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B.Sc. (Prog), Thermal Physics
Chapter - 4
Theory of Radiation

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Chapter- 4

Theory of Radiation: Blackbody radiation, Spectral distribution, Concept of Energy Density, Derivation of Planck's law, Deduction of Wien's distribution law, Rayleigh-Jeans Law, Stefan Boltzmann Law and Wien's displacement law from Planck's law.

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Que 1: Explain Blackbody Radiation in brief.

Ans: "Blackbody radiation" or "cavity radiation" refers to an object or system which absorbs all radiation incident upon it and re-radiates energy which is characteristic of this radiating system only, not dependent upon the type of radiation which is incident upon it. The radiated energy can be considered to be produced by standing wave or resonant modes of the cavity which is radiating.

Que 2: Briefly explain Energy density, Spectral energy density, Emissive power and Spectral emissive power.

Energy Density

At a particular temperature T energy in the cavity per unit volume is called energy density. It is denoted by u .

Spectral energy Density

At a certain temperature the average energy density between wavelength λ and $\lambda + d\lambda$ is given by $u_\lambda d\lambda$. where $u - \lambda$ is called spectral energy density. the relation between u and u_λ is

$$U = \int_0^\infty u_\lambda d\lambda$$

One can use u_ν instead of u_λ . Which is defined between frequency range ν and $\nu + d\nu$.

Emissive Power

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Total radiation energy emitted by unit surface area of the blackbody at a certain temperature is called Emmisive power of the blackbody. It is denoted by E.

Spectral Emissive Power

Total radiated energy between λ and $\lambda + d\lambda$ range emitted by unit surface area of blackbody at certain temperature is given by

$$E_\lambda d\lambda$$

Where E_λ is called spectral emissive power of blackbody. It is clear from the definition that

$$E = \int_0^\infty E_\lambda d\lambda$$

If we know E_λ at a certain temperature we can find u_λ by the following relation

$$U = \frac{4}{c} E_\lambda$$

Where c is the speed of light.

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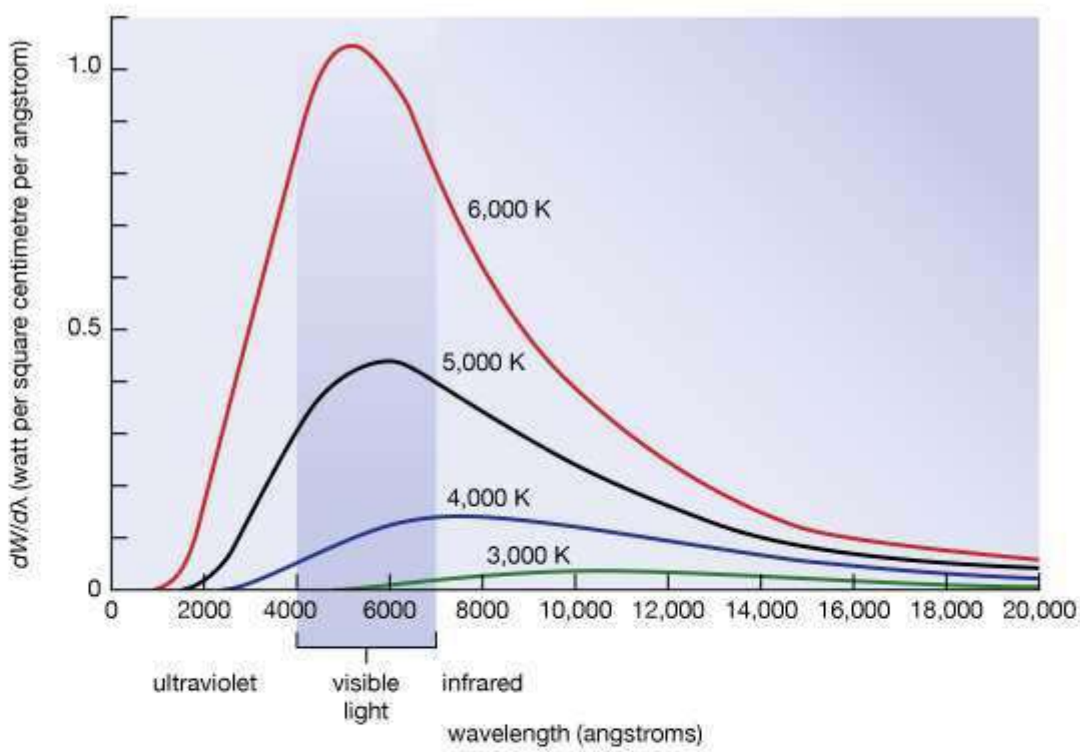
Que 3: Explain the characteristics of Blackbody Radiation using characteristic experimental curve.

Ans:

Characteristics of Blackbody Radiation



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1. As it is clear from the figure the graph is continuous which means that at every temperature radiation for all wavelengths emitted but the spectral emissive power is different for different wavelength.

2. Spectral energy density $E\lambda$ for each λ increases with temperature.

3. At a particular temperature at first $E\lambda$ increases with λ but after reaching a certain highest value it goes on decreasing. That highest value is denoted by E_{λ_m} and the wavelength at which E_λ is maximum is denoted by λ_m

4. Wien's Displacement Law-

   As we see from the graph λ_m (corresponding wavelength for maximum emission) decreases with temperature. It was Wien who first discovered mathematically that

$$\lambda_m \propto \frac{1}{T}$$

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or

$$\lambda_m = \frac{b}{T}$$

Where b is called Wien's constant its value is $b = 2.898 * 10^{-3} meter Kelvin$ The above law is known as Wien's displacement law. This is very important law as it law helps us to find the temperature of stars(hot bodies).

5. We also see that the peak of graph increases rapidly with temperature. It is found that

$$E\lambda_m \propto T^5$$

6. Stephan Boltzmann's Law- At a particular temperature the area under the curve is given by

$$\int_0^\infty E_\lambda d\lambda$$

Which is the total emissive power of blackbody. Hence the area of the curve represents the total emissive power. It is found to be proportional $to T^4$ i.e.

$$E \propto T^4$$

or

$$E = \sigma T^4$$

Where σ is known as Stephan's constant having value

$$\sigma = 5.67 * 10^{-8} \texttt{ watt}/m^2/K^4$$

This law is known as Stephan Boltzmann's law.

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Que 4: Explain and derive Planck's distribution law of Blackbody radiation.

Ans:

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Planck used Maxwell Boltzmann statistics to calculate radiation energy. According to Planck at temperature T the number of oscillators having energy $nh\nu$ is

$$N_n = Aexp(\frac{-nh\nu}{kT})$$

Where k is Boltzmann constant.

The total energy of oscillators having energy $nh\nu$ is

$$E_n = nh\nu N_n$$

Hence the total energy of all oscillators in the blackbody

$$E = \Sigma E_n = \Sigma nh\nu N_n$$

Total number of oscillators in the blackbody is

$$N = \Sigma N_n$$

$\Rightarrow$ average energy of oscillator is

$$\langle E \rangle = \frac{\texttt{total energy of oscillator}}{\texttt{total number of oscillator}}$$

or

$$\langle E \rangle = \frac{\Sigma nh\nu N_n}{\Sigma N_n}$$

$$= \frac{\Sigma nh\nu exp(\frac{-nh\nu}{kT})}{\Sigma exp(\frac{-nh\nu}{kT})}$$

let

$$x = \frac{h\nu}{kT}$$

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$$\Rightarrow \langle E \rangle = kT \frac{\Sigma n x e^{-nx}}{\Sigma e^{-nx}}$$

$$= -xkT \frac{\frac{d}{dx}[\Sigma e^{-nx}]}{\Sigma e^{-nx}}$$

but

$$\Sigma e^{-nx} = \frac{1}{1 - e^{-x}}$$

and

$$\frac{d}{dx}[\Sigma e^{-nx}] = \frac{-e^{-x}}{(1 - e^{-x})^2}$$

$$\Rightarrow \langle E \rangle = \frac{xkT}{(e^x - 1)}$$

Hence average energy of an oscillator is

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$$\langle E \rangle = \frac{h\nu}{e^{h\nu/kT} - 1}$$

Or in terms of wave length

$$\langle E \rangle = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1}$$

According to Planck energy density between range λ to $\lambda + d\lambda$ is

$$u_\lambda d\lambda = N_\lambda d\lambda \langle E \rangle$$

Planck used the calculation made by Rayleigh Jeans for number of oscillations. Hence

$$N_\lambda d\lambda = \frac{8\pi}{\lambda^4} d\lambda$$

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thus

$$u_\lambda d\lambda = \frac{8\pi hcd\lambda}{\lambda^5 exp(\frac{hc}{\lambda kT}) - 1}$$

In terms of frequency

$$U_\nu d\nu = \frac{8\pi h\nu^3 d\nu}{c^3 exp(\frac{h\nu}{kT}) - 1}$$

The above equation is called Planck,s distribution law.

Explanation in small λ range

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for $\frac{hc}{kT} \gg \lambda$

$$exp(\frac{hc}{kT}) \gg 1$$

using this in Planck's distribution law

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}}d\lambda$$

$$= \frac{8\pi hc}{\lambda^5}e^{-hc/\lambda kT}d\lambda$$

Explanation in large wavelength region

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if $\frac{hc}{kT} \ll \lambda$ then

$$exp(\frac{hc}{kT}) \approx 1 + \frac{hc}{\lambda}$$

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Using this in Planck's law we get

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{1 + hc/\lambda kT - 1}d\lambda$$

or

$$u_\lambda d\lambda = \frac{8\pi kT}{\lambda^4}d\lambda$$

Which is Rayleigh Jeans distribution law. It can explain all the properties of black-body radiation at large wavelengths.

Que5: Calculate total energy density of blackbody radiation using Planck's law.

Ans:

Calculation of Total Energy Density

Inside the blackbody total energy density is defined by

$$U = \int_0^\infty u_\lambda d\lambda$$

using plank distribution law

$$U = \int_0^\infty \frac{8\pi hc}{\lambda^5(e^{hc/\lambda kT} - 1)}d\lambda$$

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let $\frac{hc}{\lambda kT} = x$ then differentiating it

$$d\lambda = \frac{-hc}{kTx^2}dx$$

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using this

$$U = 8\pi hc \int_0^\infty \frac{-hc/kTx^2}{(hc/xkT)^5(e^x - 1)}dx$$

$$= \frac{8\pi k^4 T^4}{c^3 h^3} \int_0^\infty \frac{x^3}{e^x - 1}dx$$

but the standard integration

$$\int_0^\infty \frac{x^3}{e^x - 1}dx = \frac{\pi^4}{15}$$

$$\Rightarrow U = \left[\frac{8\pi^5 k^4}{15h^3c^3}\right] T^4$$

This equation represents the total energy density of blackbody.

Que 6: Deduce Stephan Boltzmann's law and Wein's displacement law using Planck's distribution law.

Ans:

Deduction of Stefan Boltzmann's law

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The emissive power i.e. the energy radiated per second by unit surface area of the blackbody is

$$E = \frac{cU}{4}$$

$$= \frac{2\pi^5 k^4}{15h^3c^2}T^4$$

or

$$E = \sigma T^4$$

Which is Stefan's law. Where

$$\sigma = \frac{2\pi^5 k^4}{15h^3c^2}$$

Deduction of Wien's Displacement Law

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Planck's distribution law is

$$u_\lambda = \frac{8\pi hc}{\lambda^5(e^{hc/\lambda kT} - 1)}$$

u_λ is maximum at $\lambda = \lambda_m$ then

$$\left[\frac{du_\lambda}{d\lambda}\right]_{\lambda_m} = 0$$

This gives

$$\frac{-5}{\lambda^6}\frac{1}{e^{hc/\lambda kT} - 1} - \frac{\lambda^2 kT}{\lambda^5(e^{hc/\lambda kT} - 1)^2} = 0$$

at $\lambda = \lambda_m$

or

$$5 = \frac{ch}{\lambda kT} \frac{e^{hc/\lambda kT}}{e^{hc/\lambda kT} - 1}$$

at $\lambda = \lambda_m$

let $x = \frac{hc}{\lambda kT}$ then above equation reduces to

$$e^x = \frac{5}{5 - x}$$

or

$$x = ln5 - ln(5 - x)$$

This is a non algebric equation having solution

$$x \approx 4.965$$

Hence $\frac{hc}{\lambda_m kT} = 4.965$ or

$$\lambda_m T = \frac{hc}{k(4.965)}$$

Substituting the values of $h, c\&k$

$$\lambda_m T = 2.989 * 10^{-3}$$

Which is Wien's displacement law.

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