Free Study Material from

All Lab Experiments

Elements of Modern Physics

Chapter - 6, 7

6. Radioactivity

7. Fission and Fusion + Lasers

Chapter-6
Radioactivity

Radioactivity: stability of the nucleus; Law of radioactive decay; Mean life and half-life; Alpha decay; Beta decay- energy released, spectrum and Pauli's prediction of neutrino; Gamma ray emission, energy-momentum conservation: electron-positron pair creation by gamma photons in the vicinity of a nucleus. **(8 Lectures)**

Q: How can you predict stability against beta decay?

Ans:

The isobaric nuclei have the same mass number A but different atomic numbver Z. The semi-empirical mass formula can be written as,

$${}_ZM^A = Z(M_p - M_n) + A(M_n - a_v) + a_c \frac{Z^2}{A^{1/3}} + a_s A^{2/3} + a_a A - a_a 4Z + a_a \frac{4Z^2}{A} \pm \delta$$

$${}_ZM^A = \alpha A + \beta Z + \gamma Z^2 \pm \delta \qquad ...(1)$$

$$\alpha = M_n - \left(a_v - a_a - \frac{a_s}{A^{1/3}}\right)$$

$$\beta = -4a_a - (M_n - M_p)$$

$$\gamma = \left(\frac{4a_a}{A} + \frac{a_c}{A^{1/3}}\right)$$

δ = 0 for odd-Z, even N
= 0 for even-Z, odd-N
= $+\delta$ of odd-Z, odd-N
= $-\delta$ for even Z, even-N

$\pm\delta$ is the pairing energy and does not contain terms in Z.

When A is constant, Eq. (1) is the equation of a *parabola*. It is known as the *mass parabola*. Eq. (1) gives the dependence of nuclear mass on the nuclear charge with *constant A*. This dependence is *parabolic*.

The most stable nucleus has the *minimum* mass.

Differentiating Eq. (1) for constant A and equating it to zero, we have

$$\frac{\partial}{\partial Z}\left({}_{Z}M^{A}\right) = 0 = \beta + 2\gamma Z_0 \text{ at the point } Z = Z_0.$$

$$Z_0 = \text{nuclear charge of the “}\textit{most stable}\text{” isobar} = -\frac{\beta}{2\gamma} \qquad \ldots(2)$$

(*i*) Mass parabolas of odd *A* isobars

For Odd A nuclei, $\delta = 0$ and thus there is only one parabola, implying that there is only one stable nucleus.

The isobar $Z = Z_0 + 1$ lying on the right hand side of the parabola (Fig. 17.12), has greater mass. It will undergo β^+ decay, giving rise to a stable isobar with $Z = Z_0$.

Similarly, the isobar lying on the left of the curve will undergo β^- decay.

The situation is similar for two isobars $(A, Z_0 + 2)$ and $(A, Z_0 - 2)$ which can decay to $(A, Z_0 + 1)$ and $(A, Z_0 - 1)$, respectively; and so on.

$${}^{73}_{34}\text{Se} \xrightarrow{\beta^+, K} {}^{73}_{33}\text{As} \xrightarrow{K} {}^{73}_{32}\text{Ge} \text{ (stable)}$$

$${}^{73}_{30}\text{Zn} \xrightarrow{\beta^-} {}^{73}_{31}\text{Ga} \xrightarrow{\beta^-} {}^{73}_{32}\text{Ge} \text{ (stable)}$$

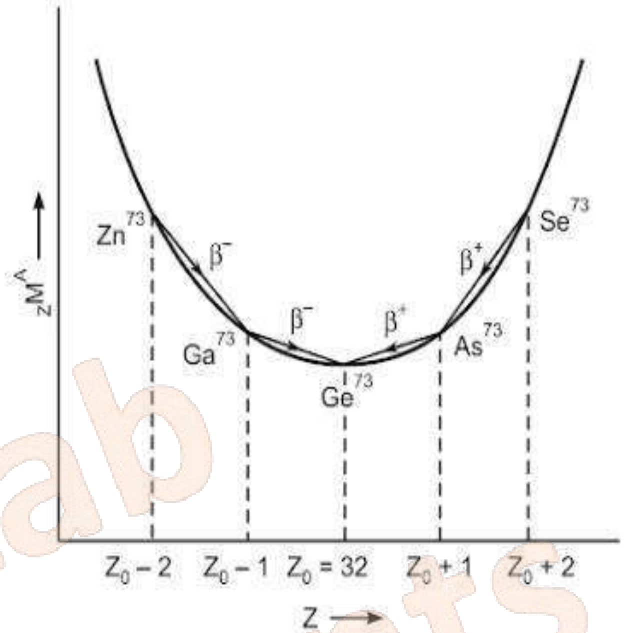


Fig. 17.12

Transition Energies

$${}_{Z_0}M^{A} = \alpha A + \beta Z_0 + \gamma Z_0^2 = \alpha A - \gamma Z_0^2 \qquad \text{(using Eq. 2)}$$

$${}_{Z}M^{A} - {}_{Z_0}M^{A} = (\alpha A + \beta Z + \gamma Z^2) - (\alpha A - \gamma Z_0^2) = \gamma (Z - Z_0)^2 \qquad \ldots(3)$$

Energy released in $\{Z \to (Z + 1)\}$ transition is

$$Q_{\beta^-} = {}_{Z}M^{A} - {}_{Z+1}M^{A} = \gamma\,[(Z - Z_0)^2 - \{(Z + 1) - Z_0\}^2] = 2\gamma\,(Z_0 - Z - 1/2)$$

The Q value for $Z \to (Z - 1)$ transition is given by

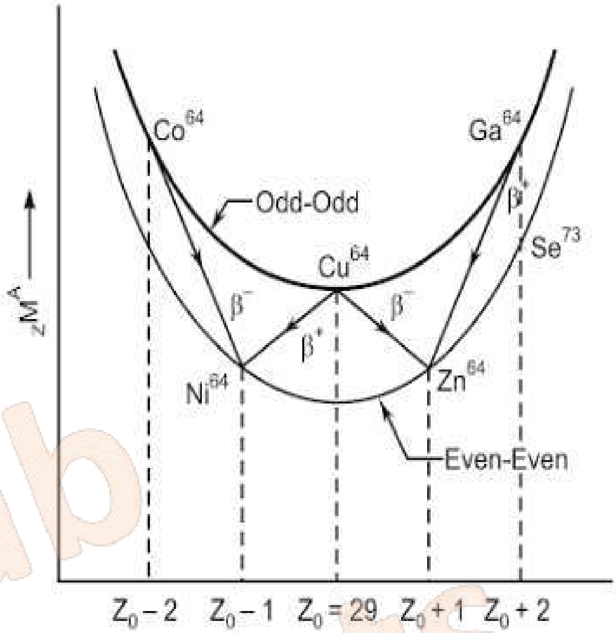
$$Q_{\beta^+} = {}_{Z}M^{A} - {}_{Z-1}M^{A} = \gamma\,\{(Z - Z_0)^2 - (Z - 1 - Z_0)^2\} = 2\gamma\,[Z - Z_0 - 1/2]$$

(*ii*) Mass parabolas of even A Isobars

Here the pairing term $\delta \neq 0$. Since both odd-odd and even-even nuclei are included, we have two parabolas. The lower parabola corresponds to more stable nuclei with even Z, while the upper parabola to less stable nuclei with odd Z. The vertical separation between two parabolas is 2δ due to the opposite sign of δ in two cases.

Fig. 17.13 shows the decay scheme. The reaction energy Q for transition $Z \rightarrow Z \pm 1$ for isobars of mass number A is

$$Q_\beta = 2\gamma \{\pm[Z_0 - Z] - 1/2\} \pm 2\delta \begin{vmatrix} + 2\delta \text{ for odd } Z \\ - 2\delta \text{ for even } Z \end{vmatrix}$$



~~2. Alpha Decay:~~

Q: demonstrate the existence of different types of radiation.

Ans:

The existence of three distinct types of radiation is demonstrated by the following simple experiment. A small quantity of radium (R) is placed at the bottom of a small hole drilled in a lead block [Fig. 20.1] A fairly parallel beam of radiation from R will issue through the hole. This lead block is placed inside an evacuated chamber to avoid absorption of the rays. A photographic plate (P) is placed at a short distance above the lead block to receive the rays. A strong magnetic field is applied at right angles to the plane of the figure and directed away from the reader. After a fairly long exposure, the photographic plate is developed. Three distinct lines will be found on the photographic plate.

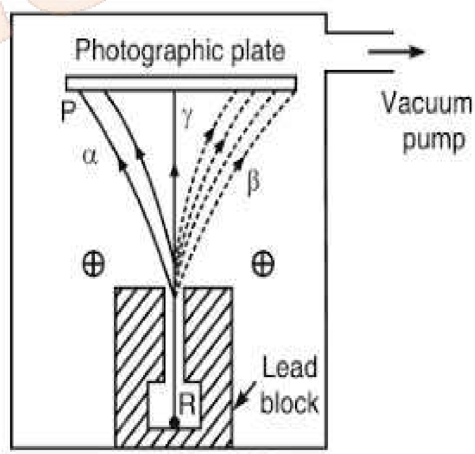


Fig. 20.1

- The α–particles will be deflected towards the left, indicating that they are *positively* charged.
- The β-particles will be deflected towards the right, showing that they are *negatively* charged.
- The γ-rays are not deflected and hit the plate P straight. This shows that the γ-rays are uncharged or neutral rays.

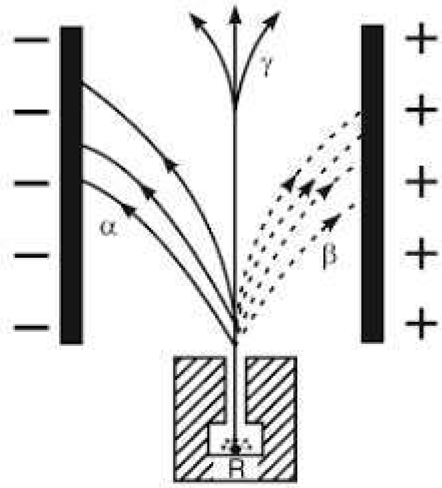


Fig. 20.2

If an electrostatic field is applied, in place of the magnetic field (Fig. 20.2), the β-rays are deflected towards the positive plate, the α-rays towards the negative and γ-rays do not bend at all.

Q: Explain half life and mean life.

Ans:

The rapidity of decay of a particular radioactive sample is usually measured by the *half-life*, $T_{1/2}$, defined as the time interval in which the number of parent nuclei at the beginning of the interval is reduced by a factor of one-half. The half-life is readily obtained in terms of λ as

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Thus, starting initially with N_0 nuclei, $N_0/2$ will be left after a time $T_{1/2}$, $N_0/4$ will remain after a time $2T_{1/2}$, etc.

Another quantity that measures how fast a sample decays is the *average* or *mean lifetime* of a nucleus, T_m, given by

$$T_m = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2}$$

Q: Calculate disintegration energy for alpha particle.

Ans:

When an α-particle is emitted from a nucleus, the nucleus recoils in order to conserve momentum. Let m and v be the mass and velocity of the α-particle. Let M and V be the mass and velocity of the daughter nucleus. According to the law of conservation of momentum,

$$mv = MV.$$

The sum of the kinetic energies of the α-particle and the product nucleus is called alpha disintegration energy (E).

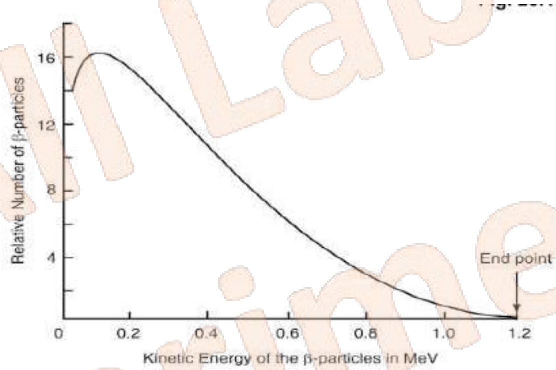
$$E = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2$$

$$E = \frac{1}{2}mv^2\left(1 + \frac{m}{M}\right).$$

The α disintegration energy can thus be easily obtained by multiplying the K.E. of the α-particles by the factor $(1 + m/M)$.

Q: Explain beta ray spectrum.

Ans:



Theory. The energies of β-particles from radio-active elements are determined by measuring the radii of curvature of their paths in a magnetic field of known flux density B. The circular path traversed by the β-particles of velocity v is governed by the relation

$$Bev = \frac{mv^2}{r}.$$

$$\therefore \quad v = Br\,(e/m).$$

From the geometry of the arrangement, the radius of the circular path r can be found.

The value of e/m can be assumed. Hence, the velocity v can be calculated.

For particles moving with very high velocities, the kinetic energy of the particle,

$$E_k = mc^2 - m_0c^2 = m_0c^2\left[\frac{1}{(1 - v^2/c^2)^{1/2}} - 1\right]$$

Q: Explain the origin of continuous spectrum of beta particles.

Ans:

(1) Law of conservation of energy. When a nucleus emits a β-particle, a neutron in the nucleus changes to a proton. Hence the atomic number increases by unity and the mass number remains the same.

Let M_1 = mass of the *neutral parent atom* of atomic number Z,
M_2 = mass of *neutral daughter atom of atomic number* $(Z + 1)$,
m = mass of the β-particle and e = Charge on the β-particle.

Then, according to the principle of mass energy,

$$\left.\begin{matrix}\text{Rest mass of} \\ \text{the parent nucleus}\end{matrix}\right\} = \text{Rest mass of daughter nucleus} + \text{Rest mass of electron} + \text{Energy of the electron.}$$

or $$(M_1 - Zm)\,c^2 = [M_2 - (Z+1)\,m]\,c^2 + mc^2 + Q$$

$\therefore$ Energy of the electron $= Q = (M_1 - M_2)\,c^2$.

Hence all the β-particles from a given radioactive substance must be emitted with the same K.E. But actual measurements show that only a few β-particles are emitted with this maximum value of energy. The majority of β-particles are emitted with smaller energies. What happens to the remaining energy?

(2) Law of conservation of angular momentum. Another difficulty comes in the conservation of angular momentum. Every nucleus has an angular momentum (nuclear spin) which is an odd multiple of $\frac{1}{2}\hbar$ for nuclei of odd mass number and an even multiple of $\frac{1}{2}\hbar$ for nuclei of even mass number. The electron has an angular momentum $\frac{1}{2}\hbar$. In β-decay, mass number remains unchanged. How is it possible for a nucleus of even mass number and therefore an integral spin to give rise to a daughter nucleus of the same mass number and also an integral spin and yet emit an electron of spin $\frac{1}{2}\hbar$? The same is the difficulty for a nucleus of odd mass number.

(3) There is also an apparent failure to conserve linear momentum in β-decay.

Pauli's Neutrino Hypothesis

In 1930, Pauli proposed that if an uncharged particle of zero mass and spin $\frac{1}{2}$ is emitted in β-decay together with the electron, the energy, angular momentum and linear momentum discrepancies discussed above would be removed. The particle was named *neutrino*. It was supposed that neutrino carries off an energy equal to the difference between Q and the actual electron K.E. Subsequently it was found that there are two kinds of neutrino involved in β-decay, the neutrino itself (symbol ν) and the anti-neutrino (symbol $\bar{\nu}$). The reason neutrinos were not experimentally detected until recently is that their interaction with matter is extremely feeble. Lacking charge and mass, and not electromagnetic in nature, the neutrino can pass unimpeded through vast amounts of matter. A neutrino would have to pass through over 100 light-years of solid iron on the average before interacting.

Q: Explain nuclear isomerism.

Ans:

There are nuclei which have the same atomic and mass numbers (same *Z* and same *A*) but differ from one another in their nuclear energy states and exhibit differences in their internal structure. These are called *nuclear isomers*.

The existence of nuclear isomers is called *nuclear isomerism*. The excited nucleus ${}_{38}Sr^{87*}$ is an isomer of ${}_{38}Sr^{87}$. The difference between the nuclear isomers is attributed to a difference of nuclear energy states. One isomer represents the nucleus in its ground state, whereas the other is the same nucleus in an excited state of higher energy. The phenomenon of nuclear isomerism was discovered by *O. Hahn* in 1921. He found that UX_2 and UZ both have the same atomic number and the same mass number but have different half-lives and emit different radiations. UX_2 has 0.394 MeV more energy in its nucleus than UZ. Both these nuclei are formed out of UX_1 by β–decay. UX_2 has half-life of 1.17 minutes and UZ has a half-life of 6.7 hours. The higher energy isomer UX_2 may directly decay to UII by β emission with a half-life of 1.17 minutes, or it may first come to the lower energy isomer by emitting a γ-ray of energy 0.394 MeV and then decay to UII by β-emission with a half-life of 6.7 hours.

Nuclear isomerism has also been detected in artificial radioactive substances. Many *isomeric pairs* have been produced by bombarding radionuclides with neutrons.

Q: Explain internal conversion in a nuclei.

Ans:

When a nucleus passes from a higher excited state to the ground state, the difference in energy of the two states is emitted as a γ-ray. As an alternative to γ-decay, an excited nucleus, in some cases, may return to its ground state by giving up its excitation energy to one of the orbital electrons around it. The emitted electron has a K.E. equal to the lost nuclear excitation energy minus the binding energy of the electron in the atom. *i.e.*,

K.E. of the ejected electron $= E_e - W$.

Here, E_e = the available excitation energy and

W = binding energy of the ejected electron in its shell of origin.

This process is called *internal conversion*. The emitted electron is called a *conversion electron*. Thus internal conversion and emission of a γ-ray from the nucleus are two alternate ways of accomplishing the same nuclear transition. The internal conversion is not a *two step* process in which a γ-ray photon is first emitted and then it knocks out an orbital electron. It is in better accord with experiment to regard internal conversion as representing a direct transfer of excitation energy from a nucleus to an orbital electron.

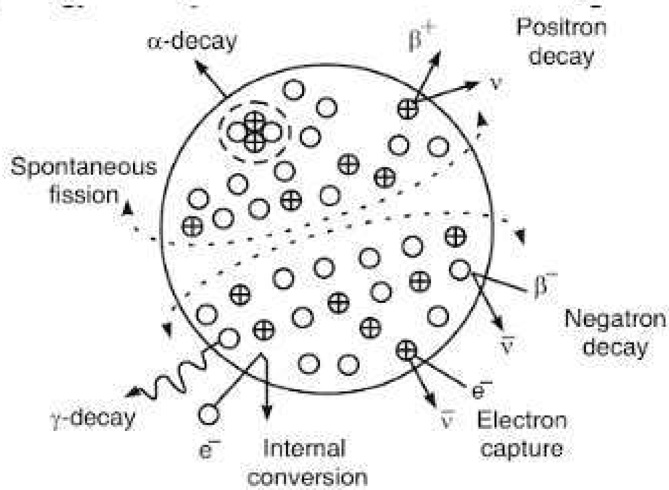


Fig. 20.26

Hence, internal conversion is a *single step* process in which the excited nucleus interacts directly with the orbital electron. The energy of the ejected electron (β-particle) has discrete values. Therefore, the corresponding β-particle energy spectrum is a line spectrum having discrete energies.

Fig. 20.26 illustrates the various kinds of disintegration processes that radioactive nuclei may ungergo. The nucleus is represented as an assembly of protons and neutrons. A proton is indicated by a cross, and a neutron by an open circle.

Q:

The half-value period of radium is 1590 *years. In how many years will one gram of pure element* (*a*) *lose one centigram, and* (*b*) be *reduced to one centigram* ?

SOL. Here, half-life period of radium = $T_{1/2}$ = 1590 years.

$$\therefore \quad \text{Radioactive constant} = \lambda = \frac{0.6931}{T_{\frac{1}{2}}} = \frac{0.6931}{1590}.$$

(*a*) Let t be the time in which one gram of radium loses one centigram (0.01g).

∴ Radium left behind = 1 − 0.01 = 0.99 gram.

Now, $N = N_0 e^{-\lambda t}$ or $\log_e N = \log_e N_0 - \lambda t$

$$\lambda t = \log_e\left(\frac{N_0}{N}\right)$$

$$t = \frac{1}{\lambda}\log_e\left(\frac{N_0}{N}\right) = \frac{1590}{0.6931}\log_e\left(\frac{1}{0.99}\right)$$

= 23.25 years.

(*b*) Here, $N = 0.01$ gram; $N_0 = 1$ gram; t = ?

$$t = \frac{1}{\lambda}\log_e\left(\frac{N_0}{N}\right) = \frac{1590}{0.6931}\log_e\left(\frac{1}{0.01}\right) = 10560 \text{ years.}$$

Chapter-7
Fission and Fusion

Fission and fusion- mass deficit, relativity and generation of energy; Fission- nature of fragments and emission of neutrons. Fusion and thermonuclear reactions driving stellar energy (brief qualitative discussions). **(3 Lectures)**

Lasers: Metastable states. Spontaneous and Stimulated emissions. Optical Pumping and Population Inversion. Basic lasing. **(4 Lectures)**

Q: Define nuclear fission.

Ans:

The process of breaking up of the nucleus of a heavy atom into two, more or less equal fragments with the release of a large amount of energy is called fission.

When uranium is bombarded with neutrons, a uranium nucleus captures a slow neutron, forming an unstable compound nucleus. The compound nucleus splits into two nearly equal parts. Some neutrons are also released in this process.

The schematic equation for the fission process is

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236*} \rightarrow X + Y + neutrons \quad ...(1)$$

$_{92}U^{236*}$ is a highly unstable isotope, and X and Y are the fission fragments. The fragments are not uniquely determined, because there are various combinations of fragments possible and a number of neutrons are given off. Typical fission reactions are

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236*} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q \quad ...(2)$$

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236*} \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_0n^1 + Q \quad ...(3)$$

Here, Q is the energy released in the reaction.

According to Eqn. (2), when $_{92}U^{235}$ is bombarded by a slow moving neutron, the nucleus becomes unstable ($_{92}U^{236*}$) and splits into $_{56}Ba^{141}$ and $_{36}Kr^{92}$ releasing 3 neutrons and energy Q (Fig. 22.1).

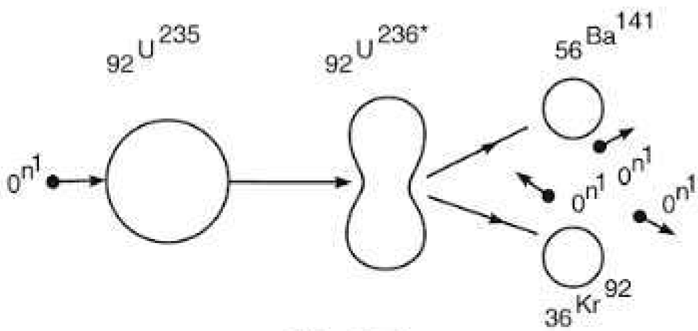


Fig. 22.1

Q: What are the features of fission that makes it useful for electrical properties.

Ans:

Three features of the fission reaction make it useful as a means to generate electrical energy:

(1) **Energy dissipation.** Most of the energy is released as kinetic energy of the fission fragments. These relatively heavy fragments do not travel very far through the reactor fuel element before they dissipate most of their kinetic energy in collisions with the atoms of the fuel element. The energy can be extracted as heat and used to boil water. The resulting steam can then be used in a conventional way to drive a turbine to generate electricity.

(2) **Neutron multiplicity.** The average number of neutrons produced is greater than one, making possible the chain reaction. How much greater than one it must be, in order to achieve a chain reaction, depends on the construction of the reactor.

(3) **Delayed neutrons.** The two neutrons emitted in the fission process

$$^{235}_{92}U_{143} + n \rightarrow \, ^{93}_{37}Rb_{56} + \, ^{141}_{55}Cs_{86} + 2n$$

are *prompt neutrons*—they are emitted essentially at the instant of fission.

Q: Explain chain reaction:

Ans:

A chain reaction is a self-propagating process in which number of neutrons goes on multiplying rapidly almost in geometrical progression during fission till whole of fissile material is disintegrated.

Q: What is nuclear fusion.

Ans:

Nuclear fusion. In this process, two or more light nuclei combine together to form a single heavy nucleus. For example, when four hydrogen nuclei are fused together, a helium nucleus is formed. The mass of the single nuleus formed is always less than the sum of the masses of the individual light nuclei. The difference in mass is converted into energy according to Einstein's equation $E = mc^2$.

Q: What are the sources of stellar energy.

Ans:

The temperature of the stars are very high and they radiate tremendous amount of energy. The sun is one of the innumerable stars. The sun radiates 3.8×10^{26} joules of energy each second. The origin of such a tremendous amount of energy is neither chemical nor gravitational. The fusion of protons is supposed to release the energy in the sun and in other stars. Bethe suggested the following carbon-nitrogen cycle as one of the most important nuclear reactions for release of energy by fusion.

Carbon-Nitrogen Cycle. The cycle is as follows (Fig. 22.10).

$$_6C^{12} + {_1H^1} \rightarrow {_7N^{13*}} + \gamma \quad ...(1)$$

$$_7N^{13*} \rightarrow {_6C^{13}} + {_1e^0} + \nu \quad ...(2)$$

$$_6C^{13} + {_1H^1} \rightarrow {_7N^{14}} + \gamma \quad ...(3)$$

$$_7N^{14} + {_1H^1} \rightarrow {_8O^{15*}} + \gamma \quad ...(4)$$

$$_8O^{15*} \rightarrow {_7N^{15}} + {_1e^0} + \nu \quad ...(5)$$

$$_7N^{15} + {_1H^1} \rightarrow {_6C^{12}} + {_2He^4} \quad ...(6)$$

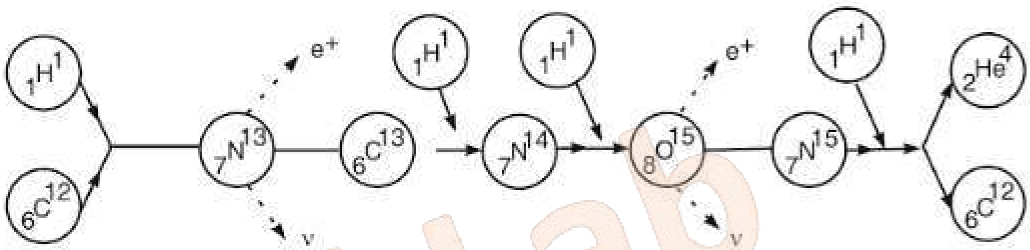


Fig. 22.10

In this cycle C^{12} acts like a catalyst.

The reaction cycle is essentially the reaction

$$4\,{_1H^1} \rightarrow {_2He^4} + 2\,{_1e^0} + 2\nu + Q$$

The loss in mass is calculated as follows :

$$4\,{_1H^1} = 4.031300;\ {_2He^4} = 4.002603 \text{ and } 2\,{_1e^0} = 0.001098.$$

$\therefore$ Loss in mass $= 0.02756\,u$.

$\therefore$ Energy released $= 0.02756 \times 931 = 27.5$ MeV.

It is found that in one million years the sun loses about 10^{-7} of its mass by the above process. Taking mass of the sun as 2×10^{30} kg and its present age as 10^{10} years, it is estimated that the C–N cycle may keep going for another 30 billion years.

Proton-Proton Cycle. Recent modification of the estimates of the central temperature of the sun now favour the proton-proton chain. In the p–p chain, two protons first fuse to produce a deuterium nucleus which combines with another proton to yield He^3. Two He^3 nuclei interact and form He^4 and two protons. These reactions can be represented by the equations,

$$_1H^1 + {_1H^1} \rightarrow ({_2He^2}) \rightarrow {_1H^2} + {_1e^0} + \nu + 0.42\,\text{MeV}$$

$$_1H^2 + {_1H^1} \rightarrow ({_2He^3}) \rightarrow {_2He^3} + \gamma + 5.5\,\text{MeV}$$

$$_2He^3 + {_2He^3} \rightarrow ({_4Be^6}) \rightarrow {_2He^4} + {_1H^1} + {_1H^1} + 12.8\,\text{MeV}$$

$$\overline{4\,{_1H^1} \rightarrow {_2He^4} + 2\,{_{+1}e^0} + 2\nu + 2\gamma + 26.7\,\text{MeV}}$$

Q: Explain metastable states.

Ans: Metastable states: It is excited state of an atom, nucleus, or other system that has a longer lifetime than the ordinary excited states and that generally has a shorter lifetime than the lowest, often stable, energy state, called the ground state. A metastable state may thus be considered a kind of temporary energy trap or a somewhat stable intermediate stage of a system the energy of which may be lost in discrete amounts. In quantum mechanical terms, transitions from metastable states are "forbidden" and are much less probable than the "allowed" transitions from other excited states.

Q: Explain spontaneous and stimulated emission.

Ans:

Spontaneous emission: electron drops from an excited state to a lower state (no outside mechanism) - emitting a photon.

Stimulated emission (lasers): photon of the same frequency interacts with electron in excited state which drops to lower state - the emitted photon is coherent with the incoming photon .

Spontaneous emissionis the process by which a quantum system such as an atom, molecule, nanocrystal or nucleus in an excited state undergoes a transition to a state with a lower energy (e.g., the ground state) and emits quanta of energy. Light or luminescence from an atom is a fundamental process that plays an essential role in many phenomena in nature and forms the basis of many applications, such as fluorescent tubes, older television screens (cathode ray tubes), plasma display panels, lasers, and light emitting diodes. Lasers start by spontaneous emission, and then normal continuous operation works by stimulated emission.

Stimulated emission is the process by which an atomic electron (or an excited molecular state) interacting with an electromagnetic wave of a certain frequency may drop to a lower energy level, transferring its energy to that field. A new photon created in this manner has the same phase, frequency, polarization, and direction of travel as the photons of the incident wave. This is in contrast to spontaneous emission which occurs without regard to the ambient electromagnetic field.

Q: What is optical pumping and population inversion

Ans: Optical pumping: It is a process in which light is used to raise (or "pump") electrons from a lower energy level in an atom or molecule to a higher one. It is commonly used in laser construction, to pump the active laser medium so as to achieve population inversion.

Population inversion: A population inversion occurs while a system exists in a state in which more members of the system are in higher, excited states than in lower, unexcited energy states. The concept is of fundamental importance in laser science because the production of a population inversion is a necessary step in the workings of a standard laser.