This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6673**

Unique Paper Code : **32221301** **HC**

Name of the Paper : **Mathematical Physics—II**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is compulsory.

Attempt *five* questions in all.

In the question paper $y \equiv y(x)$, $y' \equiv \dfrac{dy}{dx}$ and $y'' \equiv \dfrac{d^2y}{dx^2}$.

1. Attempt any *five* questions :

(*a*) Evaluate the integral :

$$I = \int_0^1 \sqrt{y(1-y)}\,dy.$$

(*b*) Identify and name the singularities (if any) of the following differential equations :

(*i*) $(1 - x)y'' - (2x - 1 - x^2)y' + (x - 1)y = 0$

(*ii*) $y'' - \frac{x}{x-1}y' + \frac{3x}{(1-x)^2}y = 0.$

(*c*) Show that for Legendre polynomials :

$$xP'_n = nP_n + P'_{n-1}.$$

(*d*) Demonstrate the linear dependence of $J_n(x)$ and $J_{-n}(x)$ where n is an integer.

(*e*) Determine if the following functions are odd, even or neither of them :

(*i*) $f(x) = |x|$ if $-5 < x < 5$

(*ii*) $g(x) = \begin{cases} \cos(-x) & \text{if } -\pi < x < 0 \\ \cos(x) & \text{if } 0 < x < \pi \end{cases}$

(*iii*) $h(x) = \sin(x)$ if $-\pi < x < \pi$.

(*f*) A guitar has six strings of equal length. They are arranged in such a way that the mass of each string is larger than the previous one. Also the tension in each of them can be controlled. Which string will produce the sound of highest pitch ? How can one manipulate the frequency of the sound emanating from each string ?

(*g*) Find the value of y if :

$$y'' = -y.$$

5×3=15

2. (*a*) The 1-D wave equation is given as :

$$\frac{\delta^2 y(x, t)}{\delta x^2} = c^2 \frac{\delta^2 (x, t)}{\delta t^2}.$$

Derive the same for a stretched string clearly mentioning the necessary assumptions.

(*b*) Evaluate :

$$\int_0^{\pi/2} \sin^4(\theta) \cos^5(\theta)\, d\theta.$$

12+3=15

3. (*a*) The Rodrigue's formula for Legendre polynomials is given as :

$$P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}\left(x^2 - 1\right)^n.$$

Prove the validity of the entity.

(*b*) Find the value of :

$$\int_{-1}^{1} x^2 P_5(x)\,dx.$$

10+5=15

4. The general Bessel's equation is given as :

$$x^2y'' + xy' + (x^2 - v^2)y = 0.$$

Starting from the Bessel's equation, obtain the expression of the Bessel's function of first kind. Also obtain the second solution of the Bessel's equation if v is not an integer. 15

5. (*a*) State if the given function $f(x)$ is an odd function.

Find its Fourier series expansion :

$$f(x) = \frac{x^2}{2} \text{ if } -\pi < x < \pi.$$

(*b*) Determine the value of 'D' and 'E' if :

$$D = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \text{..........} \text{ and}$$

$$E = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + - \text{..........}$$ 9+6=15

6. (*a*) Show that :

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$

(*b*) Demonstrate that :

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$$

(c) State and prove the Parseval's identity associated with Fourier series. 3×5=15

7. Solve the following differential equation :

$$xy'' + (1 - x)y' + \lambda y = 0.$$

Here λ is a real constant. Show that one of the solutions of this equation becomes a polynomial of order 'n' if $\lambda = n = 0, 1, 2, ...$ Name the Polynomial. 15