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# All Lab Experiments

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## Quantum Mechanics & Applications
## Chapter - 5
## Atoms in Electric and Magnetic Fields

# 5 Atoms in Electric and Magnetic Fields

**Syallbus:** Electron angular momentum. Space quantization. Electron Spin and Spin Angular Momentum. Larnor's Theorem. Spin Magnetic Moment. Stern-Gerlach Experiment. Normal Zeeman Effect: Electron Magnetic Moment and Magnetic Energy.

**Q 1. Write down the angular momentum mathematics for a particle in quantum mechanics.**

**Ans. The orbital angular momentum** of electrons in atoms associated with a given quantum state is found to be quantized in the form

$$L = \sqrt{\ell(\ell+1)}\hbar \qquad \ell = \text{angular momentum quantum number}$$

$$\ell = 0,1,2,...n-1$$

This is the result of applying quantum theory to the orbit of the electron. The solution of the Schrodinger equation yields the angular momentum quantum number. Even in the case of the classical angular momentum of a particle in orbit,

$$L = mvr\sin\theta$$

the angular momentum is conserved. The Bohr theory proposed the quantization of the angular momentum in the form

$$L = m\text{v}r = \frac{nh}{2\pi}$$

and the subsequent application of the Schrodinger equation confirmed that form for the orbital angular momentum. In the process of solving the Schrodinger equation for the hydrogen atom, it is found that the orbital angular momentum is quantized according to the relationship:

$$L^2 = \ell(\ell+1)\hbar^2 \quad \ell = \text{angular momentum quantum number}$$

It is a characteristic of angular momenta in quantum mechanics that the magnitude of the angular momentum in terms of the orbital quantum number is of the form

$$L = \sqrt{\ell(\ell+1)}\,\hbar$$

and that the z-component of the angular momentum in terms of the magnetic quantum number takes the form

$$L_z = m_\ell \hbar$$

An electron has an intrinsic angular momentum (called as spin angular momentum), independent of its orbital angular momentum. These experiments suggest just two possible states for this angular momentum, and following the pattern of quantized angular momentum, this requires an angular momentum quantum number of 1/2.



$$S = \sqrt{s(s+1)}\hbar,\ s = \frac{1}{2}\ m_s = \pm\frac{1}{2}$$

**Q 2. What is space quantization? Explain electron spin and spin angular momentum.**

**Ans.** The orbital angular momentum for an atomic electron can be visualized in terms of a vector model where the angular momentum vector is seen as precessing about a direction in space. While the angular momentum vector has the magnitude shown, only a maximum of l units can be measured along a given direction, where l is the orbital quantum number.

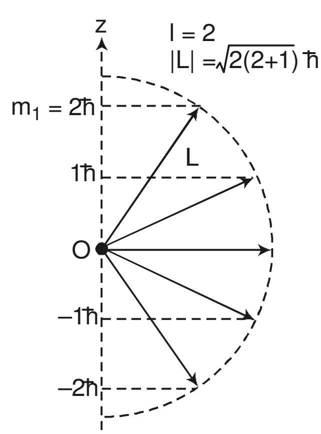
Since there is a magnetic moment associated with the orbital angular momentum, the precession can be compared to the precession of a classical magnetic moment caused by the torque exerted by a magnetic field. This precession is called Larmor precession and has a characteristic frequency called the Larmor frequency.

$$l = 2$$

$$|L| = \sqrt{2(2+1)}\,\hbar$$

While called a "vector", it is a special kind of vector because its projection along a direction in space is quantized to values one unit of angular momentum apart. The diagram shows that the possible values for the "magnetic quantum number ml for l=2 can take the values





$$m_p = -2, -1, 0, 1, 2$$

or, in general

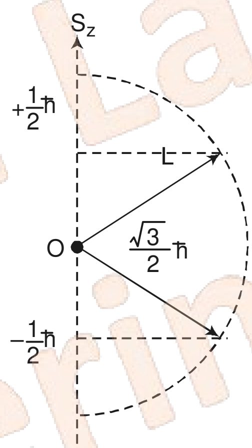
$$m_p = -/, -/+1, \ldots, /-1, / .$$

**Quantized Angular Momentum -**

This general form applies to orbital angular momentum, spin angular momentum, and the total angular momentum for an atomic system.

**Electron Spin**

An electron spin s = 1/2 is an intrinsic property of electrons. Electrons have intrinsic angular momentum characterized by quantum number 1/2. In the pattern of other quantized angular momenta, this gives total angular momentum






$$S = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar = \frac{\sqrt{3}}{2}\hbar$$

The resulting fine structure which is observed corresponds to two possibilities for the z-component of the angular momentum. Spin "up"and "down "allows two electrons for each set of spatial quantum numbers,

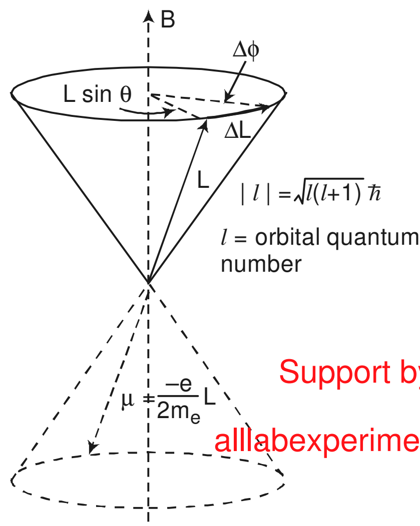
$$S_z = \pm\frac{1}{2}\hbar$$

$$n, \ell, m_\ell$$

**Q 3. State and derive Larmor's theorem.**

**Ans. Larmor Precession:** When a magnetic moment ? is placed in a magnetic field B, it experiences a torque which can be expressed in the form of a vector product

$$\tau = \mu \times B$$






For a static magnetic moment or a classical current loop, this torque tends to line up the magnetic moment with the magnetic field B, so this represents its lowest energy configuration. But if the magnetic moment arises from the motion of an electron in orbit around a nucleus, the magnetic moment is proportional to the angular momentum of the electron. The torque exerted then produces a change in angular momentum which is perpendicular to that angular momentum, causing the magnetic moment to precess around the direction of the magnetic field rather than settle down in the direction of the magnetic field. This is called Larmor precession.

When a torque is exerted perpendicular to the angular momentum L, it produces a change in angular momentum ?L which is perpendicular to L, causing it to precess about the z axis. Labeling the precession angle as ?, we can describe the effect of the torque as follows:

$$\tau = \frac{\Delta L}{\Delta t} = \frac{L\sin\theta\Delta\phi}{\Delta t} = |\mu B\sin\theta| = \frac{e}{2m_e}LB\sin\theta$$

The precession angular velocity (Larmor frequency) is

$$\omega_{Larmor} = \frac{d\phi}{dt} = \frac{e}{2m_e}B$$

These relationships for a finite current loop extend to the magnetic dipoles of electron orbits and to the intrinsic magnetic moment associated with electron spin. There is also a characteristic Larmor frequency for nuclear spins.

In the case of the electron spin precession, the angular frequency associated with the spin transition is usually written in the general form

$$\omega = \gamma B$$

where is called the gyromagnetic ratio (sometimes the magnetogyric ratio). This angular frequency is associated with the "spin flip" or spin transition, involving an energy change of 2?B. An example for magnetic field 1 Tesla follows.

$$\omega_{\text{electron spin}} = \frac{2\mu_e B}{\hbar} = \frac{2.2.\frac{1}{2}(5.79\times10^{-5} eV/T)(1T)}{6.58\times10^{-16} eV.s} = 1.7608x10^{11} s^{-1}$$

$$\nu = \frac{\omega}{2\pi} = 28.025\, GHz \;\; Larmor\;\; frequency$$

$$\omega_{\text{proton spin}} = \frac{2\mu_p B}{\hbar} = \frac{2(2.79)(3.15x10^{-8} eV/T)(1T)}{6.58x10^{-16} eV.s} = 2.6753x10^{8} s^{-1}$$

$$\nu = \frac{\omega}{2\pi} = 42.5781\, MHz \;\; Larmor\;\; frequency$$

**Q 4. What is spin magnetic moment? Explain it by Stern-Gerlach experiment.**

**Ans. Electron Spin Magnetic Moment -** Since the electron displays an intrinsic angular momentum, one might expect a magnetic moment which follows the form of that for an electron orbit. The z-component of magnetic moment associated with the electron spin would then be expected to have no magnetic moment but the measured value turns out to be about twice that. The measured value is written

$$\mu_z = \pm\frac{1}{2} g\mu_B$$

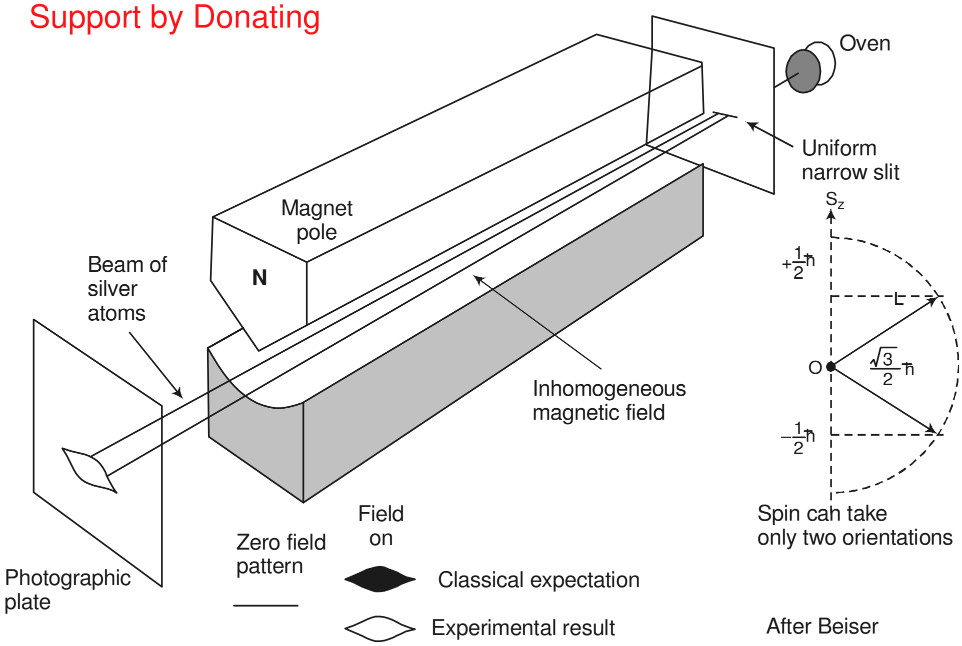
where g is called the gyromagnetic ratio and the electron spin g-factor has the value g = 2.00232 and g=1 for orbital angular momentum. The precise value of g was predicted by relativistic quantum mechanics in the Dirac equation and was measured in the Lamb shift experiment. A natural constant which arises in the treatment of magnetic effects is called the Bohr magneton. The magnetic moment is usually expressed as a multiple of the Bohr magneton.

$$\mu_B = \frac{e\hbar}{2m_e} = 9.2740154\times10^{-24} J/T = 5.7883826x10^{-5} eV/T$$

*Bohr magnrton*



**Stern-Gerlach Experiment**




This experiment confirmed the quantization of electron spin into two orientations. This made a major contribution to the development of the quantum theory of the atom.

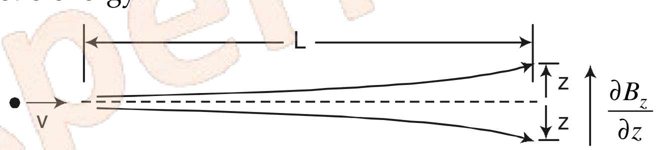
The potential energy of the electron spin magnetic moment in a magnetic field applied in the z direction is given by

$$U = -\mu.B = -\mu_B \frac{g}{2} B_z = \pm\mu_B B_z$$

where g is the electron spin g-factor and mB is the Bohr magneton. Using the relationship of force to potential energy gives

$$F_z = -\frac{\partial U}{\partial z} = \pm\mu_B \frac{\partial B_z}{\partial z}$$

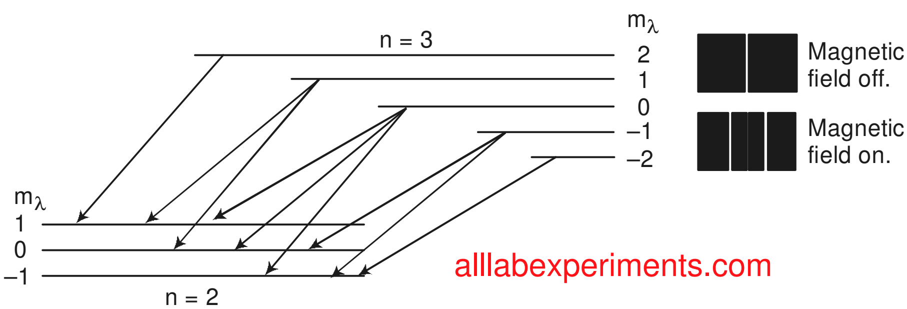
The deflection can be shown to be proportional to the spin and to the magnitude of the magnetic field gradient. It is inversely proportional to the particle kinetic energy.




$$z = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m}\left[\frac{L}{v}\right]^2 = \pm\frac{\mu_B L^2}{4KE}\frac{\partial B_z}{\partial z}$$

**Q 5. Explain normal Zeeman effect - Electron magnetic moment and magnetic energy.**

When an external magnetic field is applied, sharp spectral lines like the n=3 $\rightarrow$ 2 transition of hydrogen split into multiple closely spaced lines. First observed by pieter zeeman, this splitting is attributed to the interaction between the magnetic field and the magnetic dipole moment associated with the orbital angular momentum. In the absence of the magnetic field, the hydrogen energies depend only upon the principal quantum number n, and the emissions occur at a single wavelength.






Note that the transitions shown follow the selection rule which does not allow a change of more than one unit in the quantum number m.

An external magnetic field will exert a torque on a magnetic dipole and the magnetic potential energy which results is

$$U(\theta) = -\mu.B$$

The magnetic dipole moment associated with the orbital angular momentum is given by

$$\mu_{orbital} = \frac{-e}{2m_e} L$$

For magnetic field in the z-direction this gives

$$U = \frac{e}{2m} L_z B = m_\ell \frac{e\hbar}{2m} B$$

Considering the quantization of angular momentum, this gives equally spaced energy levels displaced from the zero field level by

$$\Delta E = m\ell \frac{e\hbar}{2m} B = m_\ell \mu_B B \quad \mu_B = \text{Bohr magnton}$$

$$\mu_B = \frac{e\hbar}{2m_e} = 9.2740154x10^{-24} J/T = 5.788382x10^{-5} eV/T \text{ [bohr magnton]}$$

This displacement of the energy levels gives the uniformly spaced multiplet splitting of the spectral lines which is called the zeeman effect.

The magnetic field also interacts with the electron spin moment, so it contribute to the zeeman effect in many cases. The electron spin had not been discovered at the time of zeeman's original experiments, so the cases where it contributed were considered to be anomalous. The term" anomalous zeeman effect " has persisted for the cases where spin contributes. In general, both orbital and spin moments are involved, and the zeeman interaction takes the form

$$\Delta E = \frac{e}{2m}(L+2S).\vec{B} = g_L \mu_B m_i \, B \qquad \text{Magnetic interaction energy}$$

The factor of two multiplying the electron spin angular momentum comes from the fact that it is twice as effective in producing magnetic. This factor is called the spin g-factor or gyromagnetic ratio. The evaluation of the scalar product between the angular.

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