

# Free Study Material from All Lab Experiments



**B.Sc. => Solid-State Physics  
Chapter - 4  
Dielectric Properties of Materials**

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## Dielectric Properties of Materials

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**Syllabus:** Polarization Local Electric Field at an Atom. Depolarization Field. Electric Susceptibility. Polarizability. Clausius Mosotti Equation. Classical Theory of Electric Polarizability. Normal and Anomalous Dispersion. Cauchy and Sellmeier relations. Langevin - Debye equation. Complex Dielectric Constant. Optical Phenomena. Application: Plasma Oscillations, Plasma Frequency, Plasmons, TO modes.

**Q.1. Obtain an expression for local electric field inside a dielectric with cubic symmetry.**

**Ans. Internal field or Local field in solids:** Consider a dielectric material and is subjected to external field of intensity  $E_1$ . The charges are induced on the dielectric plate and the induced electric field intensity is taken as  $E_2$ . Let  $E_3$  be the field at the center of the material.  $E_4$  be the induced field due to the charges on the spherical cavity. The total internal field of the material is

$$E_i = E_1 + E_2 + E_3 + E_4$$

Now consider the Electric field intensity applied  $E_1$

$$E_1 = \frac{D}{\epsilon_0}$$

$$D = E\epsilon + P$$

Substituting the Electric flux density  $D$  in  $E_1$ , we get

$$E_1 = \frac{E\epsilon_0 + P}{\epsilon_0}$$

$$E_1 = E + \frac{P}{\epsilon_0}$$

$E_2$  is the Electric field intensity due to induced or polarized charges

$$E_2 = \frac{-D}{\epsilon_0} \quad \text{alllabexperiments.com}$$

Here the charge is induced due to the induced field so the electric flux density  $D$  changes to the electric polarization  $P$

$$E_2 = \frac{-P}{\epsilon_0} \quad \text{Support by Donating}$$

Since we have considered that the specimen is non polar dielectric material, at the center of the specimen the dipole moment is zero and hence the electric field intensity at the center is zero due to symmetric structure.

$$E_3 = 0$$

Now consider a circle from the center of the dielectric material. In order to calculate the electric field intensity  $E_4$  on the surface of spherical cavity, the polarization should be calculated by resolving it into two components, as shown in the following figure.

As we know the polarization  $P$  is the induced charge per unit area

$$p = \frac{q}{A}$$

Here the polarization changes to its component we have divide in the figure and the charge changes to  $dq$  where the area of cross section changes to  $ds$ .

$$p \cos \theta = \frac{dq}{ds}$$

$$dq = p \cos \theta ds$$

Now this equation can be solved by finding out the values of the charge  $dq$  in the surface are  $ds$ . We know the Electric field intensity  $E$ .

$$E = \frac{1}{4\pi\epsilon_0 r^2} \frac{q}{r^2} \quad \text{alllabexperiments.com}$$

Multiplying with the cosine angle on both the sides we get

$$E_4 \cos \theta = \frac{1}{4\pi\epsilon_0} \times \frac{q \cos \theta}{r^2}$$

Now by applying all the present condition for the above equation we

$$E_4 = \frac{1}{4\pi\epsilon_0} \times \frac{dq \cos \theta}{r^2}$$

Now substituting the charge  $dq$  in the above equation we get

$$E_4 = \frac{1}{4\pi\epsilon_0} \times \frac{d \cos \theta ds \times \cos \theta}{r^2}$$

In the above equation  $ds$  should be calculated. Consider the spherical cavity and mark the points  $R$  and  $Q$  on the sphere of radius ' $r$ ' as shown in the above

figure and drop vertical line from Q and mark it as S. By applying the surface area of a sphere formula we get,

$$ds = 2\pi \times (RQ) \times (QS)$$

We know that angle of the sector  $\theta = \frac{\text{length of the arc}(l)}{\text{radius of the circle}(r)}$

$$d\theta = \frac{RQ}{r}$$

$$RQ = rd\theta.$$

Now consider the right angled triangle OQS from th

$$\sin \theta = \frac{QS}{OQ}$$

$$QS = r \sin \theta$$

Now substituting the values RQ and QS in the surface area we get,

$$ds = 2\pi(rd\theta)(r \sin \theta)$$

$$ds = 2\pi r^2 \sin \theta d\theta$$

Now substituting all the values in the electric field intensity on the spherical cavity  $E_4$  we get,

$$E_4 = \frac{1}{4\pi\epsilon_0} \times \frac{P \cos \theta ds \times \cos \theta}{r^2}$$

$$E_4 = \frac{1}{4\pi\epsilon_0} \times \frac{P \cos \theta}{r^2} \times 2\pi r^2 \sin \theta d\theta \cos \theta$$

$$E_4 = \frac{P}{2\epsilon_0} \sin \theta \cos^2 \theta d\theta$$

Integrating with in the limits 0 to  $\pi$

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$$E_4 = \int_0^\pi \frac{P}{2\epsilon_0} \sin \theta \cos^2 \theta d\theta$$

$$E_4 = \frac{P}{2\epsilon_0} \int_0^\pi \sin \theta \cos^2 \theta d\theta$$

On solving the integration we get,

$$E_4 = \frac{P}{2\epsilon_0} \times \left(\frac{2}{3}\right)$$

$$E_4 = \frac{P}{3\epsilon_0}$$

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So the total electric field

$$E_i = E_1 + E_2 + E_3 + E_4$$

$$E_i = E = \frac{P}{\epsilon_0} - \frac{P}{\epsilon_0} + 0 + \frac{P}{3\epsilon_0}$$

Hence the Internal field obtained is

$$E_i = E + \frac{P}{3\epsilon_0}$$

### Q.2. Derive Clausius Mossotti equation?

**Ans.** Clausius Mossotti's equation gives the relation between the dielectric constant and the ionic polarizability of atoms in dielectric material. If there are  $N$  number of atoms, the dipole moment per unit volume which is called Polarization is given by,

$$P = NaE_i$$

we know internal field

$$E_i = \left( E + \frac{P}{3\epsilon_0} \right)$$

From above equations

$$P = \left( E + \frac{P}{3\epsilon_0} \right)$$

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$$P = \frac{Na_e}{3\epsilon_0} (3E\epsilon_0 + P)$$

$$\frac{Na_e}{3\epsilon_0} = \frac{P}{(3E\epsilon_0 + P)}$$

we know polarization from the relation between polarization and dielectric constant

$$P = E\epsilon_0(\epsilon_r - 1)$$

from the above two equations we get,

$$\frac{Na_e}{3\epsilon_0} = \frac{E\epsilon_0(\epsilon_r - 1)}{3E\epsilon_0 + E\epsilon_0(\epsilon_r - 1)}$$

$$\frac{Na_e}{3\epsilon_0} = \frac{E\epsilon_0(\epsilon_r - 1)}{E\epsilon_0(\epsilon_r + 2)}$$

$$\frac{Na_e}{3\epsilon_0} = \frac{(\epsilon_r - 1)}{(\epsilon_r + 2)}$$

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### Q.3. What is Electric polarizability?

**Ans.** Electric polarizability is the relative tendency of a charge distribution, like the electron cloud of an atom or molecule, and consequently of any material body, to have its charges displaced by any external electric field, which in the uniform case is applied typically by a charged parallel-plate capacitor. The polarizability in isotropic media is defined as the ratio of the induced dipole moment of an atom to the electric field that produces this dipole moment.  $P = \alpha E$

Polarizability has the SI units of  $C \cdot m^2 \cdot V^{-1} = A^2 \cdot s^4 \cdot kg^{-1}$  while its cgs unit is  $cm^3$ . Usually it is expressed in cgs units as a so-called polarizability volume, sometimes expressed in  $\text{Å}^3 = 10^{-24} cm^3$

The polarizability of individual particles is related to the average electric susceptibility of the medium by the Clausius-Mossotti relation. For example, an electric field in the  $x$ -direction can only produce an  $x$  component in  $P$  and if that same electric field were applied in the  $y$ -direction the induced polarization would be the same in magnitude but appear in the  $y$  component of  $P$ .

### Q.4. What is Electric susceptibility?

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**Ans.** It is a dimensionless proportionality constant that indicates the degree of polarization of a dielectric material in response to an applied electric field.

$$P = \epsilon_0 \chi_e E$$

Where  $P$  is polarization density,  $\chi_e$  is electric susceptibility,  $E$  is electric field.

The greater the electric susceptibility, the greater the ability of a material to polarize in response to the field, and thereby reduce the total electric field inside the material (and store energy). It is in this way that the electric susceptibility influences the electric permittivity of the material and thus influences many other phenomena in that medium, from the capacitance of capacitors to the speed of light.

### Q.5. Make difference between Normal and anomalous dispersion.

**Ans.** When a white light (eg. Sunlight, or light from an incandescent lamp) is passed through a prism we observe colour separation. Light being electromagnetic oscillations, the different colours have different wavelengths and different frequencies in vacuum for all of them move with the same speed  $c$ . Our eyes sense (map) different wavelengths of visible spectrum with

different colours. The separation happens because the different wavelengths have different refractive indices. Whenever light enters a dielectric medium this separation happens and the phenomenon is known as dispersion. In simple term it is the variation of refractive index with the wavelength. In other words the variation of the frequency with the wavelength in a medium is dispersion. Cauchy studied dispersion and gave a formula which described the dispersion in the visible range quite well. The following formula is known as Cauchy's dispersion formula

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}, \quad \dots(1)$$

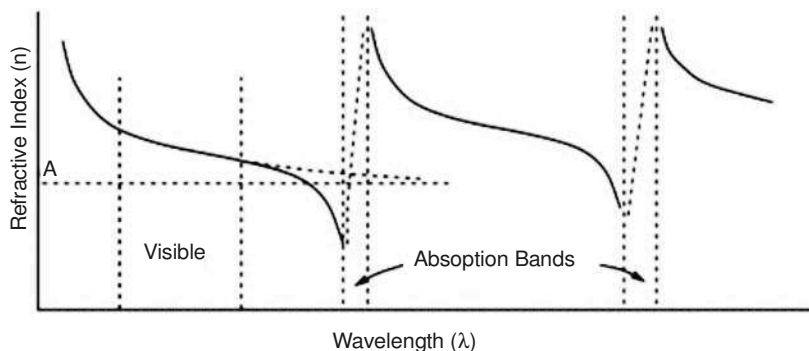
where  $A$ ,  $B$  and  $C$  are constants which depend on the medium. Experimentally the constants can be determined by measuring the refractive index for three wavelengths. In usual condition the first two terms would suffice to give an accurate value of  $n$ . The derivative of the refractive index is given by

$$dn/d\lambda = -2B/\lambda^3 \quad \dots(2)$$

to a good accuracy. Since  $A$  and  $B$  both are positive the refractive index decreases as the wavelength increases.

### Anomalous Dispersion

For material transparent to the visible region Cauchy's formula works very well but if one further increases the wavelength say to the infrared, one finds the refractive index suddenly decreases very fast and does not obey Cauchy's law. One now approaches the absorption region. Further increasing the wavelength once again the refractive index becomes large. Again the behaviour is quite similar to the visible region for the increase in wavelength. If the range is increased further one again observes another absorption band as shown in the figure.1 below. The pattern may repeat further as shown, giving many absorption bands. This dispersion is known as anomalous dispersion.



The first theory of it came from Sellmeier who assumed that all elastically bound particles in the medium oscillate with a natural frequency  $\omega_0$  which correspond to a wavelength  $\lambda_0$  in the vacuum. Sellmeier's formula gave,

$$n^2 = 1 + \frac{A\lambda^2}{\lambda^2 - \lambda_0^2} \quad \dots(3)$$

Where  $A$  is a constant. If one is away from  $\lambda_0$  it can be expanded in powers of  $\lambda_0/\lambda$  and one would get a formula of the Cauchy type.

**Q.6. What is basic difference between Cauchy's and Sellmeier equation?**

**Ans. Cauchy's equation:** It is an empirical relationship between the refractive index and wavelength of light for a particular transparent material. The most general form of Cauchy's equation is

$$n(\lambda) = B + \frac{C}{\lambda^2} + \frac{D}{\lambda^4} + \dots,$$

where  $n$  is the refractive index,  $\lambda$  is the wavelength,  $B, C, D$ , etc., are coefficients that can be determined for a material by fitting the equation to measured refractive indices at known wavelengths. The coefficients are usually quoted for  $\lambda$  as the vacuum wavelength in micrometers. Usually, it is sufficient to use a two-term form of the equation:

$$n(\lambda) = B + \frac{C}{\lambda^2},$$

where the coefficients  $B$  and  $C$  are determined specifically for this form of the equation. The theory of light-matter interaction on which Cauchy based this equation was later found to be incorrect; in particular, the equation is only valid for regions of normal dispersion in the visible wavelength region. In the infrared, the equation becomes inaccurate, and it cannot represent regions of anomalous dispersion, despite this, its mathematical simplicity makes it useful in some applications.

The Sellmeier equation is a later development of Cauchy's work that handles anomalously dispersive regions, and more accurately models a material's refractive index across the ultraviolet, visible, and infrared spectrum.

**Sellmeier equation:** It is an empirical relationship between refractive index and wavelength for a particular transparent medium. The equation is used to determine the dispersion of light in the medium. The usual form of the equation for glasses is

$$n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3},$$

where  $n$  is the refractive index,  $\lambda$  is the wavelength, and  $B_1, B_2, B_3$  and  $C_1, C_2, C_3$  are experimentally determined Sellmeier coefficients. These coefficients are usually quoted for  $\lambda$  in micrometers. Note that this  $\lambda$  is the vacuum wavelength, not that in the material itself, which is  $\lambda/n$ . A different form of the equation is sometimes used for certain types of materials, e.g. crystals.

For common optical glasses, the refractive index calculated with the three-term Sellmeier equation deviates from the actual refractive index by less than  $5 \times 10^{-6}$  over the wavelengths' range of 365 nm to 2.3  $\mu\text{m}$ , which is of the order of the homogeneity of a glass sample.

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**Q.7. What is Langevin-Debye formula?**

**Ans.** A formula for the polarizability of a dielectric material or the paramagnetic susceptibility of a magnetic material, in which these quantities are the sum of a temperature independent contribution and a contribution arising from the partial orientation of permanent electric or magnetic dipole moments which varies inversely with the temperature. Also known as Langevin-Debye law.

Although the dielectric materials are classified in different groups based in the their mode of polarization, if a material can experience all forms of polarization, then its total polarizability can be given as the sum of electronic, ionic, and orientation polarizability, i.e.

$$\therefore \alpha = \alpha_{elec} + \alpha_{ionic} + \alpha_{ori} = 4\pi\epsilon_0 R^3 + \frac{e^2}{\omega_0^2} \left[ \frac{1}{M} + \frac{1}{m} \right] + \frac{\mu_{ori}^2}{3k}$$

This is called Langevin-debye equation for total polarizability in dielectrics.

Here the contribution due to space charge polarization is not considered because it is almost negligible in most common dielectrics. In the above equations, first two terms on the right hand side are the functions of molecular structure of the dielectronic material which are usually independent of temperature. Due to this reason, they are also known as deformation polarizability.

**Q.8. Discuss about complex dielectric constant.**

**Ans.** It is used to describe the dielectric constant during a periodic variation of the electric field, where the field variation is described by a sine-shaped waveform. It is written in the form  $\epsilon = \epsilon_1 + i\epsilon_2$

Where the real part,  $\epsilon_1$ , is the permittivity component quantifying the stored energy (i.e. the part directly proportional to the field amplitude) and the imaginary part,  $\epsilon_2$  is the dielectric loss factor, which describes the part of the electric energy that is lost through movement of molecules/ions as a result of the continually changing field (the contribution from this component is proportional to the rate of the electric field change, or the first derivative of the electrical field function). This component also gives rise to a phase difference between the field function and the resulting polarization. Without this component both would be perfectly in phase. Both components depend on the frequency of the field variation.

**Q.9. What is Plasma Frequency ,correlate between plasma and Plasmons?**

**Ans. Plasma oscillations:** Also known as Langmuir waves (after Irving Langmuir), are rapid oscillations of the electron density in conducting media such as plasmas or metals in the ultraviolet region. The oscillations can be described as an instability in the dielectric function of a free electron gas. The frequency only depends weakly on the wavelength of the oscillation. The quasiparticle resulting from the quantization of these oscillations is the plasmon.

Consider an electrically neutral plasma in equilibrium, consisting of a gas of positively charged ions and negatively charged electrons. If one displaces by a tiny amount an electron or a group of electrons with respect to the ions, the Coulomb force pulls the electrons back, acting as a restoring force. If the thermal motion of the electrons is ignored, it is possible to show that the charge density oscillates at the plasma frequency

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{m^* \epsilon_0}}, \text{ [rad/s] (SI units),}$$

$$\omega_{pe} = \sqrt{\frac{4\pi n_e e^2}{m^*}}, \text{ (cgs units)}$$

where  $n_e$  is the number density of electrons,  $e$  is the electric charge,  $m^*$  is the effective mass of the electron, and  $\epsilon_0$  is the permittivity of free space. Note that the above formula is derived under the approximation that the ion mass is infinite. This is generally a good approximation, as the electrons are so much lighter than ions. (This expression must be modified in the case of electron-positron plasmas, often encountered in astrophysics). Since the frequency is independent of the wavelength, these oscillations have an infinite phase velocity and zero group velocity.

**Plasma Frequency :** is the most fundamental time-scale in plasma physics. Clearly, there is a different plasma frequency for each species. However, the relatively fast electron frequency is, by far, the most important, and references to "the plasma frequency" in text-books invariably mean the electron plasma frequency.

$$\omega_p^2 = \frac{\eta e^2}{\epsilon_0 m}, \quad \text{alllabexperiments.com}$$

It is easily seen that  $\omega_p$  corresponds to the typical electrostatic oscillation frequency of a given species in response to a small charge separation. For instance, consider a one-dimensional situation in which a slab consisting entirely of one charge species is displaced from its quasi-neutral position by an infinitesimal distance  $\delta\chi$ . The resulting charge density which develops on the leading face of the slab is  $\sigma = e n \delta\chi$ . An equal and opposite charge density develops on the opposite face. The  $\chi$ -directed electric field generated inside the slab is of magnitude  $E_x = -\sigma/\epsilon_0 = -en\delta\chi/\epsilon_0$ . Thus, Newton's law applied to an individual particle inside the slab yields.

$$\frac{m d^2 \delta\chi}{dt^2} = e E_x = -m \omega_p^2 \delta\chi,$$

Note that plasma oscillations will only be observed if the plasma system is studied over time periods  $\tau$  longer than the plasma period  $\tau_p \equiv 1/\omega_p$ , and if external actions change the system at a rate no faster than  $\tau_p^{-1}$ . In the opposite case, one is clearly studying something other than plasma physics (e.g., nuclear reactions), and the system cannot usefully be considered to be a plasma.

Likewise, observations over length-scales  $L$  shorter than the distance  $v_i\tau_p$

traveled by a typical plasma particle during a plasma period will also not detect plasma behaviour. In this case, particles will exit the system before completing a plasma oscillation. This distance, which is the spatial equivalent to  $\tau_p$ , is called the Debye length, and takes the form

$$\lambda_D \equiv \sqrt{T/m\omega_p^{-1}}$$

Note that

$$\lambda_D = \sqrt{\frac{\epsilon_0 T}{ne^2}}$$

is independent of mass, and therefore generally comparable for different species.

Clearly, our idealized system can only usefully be considered to be a plasma provided that

$$\frac{\lambda_D}{L} \ll 1,$$

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and

$$\frac{\tau_p}{\tau} \ll 1,$$

Here,  $\tau$  and  $L$  represent the typical time-scale and length-scale of the process under investigation.

**Plasma:** It is a state of matter in which an ionized gaseous substance becomes highly electrically conductive to the point that long-range electric and magnetic fields dominate the behavior of the matter. This state can be contrasted with the other states: solid, liquid, and gas. Unlike these other states of matter, plasma mostly does not naturally exist on the Earth's surface under normal conditions, and must be artificially generated from neutral gases. Plasma is an electrically neutral medium of unbound positive and negative particles (i.e. the overall charge of a plasma is roughly zero). Although these particles are unbound, they are not 'free' in the sense of not experiencing forces. Moving charged particles generate an electric current within a magnetic field, and any movement of a charged plasma particle affects and is affected by the fields created by the other charges.

A plasmon is a quantum of plasma oscillation. Just as light (an optical oscillation) consists of photons, the plasma oscillation consists of plasmons. The plasmon can be considered as a quasiparticle since it arises from the

quantization of plasma oscillations, just like phonons are quantizations of mechanical vibrations. Thus, plasmons are collective (a discrete number) oscillations of the free electron gas density. For example, at optical frequencies, plasmons can couple with a photon to create another quasiparticle called a plasmon polarizing. Plasmons can be described in the classical picture as an oscillation of electron density with respect to the fixed positive ions in a metal.



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