Free Study Material from

<u>All Lab Experiments</u>

Waves & Optics (B.Sc.)

Chapter - 6, 7

6. Interference

7. Michelson's Iterferometer

Chapter 6
Interference

Interference: Interference: Division of amplitude and division of wavefront. Young's Double Slit experiment. Lloyd's Mirror & Fresnel's Biprism.Phase change on reflection: Stokes' treatment. Interference in Thin Films: parallel and wedge-shaped films. Fringes of equal inclination (Haidinger Fringes); Fringes of equal thickness (Fizeau Fringes). Newton's Rings: measurement of wavelength and refractive index. **(12 Lectures)**

Q: Discuss coherent and incoherent sources.

Ans: whenever the two needles vibrate with a constant phase difference, a stationary interference pattern is produced. The positions of the maxima and minima will, however, depend on the phase difference in the vibration of the two needles. Two sources which vibrate with a fixed phase difference between them are said to be coherent.

the two needles are sometimes vibrating in phase, sometimes vibrating out of phase, sometimes vibrating with a phase difference of p/3, etc.; then the interference pattern will keep on changing. If the phase difference changes with such great rapidity that a stationary interference cannot be observed, then the sources are said to be incoherent.

Q:derive the expression for intensity by superposition of two waves.

Ans:

Let the displacement produced by the sources at S_1 and S_2 be given by

$$y_1 = a\cos\omega t$$
$$y_2 = a\cos(\omega t + \phi) \qquad (11)$$

Then the resultant displacement is

$$y = y_1 + y_2 = 2a\cos\frac{\phi}{2}\cos\left(\omega t + \frac{\phi}{2}\right) \qquad (12)$$

The intensity I which is proportional to the square of the amplitude can be written in the form

$$I = 4I_0 \cos^2 \frac{\phi}{2} \quad (13)$$

where I_0 is the intensity produced by each one of the sources individually. Clearly if $\phi = \pm\pi, \pm 3\pi, \ldots$, the resultant intensity will be zero and we will have minima. On the other hand, when $\phi = 0, \pm 2\pi, \pm 4\pi, \ldots$, the intensity will be maximum $(= 4I_0)$. However, if the phase difference between sources S_1 and S_2 (i.e., ϕ) is changing with time, the observed intensity is given by

$$I = 4I_0 \left\langle \cos^2 \frac{\phi}{2} \right\rangle \quad (14)$$

where $\langle \ldots \rangle$ denotes the time average of the quantity inside the angular brackets; the time average of a time-dependent function is defined by the relation

$$\langle f(t) \rangle = \frac{1}{\tau} \int_{-\tau/2}^{+\tau/2} f(t)\,dt \quad (15)$$

where τ represents the time over which the averaging **is carried out.**

Q:describe youngs double slit experiment.

Ans:

Let S_1 and S_2 represent the two pinholes of Young's interference experiment. We want to determine the positions of maxima and of minima on line LL' which is parallel to the y axis and lies in the plane containing points S, S_1, and S_2 (see Fig. 14.8). We will show that the interference pattern (around point O) consists of a series of dark and bright lines perpendicular to the plane of Fig. 14.8; point O is the foot of the perpendicular from point S on the screen.

For an arbitrary point P (on line LL') to correspond to a maximum, we must have

$$S_2P - S_1P = n\lambda \qquad n = 0, 1, 2, \ldots \qquad \textbf{(17)}$$

Now,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(y_n + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(y_n - \frac{d}{2}\right)^2\right] = 2y_n d$$

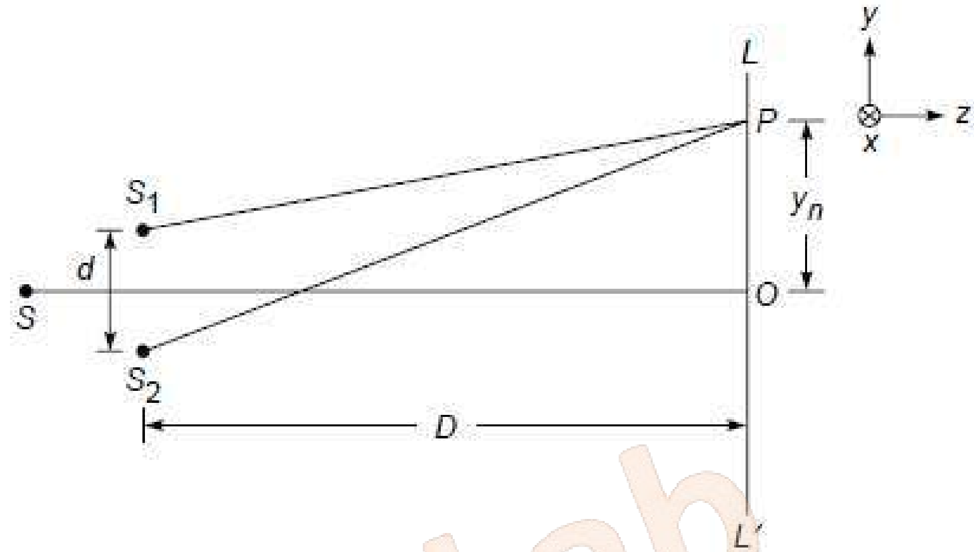


Fig. 14.8 Arrangement for producing Young's interference pattern.

where

$$S_1S_2 = d \qquad \text{and} \qquad OP = y_n$$

Thus

$$S_2P - S_1P = \frac{2y_nd}{S_2P + S_1P} \tag{18}$$

If y_n, $d << D$, then negligible error will be introduced if $S_2P + S_1P$ is replaced by $2D$. For example, for $d = 0.02$ cm, D=50 cm, and $OP = 0.5$ cm (which corresponds to typical values for a light interference experiment)

$$S_2P + S_1P = [(50)^2 + (0.51)^2]^{1/2} + [(50)^2 + (0.49)^2]^{1/2}$$
$$\approx 100.005 \text{ cm}$$

Thus if we replace $S_2P + S_1P$ by $2D$, the error involved is about 0.005%. In this approximation, Eq. (18) becomes

$$S_2P - S_1P \approx \frac{y_n d}{D} \tag{19}$$

Using Eq. (17), we obtain

$$y_n = \frac{n\lambda D}{d} \tag{20}$$

Thus the dark and bright fringes are equally spaced, and the distance between two consecutive dark (or bright) fringes is given by

$$\beta = y_{n+1} - y_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

or

$$\beta = \frac{\lambda D}{d} \tag{21}$$

which is the expression for the fringe width.

To determine the shape of the interference pattern, we first note that the locus of point P such that

$$S_2P - S_1P = \Delta \tag{22}$$

is a hyperbola in any plane containing points S_1 and S_2 (see Example 14.2). Consequently, the locus is a hyperbola of revolution obtained by rotating the hyperbola about the axis S_1S_2. To find the shape of the fringe on the screen, we assume the origin to be at point O and the z axis to be perpendicular to the plane of the screen as shown in Fig. 14.6. The y axis is assumed to be parallel to S_2S_1. We consider an arbitrary point P on the plane of the screen (i.e., $z = 0$) (see Fig. 14.6). Let its coordinates be $(x, y, 0)$. The coordinates of points S_1 and S_2 are $(0, d/2, D)$ and $(0, -d/2, D)$ respectively. Thus

$$S_2P - S_1P = \left[x^2 + \left(y + \frac{d}{2}\right)^2 + D^2\right]^{1/2} - \left[x^2 + \left(y - \frac{d}{2}\right)^2 + D^2\right]^{1/2}$$

$$= \Delta \quad \text{(say)}$$

or

$$\left[x^2 + \left(y + \frac{d}{2}\right)^2 + D^2\right] = \left\{\Delta + \left[x^2 + \left(y - \frac{d}{2}\right)^2 + D^2\right]^{1/2}\right\}^2$$

or $$(2yd - \Delta^2)^2 = (2\Delta)^2\left[x^2 + \left(y - \frac{d}{2}\right)^2 + D^2\right]$$

Hence,

$$(d^2 - \Delta^2)y^2 - \Delta^2x^2 = \Delta^2\left[D^2 + \frac{1}{4}(d^2 - \Delta^2)\right]$$

which is the equation of a hyperbola. Thus the shape of the fringes is hyperbolic. On rearranging, we get

$$y = \pm\left(\frac{\Delta^2}{d^2 - \Delta^2}\right)^{1/2}\left[x^2 + D^2 + \frac{1}{4}(d^2 - \Delta^2)\right]^{1/2} \qquad \textbf{(23)}$$

Q:explain llyods mirror arrangement.

Ans:

In this arrangement, light from a slit S_1 is allowed to fall on a plane mirror at grazing incidence (see Fig. 14.21). The light directly coming from slit S_1 interferes with the light reflected from the mirror, forming an interference pattern in the region BC of the screen. One may thus consider slit S_1 and its virtual image S_2 to form two coherent sources which produce the interference pattern. Note that at grazing incidence one really need not have a mirror; even a dielectric surface has very high reflectivity ([redacted]).

As can be seen from Fig. 14.21, the central fringe cannot be observed on the screen unless the latter is moved to the position $L'_1L'_2$, where it touches the end of the reflector. Alternatively, one may introduce a thin mica sheet in the path of the direct beam so that the central fringe appears in the region BC. (This is discussed in detail in Prob. 14.2.) Indeed, if the central fringe is observed with white light, it is found to be dark. This implies that the reflected beam undergoes a sudden phase change of π on reflection. Consequently, when point P on the screen is such that

$$S_2P - S_1P = n\lambda \qquad n = 0, 1, 2, 3, \ldots$$

we will get minima (i.e., destructive interference). On the other hand, if

$$S_2P - S_1P = \left(n + \frac{1}{2}\right)\lambda$$

we will get maxima.

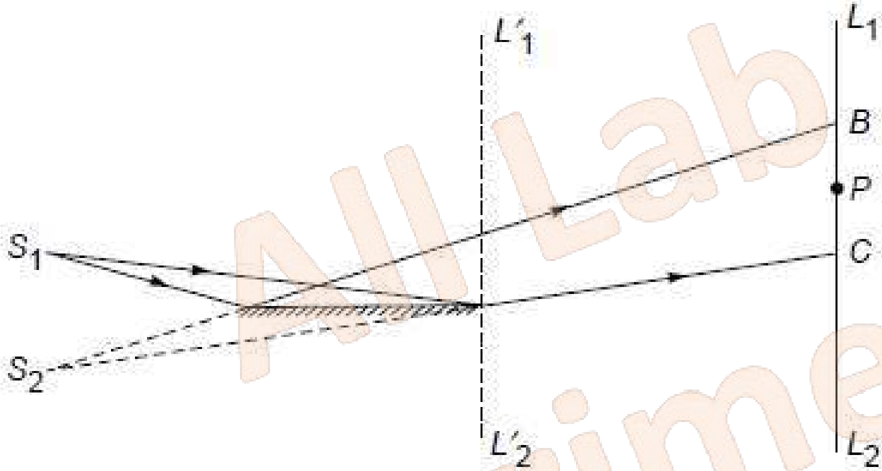


Fig. 14.21 The Lloyd's mirror arrangement.

Q: discuss fresnel's biprism.

Ans: Fresnel devised yet another simple arrangement for the production of interference pattern. He used a biprism, which was actually a simple prism, the base angles of which are extremely small (~20'). The base of the prism is shown in Fig. 14.19, and the prism is assumed to stand perpendicular to the plane of the paper. Point S represents the slit which is also placed perpendicular to the plane of the paper. Light from slit S gets refracted by the prism and produces two virtual images S1 and S2. These images act as coherent sources and produce interference fringes on the right of the biprism. The fringes can be viewed through an eyepiece. If n represents the refractive index of the material of the biprism and α the base angle, then $(n-1)\alpha$ is approximately the angular deviation produced by the prism, and therefore the distance S_1S_2 is $2a(n-1)\alpha$, where a represents the distance from S to the base of the prism. Thus, for $n = 1.5$, $\alpha \approx 20' \approx 5.8 \times 10^{-3}$ radians, $a \approx 2$ cm, one gets $d = 0.012$ cm.

The biprism arrangement can be used for the determination of wavelength of an almost monochromatic light such as the one coming from a sodium lamp. Light from the sodium lamp illuminates slit S, and interference fringes can be easily viewed through the eyepiece. The fringe width β can be

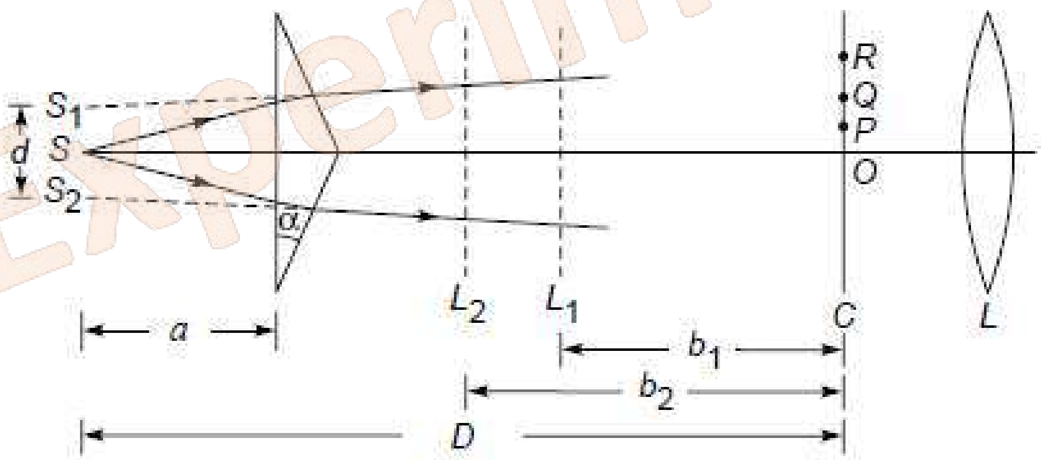


Fig. 14.19 Fresnel's biprism arrangement. Points C and L represent the positions of the crosswires and the eyepiece, respectively. To determine d, one introduces a lens between the biprism and the crosswires; L_1 and L_2 represent the two positions of the lens where the slits are clearly seen.

determined by means of a micrometer attached to the eye-piece. Once β is known, λ can be determined by using the following relation:

$$\lambda = \frac{d\beta}{D} \qquad \textbf{(34)}$$

To determine d, one need not measure the value of α. In fact the distances d and D can be easily determined by placing a convex lens between the biprism and the eyepiece. For a fixed position of the eyepiece there will be two positions of the lens (shown as L_1 and L_2 in Fig. 14.19) where the images of S_1 and S_2 can be seen at the eyepiece. Let d_1 be the distance between the two images when the lens is at position L_1 (at a distance b_1 from the eyepiece). Let d_2 and b_2 be the corresponding distances when the lens is at L_2. Then it can be easily shown that

$$d = \sqrt{d_1 d_2}$$

and

$$D = b_1 + b_2$$

Typically for $d \approx 0.01$ cm, $\lambda \approx 6 \times 10^{-5}$ cm, $D \approx 50$ cm, and $\beta \approx 0.3$ cm.

In the above we considered here a slit instead of a point source. Since each pair of points S_1 and S_2 produces (approximately) straight-line fringes, the slit will also produce straight-line fringes of increased intensity.

Q:Describe the phase change on reflection.hence derive stoke's relation.

Ans: We will now investigate the reflection of light at an interface between two media, using the principle of optical reversibility. According to this principle, in the absence of any absorption, a light ray that is reflected or refracted will retrace its original path if its direction is reversed.

Consider a light ray incident on an interface of two media of refractive indices n_1 and n_2 as shown in Fig. 14.22(a). Let the amplitude reflection and transmission coefficients be r_1 and t_1, respectively. Thus, if the amplitude of the incident ray is a, then the amplitudes of the reflected and refracted rays are ar_1 and at_1, respectively.

We now reverse the rays, and we consider a ray of amplitude at1 incident on medium 1 and a ray of amplitude ar_1 incident on medium 2 as shown in Fig. 14.22(b). The ray of amplitude at_1 will give rise to a reflected ray of amplitude at_1r_2 and a transmitted ray of amplitude at_1t_2, where r_2 and t_2 are the amplitude reflection and transmission coefficients, respectively, when a ray is incident from medium 2 on medium 1. Similarly, the ray of amplitude ar_1 will give rise to a ray of amplitude ar_2 1 and a refracted ray of amplitude ar_1t_1. According to the principle of optical reversibility, the two rays of amplitudes ar_2 1 and at_1t_2 must combine to give the incident ray of Fig. 14.22(a); thus

$$ar_1^2 + at_1t_2 = a$$

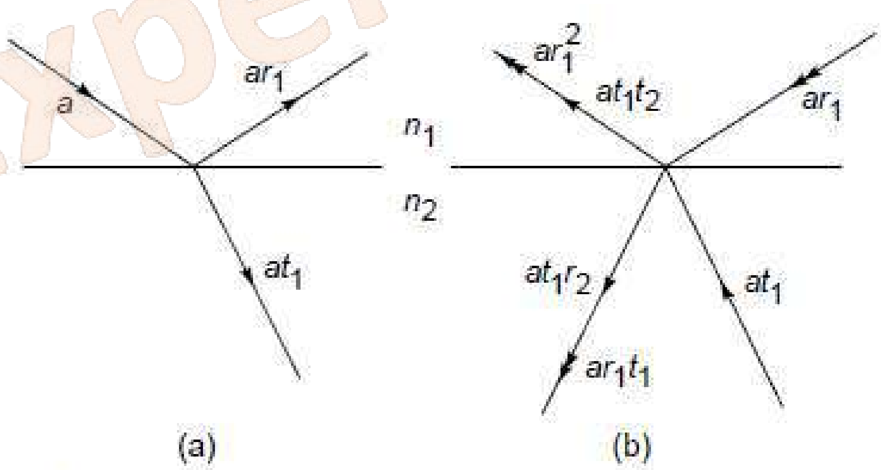


Fig. 14.22 (a) A ray traveling in a medium of refractive index n_1 incident on a medium of refractive index n_2. (b) Rays of amplitude ar_1 and at_1 incident on a medium of refractive index n_1.

or

$$t_1 t_2 = 1 - r_1^2 \quad (38)$$

Further, the two rays of amplitudes at_1r_2 and ar_1t_1 must cancel each other, i.e.,

$$at_1r_2 + ar_1t_1 = 0$$

or

$$r_2 = -r_1 \quad (39)$$

Since we know from Lloyd's mirror experiment that an abrupt phase change of π occurs when light gets reflected by a denser medium, we may infer from Eq. (39) that no such abrupt phase change occurs when light gets reflected by a rarer medium. This is indeed borne out by experiments. Equations (38) and (39) are known as Stokes' relations.

Q:Discuss interference by thin films.

Ans:

Suppose monochromatic light is incident on a thin film of thickness t and having an index of refraction n. Part of this light reflects back toward the observer from the top and bottom surfaces of the thin film toward the observer, and part is transmitted through the thin film. ~~(Refer,~~

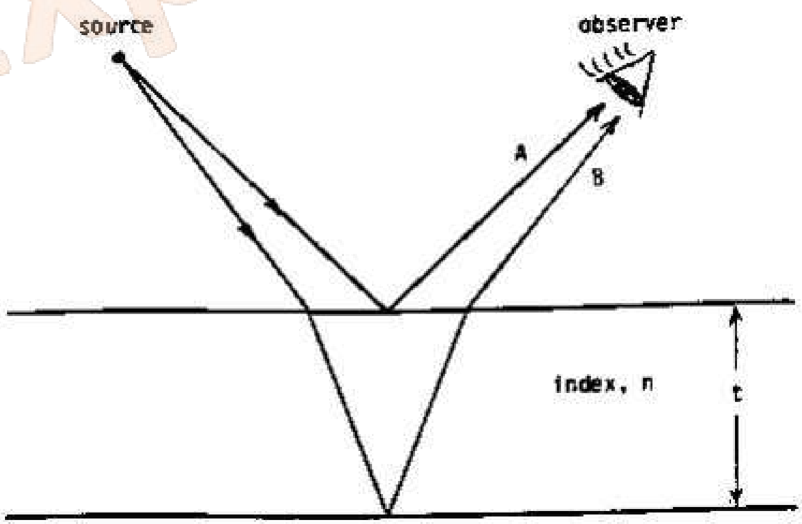


Figure 1. Monochromatic light shown reflecting off the top and bottom surfaces of the thin film. The transmitted light is not shown.

The light leaving the source is in phase. However, upon reflecting from the thin film back to the observer, the light will necessarily be out of phase due to the extra distance ray B travels as it reflects off the bottom surface of the thin film. If the light hits the surface perpendicularly or almost perpendicularly, then the extra distance ray B travels is twice the thickness of the thin film at the point where the light is incident. This extra distance and the corresponding phase differences are related to each other as

$$\frac{\text{phase difference}}{2\pi} = \frac{\text{path difference}}{\text{wavelength with in the thin film}}, \qquad (1)$$

$$\frac{\phi}{2\pi} = \frac{2t}{\lambda_n}, \qquad (2)$$

where ϕ is the phase difference, t is the thickness of the film, and λ_n is the wavelength of the light in the thin film. In this case, the thin film has an index of refraction n and $\lambda_n = \lambda/n$, where λ is the wavelength of the light in air. Equation (2) then becomes

$$\frac{\phi}{2\pi} = \frac{2t\eta}{\lambda}, \qquad (3)$$

If two waves interfere and the phase difference between them is $2m\pi$, where m = 0, 1, 2, ..., then constructive interference results. And, if the phase difference is $2(m - ½)\pi$, where m = 0, 1, 2, ..., then destructive interference results. Other phase differences give partial constructive or destructive interference.

When the phase difference, ϕ, in (3) is replaced by the values of the phase difference, the results are

$$2t\eta = m\lambda. \quad m = 0,1,2,... \qquad (4)$$

for constructive interference, and

$$2t\eta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0,1,2,... \qquad (5)$$

for destructive interference.

An added complication arises in thin film interference. It is found that when light traveling in a medium of one index of refraction is reflected from a surface of higher index, a $-\pi$ phase shift occurs. No phase shift occurs at the surface when the reflecting surface has a lower index of refraction. The result is that if the total number of phase shifts that rays A and B undergo is zero or an even number, then (4) and (5) hold as above. But, if an odd number of phase shifts occur, then (4) corresponds to destructive interference and (5) corresponds to constructive interference. That is,

$$2t\eta = m\lambda. \quad m = 0,1,2,... \qquad (6)$$

for destructive interference, and

$$2t\eta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0,1,2,... \qquad (7)$$

for constructive interference.

Q: what are haidinger fringes.

Ans: Haidinger fringes are interference fringes formed by the interference of monochromatic and coherent light to form visible dark and bright fringes. Also known as fringes of equal inclination, these fringes result when light from an extended source falls on a thin film made of an optically denser medium.

They are also known as fringes of equal inclination because the changes in the optical path are due to the changes in the direction of incidence and hence in the value of ϴ

Q:what are fizeau fringes.

Ans: Interference fringes of monochromatic light from interference in a geometrical situation other than plane parallel plates are known asFizeau fringes.they are also known as fringes of equal thickness. Interference fringes in light from a Fizeau interferometer.

Q:derive an expression for refractive index index of liquid from newton's ring experiment.

Ans: The experiment is performed when there is an air film between the plano-convex lens and the optically plane glass plate.The diameter of the mth and the (m+p)th dark rings are determined with the help of a travelling microscope.

For air

$$D^2_{m+p} = 4(m+p)\lambda R, D^2_{m} = 4m\lambda R$$

$$D^2_{m+p} - D^2_{m} = 4p\lambda R$$

As shown in figure arrange the lens with glass plate. Pour one or two drops of liquid whose refractive index is to be determined without disturbing the arrangement. Now the air film between the lens and glass plate is replaced by the liquid. The diameters of m+pth and mth rings are determined.

For liquids,

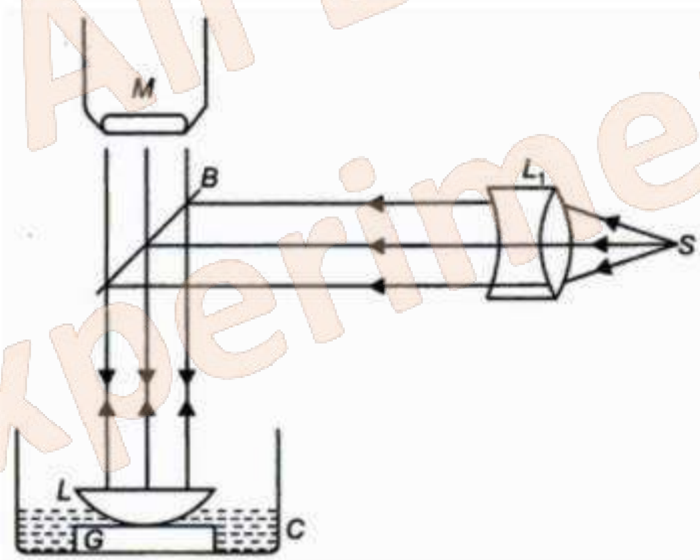
$2\mu t \cos r = m\lambda$,for dark rings

For normal incidence cosr =1,so

$2\mu t = m\lambda$

But $t = \frac{r^2}{2R}, r = \frac{D}{2}$

Rearranging the above equation ,we get



$$D_m^2 = \frac{4m\lambda R}{\mu}$$

We have $D_{m+p}^2 - D_m^2 = 4p\lambda R$

for liquids, $D'^2_{m+p} - D'^2_m = \frac{4p\lambda R}{\mu}$

From these two euations the refractive index of the given liquids is given by

$$\mu = \frac{D^2{}_{m+p} - D^2{}_m}{D'^2{}_{m+p} - D'^2{}_m}$$

Chapter-7
Michelson's interferometer

Michelson's Interferometer: Construction and working. Idea of form of fringes (no theory needed), Determination of wavelength, Wavelength difference, Refractive index, and Visibility of fringes. **(4 Lectures)**

Q: Describe the construction and working of Michelson Interferometer. Explain how you will determine the wavelength difference of two components of a line by Michelson Interferometer.

OR

Explain the working of Michelson's interferometer. How circular fringes he produced with it.

Ans: Working of Michelson's Interferometer :

Michelson designed an instrument for the measurement of wavelength of sodium light, thickness of thin film and for many applications. The instrument is based on principle of interference of light known as Michelson's Interferometer.

It is based on principle of interference of light by the way of division of amplitude. According to this the incident beam is divided into two parts and sent into two perpendicular directions and brought back together by using plane mirror to interfere each other.

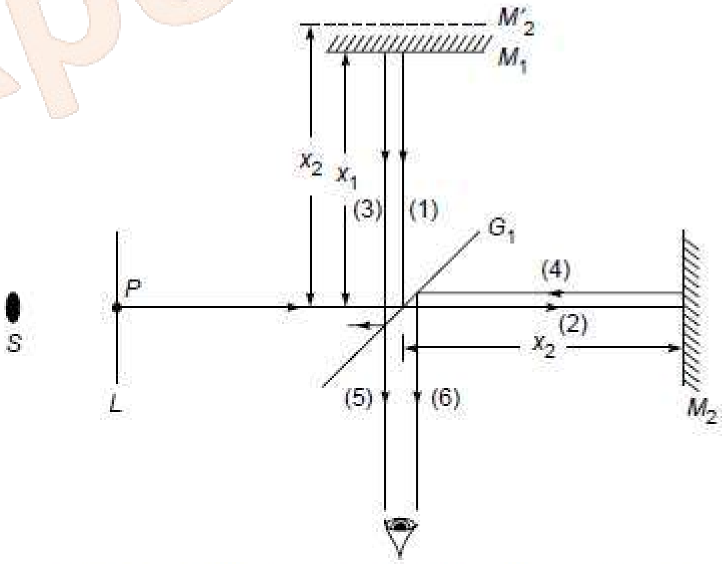


Fig. 15.34 Schematic of the Michelson interferometer.

Construction:

It consists of two highly polished plane mirror M1 and M2, with two optically plane glass plate G1 and G2 which are of same material and same thickness. The mirror M1 and M2 are adjusted in such a way that they are mutually perpendicular to each other. The plate G1 and G2 are exactly parallel to each other and placed at 45° to mirror M1 and M2. Plate G1 is half silvered from its back while G2 is plane and act as compensating plate. Plate G1 is known as beam-splitter plate.

The mirror M2 with screw on its back can slightly titled about vertical and horizontal direction to make it exactly perpendicular to mirror M1. The mirror M1 can be moved forward or backward with the help of micrometer screw and this movement can be measured very accurately.

Working:

Light from a broad source is made paralied by using a convex lens L. Light from lens L is made to fall on glass plate G_1 which is half silver polished from its back. This plate divides the incident beam into two light rays by the partial reflection and partial transmission, known as Beam splitter plate. The reflected ray travels towards mirror M_1 and transmitted ray towards mirror M_2. These rays after reflection from their respective mirrors meet again at 'O' and superpose to each other to produce interference fringes. This firings pattern is observed by using telescope.

Functioning of Compensating Plate:

In absence of plate G_2 the reflected ray passes the plate G_1 twice, whereas the transmitted ray does not passes even once. Therefore, the optical paths of the two rays are not equal. To equalize this path the plate G_2 which is exactly same as the plate G_1 is introduced in path of the ray proceeding towards mirror M_2 that is why this plate is called compensating plate because it compensate the additional path difference.

Measurement of Wavelength of Monochromatic Light:

Monochromatic Light is allowed to fall on plate G_1. The M.I. is adjusted for circular fringes. For this mirror M_1 is made exactly perpendicular to mirror M_2 with the help of leveling screw and movable mirror M_1 is moved in such a way so that $G_1M_2 = G_1M_1$.

Consequently circular fringes are observed when viewed through telescope. Suppose the separation between real mirror M_1 and image of mirror M_2, M_2' is such that a n^{th} order dark ring is formed at the centre in the field of view. Thus the conditions for dark fringe at centre.

Formation of Circular Fringes:

The shape of fringes in M.I. depends on inclination of mirror M_1 and M_2.

Circular fringes are produced with monochromatic light, if the mirror M_1 and M_2 are perfectly perpendicular to each other. In this position an image of mirror M_2, M_2' is formed due to half silvered polished plate G_1, just below the mirror M_1. The virtual image of mirror M_2 and the mirror M_1 must be parallel. Therefore it is assumed that an imaginary air film is formed in between mirror M1 and virtual image mirror M'_2.

The interference pattern can be considered as the rays of light reflected, from the surface of mirror M_1 (real) and mirror M_2 (virtual). Therefore, the interference pattern will ve obtained due to imaginary air film enclosed between M_1 andM'_2.

From Fig. if the distance M_1 and M_2 and M'_2 is'd', the distance between S'_1 and S'_2 will be 2D.

These circular fringes which are due to interference with a phase difference with a phase difference determined by the inclination 'ð' are known as fringes of equal inclination.

Circular fringes can be seen by telescope because they are formed at infinity because they are formed due to two parallel interfering rays. When d becomes zero, the whole pattern becomes dark. Since a circular fringe is formed at the same inclination so they are called fringe of equal inclination and also called Haidinger's fringes.