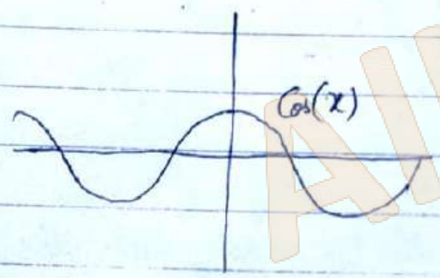
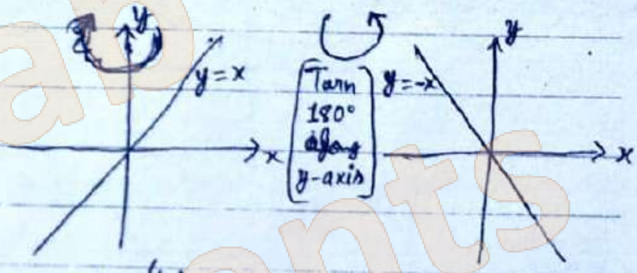
How to Plot (Basic Curves) → Many problems in Physics are solved by plotting the curves or many times it gives us basic idea about the problem. We can use some basic methodologies to plot typical curves.

# Use symmetries → i) $(x \to -x) \Rightarrow$ If in a function there is no difference when we replace $x$ by $-x$ then ~~it is~~ function is symmetrical w.r.t. y-axis. e.g. $\cos(-x) = \cos x$

So when ever we change $x$ to $-x$. ~~in a function~~ The new function generated is such that as we rotate old function by 180° along y-axis.

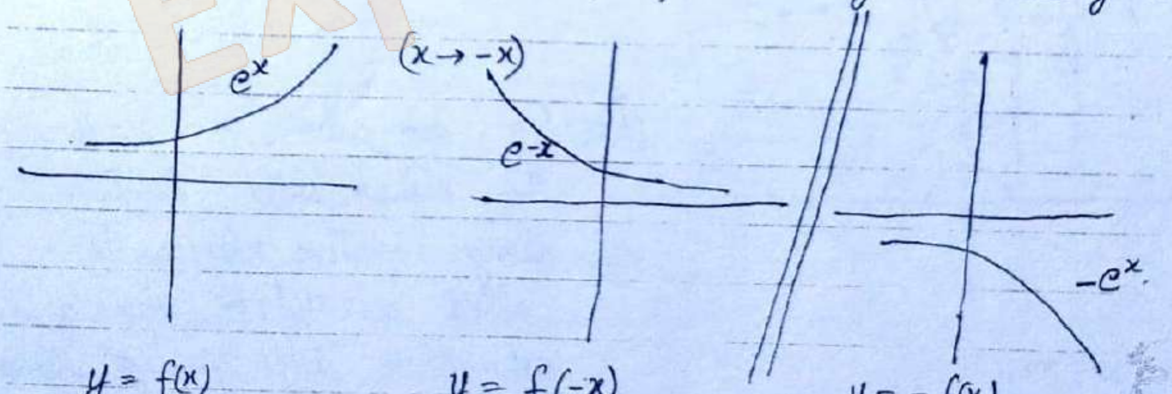



$y = f(x) = x$ $\qquad$ $y = f(-x) = -x$

ii) $(y \to -y) \Rightarrow$ If there is no difference when we replace $y$ by $-y$ then function is symmetrical along x-axis.

Same again change $y$ to $-y$. The new function is such that as we rotate old function by 180° along x-axis.




$y = f(x)$ $\qquad$ $y = f(-x)$ $\qquad$ $y = -f(x)$

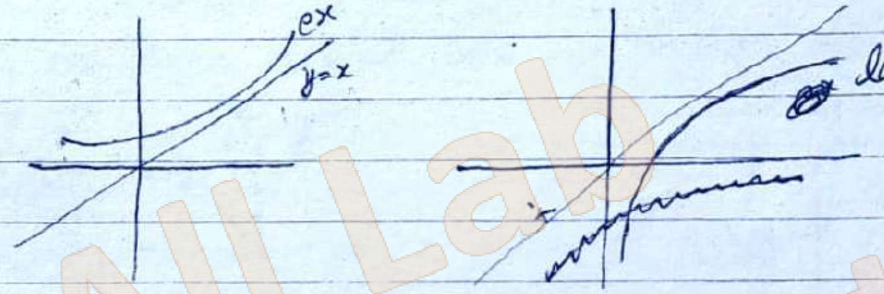
iii) $(x \to -x)$ & $(y \to -y) \Rightarrow$ If function doesn't show any change on applying these transformations then function is symmetrical along origin or say both cases discussed above work together.

$(x \leftrightarrow y)$

iv) $(x \to y)$ & $(y \to x)$ → on ~~changing~~ replacing $x$ ~~by~~ and $y$ ~~&~~ ~~y by x~~ by $y$ & $x$, if there is no difference in function then we get a new function which is symmetrical to $y=x$ line.

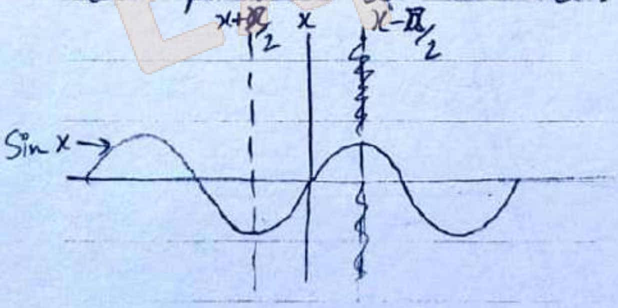
Similarly the new function generated when we ~~replace~~ change the places of $y$ and $x$ can be obtained by rotating 180° along $y=x$ line.

$y = e^x$ $(x \leftrightarrow y)$ $x = e^y \Rightarrow y = \log_e x$




Translation → i) $x \to x \pm a$ → ~~Shifting~~ changing $x$ by $x+a$ shifts the function by $a$. The value of function which was present at $x=0$ is now present at $x=-a$. So shift the y-axis on ~~positive~~ x-axis upto point $-a$ or shift function upto point $a$.

Similarly $x \to x-a$, the value present at $x=0$ is now present at $x=+a$.




$x \to x+a$

$\sin(x) \to \sin(x+\pi/2) = \cos x$

~~Sin x~~ $x \to x-a$

$\sin(x) \to \sin(x-\pi/2) = \cos x$

(k) ⇒ So, ~~we~~ when $x \to x-a$ then either shift function along negative x-axis is called as active view and when we shift ~~frame~~ along positive x-axis is called passive view.

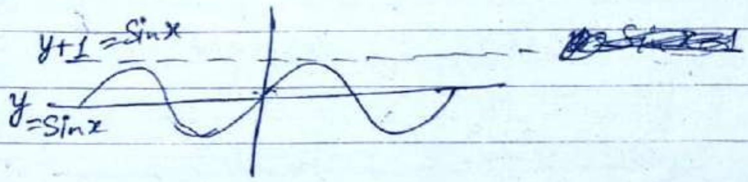
ii) $y \to y \pm a$ → This is exactly same as the last description. ~~which~~ when a constant is added ~~or~~ subtracted to $y$ then function shifts by that constant upward (for $-a$) or downward (for $+a$) along $y$-axis.

$y = \sin x$

$y \to y + 1$

then

$y = \sin x - 1$



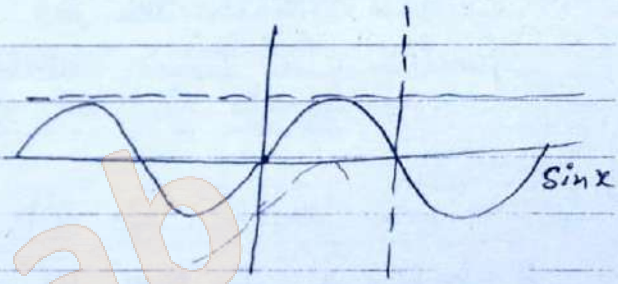

iii) $x \to x \pm a$ & $y \to y \pm a$ → Now apply both the transformations to get the required plot.

Let $y = \sin x$

$y \to y+1, \quad x \to x - \pi$

$y + 1 = \sin(x - \pi)$

$y = \sin(x-\pi) - 1$

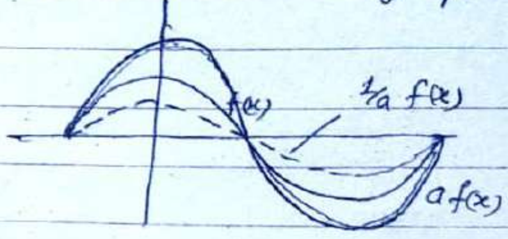



dotted lines give us new set of axis.

Stretching & Shrinking → $f(x) \to a\, f(x)$ ; $a > 1$

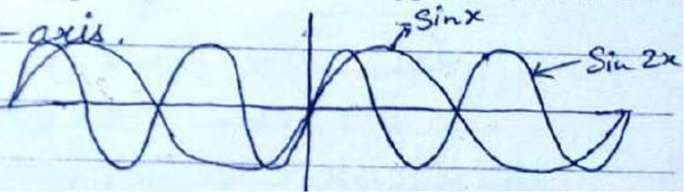
for plotting this kind of graphs in which we multiply by a constant greater than 1, we need to stretch the graph of $f(x)$ 'a' times along $y$-axis.

$f(x) \to \frac{1}{a} f(x)$ ; $a > 1$ then shrink the graph of $f(x)$ 'a'-times along $y$-axis.




when $x \to ax$ ; $a > 1$

Again we play the same game of stretch & shrink. for transformation like $x \to ax$ we ~~stretch~~ shrink the function 'a' times along $x$-axis otherwise for $x \to \frac{1}{a}x$ ; $a>1$ we stretch the function 'a' times along $x$-axis.



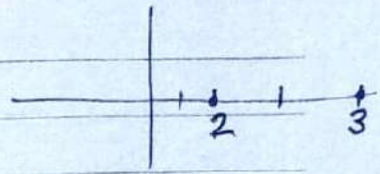
# Working on function, its slope & curvature →

Here in this part of How to plot we will first work on function to get some of its knowledge. Then its slope (differentiation) and curvature (double-differentiation) gives us full knowledge of how to plot that curve.

① Now we are targeting quadratic, cubic, biquadratic or further polynomials. ~~Every polynomial have some~~ so first we find out the roots of the equation. Roots of an equation are the values of $x$ where function vanishes. when $f(x) = 0$ the values of $x$ are known as roots.

let an example $x^2 - 5x + 6 = f(x)$. The roots of this equation are $x = 2, 3$. So these are the values where $f(x)$ is zero.

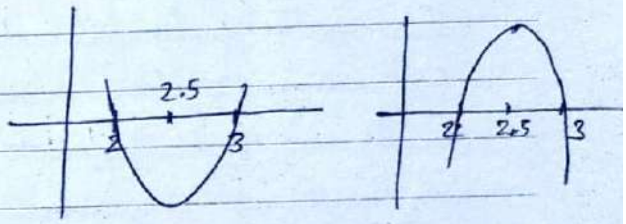
2. In 2nd step we differentiate the function. i)




It gives us the slope of the function. It also (equation of) gives us ~~stable~~ point (maximum or minimum) where differentiation vanishes.

$$\frac{d\,f(x)}{dx} = 2x - 5 = 0$$

$$\Rightarrow x = \frac{5}{2}$$




This curve may represent maximum or minimum so now our third step of analysis gives us this answer –

3. 2nd differentiation of this function gives us clue that either it is maximum or minimum.

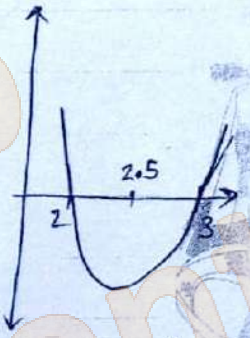
$\frac{d^2y}{dx^2}$ tells us the change in $\frac{dy}{dx}$ w.r.t. change in $x$.

Concave up. Concave down.

$\frac{dy}{dx}$ ~~increases~~ decreases as $x$ decreases $\Rightarrow \frac{d^2y}{dx^2} < 0$ shows concave down. (maximum)

$$\frac{d^2f(x)}{dx^2} = 2 > 0$$

then it will show a minimum.




# point of inflexion → The point where ~~convex~~ curvature changes from concave up to concave down is known as point of inflexion.

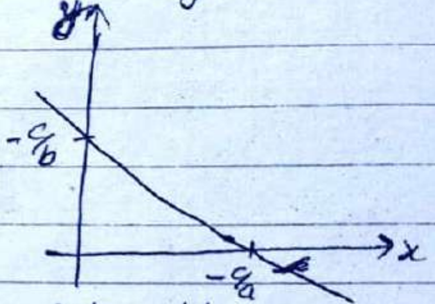
$$d^2y/_{dx^2} = 0 \quad \text{at point of inflexion.}$$

In this way by knowing the curvature of a curve we may analyse its nature at certain place.

# Plot straight line → ~~ax+b~~ We use intercept form of the equation to plot it. Let an equation $ax+by+c=0$ then we can write it as

$$\frac{ax}{-c} + \frac{by}{-c} = 1$$

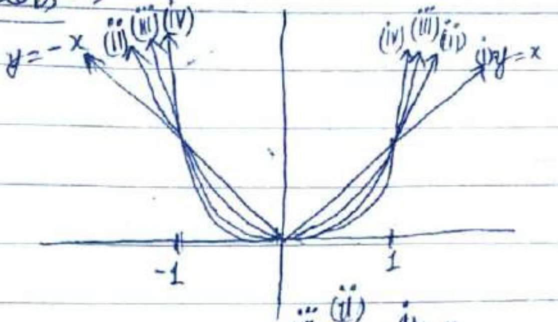
$$\Rightarrow \frac{x}{(-c/a)} + \frac{y}{(-c/b)} = 1$$




The terms divided to $x$ & $y$ are the intercepts on co-ordinate axes. Then it is very easy to plot a straight line.

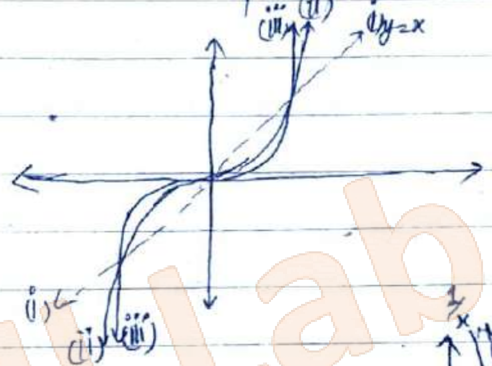
Some Sample Plots →

i) $y = x$
ii) $y = x^2$
iii) $y = x^4$
iv) $y = x^6$




fun. (ii), (iii), (iv) are all even functions.

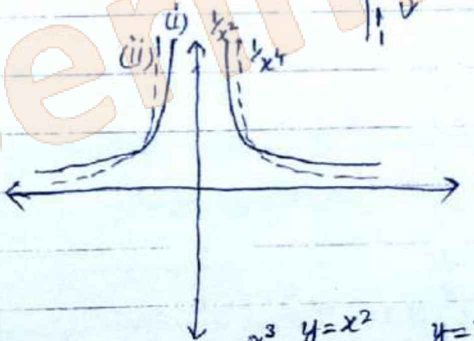
i) $y = x$
ii) $y = x^3$
iii) $y = x^5$



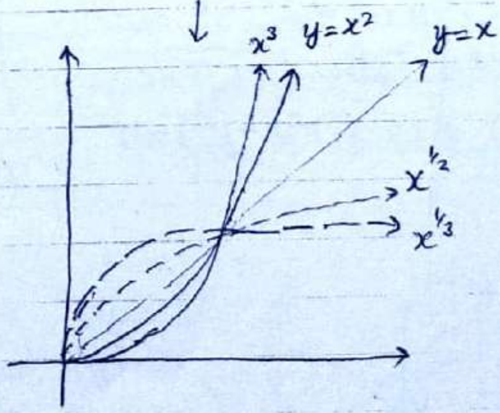

fun. (i), (ii), (iii) are all odd functions.

(i) $y = 1/x$
ii) $y = 1/x^3$


i) $y = 1/x^2$
ii) $y = 1/x^4$




i) $y = x$
ii) $y = x^2$
iii) $y = x^3$
iv) $y = x^{1/2}$
v) $y = x^{1/3}$



⑥ i) $\sin x$

~~ii) Cos x~~

ii) $\operatorname{Cosec} x$

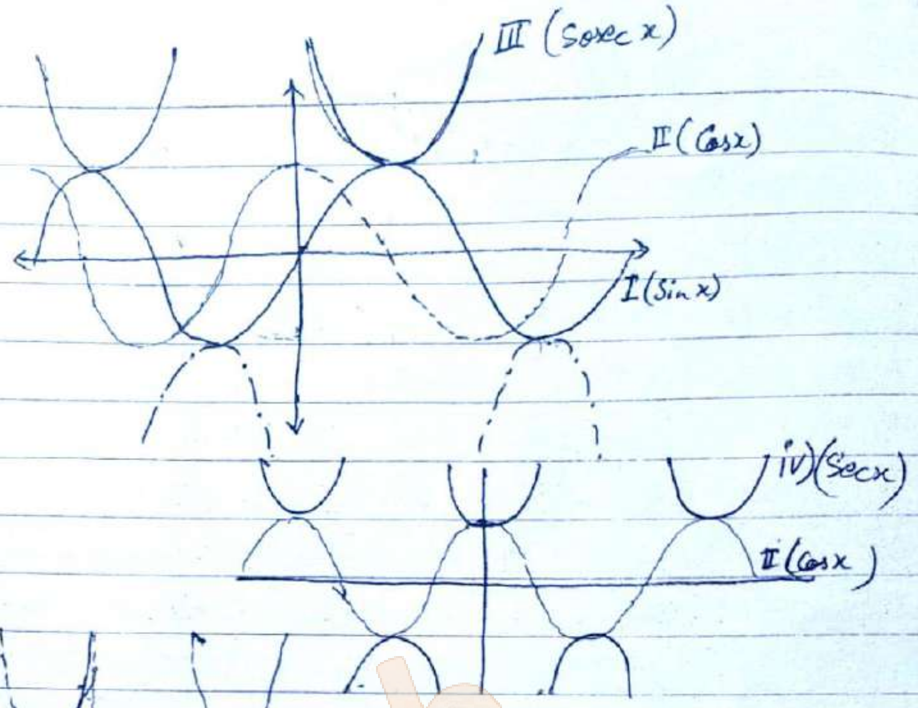
~~iv) Sec x~~

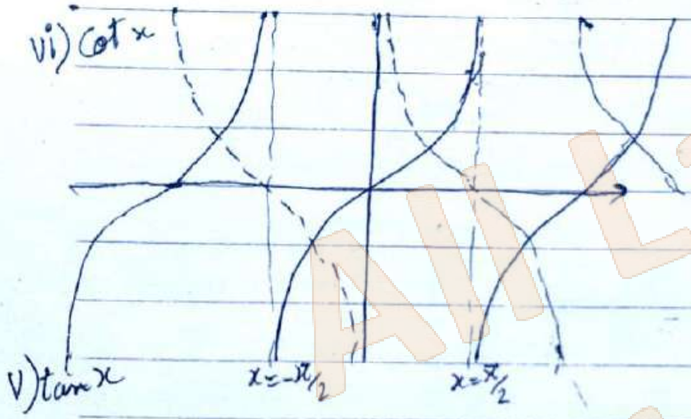
iii) $\cos x$

iv) $\sec x$

v) $\tan x$

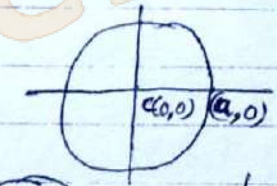
vi) $\cot x$



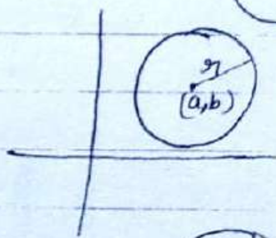




⑦ Circle →

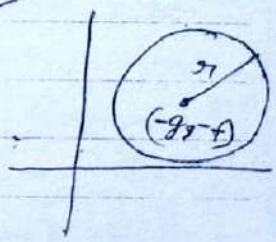
i) $x^2+y^2=a^2$ centre at $(0,0)$ and radius $a$.



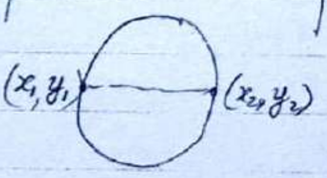

ii) $(x-a)^2+(y-b)^2=r^2$ centre at $(a,b)$ and radius $r$.




iii) $x^2+y^2+2gx+2fy+c=0$ centre $(-g,-f)$ & radius $\sqrt{g^2+f^2-c}$




iv) $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$




⑧ Parabola →

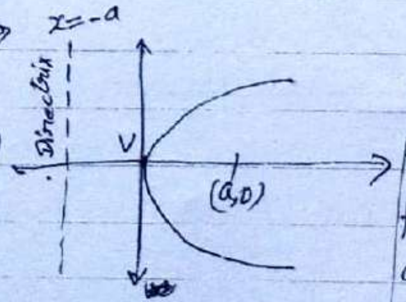
i) $y^2=4ax$

vertex $(0,0)$

focus $(a,0)$

axis → $y=0$

Directrix $x=-a$



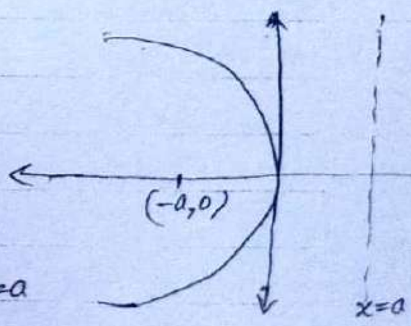

ii) $y^2=-4ax$

vertex $(0,0)$

focus $(-a,0)$ ←

axis $y=0$

directrix $x=a$



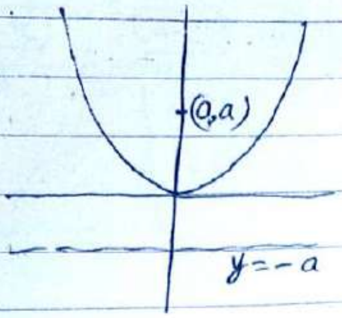
iii) $x^2 = 4ay$

vertex $(0,0)$

focus $(0,a)$

axis $x=0$

Directrix $y=-a$



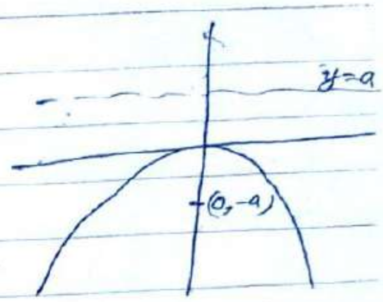

iv) $x^2 = -4ay$

vertex $(0,0)$

focus $(0,-a)$

axis $x=0$

directrix $y=a$



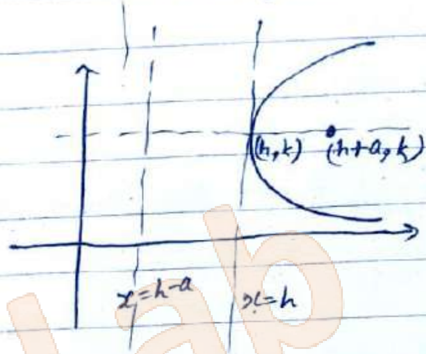

V) $(y-k)^2 = 4a(x-h)$

Vertex $(h,k)$

focus $(h+a,k)$

axis $x=h$

directrix $x=h-a$




Ellipse →

i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a^2 > b^2)$

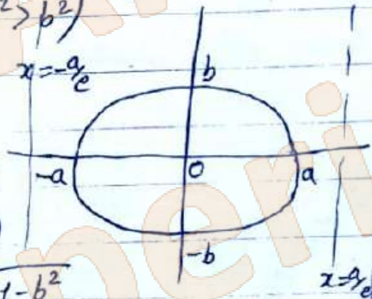
Centre : $(0,0)$

focus : $(\pm ae, 0)$

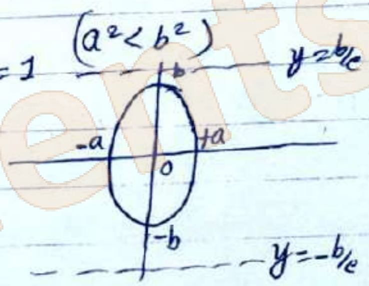
vertex : $(\pm a, 0)$

eccentricity : $e = \sqrt{1 - \frac{b^2}{a^2}}$

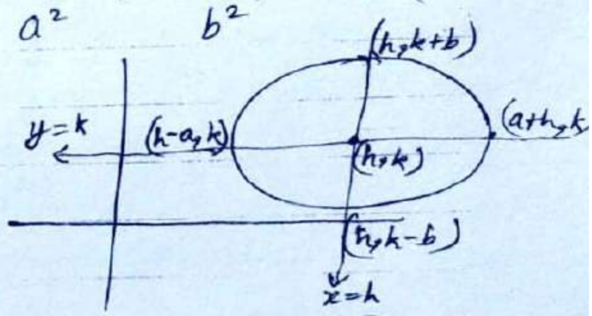
directrix : $x = \pm a/e$




ii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a^2 < b^2)$




iii) $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad (a^2 > b^2)$




Hyperbola →

i) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

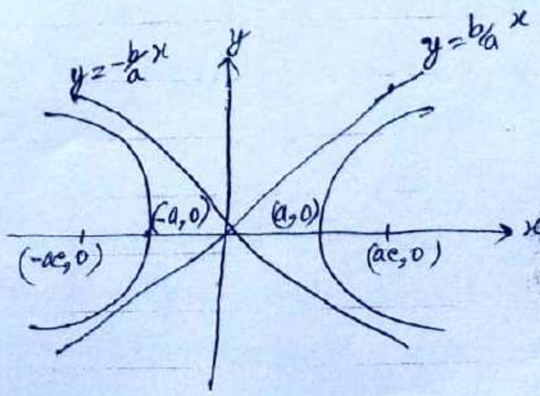
Centre $(0,0)$

focus $(\pm ae, 0)$

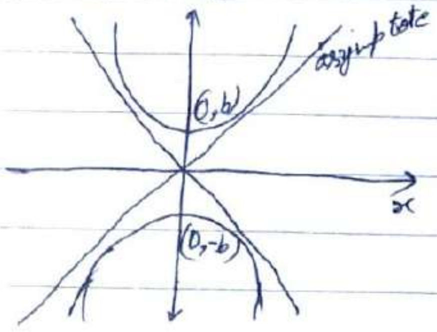
vertices $(\pm a, 0)$

eccentricity $e = \sqrt{1 + \frac{b^2}{a^2}}$

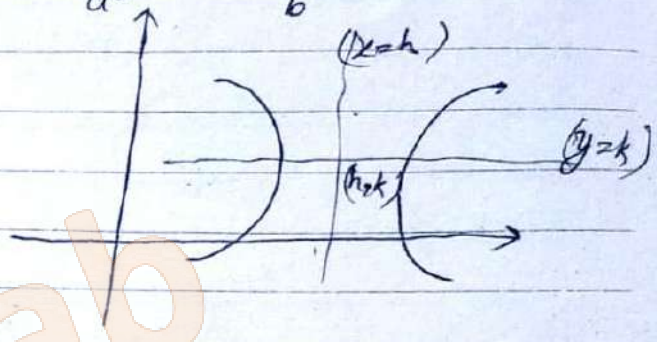
directrix $x = \pm a$



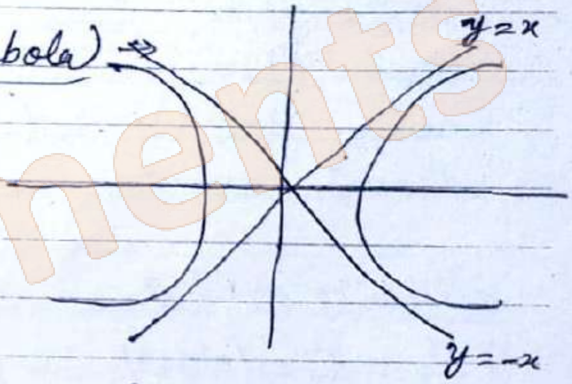
ii) $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



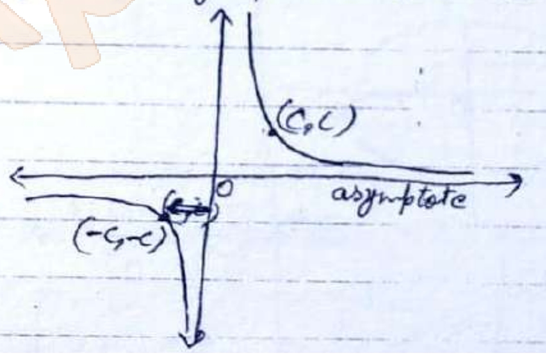

iii) $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$




iv) $x^2 - y^2 = a^2$ (Rectangular Hyperbola)




v) $xy = c^2$ (Here asymptotes are x-axis & y-axis.



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**# Hopefully, I will turn these notes into a clean article soon. It depends on my time and the financial support.**

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