^2} = \frac{1}{\lambda_L^2}B \quad ...(6)$$

The solution of this equation is

$$B_z = B_z(0)e^{-x/\lambda_L} \quad ...(7)$$

Here $B_z(0)$ is the value of magnetic field at the boundary of slab. Hence the field decay exponentially and the parameter λ_L is called penetrates depth. Hence penetration depth(λ_L) is the effective depth up to which the magnetic field penetrates a superconductor. Fig. shows the obliteration of magnetic field into a bulk specimen and a thin film, whose thickness is of the order of penetration depth. In bulk specimen, the thickness of the specimen is much larger than penetration depth. So in this case field does not penetrate a larger portion of the specimen. Hence, in such a situation $\vec{M} \cong -\vec{H}$ and so $\chi = -1$, which proves Meissner effect. A thin film will not exhibit the same diamagnetism ($\chi = -1$) as a bulk specimen since it is partially penetrated by the magnetic field.



(a) Bulk specimen (b) Thin film

$$\therefore \quad \frac{d\vec{J}_s}{dt} = -n_s e \frac{d\vec{v}}{dt}$$

$$\text{or} \quad \frac{d\vec{J}_s}{dt} = -n_s e \left(-\frac{e}{m}\vec{E}\right)$$

$$\text{or} \quad \frac{d\vec{J}_s}{dt} = \frac{n_s e^2}{m}\vec{E} \qquad \ldots(8)$$

Taking curl of both sides of equation (2)

$$\left(\nabla^2 \frac{d\vec{J}_s}{dt}\right) = \frac{n_s e^2}{m}\nabla^2 \times \vec{E}$$

$$\text{or} \quad \frac{d}{dt}(\vec{\nabla}\times\vec{J}_s) = \frac{n_s e^2}{m}\nabla^2 \times \vec{E}$$

$$\text{or} \quad \frac{d}{dt}(\vec{\nabla}\times\vec{J}_s) = \frac{n_s e^2}{m}\left(-\frac{d}{dt}\vec{B}\right)$$

$$\text{or} \quad \frac{d}{dt}(\vec{\nabla}\times\vec{J}_s) = -\frac{d}{dt}\left(\frac{n_s e^2}{m}\vec{B}\right)$$

$$\therefore \quad \vec{\nabla}\times\vec{J}_s = -\frac{n_s e^2}{m}\vec{B}$$

$$\text{or} \quad \vec{\nabla}\times\vec{J}_s = -\frac{n_s e^2}{m}(\vec{\nabla}\times\vec{A}) \qquad (\because \vec{B} = \vec{\nabla}\times\vec{A})$$

$$\Rightarrow \quad \vec{J}_s = -\frac{n_s e^2}{m}\vec{A} \qquad \ldots(9)$$

Also $$\vec{J}_s = -\frac{1}{\mu_{02}\lambda_L^2}\vec{A} \qquad ...(10)$$

Compare equation (9) and (10)

$$\therefore \qquad -\frac{n_s e^2}{m} = -\frac{1}{\mu_0 \lambda_L^2}$$

or $$\lambda_L^2 = \frac{m}{\mu_0 n_s e^2}$$

Now $$c = \frac{1}{\sqrt{\mu_0 \in_0}} \therefore \mu_0 = \frac{1}{c^2 \in_0}$$

$$\therefore \qquad \lambda_L^2 = \frac{m \in_0 c^2}{n_s e^2}$$

or $$\lambda_L = \left(\frac{m \in_0 c^2}{n_s e^2}\right)^{1/2} \qquad ...(11)$$

This is the expression for London's penetration depth. It is clear from equation (11) that the impurities in a material should not effect the penetration depth. But papered observed that penetration depth changes roughly by a factor of two at low temperature. This leads to the non-local generalization of London theory.

Q.3. Make difference between Type I and Type II Semiconductors

Ans. SoftSuperconductors (Type1):

1. Soft superconductors are those which can tolerate impurities without affecting the superconducting properties.
2. They have low critical field.
3. Show complete Meissner effect.
4. The current flows through the surface only.
5. E.g. Tin, Aluminium

Hard Superconductors (Type II):

1. Hard superconductors are those which cannot tolerate impurities, i.e., the impurity affects the superconducting property.
2. They have high critical field.
3. Hard super conductors trap magnetic flux and hence Meissner effect is not complete.
4. It is found that current flows throughout the material.
5. E.g. Tantalum, Neobium.

Q.4. Give Brief Introduction on BCS Theory. [Important]

Ans. (i). The situation is shown in Fig. The whole process is virtual and the phonons involved are known as virtual phonons, because of very short life of these phonons the energy may not be conserved (which is in accordance with uncertainty principle.)

Now it is clear that lattice vibrations play an important role in the phenomenon of super conductivity, hence materials with large amplitude lattice vibrations are good super conductor though they may not be good conductors at room temperature. Good conductors such as copper, hence they are not superconductors. Metals like mercury, tin, lead, etc. Have large amplitude lattice vibrations hence these are good superconductors.

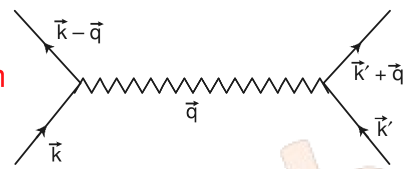


Fig. Electron phonon interaction

(ii) Cooper pair: As already described, a cooper pair is produced when two electrons interact with each other via phonon attractively and by overcoming the Coulomb's repulsive forces. The binding energy of a cooper pair is called energy gap E_g, which is of the order of 10^{-3} eV, which is less than the energy of the unbounded (free) electrons. Since the superconductivity phenomenon is due to these cooper pairs, which have binding energy only of the order of 10 kelvin, hence superconductivity is a low temperature phenomenon.

Although the two electrons are bound in a cooper pair but they are weakly bound and are in a continuous process of breaking and making new pairs with new partners. The electrons in a cooper pair have opposite spin and hence the total spin of the pair is zero. As a result pairs behave as bosons (instead of fermions as individual electrons in a normal conductor) and hence they follow Bose-Einstein statistics i.e. many pairs can live together. Also when there is current in the superconductor, the linear momentum of each electron of the cooper pair is equal and opposite, to make the sum zero. If all the pairs have same constant total momentum, then there will be no inhibition to the unavoidable process of old pairs by breaking up and new pairs reforming since in this situation any pair can be converted to any other pair by phonon exchange and hence a very large number of pairs are present. All the pairs are in the same ground state of pairs. Each pair is now having non-zero momentum. To alter requires a very large amount of energy and as a consequence the current persists indefinitely if undisturbed.

(iii) BCS ground state and existence of energy gap: The BCS ground state is different from the normal ground state. In the normal conductors the electron (fermi gas) do not interact with each other but in case of superconductivity the electrons interact and form cooper pairs which are responsible for the phenomenon of superconductivity. In case of non-interacting fermi gas all the energy states above the fermi surface are vacant and below it, all states are filled. This filled state blow fermi surface is the ground state for normal conductors. To form an excited state, arbitrarily very small energy is required as the first excited state is just above the fermi surface.

Now in the case of superconductivity the electrons with each other attractively and phonons mediate this interaction (as already described). This interacting fermi gas from the BCS ground state. This ground state is separated from lowest excited state by energy gap E_g, which is the binding energy of the cooper pair. This energy gap is a function of temperature unlike the energy gap of semiconductors and insulators. The maximum energy gap in superconducting state occurs at 0 K, where pairing is complete and vanishes at $T = T_c$, where pairing also vanishes.

The formation of BCS ground state is shown in Fig. A look at the BCS state shows that it has more kinetic energy than the fermi state, but this is not the case since the attractive potential energy (not shown) of BCS state lowers the BCS state from that of fermi state. As a consequence the BCS state is more stable. The occupation of one-particle orbital in BCS ground state near Ef are filled like Fermi-Dirac distribution for some finite temperature. However, in BCS ground state one particle orbitals are occupied by cooper pairs. If an orbital with wave vector $\vec{K}$ and spin up is occupied then the orbital with wave vector $-\vec{K}$ and spin down is also occupied and if $K_1 \uparrow$ is vacant then $-K_1 \downarrow$ is also vacant.

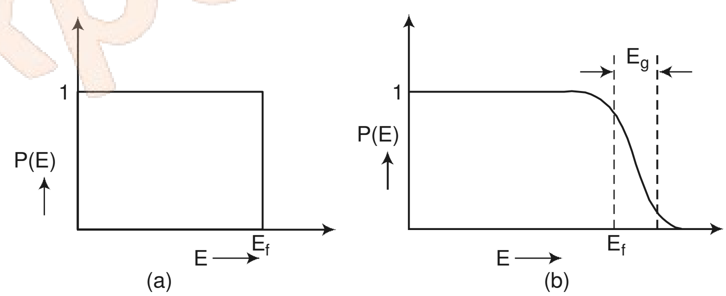


Fig: (a) Probability of occupation $P(E)$ as a function of I for an orbital of kinetic energy E for ground state of non-interacting fermi gas (*b*) BCS ground state differ from (*a*) only in a region of width if the order of a region of a region of width of the order of energy gap E_g at the fermi surface (both curves at 0K).

BCS state is more stable. The occupation of one-particle orbital in BCS ground state neat Ef are filled like Fermi-Dirac distribution for some finite temperature. However, in BCS ground state one-particle orbitals are occupied by cooper pairs. If an orbital with wave vector $\vec{K}$ and spin up is occupied then the orbital with wave vector $-\vec{K}$ and spin down is also occupied and if $K_1 \uparrow$ is vacant then $-K_1 \downarrow$ is also vacant.