This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper	**: 104**	**I**
Unique Paper Code	**: 32221301**	
Name of Paper	**: Mathematical Physics – II**	
Name of Course	**: B.Sc. (Hons.) Physics**	
Semester	**: III**	
Duration	**: 3 hours**	
Maximum Marks	**: 75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is compulsory.
*Attempt **five** questions in all.*

1. Attempt any *five* questions :

(a) Identify the singularities of the equation :

$$x^2\left(1-x^2\right)y'' + \frac{2}{x}y' + 4y = 0.$$

(b) Show that :

$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x.$$

(c) Prove that :

$$P_n'(1) = \frac{n(n+1)}{2}.$$

(d) Evaluate $\Gamma\left(\frac{-3}{2}\right)$.

P.T.O.

(e) State the Dirichlet conditions.

(f) Prove that :

$$\int_{-L}^{L} \sin\frac{m\pi x}{L} \sin\frac{n\pi x}{L}\, dx = \begin{cases} 0 & m \neq n \\ L & m = n \end{cases}$$

where m and n can assume any of the values 1, 2, 3, ...

(g) Show that $\beta(a, b) = \beta(a+1, b) + \beta(a, b+1)$. 5×3=15

2. Find the Fourier series of periodic function defined by

$$f(x) = x^2 \quad \text{for} \quad -\pi < x < \pi$$

and hence show that :

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$ 15

3. Using Frobenius method, obtain the solution of the following equation about $x = 0$:

$$(x-1)y'' + (3x-1)y' + y = 0.$$

(a) Evaluate :

$$\int_0^1 x^3 J_0(x)\, dx$$

and express it in terms of $J_0(x)$ and $J_1(x)$. 10,5

4. (a) Expand $x^4 - 3x^2 + x$ in a series of the form

$$\sum_{k=0}^{\infty} A_k P_k(x)$$

for at least $k = 0$, 1 and 2.

(b) Prove that :

$$(2n+1)\ P_n(x) = P_{n+1}'(x) - P_{n-1}'(x).$$ 10,5

5. (a) Prove that :

$$\int_0^1 x J_n(ax) J_n(bx)\, dx = \frac{1}{2} J_{n+1}^2(x) \delta_{ab}$$

where $J_n(a) = J_n(b) = 0$.

(b) Prove that :

$$nJ_n(x) + xJ_n'(x) = xJ_{n-1}(x).$$ 10,5

6. (a) Solve :

$$\int_0^1 \frac{dx}{\sqrt{-\ell n x}}$$

using Gamma function.

X(b) Show that :

$$\int_0^1 x^{m-1}\left(1-x^2\right)^{n-1} dx = \frac{1}{2}\beta\left(\frac{m}{2},\ n\right).$$

(c) Find out the roots of indicial equation for the Laguerre's differential equation :

$$xy'' + (1 - x)y' + \lambda y = 0$$

where λ is a constant.

7. Find the solution of 1-dimensional wave equation :

$$\frac{\partial^2 U}{\partial t^2} = c^2 \frac{\partial^2 U}{\partial x^2};\ 0 \le x \le 1,\ t > 0$$

under the boundary conditions $U(0, t) = U(l, t) = 0$ and initial conditions :

$$U(x,0)=\begin{cases} x & 0\le x\le \frac{\ell}{2} \\ l-x & \frac{\ell}{2}\le x\le \ell \end{cases} \quad \text{and} \quad \left.\frac{\partial U}{\partial t}\right|_{t=0}=0. \qquad 15$$