

Unique Paper Code : 32221502 **Solved Paper - 2017**
Name of the Paper : Solid State Physics
Name of the Course : B.Sc. (Hons.) Physics (CBCS)
Semester : V
Duration : 3 hours
Maximum Marks : 75

INSTRUCTIONS FOR CANDIDATES

Attempt any five questions. Question No. 1 is compulsory. All questions carry equal marks.

Q.1. Attempt any five of the following: **(5 × 3 = 15)**

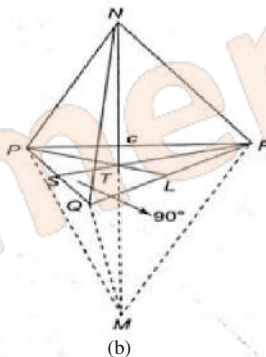
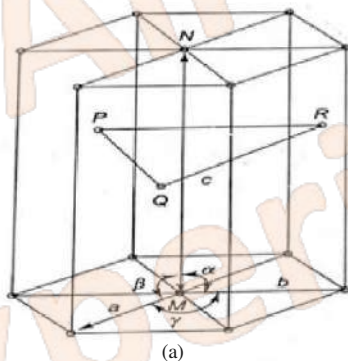
(a) Prove that c / a ratio in hcp is $\sqrt{\frac{8}{3}}$.

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Ans. $\frac{c}{a}$ ratio of hexagonal close-packed structure. In the hcp structure the

primitive cell has $a = b$ with an included angle $\lambda = 120^\circ$. The c axis is normal to the plane of a and b so that α (angle between b and c axes) = β , (angle between a and c axes) = 90° . Suppose P, Q and R be the centres of the adjacent atoms in the middle plane and N and M the centres of the adjacent atoms in the planes immediately above separately in Fig.



Join MN . Also join M and N to P, Q and R . Thus we get two tetrahedrons $NPQR$ and $MPQR$ with the common base PQR which is an equilateral triangle. The line MN passes through the plane triangle PQR at the point T which is the point of intersection of the three median.

Thus if R is joined to the mid point S of the side PQ , then RS is a median and the point T lies on it such that

$$TR = 2TS$$

As N and M are the two adjacent lattice points on the c -axis, the distance between them is the unit distance c along this axis. [Fig.(a)].

Now $PQ = QR = a$

As the medium of an equilateral triangle is also perpendicular to the opposite side. the angle $QSR = 90^\circ$

$$\therefore RS = \sqrt{a - \left(\frac{a}{2}\right)^2} = \frac{\sqrt{3}}{2}a$$

also $RT = 2TS = \frac{2}{3}(TS + TR) = \frac{2}{3}RS$

$$= \frac{2}{3} \cdot \frac{\sqrt{3}}{2}a \cdot \frac{a}{\sqrt{3}}$$

Now $NT = \sqrt{NR^2 - RT^2} = \sqrt{a^2 - \frac{a^2}{3}}$

$$= \sqrt{\frac{2}{3}}a \quad \text{alllabexperiments.com}$$

because $NR = NQ = NP = a$

Again $c = 2NT = 2a\sqrt{\frac{2}{3}} = \sqrt{\frac{8}{3}}a$

or $\frac{c}{a} = \sqrt{\frac{8}{3}} = 1.633$

(b) Sketch the dependence of polarization in dielectrics on the frequency of the applied electric field clearly indicating frequency ranges for the electronic, ionic, and dipolar polarization.

Ans. Polarization mechanisms have frequency dependence and that is because of the mass associated with the charge dipoles and hence inherent inertia to movement. This also means that if the frequency of the field is very high, say above 10^{15} Hz, no dipole system will be able to follow the field oscillators and in such a situation, no mechanism will contribute and hence $\epsilon_r \sim 1$. However, at frequencies below 10^{12} Hz, the dielectric properties are affected by the frequency of the field. In the electronic and ionic polarization, the charge dipoles can be considered behaving as mechanical where charge are connected with linear springs whose restoring force balances the force induced via applied electric field. The characteristic of any such mechanically

oscillating system is resonance at a certain frequency. In case of orientation polarization, there is no direct mechanical restoring force. Instead we have many statistical events, that respond in their average results to the driving forces of electrical fields. In other words, if a driving force is present, there is an equilibrium state with an (average) net dipole moment. If the driving force were to disappear suddenly, the ensemble of dipoles will assume as new equilibrium state (random distribution of the dipoles) within some characteristic time called relaxation time. This does not show any resonance phenomena and is characterized by its relaxation time of a resonance frequency.

(c) X-rays of wavelength 1.75 \AA undergoes first order Bragg reflection from (111) plane at 30° . What is inter - atomic spacing of the crystal?

Ans. Given $A = 1.75 \text{ \AA} = 1.75 \times 10^{-10} m.$

Angle $= 30^\circ$

$n = 1$

Now $d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$

where $a =$ interatomic spacing,

$h, k, l \rightarrow$ Miller indices

$d =$ inter planes distance

so $d_{111} = \frac{a}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{a}{\sqrt{3}}$

Using Bragg's equation $2d \sin \theta = n\lambda$ alllabexperiments.com

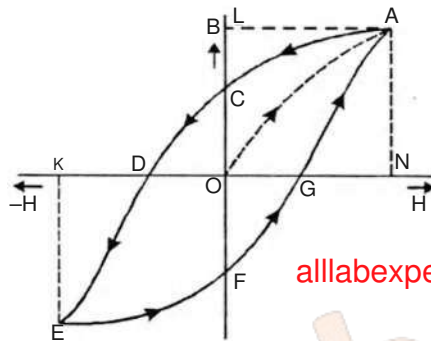
$$2 \times \frac{a}{\sqrt{3}} \times \sin 30^\circ = 1 \times 1.75 \times 10^{-10} \quad \text{Support by Donating}$$

$$\Rightarrow a = \frac{\sqrt{3}}{2} \times 1.75 \times 10^{-10} \times 2 = 3.03 \times 10^{-10} m$$

(d) Draw B-H hysteresis curve of a ferromagnetic material on the basis of domain theory. What are hard and soft semi-conductors?

Ans. Magnetic hysteresis occurs when an external magnetic field is applied to a ferromagnet such as iron and the domains align themselves with it. Even when the field is removed, part of the alignment will be retained, the material has become magnetized, the magnet will stay magnetized indefinitely. To demagnetize it requires heat or a magnetic field in the opposite direction. This is the effect that provides the element of memory in a hard disk drive. The relationship between field strength H and magnetization M is not linear in such materials. At zero field strength, the magnetization is offset from the origin by an amount called the remanence. If the H - M

relationship is plotted for all strengths of applied magnetic field the result is a hysteresis loop called the main loop. The width of the middle section along the H axis is twice the coercivity of the material.

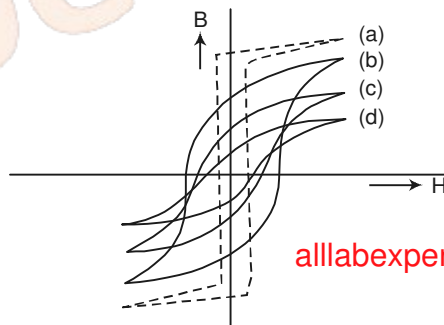


Difference Between Soft And Hard Magnetic Materials

The shape and area of the hysteresis loop (I-H or B-H) is characteristic of material. So hysteresis loops of different materials have different shape and area depending upon their properties. The properties which are illustrated by a cycle of magnetisation are

- (i) maximum magnetic induction or permeability
- (ii) maximum magnetisation or susceptibility
- (iii) retentivity
- (iv) coercivity
- (v) hysteresis

For a few materials the hysteresis loops are shown in Fig. Curve (a), (b), (c) and (d) are respectively for the specimen of soft iron, steel, cobalt and nickel materials. The study of these curves gives the following information.



- (i) **Susceptibility:** The susceptibility, the intensity of magnetisation per unit magnetising field (i.e. I/H) is greater for soft materials than for hard materials, because it is clear from curve (a) that I is larger in case of (a) and is smaller in curves (b), (c) and (d).

- (ii) **Permeability:** The permeability, the magnetic induction per unit magnetism field (i.e. B/H) is greater for soft materials than for hard materials. alllabexperiments.com
- (iii) **Retentivity:** When a magnetic material is magnetised first by increasing H , and then the magnetizing field is decreased to zero, the magnetic material retains intensity of magnetisation I (or magnetic induction B), which is known as returnability (or residual magnetism). The retentivity is greater for soft materials than for hard materials.
- (iv) **Coercivity:** For demagnetizes the magnetic material completely, a reverse magnetising field is applied. The value of reverse field H for reducing I or B to zero, is known as coercivity. It is less for soft-material than for hard material as is clear from curves (a), (b), (c) and (d).
- (v) **Hysteresis loss:** The area of the hysteresis curve and hence the hysteresis loss per unit volume per cycle is less for soft material than for hard materials.

(e) **What is the effect of temperature and doping concentration on mobility of semiconductors?**

Ans. Mobility (μ): The mobility means the movement of charge carriers. The mobility of intrinsic semiconductor decreases with increases in temperature because at higher temperature, the numbers of carriers are more and they are energetic also. This causes an increased number of collisions of charge carriers with the atoms and thus the mobility decreases.

- The two main scattering events that influence electron and hole motion (and, thus, mobility) are the lattice scattering and the impurity scattering.
- All lattice atoms vibrate due to temperature and can scatter carriers due to collisions.
- These collective vibrations are called phonons, thus lattice scattering is also known as phonon scattering.
- With increasing temperature, lattice vibrations increase, and the mean free time between collisions decreases \Rightarrow mobility decreases (typical dependence $\sim T^{-3/2}$).
- Scattering from crystal defects and ionized impurities dominate at low temperatures.
- Since carriers moving with low velocity (at low temperature) can get scattered more easily by ionized impurities, this kind of scattering causes a decrease in mobility with decreasing temperature (typical dependence $\sim T^{-3/2}$).
- Note: the scattering probability is inversely proportional to the first time (and to mobility), hence, the mobilities due to two or more scattering events and inversely.

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \dots$$

- Thus, the mechanism causing the lowest mobility value dominates.
- Mobility also decreases with increasing doping, since the ionized impurities scatter carriers more (e.g., μ_n for intrinsic Si is $1350 \text{ cm}^2/\text{V}\cdot\text{sec}$ at 300 K , where as with a donor doping of $10^{17}/\text{cm}^3$, n drops to $700 \text{ cm}^2/\text{V}\cdot\text{sec}$).

(f) Write the expression for the plasma frequency in a material. What is the differences between plasma and plasmons?

Ans. Plasma is a state of matter in which an ionized gaseous substance becomes highly electrically conductive to the point that long-range electric and magnetic fields dominate the behavior of the matter. This state can be contrasted with the other state, solid, liquid, and gas. Unlike these other states of matter, plasma mostly does not naturally exist on the Earth's surface under normal conditions, and must be artificially generated from neutral gases. Plasma is an electrically neutral medium of unbound positive and negative particles (i.e. the overall charges of a plasma is roughly zero). Although these particles are unbound, they are not 'free' in the sense of not exercising forces. Moving charged particles generate an electric current within a magnetic field, and any movement of a charged plasma particle affects and is affected by the fields created by the other charges.

A plasmon is a quantum of plasma oscillation. Just as light (an optical oscillation) consists of photons, the plasma oscillation consists of plasmon. The plasmon can be considered as a quasiparticle since it arises from the quantization of plasma oscillations, just like phonons are quantizations of mechanical vibrations. Thus, plasmons are collective (a discrete number) oscillations of the free electron gas density. For example, at optical frequencies, plasmons can couple with a photon to create another quasiparticle called a plasmon polartion. Plasmons can be described in the classical picture as an oscillation of electron density with respect to the fixed positive ions in a metal.

(g) What is the difference between Type-I and Type-II superconductors? Give one example of each.

Ans. Soft Superconductors (Type 1):

1. Soft superconductors are those which can tolerate impurities without affecting the superconducting properties.
2. They have low critical field.
3. Show complete Meissner effect.
4. The current flows through the surface only.
5. E.g. Tin, Aluminium.

Hard Superconductors (Type II):

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1. Hard superconductors are those which cannot tolerate impurities, i.e., the impurity effects the superconducting property.
2. They have high critical field.
3. Hard super conductors trap magnetic flux and hence Meissner effect is not complete.
4. It is found that current flows throughout the material.
5. E.g. Tantalum, Neobium.

(h) Calculate the Debye cut-off frequency of Be, whose Debye temperature is 1440 K.

Ans. Given Debye Temp $\theta_D = 1440 \text{ K}$

from Debye model highest or Debye cut-off frequency

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$$V_m = \frac{K_s \theta_D}{h} \text{ where } K_s \rightarrow \text{Boultzman Constant}$$

$$K_s = 1.4 \times 10^{-23} \text{ JK}^{-1}$$

So
$$V_m = \frac{1.4 \times 10^{-23} \times 1440}{6.6 \times 10^{-34}} = 3.05 \times 10^{13} \text{ s}^{-1}$$

Q.2. (a) What is a Brillouin Zone? Draw and explain first and second Brillouin zones of a square lattice.

Ans. Find its answer in the book.

(b) Calculate the geometrical structure factor of a bcc lattice. Name two planes which will be missing from the X-ray diffraction spectrum. (5)

Ans. Let us consider a unit cell of a crystal, having n number of atoms per unit cell.

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Let the position vector of jth atom is given by

$$\vec{r}_j = u_j \vec{a} + v_j \vec{b} + w_j \vec{c} \quad \dots(1)$$

Here u_j , v_j and w_j are constants. Suppose that all the electrons of jth atom are concentrated at the position \vec{r}_j . Let f_j be the scattering power of jth atom relative to single electron when both are located at the space point.

Geometrical structure factor.

$$\therefore S = f \sum_j e^{-2\pi i [u_j h + v_j k + w_j l]} \quad \dots(2)$$

The structure factor need not be real, the involves the absolute value S i.e. S^*S , where S^* is the complex conjugate of S . The structure factor can cancel some of the reflection allowed by the space lattice, and the missing reflections

help us in the determination of the structure. Let us calculate geometrical structure factor for some simple crystals and the intensity of various order reflections associated with them. Enough, so that the intensity peaks may be seen.

(ii) Structure factor for b.c.c. lattice: A bcc lattice has two identical atoms per unit cell, one contributed by the corner atoms of the cell (0, 0, 0) and the

centre of the cell $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$.

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Hence from equation (2), put $uj = vj$ and wj equal to (0, 0, 0) and $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

$$S = f \left[e^0 + e^{-2i\left(\frac{h}{2} + \frac{k}{2} + \frac{l}{2}\right)} \right]$$

or
$$S = f \left[1 + e^{-\pi i(h+k+l)} \right] \quad \dots(3)$$

Now it is obvious that

(i)
$$S = 0 \text{ for } (h + k + l) = \text{odd integer}$$

$$[\because e^{-\pi i(h+k+l)} = -1 \text{ for } (h + k + l) \text{ odd}]$$

So in *b.c.c.* certain reflections are missing e.g.

(100), (300), (111) and (221)

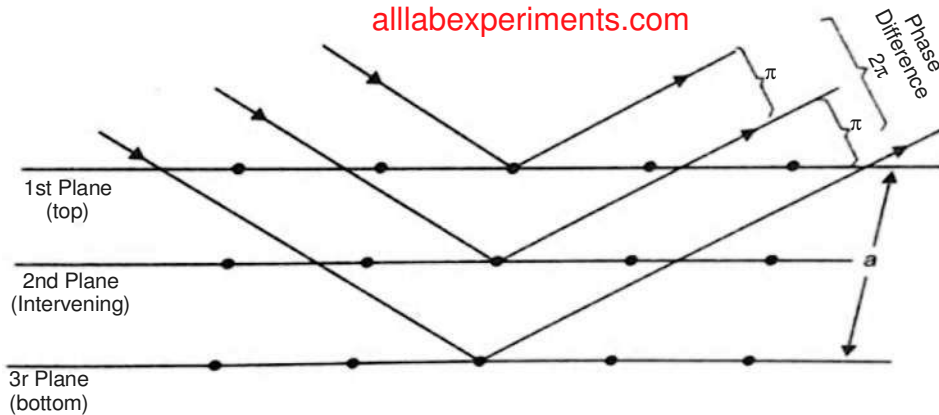
(ii)
$$S = 2f \text{ for } (h + k + l) = \text{even integer}$$

$$[\because e^{-\pi i(h+k+l)} = 1 \text{ for } (h + k + l) \text{ even}]$$

So the reflection (200), (110), (222) are present.

Reason for absence of (100) reflection and presence of (200) reflection.

The absence or presence of a reflection is considered only in terms of the first reflection. The (100) reflection normally occurs when reflection from the planes that bound the cubic cell differ in phase by 2π . In *b.c.c.* lattice there is an intervening plane (fig.) of atoms between the top and bottom cube faces due to body centre atoms of the unit cells. This intervening plane gives a reflection retarded in phase by π with respect to the first plane, hence cancelling the contribution from that plane. The phase difference between the radiations scattered from first plane (top plane) and the second plane (intervening plane due to body atoms) is π radians. Similarly, the phase difference between the second plane (intervening plane) and third plane (bottom plane) is π radians. Therefore, radiations from top and intervening plane and also from bottom and intervening plane, interfere destructively occur in the *b.c.c.* lattice because the planes are identical in composition. A similar cancellation can easily be found in the hcp structure.



Q.3. (a) Distinguish between crystalline and amorphous solids with one example, each. (3)

Ans. Crystalline Solids: The solids having the property of incompressibility and rigidity are known as crystalline substances. The crystalline solids are those in which group of atoms, molecules or ions are arranged in a definite regular manner forming three dimensional pattern which may be obtained by a three-dimensional repetition of a certain pattern unit. The regular structural pattern is called crystalline structure. A crystal is, thus a body whose faces are regular polygons and their shapes depend upon the conditions in which they are formed. In crystals between the edges of the faces and between the faces themselves are constant. The majority of solid are crystalline. If we examine the grains of a common salt through a lens, we find them to have shapes of regular cube. But in most of the cases, we cannot see crystalline structure of solid, because in these cases, the material is composed of large number of very small tightly etc. Two dimensional picture of a crystal is shown in fig.

The crystalline solids have the following characteristics:

- (i) The atoms, ions or molecules of crystalline solid are arranged in a definite geometrical order.
- (ii) The orderly arrangement of atoms, ions and molecules in a crystal is extended over a large volume of the crystal. It means that crystalline solids show a long range order by the atoms, ions and molecules.
- (iii) Crystalline solids are bounded by flat surfaces.
- (iv) Crystalline solids have uniform chemical composition throughout.
- (v) Crystalline solids have the property of anisotropy i.e. their physical properties are not the same in all directions e.g. when a ray of light entering a crystal splits in two components, it may travel with different speed in two directions. Other properties of crystals such as thermal expansion, compressibility, refractive index, electrical and thermal

conductivities etc, have different values, when measured in different directions.

- (vi) Crystalline solids has a sharp melting point because the bonds between the various atoms of a crystalline solid are equally strong and all of them get ruptured at the same temperature when solid is heated.
- (vii) Below the temperature of crystallization, the crystalline materials are in stable state. Since a stable state is the state of minimum energy, materials acquire a state of minimum energy on crystallization.



Fig. Two dimensional illustration of crystalline solid.

Amorphous (Glassy Solids): There are certain substances such as ordinary glass, plastics, lamp black, which look like solids in appearance but as far as their structure (arrangement of their atoms) is concerned, they cannot be considered as such. They can be considered as viscous liquids. The atoms of such substances are arranged in an irregular manner as shown in Fig. for two dimensions but they have strong bonds between them. Such solids are called amorphous or glassy. As regards structure, glassy solids are like liquids.



Fig. Two dimensional illustration of amorphous solid.

The amorphous or glassy solids have the following characteristics:

- (i) The atoms or molecules in an amorphous solid are arranged in an irregular manner.

- (ii) As the atoms or molecules in such solids are distributed randomly, so such solids are not bound by flat surface.
- (iii) The glassy solids are isotropic. It means that the value of their physical properties measured in different directions are same.
- (iv) Glassy solids do not have sharp melting point. It is because the bonds between atoms in such solids are not equally strong. So when heated, first the weakest bonds get ruptured at lower temperature and then the stronger one at higher temperatures. Thus on heating, glassy solid behaves like a viscous liquid. The temperature at which this occurs is called glassy transition temperature.

(b) Show that in Einstein model of specific heat.

$$C_v = 3R \left(\frac{\theta_E}{T} \right)^2 e^{-\frac{\theta_E}{T}}$$

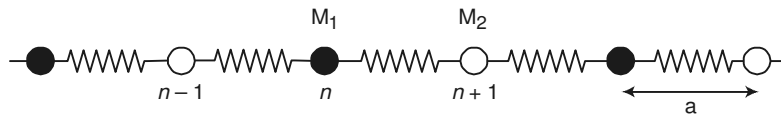
where symbols have their usual meanings. (12)

Ans. The expression of Einstein specific heat theory is in text.

Q.4. (a) Obtain the dispersion relation in one dimensional diatomic lattice. (7)

Ans. The “beads” of an electromagnetic wave have no rest mass, so they will be propagating at the speed of light. However, atoms in a crystal have a finite mass; therefore they travel at a lower and the frequency of oscillation will become dependent on the wave number (and also on the direction of the wave vector). The relationship between frequency (usually expressed as an angular frequency, ω) and wave number is known as a dispersion relation.

Diatomic 1D lattice: Now we consider a one-dimensional lattice with two non-equivalent atoms in a unit cell. It appears that the diatomic lattice exhibit important features different from the monoatomic case. Fig. shows a diatomic lattice with the unit cell composed of two atoms of masses M_1 and M_2 with the distance between two neighboring atoms a .



We can treat the motion of this lattice in a similar fashion as for monoatomic lattice. However, in this case because we have two different kinds of atoms, we should write two equations of motion.

$$M_1 \frac{d^2 u_n}{dt^2} = -c(2u_n - u_{n+1} - u_{n-1}) \quad \dots(1)$$

$$M_2 \frac{d^2 u_{n+1}}{dt^2} = -c(2u_{n+1} - u_{n+2} - u_n)$$

In analogy with the monoatomic lattice we are looking for the solution in the form of traveling mode for the two atoms.

$$\begin{bmatrix} u_n \\ u_{n+1} \end{bmatrix} = \begin{bmatrix} A_1 e^{iqax} \\ A_2 e^{iq(n+1)a} \end{bmatrix} e^{-iax} \quad \dots(2)$$

which is written in the obvious matrix form. Substituting this solution to Eq. (1) we obtain

$$\begin{bmatrix} 2C - M_1 \omega^2 & -2C \cos qa \\ -2C \cos qa & 2C - M_2 \omega^2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0 \quad \text{alllabexperiments.com} \quad \dots(3)$$

This is a system of linear homogeneous equation for the unknowns A_1 and A_2 . A nontrivial solution exists only if the determinant of the matrix is zero. This leads to the secular equation.

$$\begin{bmatrix} 2C - M_1 \omega^2 & -2C \cos qa \\ -2C \cos qa & 2C - M_2 \omega^2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0 \quad \dots(4)$$

This is a quadratic equation, which can be readily solved.

$$\omega^2 = C \left(\frac{1}{M_1} + \frac{1}{M_2} \right) \pm \sqrt{\left(\frac{1}{M_1} + \frac{1}{M_2} \right)^2 - \frac{4 \sin^2 qa}{M_1 M_2}} \quad \dots(5)$$

Finally take the root to produce dispersion relation for linear chain of diatomic lattice.

$$\omega = \sqrt{c \left(\frac{1}{M_1} + \frac{1}{M_2} \right) \pm C \sqrt{\left(\frac{1}{M_1} + \frac{1}{M_2} \right)^2 - \frac{4 \sin^2 qa}{M_1 M_2}}}$$

(b) Derive an expression for specific heat of solids on the basis of Debye model. How does Debye model differ from Einstein model? Discuss the variation of specific heat with temperature.

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Obtain the values W_+ and W_- in the limits

(i) $K \rightarrow 0$

(ii) $K \rightarrow \pi/2a$ where K is the wave vector and ' c ' is the separation between two consecutive atoms. (8)

Ans. In Debye model the following simplifying assumptions are made:

1. In Einstein theory of specific heat of solids, each atom is supposed to vibrate as a single unit/oscillator independent of its neighbouring atoms. But the situation is not so simple because each atom is under the influence of the field of force of a number of other vibrating atoms. Debye took into solid as a continuous elastic body.

2. According to Debye the internal energy of a solid instead of residing in the vibrations of individual atoms was present in the elastic standing waves being produced because of the vibrations of individual atoms.
3. Further Debye assumed that the energy of these elastic standing waves was not continuous but quantised. A quantum vibrational energy is called phonon analogous to photon in e.m. waves. A phonon has energy $h\nu$ similar to that of a photon where ν is frequency of variation and h is Planck's constant. A phonon travels with the velocity of sound waves are elastic waves in nature.
4. The number of vibrational modes of any kind in the frequency range ν and $(\nu + d\nu)$ is given by $Z(\nu) d\nu = \frac{4\pi V}{\nu^3} \nu^2 d\nu$ where $Z(\nu)$ is number of vibrational modes per unit frequency range and is known as density of modes, $V = L^3$ is the volume of the solid taken in the form of a having each side $= L$ and ν is the velocity of propagation of the wave through the medium.
5. Debye further assumed that the phonon gas behaves like a boson gas which obeys Bose-Einstein statistics and the average energy $\langle \epsilon \rangle$ per standing wave at a temperature T is given by

$$\langle \epsilon \rangle = \frac{h\nu}{e^{h\nu/k_B T} - 1}$$

If a quantum oscillator is associated with each vibration mode of the same frequency, the vibrational frequency of the crystal is given by

$$E = \int_0^{\nu_D} \langle \epsilon \rangle D(\nu) d\nu \quad \dots(1)$$

Substituting the values of $D(\nu)d\nu$ from eqn. (4.29), we have

$$\begin{aligned} E &= \int_0^{\nu_D} 4\pi V \left[\frac{1}{\nu_t^3} + \frac{2}{\nu_l^3} \right] \frac{h\nu^3}{e^{h\nu/k_B T} - 1} d\nu \\ &= 4\pi V \left[\frac{1}{\nu_t^3} + \frac{2}{\nu_l^3} \right] \int_0^{\nu_D} \frac{h\nu^3}{e^{h\nu/k_B T} - 1} d\nu \quad \dots(2) \end{aligned}$$

From eqn. (1)

$$4\pi V \left[\frac{1}{\nu_t^3} + \frac{2}{\nu_l^3} \right] = \frac{9N}{\nu_D^3}$$

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Substituting in eqn. (2), we have

$$E = \frac{9N}{v_D^3} \int_0^{v_D} \frac{h\nu^3}{e^{h\nu/k_B T} - 1} d\nu \quad \dots(3)$$

Substituting $x = \frac{h\nu}{k_B T}$ and $x_D = \frac{k_B T}{h}$, we get

$$\nu = \frac{x k_B T}{h} \text{ or } d\nu = \frac{k_B T}{h} dx$$

Equation (3) therefore becomes

$$\begin{aligned} E &= \frac{9Nh}{v_D^3} \int_0^{x_D} \frac{x^3 k_B^3 T^3}{h^3} \frac{k_B T dx}{h} \frac{1}{e^x - 1} \\ &= 9N \left(\frac{k_B T}{h v_D} \right) k_B T \int_0^{x_D} \frac{x^3}{e^x - 1} \end{aligned}$$

As in Einstein model, it is convenient here also to introduce a characteristic temperature, θ_D , called Debye temperature and defined as—

$$\theta_D = \frac{h\nu_D}{k_B}$$

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so that upper limit $x_D = \frac{h\nu_D}{k_B T} = \frac{\theta_D}{T}$

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Thus eqn. (4) can be written as

$$E = 9Nk_B T \left(\frac{T}{\theta_D} \right)^3 \int_0^{\theta_D/T} \frac{x^3}{e^x - 1} dx \quad \dots(5)$$

The value of heat capacity or specific heat is obtained by differentiating this energy expression *w.r.t.* absolute temperature.

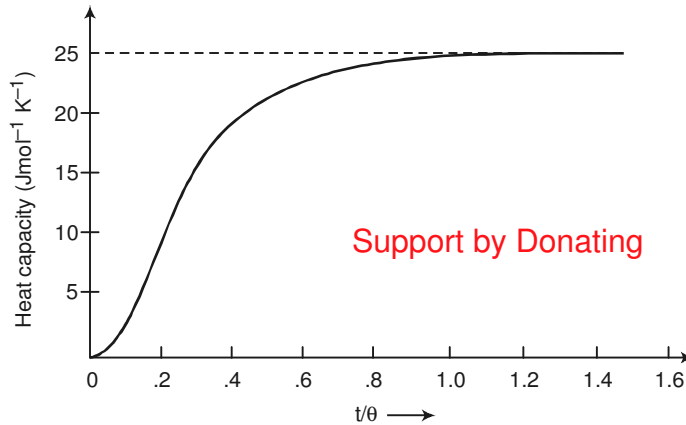
$$C_v = \left(\frac{\partial E}{\partial T} \right) = 9Nk_B \left(\frac{T}{\theta_D} \right)^3 \int_0^{\theta_D/T} \frac{e^x x^4 dx}{(e^x - 1)^2}$$

or
$$C_v = 3RF_D \times \left(\frac{T}{\theta_D} \right) \quad \dots(6)$$

where

$$R = Nk_B \text{ for one mole of the substance}$$

and $F_D = 3 \left(\frac{T}{\theta_D} \right)^4 \int_0^{\theta_D/T} \frac{x^4 e^x}{(e^x - 1)^2} dx$ is called Debye function



Expression. (6) gives the specific heat for one mole of the substance and is called the Debye formula. If we plot heat capacity as a function of T/θ , (Fig), it is seen from this plot that the heat capacity approaches the classical value $3Nk_B$ at high temperatures and zero at low temperatures. Now we examine below the heat capacities at these temperature ranges more closely.

(i) For high temperatures i.e. for $T \gg \theta_D$, x is small as compared with unity for the whole range of integration. In that case, the denominator of the integrand in eqn. (5) i.e. $e^x - 1 \approx x$, in first approximation, then we get

$$\begin{aligned}
 E &= 9N \frac{1}{x_m^3} k_B T \int_0^{x_D} \frac{x^3}{x} dx \\
 &= 9N \frac{1}{x_m^3} k_B T \frac{x_m^3}{3} \\
 E &= 3Nk_B T
 \end{aligned}$$

and the specific heat

$$C_v = \frac{dE}{dT} = 3Nk_B = 3R \text{ which is Dulong's law.}$$

This specific heat is identical as obtained both by classical theory and Einstein theory at high temperatures. Physically this means that the quantum consideration are of almost no significance at high temperatures.

(ii) For very low temperatures i.e. for $T \ll \theta_D$, the upper limit of integration of eqn. (5) becomes infinity, $(\because x_D = \frac{\theta_D}{T} \rightarrow \infty)$ thus

$$E = 9Nk_B T \left(\frac{k_B T}{h\nu_d} \right)^3 \int_0^\infty \frac{x^3 dx}{e^x - 1}$$

where
$$\int_0^\infty \frac{x^3 dx}{e^x - 1} = 6\zeta(4) = 6 \sum_{n=1}^\infty \frac{1}{n^4} = \frac{\pi^4}{15}$$

where $\zeta(4)$ is Riemann Zeta function.

\therefore
$$E = 9Nk_B T \left(\frac{k_B T}{h\nu_D} \right)^3 \times \frac{\pi^4}{15}$$

or
$$E = \frac{3}{5} \pi^4 N k_B \frac{T^4}{\theta_D^3} \quad \dots(7)$$

Thus the energy of vibration is proportional to T^4 at low temperatures. The specific heat is then given by

$$C_v = \frac{\partial E}{\partial T} = \frac{12}{15} \pi^4 N k_B \left(\frac{T}{\theta_D} \right)^3 \text{ for } T \ll \theta_D$$

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$$= 77.94 \times 3R \left(\frac{T}{\theta_D} \right)^3$$

This law is analogous to Stefan's law Black body radiation. The phonons (quantum of energy in elastic wave is called phonon) and photons (quantum of energy of electromagnetic waves) obey the same statistics with the difference that phonons obey T^3 law at low temperatures while the photons obey T^4 law at all temperatures.

Thus we find that specific heat is directly proportional to T^3 at extremely low temperature because θ_D is constant for the substance. This is called the famous Debye's T^3 Law.

Comparison of C_v from Einstein and Debye Theory. Plots of values of heat capacity obtained from Einstein and Debye models $v/s T/\theta$ are shown in Fig. The Debye curve lies above the Einstein curve. The reason is that in Debye model, the lower frequency modes are taken into account and at low temperatures these have a higher average energy and temperature derivative than the relatively high frequency Einstein oscillators.

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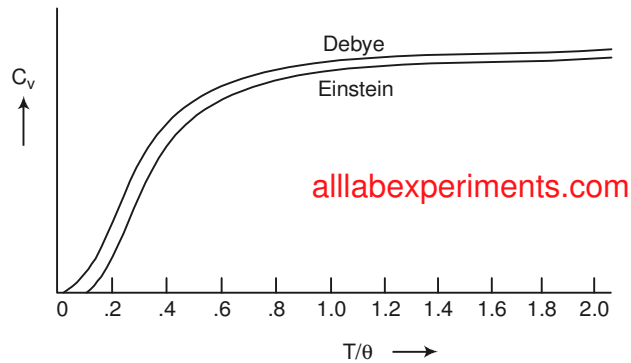


Fig. Comparison for C_v obtained from Einstein and Debye models

Q.5. (a) Derive an expression for diamagnetic susceptibility on the basis of classical Langevin's theory. (9)

Ans. Langevin's theory is given in the book.

(b) Distinguish between dia, para, ferro-and ferri-magnetic materials with examples. (6)

Ans. Faraday showed that all materials are magnetic in nature and he further established that when these substances are placed in strong magnetic fields, they were to a lesser or greater extent affected by magnetic fields. However, of all the substances, only some were found to be attracted and most of them are found to be repelled by a magnet. From amongst the attracted ones, a few are found to be strongly attracted and the rest are only feebly attracted. As the magnetic field is strong at regions close to a magnet and is weak at regions farther away from the magnet, the substances which are attracted by a magnet move from weaker to stronger parts of a magnetic

field and the substances which are repelled by a magnet move from stronger to weaker parts of a magnetic field. Faraday divided the magnetic substances into three categories.

- (i) Diamagnetic substances
- (ii) Paramagnetic substances
- (iii) Ferromagnetic substances

(i) Diamagnetic substances: are those substances which are feebly (weakly) magnetised in a direction opposite to the magnetising field in which they are placed. Thus these substances are feebly repelled by magnets and move from stronger to weaker regions of magnetic field. These substances are also simply known as diamagnetics. Some examples of diamagnetics are water, copper, lead, zinc, bismuth, sodium-chloride, gold, mercury, graphite, hydrogen, molecule, He, Ne etc.

(ii) Paramagnetic substances: are those substances which are feebly magnetised in the direction of magnetising field in which they are placed. Thus these substances are feebly attracted by magnets and move from weaker

to stronger parts of a magnetic field. These substances are also called paramagnetics. Some of paramagnetism are wood, aluminium, platinum, oxygen and cupric sulphate.

(iii) Ferromagnetic substances: are those substances which are strongly magnetised in the direction of magnetising field in which they are placed. Thus these substances are strongly attracted by magnets and move from weaker to stronger parts of a magnetic field. These substances are also called ferromagnetics. Some prominent ferromagnetics are iron, steel, nickel, cobalt, and some alloys such as permalloy (Ni and Fe), omental (Ni, Fe, Cr and Cu) and alnico (Ni, Al, Co, Cu and Fe).

Q.6. (a) Obtain an expression for local electric field inside a dielectric with cubic symmetry. (10)

Ans. Find the answer to this question in the text portion.

(b) Derive Clausius-Mossotti formula constant. (5)

Ans. Clausius Mossotti's gives the relation between the dielectric constant and the ionic polarizability of atoms in dielectric material. If there are N number of atoms, the dipole moment per unit volume which is called polarization is given by,

$$P = Na_e E_i$$

we know internal field

$$E_i = \left(E + \frac{P}{3\epsilon_0} \right)$$

From above equations

$$P = Na_e \left(E + \frac{P}{3\epsilon_0} \right)$$

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$$P = \frac{Na_e}{3\epsilon_0} (3E\epsilon_0 + P)$$

$$\frac{Na_e}{3\epsilon_0} = \frac{P}{(3E\epsilon_0 + P)}$$

we know polarization from the relation between polarization and dielectric constant

$$P = E\epsilon_0(\epsilon_r - 1)$$

from the above two equations we get,

$$\frac{Na_e}{3\epsilon_0} = \frac{E\epsilon_0(\epsilon_r - 1)}{3E\epsilon_0 + E\epsilon_0(\epsilon_r - 1)}$$

$$\frac{Na}{3\epsilon_0} = \frac{E\epsilon_0(\epsilon_r - 1)}{E\epsilon_0(\epsilon_r + 2)}$$

$$\frac{Na}{3\epsilon_0} = \frac{(\epsilon_r - 1)}{(\epsilon_r + 2)}$$

Q.7. (a) Discuss the Kronig-Penny model for a linear lattice. How does it lead to the formation of energy bands in solids? Find the energy of electron with the change in the strength of the periodic potential under following cases: (9+3+3)

(i) $V \rightarrow \infty$

(ii) $V \rightarrow 0$

Ans. Find the solution to this question in text.

Q.8. (a) Derive London equation for a superconductor and obtain an expression for penetration depth. (12)

Ans. Solution to this question is available in text.

(b) A superconducting tin has a critical temperature of 3.7 K in zero tesla at 0K. Find the critical field at 2K. (3)

Ans. Given critical field at 0k.

$$B_0 = 0.0306 \text{ T}$$

Critical Temp.

$$T_c = 3.7 \text{ K}$$

Temperature

$$T = 2 \text{ k}$$

So using relation

$$\begin{aligned} B_c &= B_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right] \\ &= 0.0306 \left[1 - \left(\frac{2}{3.7} \right)^2 \right] = 0.0217 \text{ Tesla.} \end{aligned}$$

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