Free Study Material from

All Lab Experiments

Waves & Optics (B.Sc.)

Chapter - 8, 9

8. Diffraction

9. Polarization

Chapter-8
diffraction

Diffraction: Fraunhofer diffraction: Single slit; Double Slit. Multiple slits & Diffraction grating. Fresnel Diffraction: Half-period zones. Zone plate. Fresnel Diffraction pattern of a straight edge, a slit and a wire using half-period zone analysis.
(14 Lectures)

Q:what do you understand by diffraction.

Ans:

In addition to interference, waves also exhibit another property – *diffraction*, which is the bending of waves as they pass by some objects or through an aperture. The phenomenon of diffraction can be understood using *Huygens's principle* which states that

Every unobstructed point on a wavefront will act a source of secondary spherical waves. The new wavefront is the surface tangent to all the secondary spherical waves.

Figure 14.4.1 illustrates the propagation of the wave based on Huygens's principle.

Figure 14.4.1 Propagation of wave based on Huygens's principle.

According to Huygens's principle, light waves incident on two slits will spread out and exhibit an interference pattern in the region beyond (Figure 14.4.2a). The pattern is called a diffraction pattern. On the other hand, if no bending occurs and the light wave continue to travel in straight lines, then no diffraction pattern would be observed (Figure 14.4.2b).

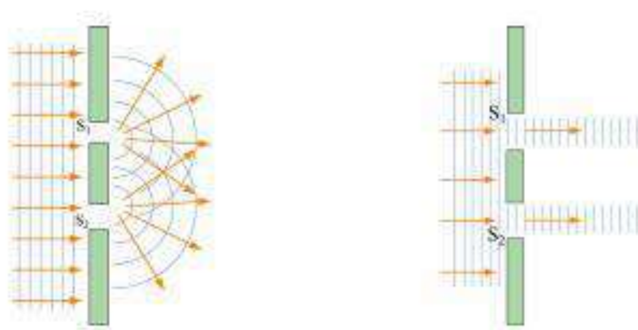


Figure 14.4.2 (a) Spreading of light leading to a diffraction pattern. (b) Absence of diffraction pattern if the paths of the light wave are straight lines.

Q:explain single slit diffraction.

Ans:

In our consideration of the Young's double-slit experiments, we have assumed the width of the slits to be so small that each slit is a point source. In this section we shall take the width of slit to be finite and see how Fraunhofer diffraction arises.

Let a source of monochromatic light be incident on a slit of finite width a, as shown in Figure 14.5.1.

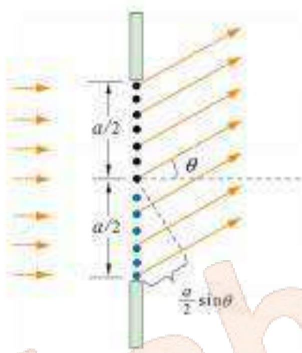


Figure 14.5.1 Diffraction of light by a slit of width a.

In diffraction of Fraunhofer type, all rays passing through the slit are approximately parallel. In addition, each portion of the slit will act as a source of light waves according to Huygens's principle. For simplicity we divide the slit into two halves. At the first minimum, each ray from the upper half will be exactly 180° out of phase with a corresponding ray form the lower half. For example, suppose there are 100 point sources, with the first 50 in the lower half, and 51 to 100 in the upper half. Source 1 and source 51 are separated by a distance $a/2$ and are out of phase with a path difference $\delta = \lambda/2$. Similar observation applies to source 2 and source 52, as well as any pair that are a distance $a/2$ apart. Thus, the condition for the first minimum is

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \qquad (14.5.1)$$

$$\sin\theta = \frac{\lambda}{a} \qquad (14.5.2)$$

Applying the same reasoning to the wavefronts from four equally spaced points a distance $a/4$ apart, the path difference would be $\delta = a\sin\theta/4$, and the condition for destructive interference is

$$\sin\theta = \frac{2\lambda}{a} \qquad (14.5.3)$$

The argument can be generalized to show that destructive interference will occur when

$$\boxed{a\sin\theta = m\lambda, \quad m = \pm1, \pm2, \pm3, \ldots \text{ (destructive interference)}} \qquad (14.5.4)$$

Figure 14.5.2 illustrates the intensity distribution for a single-slit diffraction. Note that $\theta = 0$ is a maximum.

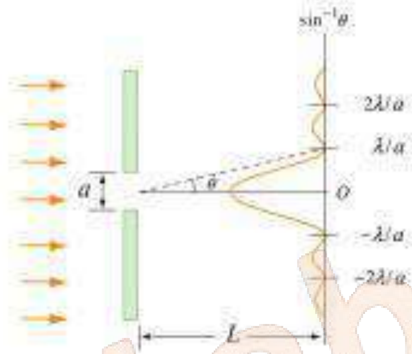


Figure 14.5.2 Intensity distribution for a single-slit diffraction.

By comparing Eq. (14.5.4) with Eq. (14.2.5), we see that the condition for *minima* of a single-slit diffraction becomes the condition for *maxima* of a double-slit interference when the width of a single slit a is replaced by the separation between the two slits d. The reason is that in the double-slit case, the slits are taken to be so small that each one is considered as a single light source, and the interference of waves originating within the same slit can be neglected. On the other hand, the minimum condition for the single-slit diffraction is obtained precisely by taking into consideration the interference of waves that originate within the *same* slit.

Q:derive the expression for intensity of single slit diffraction.

Ans:

Let's divide the single slit into N small zones each of width $\Delta y = a/N$, as shown in Figure 14.6.1. The convex lens is used to bring parallel light rays to a focal point P on the screen. We shall assume that $\Delta y \ll \lambda$ so that all the light from a given zone is in phase. Two adjacent zones have a relative path length $\delta = \Delta y \sin\theta$. The relative phase shift $\Delta\beta$ is given by the ratio

$$\frac{\Delta\beta}{2\pi} = \frac{\delta}{\lambda} = \frac{\Delta y\sin\theta}{\lambda}, \quad \Rightarrow \quad \Delta\beta = \frac{2\pi}{\lambda}\Delta y\sin\theta \qquad (14.6.1)$$

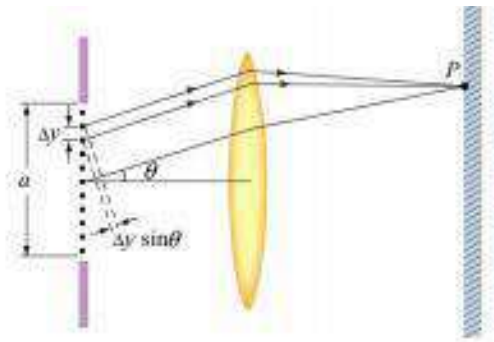


Figure 14.6.1 Single-slit Fraunhofer diffraction

Suppose the wavefront from the first point (counting from the top) arrives at the point P on the screen with an electric field given by

$$E_1 = E_{10} \sin \omega t \qquad (14.6.2)$$

The electric field from point 2 adjacent to point 1 will have a phase shift $\Delta\beta$, and the field is

$$E_2 = E_{10} \sin\left(\omega t + \Delta\beta\right) \qquad (14.6.3)$$

Since each successive component has the same phase shift relative the previous one, the electric field from point N is

$$E_N = E_{10} \sin\left(\omega t + (N-1)\Delta\beta\right) \qquad (14.6.4)$$

The total electric field is the sum of each individual contribution:

$$E = E_1 + E_2 + \cdots E_N = E_{10}\left[\sin \omega t + \sin\left(\omega t + \Delta\beta\right) + \cdots + \sin\left(\omega t + (N-1)\Delta\beta\right)\right] \qquad (14.6.5)$$

Note that total phase shift between the point N and the point 1 is

$$\beta = N\Delta\beta = \frac{2\pi}{\lambda} N\Delta y \sin\theta = \frac{2\pi}{\lambda} a \sin\theta \qquad (14.6.6)$$

where $N\Delta y = a$. The expression for the total field given in Eq. (14.6.5) can be simplified using some algebra and the trigonometric relation

$$\cos(\alpha - \beta) - \cos(\alpha + \beta) = 2\sin\alpha \sin\beta \qquad (14.6.7)$$

$$\cos(\omega t - \Delta\beta/2) - \cos(\omega t + \Delta\beta/2) = 2\sin\omega t \sin(\Delta\beta/2)$$

$$\cos(\omega t + \Delta\beta/2) - \cos(\omega t + 3\Delta\beta/2) = 2\sin(\omega t + \Delta\beta)\sin(\Delta\beta/2)$$

$$\cos(\omega t + 3\Delta\beta/2) - \cos(\omega t + 5\Delta\beta/2) = 2\sin(\omega t + 2\Delta\beta)\sin(\Delta\beta/2) \qquad (14.6.8)$$

$$\cos[\omega t + (N-1/2)\Delta\beta] - \cos[\omega t + (N-3/2)\Delta\beta] = 2\sin[\omega t + (N-1)\Delta\beta]\sin(\Delta\beta/2)$$

Adding the terms and noting that all but two terms on the left cancel leads to

$$\cos(\omega t - \Delta\beta/2) - \cos[\omega t - (N-1/2)\Delta\beta] \\ = 2\sin(\Delta\beta/2)\left[\sin\omega t + \sin\left(\omega t + \Delta\beta\right) + \cdots + \sin\left(\omega t + (N-1)\Delta\beta\right)\right] \tag{14.6.9}$$

The two terms on the left combine to

$$\cos(\omega t - \Delta\beta/2) - \cos[\omega t - (N-1/2)\Delta\beta] \\ = 2\sin(\omega t + (N-1)\Delta\beta/2)\sin(N\Delta\beta/2) \tag{14.6.10}$$

with the result that

$$\left[\sin\omega t + \sin\left(\omega t + \Delta\beta\right) + \cdots + \sin\left(\omega t + (N-1)\Delta\beta\right)\right] \\ = \frac{\sin[\omega t + (N-1)\Delta\beta/2]\sin(\beta/2)}{\sin(\Delta\beta/2)} \tag{14.6.11}$$

The total electric field then becomes

$$E = E_{10}\left[\frac{\sin(\beta/2)}{\sin(\Delta\beta/2)}\right]\sin\left(\omega t + (N-1)\Delta\beta/2\right) \tag{14.6.12}$$

The intensity I is proportional to the time average of E^2:

$$\left\langle E^2\right\rangle = E_{10}^2\left[\frac{\sin(\beta/2)}{\sin(\Delta\beta/2)}\right]^2\left\langle\sin^2\left(\omega t + (N-1)\Delta\beta/2\right)\right\rangle = \frac{1}{2}E_{10}^2\left[\frac{\sin(\beta/2)}{\sin(\Delta\beta/2)}\right]^2 \tag{14.6.13}$$

and we express I as

$$I = \frac{I_0}{N^2}\left[\frac{\sin(\beta/2)}{\sin(\Delta\beta/2)}\right]^2 \tag{14.6.14}$$

where the extra factor N^2 has been inserted to ensure that I_0 corresponds to the intensity at the central maximum $\beta = 0$ $(\theta = 0)$. In the limit where $\Delta\beta \to 0$,

$$N\sin(\Delta\beta/2) \approx N\Delta\beta/2 = \beta/2 \tag{14.6.15}$$

and the intensity becomes

$$I = I_0\left[\frac{\sin\left(\beta/2\right)}{\beta/2}\right]^2 = I_0\left[\frac{\sin\left(\pi a\sin\theta/\lambda\right)}{\pi a\sin\theta/\lambda}\right]^2 \tag{14.6.16}$$

In Figure 14.6.2, we plot the ratio of the intensity I/I_0 as a function of $\beta/2$.

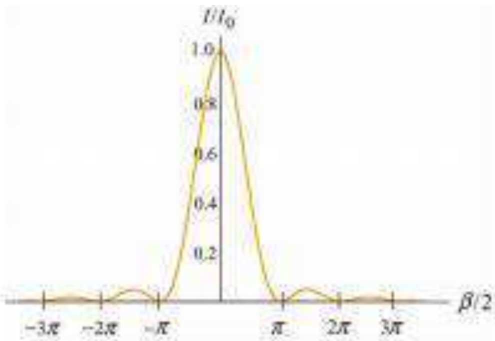


Figure 14.6.2 Intensity of the single-slit Fraunhofer diffraction pattern.

Q: explain fresnel zones.

Ans:

In the study of Fresnel diffraction it is convenient to divide the aperture into regions called Fresnel zones. Figure 1 shows a point source, S, illuminating an aperture a distance z_1 away. The observation point, P, is a distance to the right of the aperture. Let the line SP be normal to the plane containing the aperture. Then we can write

$$SQP = r_1 + r_2 = \sqrt{z_1^2 + \rho^2} + \sqrt{z_2^2 + \rho^2}$$

$$= z_1 + z_2 + \frac{1}{2}\rho^2\left(\frac{1}{z_1} + \frac{1}{z_2}\right) + \cdots$$

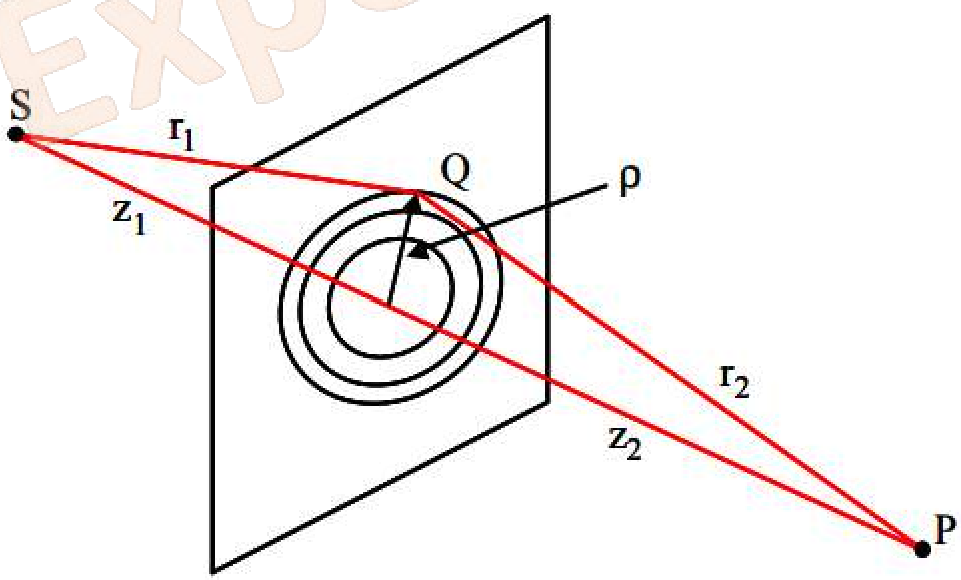


Fig. 1. Spherical wave illuminating aperture.

The aperture can be divided into regions bounded by concentric circles ρ = constant defined such that $r_1 + r_2$ differ by $\lambda/2$ in going from one boundary to the next. These regions are called Fresnel zones or half-period zones. If z_1 and z_2 are sufficiently large compared to the size of the aperture the higher order terms of the expansion can be neglected to yield the following result.

$$n \frac{\lambda}{2} = \frac{1}{2} \rho_n^{\,2} \left(\frac{1}{z_1} + \frac{1}{z_2} \right)$$

Solving for ρ_n, the radius of the nth Fresnel zone, yields

$$\rho_n = \sqrt{n \lambda L} \text{ or } \rho_1 = \sqrt{\lambda L},\ \rho_2 = \sqrt{2 \lambda L},\ \cdots,\ \text{where}$$

$$L = \frac{1}{\frac{1}{z_1} + \frac{1}{z_2}}$$

If ρ_n and ρ_{n+1} are inner and outer radii of the nth zone then the area of the nth zone is given by

$$\begin{aligned} \text{Area of nth Fresnel zone} &= \pi \rho_{n+1}^{\,2} - \pi \rho_n^{\,2} \\ &= \pi (n+1) \lambda L - \pi (n) \lambda L \\ &= \pi \lambda L = \pi \rho_1^{\,2},\ \text{independent of n.} \end{aligned}$$

That is, the area of all zones are equal. If the higher order terms in the expansion for SRQ are maintained the area of the zones would slightly increase with increasing ρ. Generally, it is assumed that z_1 and z_2 are sufficiently large compared to ρ that the higher order terms can be neglected and the area of all zones are equal.

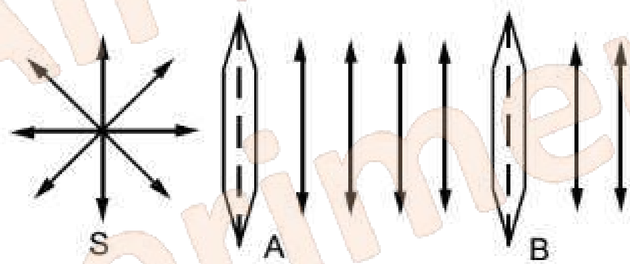
Chapter-9
Polarisation

Polarization: Transverse nature of light waves. Plane polarized light – production and analysis. Circular and elliptical polarization. **(6 Lectures)**

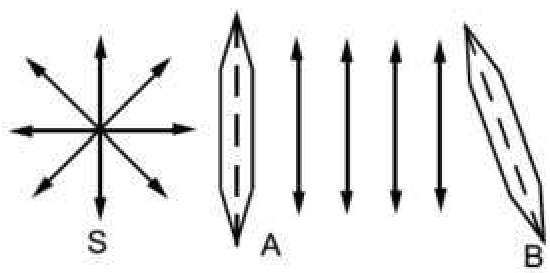
Q:prove transverse nature of light waves.

Ans:

- When light waves are passed through two crystalline slits say A and B (These slits are the tourmaline plates cut parallel to the axis of crystal). Ordinary light say from the sun is incident on the crystal A.a) When crystal A and crystal B are parallel to each other, the intensity of the light emerging from crystal A is constant at any orientation of A and passes through crystal B without any change.



- Now crystal B is rotated w.r.t. A, the intensity of emerging light from crystal B decreases and becomes zero, when crystal B is at right angle w.r.t crystal A.



- This experiment proves the transverse nature of the light waves. Crystal A is called polariser and crystal B is called analyser.

Q: state different methods by which plane polarized light can be produced.

Ans:

Different methods of production of polarized light

(i) Polarization by reflection

(ii) Polarization by refraction

(iii) Polarization by selective absorption

(iv) Polarization by double refraction

(v) Polarization by scattering

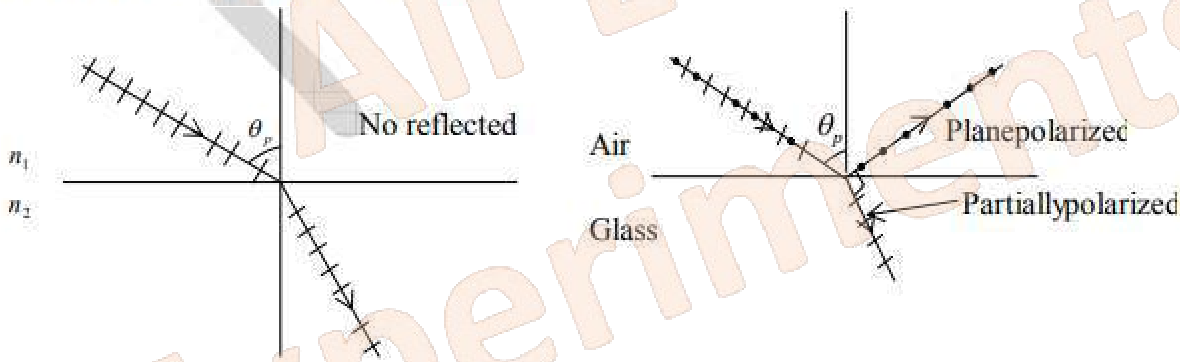
Q: how light can be produced by reflection.

Ans:

If a linearly polarized wave (Electric vector associated with the incident wave lies in the plane of incidence) is incident on the interface of two dielectrics with the angle of incidence equal to θ. If the angle of incidence θ is such that

$$\theta = \theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

then the reflection coefficient is zero.



Thus if an unpolarized beam is incident with an angle of incidence equal to θ_p, the reflected beam is plane polarized whose electric vector is perpendicular to the plane of incidence.

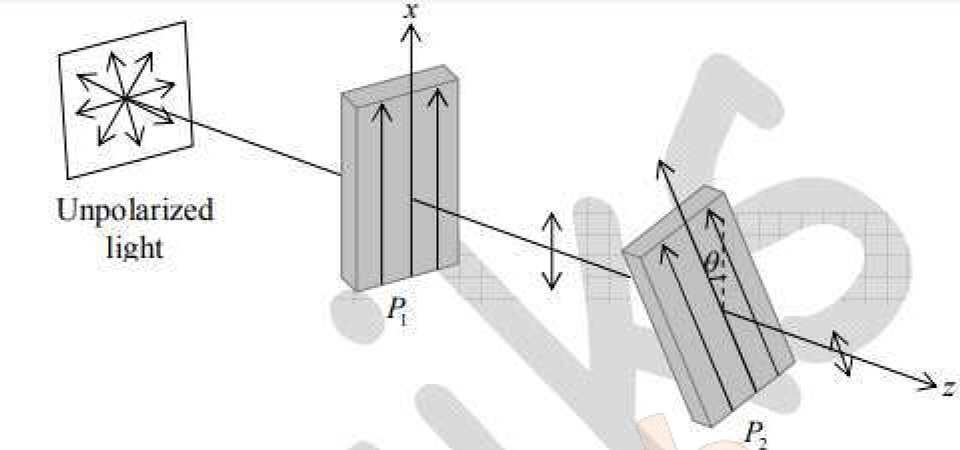
Above equation is known as Brewster's law. The angle θ_p is known as the polarizing angle or the Brewster angle. At this angle, the reflected and the refracted rays are at right angle to each other i.e. $\theta_p + r = \frac{\pi}{2} \Rightarrow n = \frac{\sin\theta_p}{\sin r} = \frac{\sin\theta_p}{\cos\theta_p} \Rightarrow \tan\theta_p = n$

For the air-glass interface, $n_1 = 1$ and $n_2 \approx 1.5$ giving $\theta_p \approx 57^0$.

Q:what is malus law?

Ans:

An unpolarized light beam gets polarized after passing through the Polaroid P_1 which has a pass axis parallel to the x axis. When this x-polarized light beam incident on the second Polaroid P_2 whose pass axis makes an angle θ with the x axis, than the intensity of the emerging beam will vary as



$$I = I_0 \cos^2 \theta$$

where I_0 represents the intensity of the emergent beam when the pass axis of P_2 is also along the x axis (i.e., when $\theta = 0$), above equation known as Malus' law.

Thus, if a linearly polarized beam is incident on a Polaroid and if the Polaroid is rotated about the z axis, then the intensity of the emergent wave will vary according to the above law.

Q:derive an expression for amplitude of circularly polarised light.

Ans: Let us consider a plane wave having electric field in x-y plane. The amplitude of electric field along x axis is E_{ox} and that of along y axis is E_{oy}. The x and y components have some phase difference given by α. The general expression for the electric field fo the em wave propagating along z axis of frequency is is given by

$$\mathbf{E} = E_{ox}\hat{i}\cos(kz-\omega t) + E_{oy}\hat{j}\cos(kz-\omega t-\alpha) \qquad (1)$$

Circularly polarized light

In equation (1), if the amplitude of the components of electric field along x axis and y axis are equal

$$E_{ox} = E_{oy} = E_o$$

And the phase difference $\alpha = \pi/2$

Than equation (1) reduces to

$$E(z) = \hat{i}E_o\cos(kz-\omega t) + \hat{j}E_o\sin(kz-\omega t) \qquad (2)$$

Let us monitor the variation of electric field as a function of time at a given location in the longitudinal direction say $z = 0$
form equation 2

$$E(z) = \hat{i}E_o \cos(-\omega t) + \hat{j}E_o \sin(-\omega t) \tag{3}$$

The magnitude of the x and y component of the electric field , the direction of the electric field and the resultant electric field as a function of time are listed in table 1.
Table 1.

t	E_x	E_y	$\lvert E \rvert$	Direction of resultant electric field
0	E_o	0	E_o	→
$\frac{\pi}{4}/\omega$	$E_o/\sqrt{2}$	$-E_o/\sqrt{2}$	E_o	↘
$\frac{2\pi}{4}/\omega$	0	$-E_o$	E_o	↓
$\frac{3\pi}{4}/\omega$	$-E_o/\sqrt{2}$	$-E_o/\sqrt{2}$	E_o	↙
$\frac{4\pi}{4}/\omega$	$-E_o$	0	E_o	←
$\frac{5\pi}{4}/\omega$	$-E_o/\sqrt{2}$	$E_o/\sqrt{2}$	E_o	↖
$\frac{6\pi}{4}/\omega$	0	E_o	E_o	↑
$\frac{7\pi}{4}/\omega$	$E_o/\sqrt{2}$	$E_o/\sqrt{2}$	E_o	↗
$2\pi/\omega$	E_o	0	E_o	→

The magnitude of the resultant electric field is constant (whch has to be as the wave is propagating in a loslee media) but the component of electric fields along x and y are changing and hence the direction of resultant electric field is changing continuously. The resultant electric field vector **E** is rotating clock wise at an angular frequency w.

From equation (2), the x component

$$E_x = E_0 \hat{i} \cos(kz - \omega t) \tag{4}$$

and y component

$$E_y = E_0 \hat{j} \sin(kz - \omega t) \tag{5}$$

the magnitude of electric field from above

$$E^2 = E_x^2 + E_y^2 = E_o^2 \tag{6}$$

Equation (6) represents the equation of circle. Therefore we can describe the wave given by equation (2) as a wave whose direction of electric field is rotating in x-y plane with the angular frequency, the frequency of the wave and the tip of the electric field is moving in a circle in **clock wise** direction as shown in the fig 1 below.

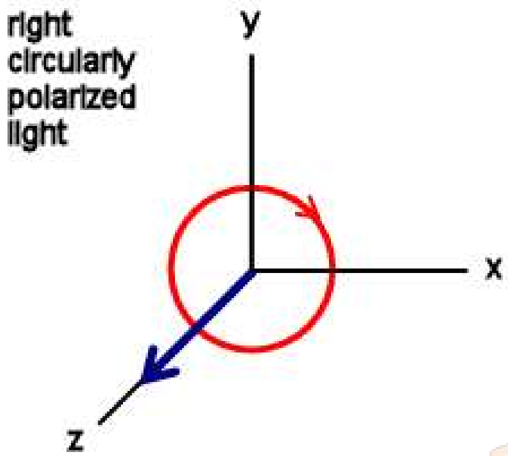


Fig 1 (animation for right circularly polarized light. This kind of wave is termed as **right circularly polarized** light. Suppose that the phase difference is such that equation 2 reduces to

$$E(z) = \hat{i}E_o \cos(kz - \omega t) - \hat{j}E_o \sin(kz - \omega t) \qquad (7)$$

In the above case the tip of the electric field will be encircling with a frequency ω but in **anti clockwise** direction as shown in fig 2. This kind of wave represented by eq 7 is termed as **left circularly polarized** light

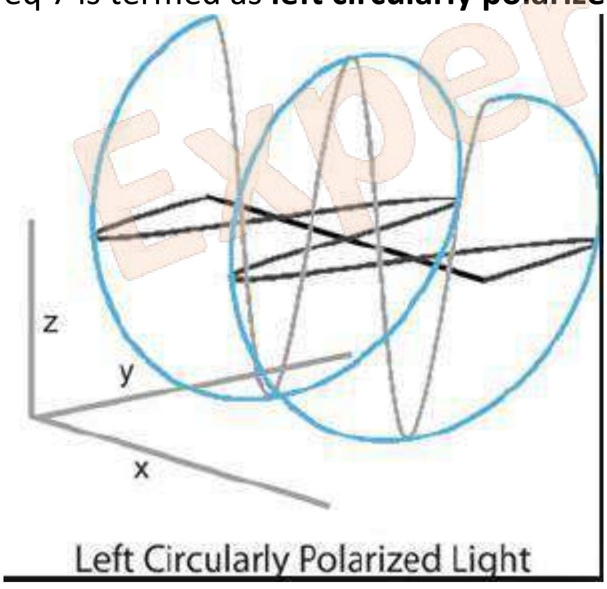


Fig 2 left circularly polarized light

From eq 3 and 7, the circularly polarized light can be generated by super position of two orthogonal polarization of same amplitude and frequency but having a phase gap of $\pm\pi/2$

Q:derive an expression for amplitude of elliptically polarised light.

Ans: **Elliptically polarized light**

Considering the general expression for electric field of a plane wave propagating along z axis

$$E = E_{ox}\hat{i}\cos(kz - \omega t) + E_{oy}\hat{j}\cos(kz - \omega t + \alpha) \qquad (9)$$

under the situation $E_{ox} \neq E_{oy}$, The x and y components of electric fields are given by

$$E_x = E_{0x}\cos(kz - \omega t) \qquad (10)$$

and

$$E_y = E_{0y}\cos(kz - \omega t + \alpha) \qquad (11)$$

With some rearrangement equation 10 and 11 can be put together in the form

$$\left(\frac{E_x}{E_{ox}}\right)^2 + \left(\frac{E_y}{E_{oy}}\right)^2 - 2\left(\frac{E_x}{E_{ox}}\right)\left(\frac{E_x}{E_{ox}}\right)\cos\alpha = \sin^2\alpha \qquad (12)$$

Equation 12 represents an equation of an ellipse with axis as E_x and E_y as shown in figure 3. The major (minor) axis of the ellipse making an angle of θ with E_x (E_y) given by

$$\tan 2\theta = 2\frac{E_{ox}E_{oy}}{E_{ox}^2 - E_{oy}^2}\cos\alpha$$

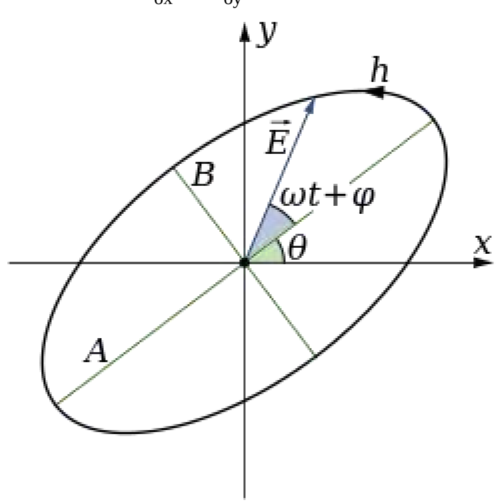


Figure 3

The tip of the electric field vector sweeps an ellipse with in one time period in clock wise direction in x-y plane. Such a wave represented by equation 11 (or eq

12) is called as **right elliptically polarized** wave. When the electric field associated with the wave is given by

$$E = E_{ox}\hat{i}\cos(kz-\omega t) - E_{oy}\hat{j}\cos(kz-\omega t+\alpha) \quad (13)$$

The tip of the electric field vector sweeps an ellipse in an anti clock wise direction and such waves are termed as **left elliptically polarized** waves.

Equation 9 and eq 13 reduces to eqs 3 and 7 respectively for $\alpha=\pm\pi/2$ and $E_{0x}=E_{oy}$. This means that circular polarized light is an special case of elliptically polarized light.